Propagator of the $SU(2)$ gauge boson on a 3-dimensional lattice in the Landau gauge

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- Motivation
- Center symmetry
- Absolute and minimal Landau gauges
- Gauge fixing algorithm and technical details
- An approach to the absolute gauge
- Momentum dependence of the propagator
- The role of Z_2^3 sectors
- The effect of Gribov copies
- The effects of finite volume

Infrared behavior of the gluon propagator in the Landau gauge
is of interest because

- Propagator is needed for calculation of physical quantities;
- The Kugo-Ojima and Gribov-Zwanziger confinement criteria
  are formulated in terms of propagator behavior
  in the Euclidean domain.

If the Osterwalder-Schrader reflection positivity
is violated for the gluon fields,
one cannot construct
the respective Hilbert space with positive metric.
The gluon fields are not associated with asymptotic states.

$\Longrightarrow$ gluons are confined

- It is of interest to compare lattice and continuum results for the propagator
- Gauge fixing on a lattice is also of intererst because the respective continuum gauge theory is defined only in a particular gauge.

The gluon propagator in the Landau gauge:

$$D^{ab}_{\mu\nu}(p) = \delta^{ab} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) D(p)$$

The Functional Renormalization Group (FRG)
and the Schwinger–Dyson Equations (SDE)
imply at $p \to 0$ [Fischer, Pawlowsky, 2006; Alkofer etc]:

- scaling solution:

$$D(p) \simeq (p^2)^{2\kappa + (2-D)/2} \qquad D_{Gh}(p) \simeq (p^2)^{-1-\kappa}, \tag{1}$$

- massive solution

$$D(p) \simeq const \qquad D_{Gh}(p) \simeq \frac{Z}{(p^2)}, \tag{2}$$

$$S = \frac{4}{g^2} \sum_{P=x,\mu,\nu} \left(1 - \frac{1}{2} \mathrm{Tr}\ U_P \right)$$

where

$$U_P = U_{x,\mu} U_{x+\hat{\mu},\nu} U^\dagger_{x+\hat{\nu},\mu} U^\dagger_{x,\nu}$$

$$U_{x,\mu} \in SU(2), \quad D = 3$$

$$U_{x,\mu} \;=\; u_0 \;+\; i \sum_{a=1}^{3} u_a \sigma_a, \quad (3)$$

$$A^a_\mu = -\frac{2 u^a_\mu}{g a}, \quad (4)$$

$$\Lambda : \quad U_{x,\mu} \to \Lambda^\dagger_x U_{x,\mu} \Lambda_{x+\hat{\mu}},$$

We fix the absolute Landau gauge by finding the global maximum of the functional

$$\mathcal{F}[\,\mathcal{U}\,] = \sum_{x,\mu} \mathrm{Tr}\, U_{x,\mu}, \quad (5)$$

Stationarity condition:

$$\partial_\nu A^a_\nu = 0.$$

Gribov copies: residual gauge orbit

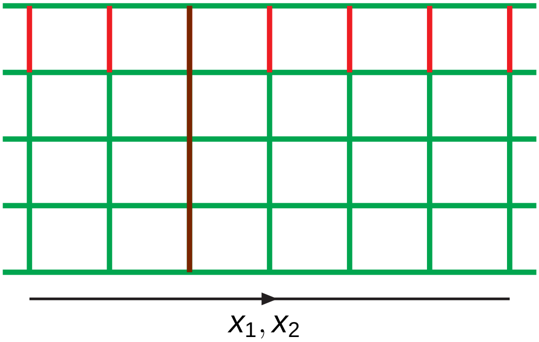
$$\mathcal{R}(\mathcal{U}) \;=\; \{\mathcal{U}_m | \mathcal{U}_m = \mathcal{U}^{g_m}, \delta\mathcal{F}[\mathcal{U}_m] = 0\}$$

.

- Minimal Landau gauge:
  to select any element $\in \mathcal{R}$
- Absolute Landau gauge:
  to select the element with the maximal value of $\mathcal{F}[\mathcal{U}_m]$.

$$D(p) \neq D(p)!!!$$

Problem of degenerate maxima.



Center symmetry:

$\mathbb{Z}_2 : U_{x,\mu} \to -\, U_{x,\mu}$

$$\boxed{L(x_1, x_2) \to -L(x_1, x_2)}$$

$$L(x_1, x_2) = Tr \prod_{j=1}^{N_\tau} U(x + j\hat{3}, 3) = P \exp\left(iga \oint A_\mu^c(z) \Gamma^c dz \right) .$$

We extend the gauge group

$$\mathcal{G} \longrightarrow \mathcal{G}_E = \mathcal{G} \times \mathbb{Z}_2^3, \qquad (6)$$

where $\mathcal{G} = \{\Omega(x)\}, \quad \Omega(x) \in SU(2)$:

$$U_{x,\mu} \rightarrow \Omega_x^\dagger U_{x,\mu} \Omega_{x+\hat{\mu}}, . \qquad (7)$$

The configuration space $\{\mathcal{U}\}$ is divided into 8 $\mathbb{Z}_2^3$ sectors, according to the signs of

$$\sum_{x_\mu=a}^{La} \sum_{x_\nu=a}^{La} L(x_\mu, x_\nu)$$

Gauge fixing algorithm

- We generate a configuration $\mathcal{U}_0$ using the heat bath method,
- perform $\mathbb{Z}_2^3$ transformations and obtain $\mathcal{U}_1, ..., \mathcal{U}_7$ associated with $\mathcal{U}_0$.
  All of them have the same Wilson action, however, they cannot be transformed into each other by a proper gauge transformation.
  Nevertheless, we consider them as Gribov copies corresponding to the extended gauge group.

- In the sth sector, we produce N_B elements $\bar{\mathcal{V}}_{sk}$ of the gauge orbit, associated with $\mathcal{U}_s$.
- $$F_{sk}(g) \equiv \mathcal{F}(\mathcal{V}_{sk}^{g}) \quad (8)$$

  is the functional on $\mathcal{G}$. Its maxima provide Gribov copies.
- we begin with the "Simulated Annealing" (SA) method and then proceed to the overrelaxation (OR) algorithm.
  SA is used for preliminary maximization of $F_{sk}(g)$,
  the OR algorithm is more efficient at the final stage.

The SA algorithm generates gauge transformations $g(x)$ by MC iterations with a statistical weight proportional to $\exp(4V\ F_{sk}[\mathcal{G}]/T)$. T is an auxiliary parameter which is gradually decreased to maximize $F_{sk}[g]$.
[Bogolubsky et al., 2007; Schemel et al., 2006]: $T_{\rm init} = 1.3$, $T_{\rm final} = 0.01$ After each quasi-equilibrium sweep, including both heatbath and microcanonical updates, T is decreased by equal intervals. The final SA temperature is fixed such that the quantity

$$\max_{x,a} \left| \sum_{\mu=1}^{3} \left(A^{a}_{x+\hat{\mu}/2;\mu} - A^{a}_{x-\hat{\mu}/2;\mu} \right) \right| \qquad (9)$$

decreases monotonously during OR for the majority of gauge fixing trials. The number of the SA steps is set equal to 3000.

We use the standard Los-Alamos type overrelaxation with the parameter value $\omega = 1.7$.
The number of iterations:
$500 \div 700$ at $L = 32$ $1500 \div 3000$ at $L = 80$;
in few cases, several times greater.

The precision of gauge fixing:

$$\max_{x,a} \left| \sum_{\mu=1}^{3} \left(A^a_{x+\hat{\mu}/2;\mu} - A^a_{x-\hat{\mu}/2;\mu} \right) \right| < 10^{-7} \qquad (10)$$

The configuration $\bar{\mathcal{V}}_{sk}$ with the greatest value of $F_{sk}[g]$ is referred to as "the kth Gribov copy in the sth sector".

- We put the configurations $\bar{\mathcal{V}}_{sk}$ in the linear order: $\bar{\mathcal{V}}_{sk} \to \bar{\mathcal{V}}_r$. There are two natural arrangements:

$$\bar{\mathcal{V}}_r^{(1)} = \mathcal{V}_{sk}\,, \qquad \text{where} \quad r = N_{copy}(s-1)+k; \qquad (11)$$

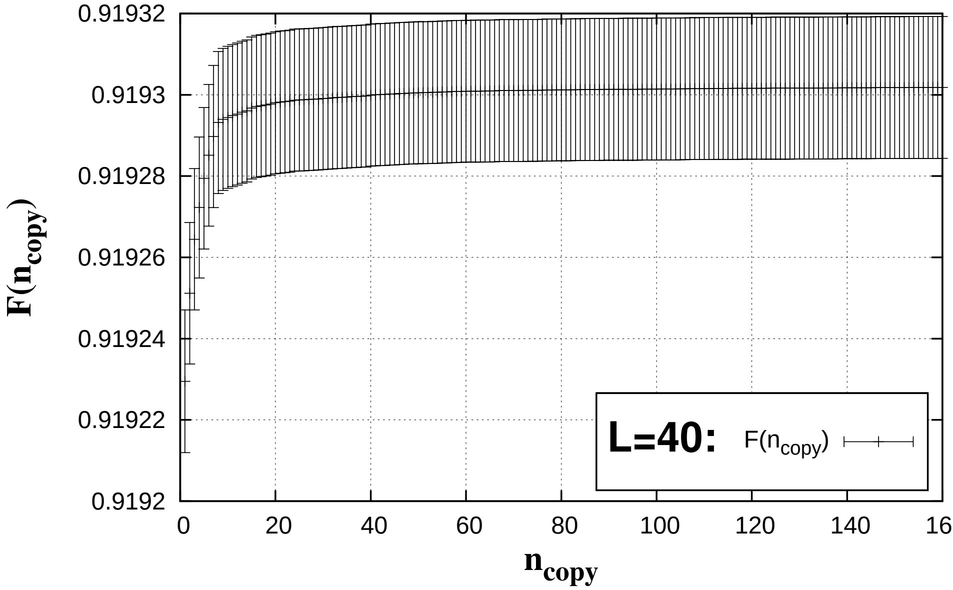
$$\bar{\mathcal{V}}_r^{(2)} = \mathcal{V}_{sk}(j), \qquad \text{where} \quad r = 8(k-1)+s; \qquad (12)$$

  r runs from 1 to $N_{copy}^{tot} = 8N_{copy}$.
- Now we can take a part $\mathcal{P}_n$ of the residual gauge orbit $\mathcal{R}(\mathcal{U}_0)$ consisting of n elements, $1 \le n \le N_{copy}^{tot}$.
- Let $\mathcal{F}[\bar{\mathcal{V}}_r]$ approaches its maximum on $\mathcal{P}_n$ at $\bar{\mathcal{V}}_{\bar{r}}$.

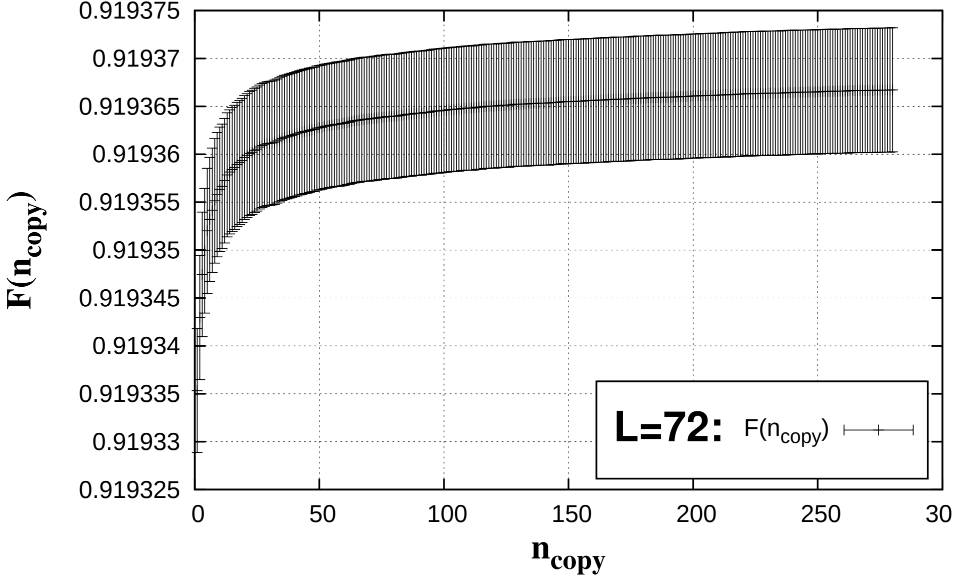
- We evaluate (measure) the value of the propagator using $\bar{\mathcal{V}}_{\bar{r}}$.
- We repeat this procedure N_{meas} times;
  the initial configuration $\mathcal{U}_0(j)$ for each measurement being separated by 200 sweeps from the previous one in order to be considered as statistically independent.
- Then we take an average over the measurements.

L	N_{meas}	N_{copy}	aL [Fm]	$\mathcal{F}_{max}$
32	800	16	5.38	0.9192939 ± 0.0000173
40	400	20	6.73	0.9193018 ± 0.0000177
48	905	20	8.08	0.9193386 ± 0.0000091
56	788	20	9.43	0.9193515 ± 0.0000080
64	474	20	10.8	0.9193404 ± 0.0000078
72	578	35	12.1	0.9193656 ± 0.0000065
80	557	20	13.5	0.9193527 ± 0.0000055

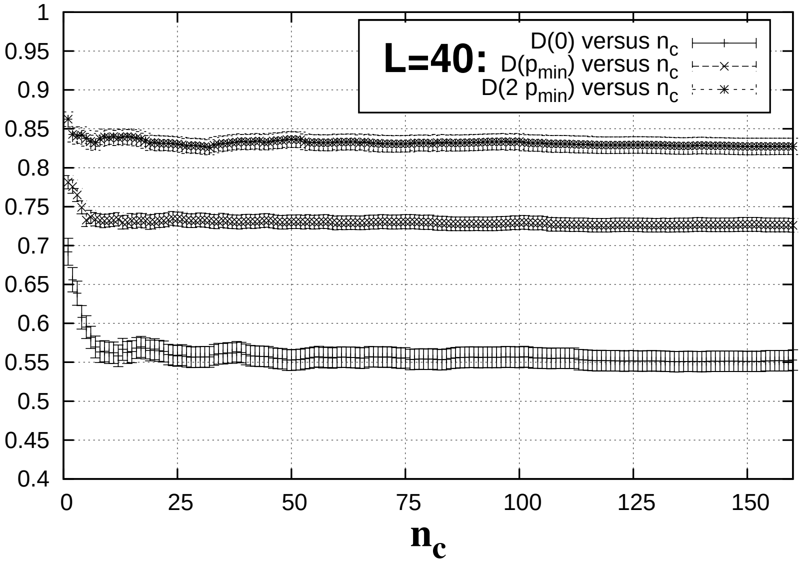
Table: $a\sqrt{\sigma} \approx 0.567$, $\sqrt{\sigma} = 440$ MeV; $a = 0.168$ Fm $\sim (1.17 \text{ GeV})^{-1}$; 1 GeV$^{-1} \simeq 0.197$ Fm.



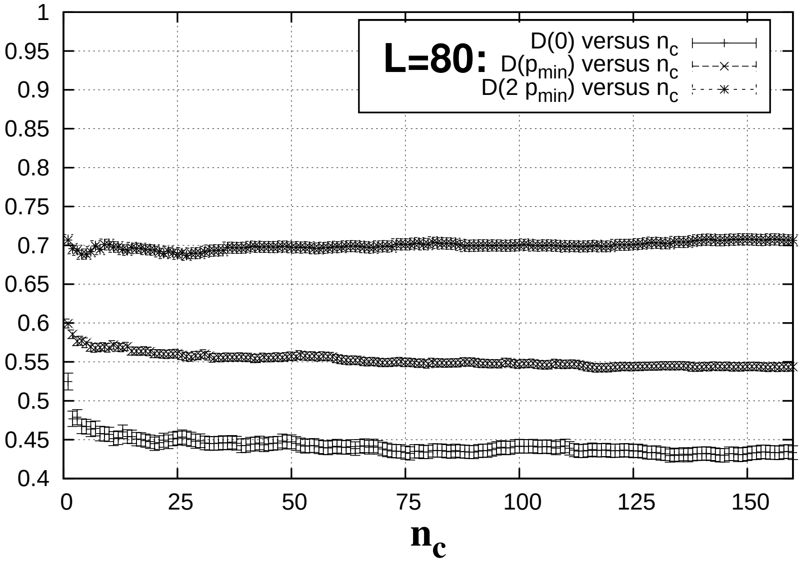
0.91932
0.9193
0.91928
0.91926
0.91924
0.91922
0.9192
F(n_copy)
0
20
40
60
80
100
120
140
16
n_copy
L=40:
F(n_copy)



0.919375
0.91937
0.919365
0.91936
0.919355
0.91935
0.919345
0.91934
0.919335
0.91933
0.919325
0
50
100
150
200
250
30
$F(n_{copy})$
n_{copy}
L=72:
$F(n_{copy})$



An approach to the absolute Landau gauge:
first we run Z_2^3 sectors



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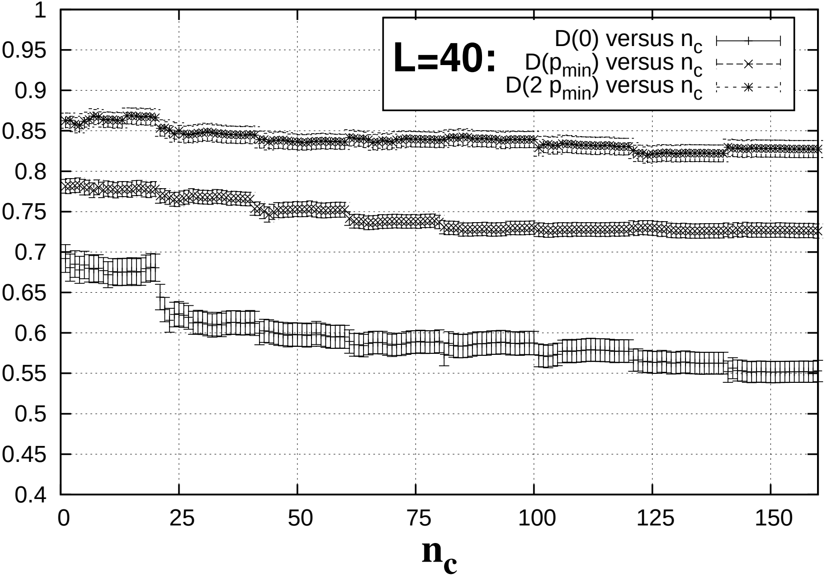
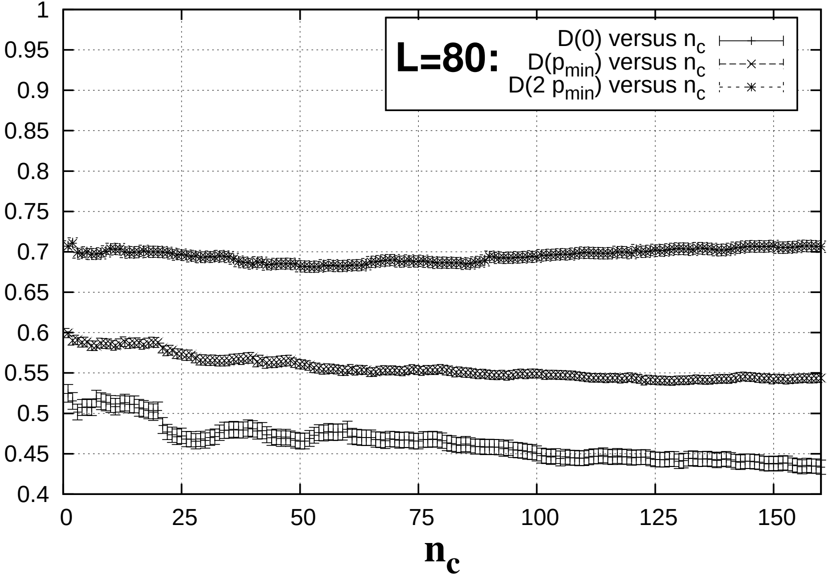


An approach to the absolute Landau gauge:
first we run within a single sector



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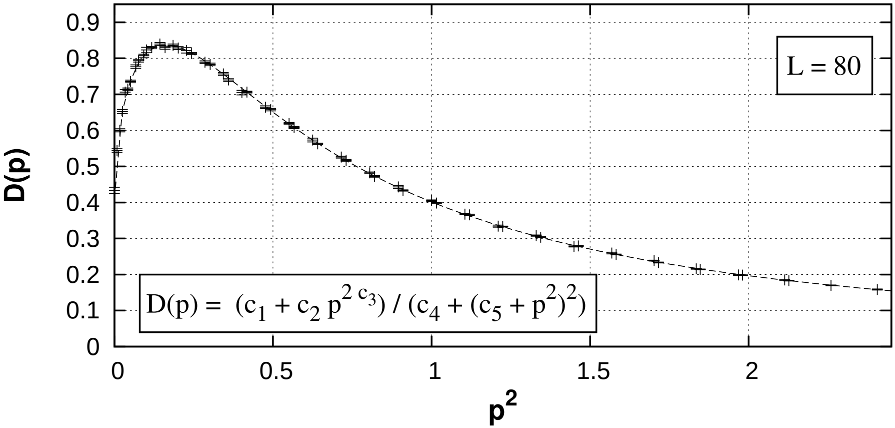
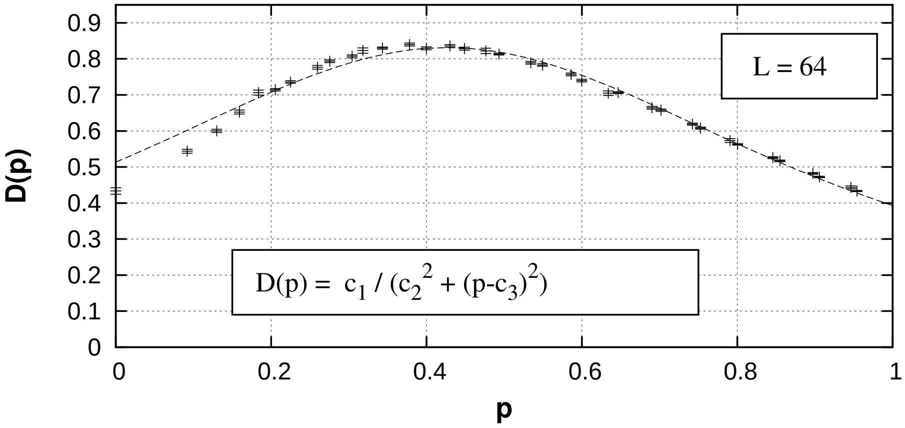


Fit parameters: $c_1 = 0.122 \pm 0.004,\ c_2 = 0.668 \pm 0.011,$
$c_3 = 0.563 \pm 0.012,\ c_4 = 0.184 \pm 0.007,\ c_5 = 0.335 \pm 0.011,$
$\frac{\chi^2}{N_{dof}} = 1.33$

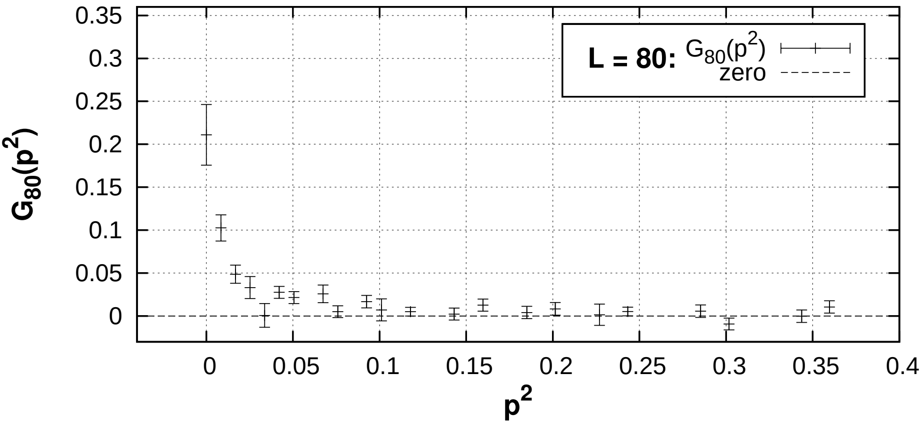


Conventional parametrization by mass

does not work

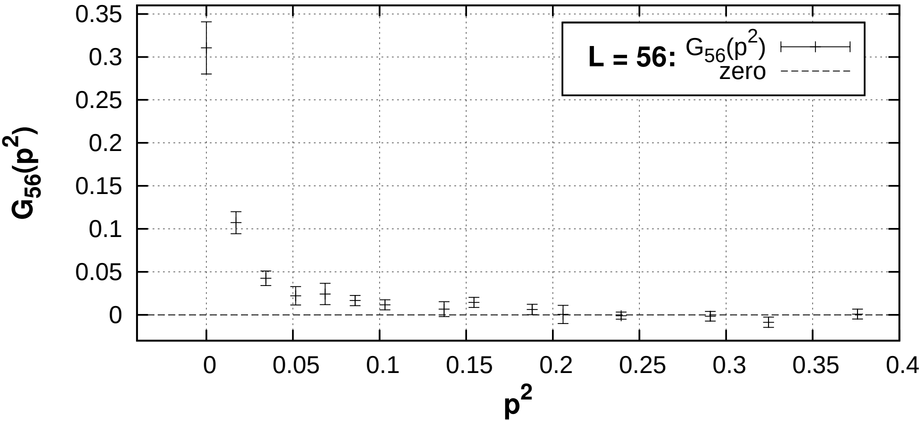
However, to study the infrared asymptotics more precisely, we should consider the infinite-volume limit. To take an example, $D(0)$ versus $L = Na$

Taking Gribov copies into account results in a substantial decrease of $D(0)$, $D(p_{min})$, $D(2p_{min})$. An analysis performed on a finite lattice with the neglect of such decrease may lead to erroneous conclusions on infrared behavior of the gluon propagator.



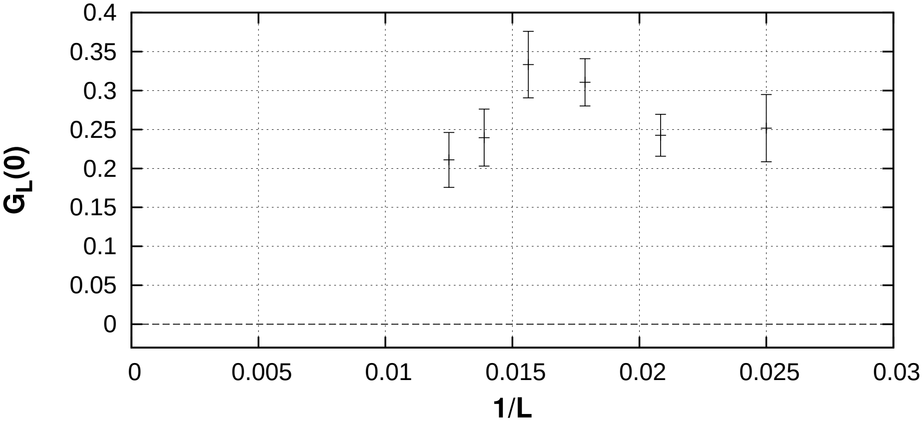
The effect of Gribov copies

$$G_L(p) = \frac{D^{(first)}(p) - D^{(best)}(p)}{D^{(best)}(p)} \qquad (13)$$



$$G_L(p) = \frac{D^{(first)}(p) - D^{(best)}(p)}{D^{(best)}(p)} \tag{14}$$

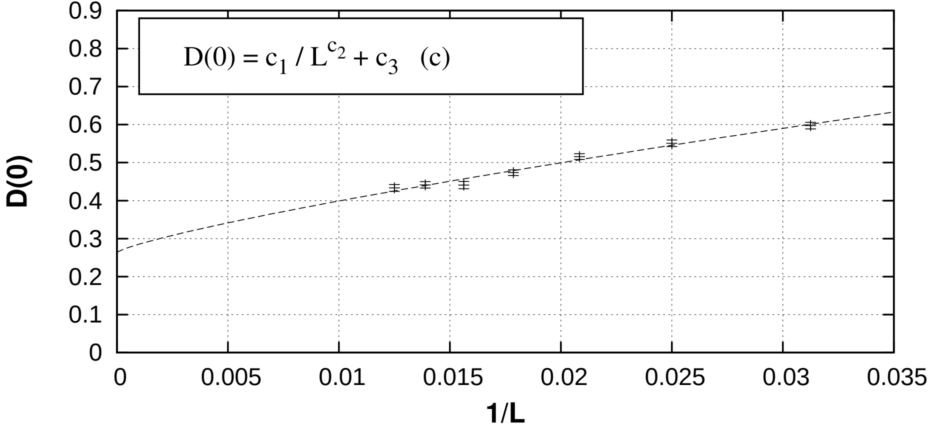
Maas [0808.3047]: $G_{56}(0) \simeq 0.1$, $\beta = 4.24$
In our study, the effect is 3 times greater.



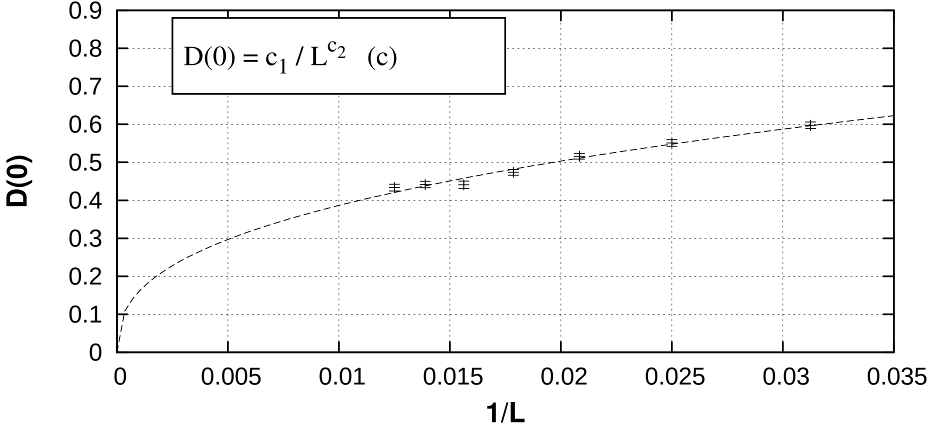
The effect of Gribov copies
on the zero-momentum propagator
as a function of volume

It is considered [Zwanziger, 1999] that, in the infinite-volume limit, Gribov copies have no effect on the gluon propagator. This statement can now be formulated more precisely:

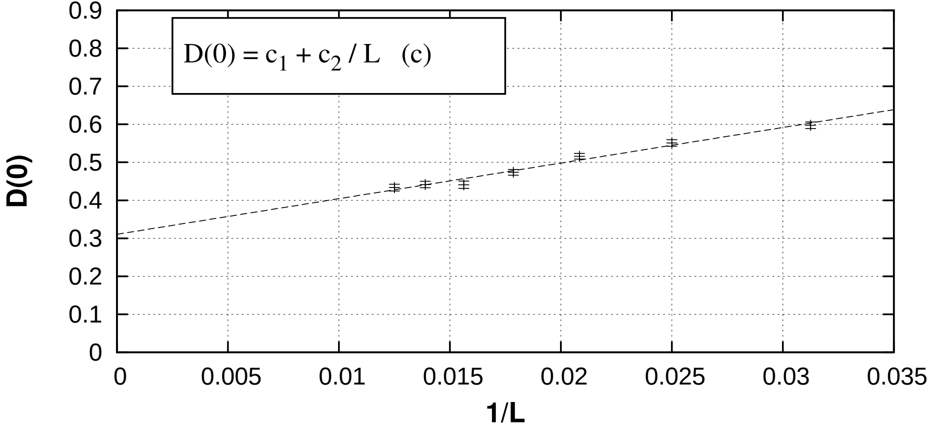
- For a fixed physical momentum ($p \neq 0$) $G_L(p) \to 0$ as $L \to \infty$
- For $p = 0,\ p = p_{min} = \dfrac{2\pi a}{L},\ p = 2p_{min}, ..., p = tp_{min}$ the effect of Gribov copies (measured by $G_L(p)$) exists and ranges up to 0.25 for $p = 0$. However, it decreases exponentially with t.



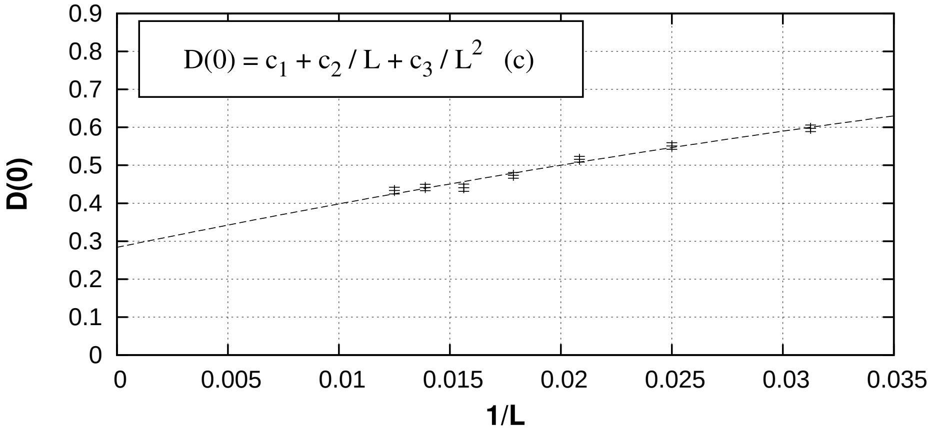
$$c_1 = 5.47 \pm 7.3,\ c_2 = 0.81 \pm 0.51,\ c_3 = 0.26 \pm 0.15, \quad \frac{\chi^2}{4} = 1.53;$$



$$c_1 = 2.23 \pm 0.23,\ c_2 = 0.38 \pm 0.03, \qquad \frac{\chi^2}{5} = 1.45$$

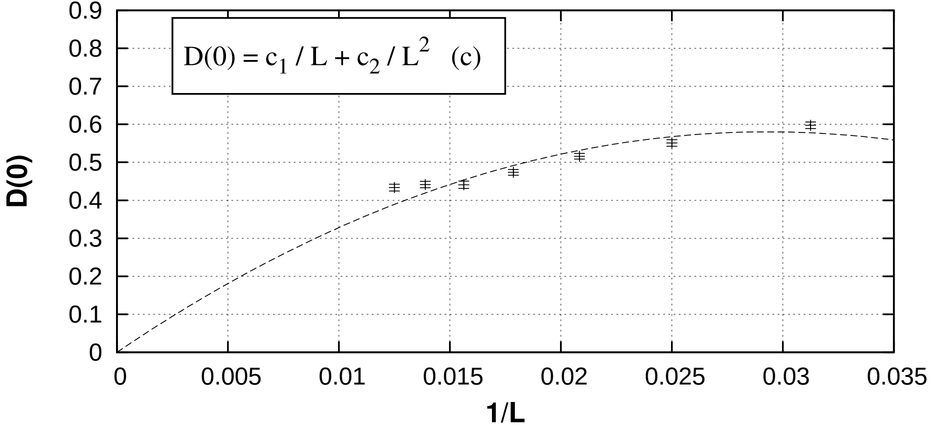


$$c_1 = 0.31 \pm 0.01, \; c_2 = 9.4 \pm 0.6, \qquad \frac{\chi^2}{5} = 1.26$$



$$c_1 = 0.28 \pm 0.05,\ c_2 = 12.1 \pm 4.9,\ c_3 = -62 \pm 113,$$

$$\frac{\chi^2}{4} = 1.47$$



$$c_1 = 39.6 \pm 1.9,\ c_2 = -675 \pm 80, \qquad \frac{\chi^2}{5} = 10.5$$

The scaling solution $D(p) \simeq (p^2)$ characterized by $D(0) = 0$ is not excluded in the absolute Landau gauge.

In agreement with Maas, 2008

In the minimal Landau gauge it is excluded

[Maas 2008; Cucchieri, Mendes et al. 2003-2010]