Landau gauge gluon and ghost propagators from the lattice point of view

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Outline of the talk

1. Introduction, motivation: the infrared QCD debate
2. How to compute Landau gauge gluon and ghost propagators on the lattice
3. Results for gluon, ghost propagators and the running coupling in lattice quenched and full QCD
4. Gribov copies, finite-volume effects, multiplicative renormalization, continuum limit
5. Conclusion and outlook

1. Introduction, motivation: the infrared debate

Landau gauge gluon, ghost, quark propagators and vertex functions:

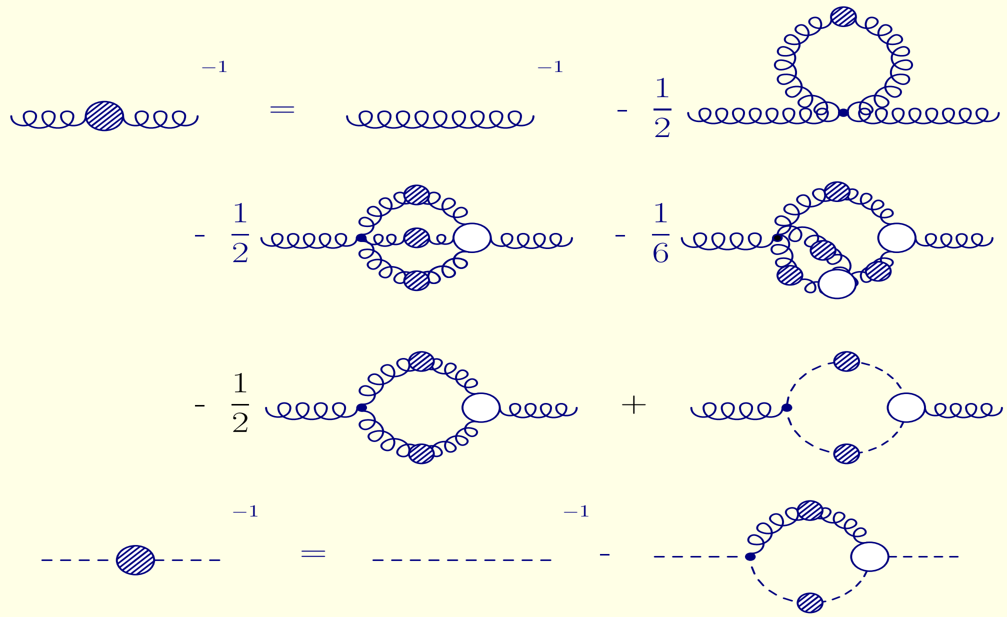
- ⇒ allow to fix phenomenologically useful parameters:
  effective (dynamical) gluon mass m_g, Λ_{QCD}, $\langle\overline{\psi}\psi\rangle$, $\langle A^2\rangle$ (?), ...;
- ⇒ can be directly used as input for hadron phenomenology:
  Bethe-Salpeter eqs. for mesons, Faddeev eqs. for baryons,
  cf. Alkofer, Eichmann, Krassnigg, Nicmorus, Chin. Phys. C34 (2010),
  arXiv:0912.3105 [hep-ph];
- ⇒ their infrared behaviour is related to confinement criteria:
  Gribov-Zwanziger, Kugo-Ojima, violation of positivity,...;
- ⇒ for $T > 0$ allow for determining screening length
  and other characteristica at T_c.

⟹ Intensive non-perturbative investigations in the continuum and on the lattice over many years.

⟹ Infrared (IR) limit of special interest.

Landau gauge Green's functions in the continuum determined from (truncated) Dyson-Schwinger (DS) and Wetterich funct. RG (FRG) eqs. taking into account Slavnov-Taylor identities (STI)

[Alkofer, Aguilar, Boucaud, Dudal, Fischer, Pawlowski, von Smekal, Zwanziger,.. ('97 - '09)]



$$\Rightarrow \quad D^{ab}_{\mu\nu} = \delta^{ab}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\frac{Z(q^2)}{q^2}$$

$$\Rightarrow \quad G^{ab} = \delta^{ab}\frac{J(q^2)}{q^2}$$

Running coupling from ghost-ghost-gluon vertex in a MOM scheme

$$\alpha_s(q^2) \equiv \frac{g^2(\mu)}{4\pi}\; Z(q^2,\mu^2)\cdot[J(q^2,\mu^2)]^2$$

Renorm. group invariant quantity.

Infrared "scaling" solution of DS and FRG eqs.

[Alkofer, Fischer, Lerche, Maas, Pawlowski, von Smekal, Zwanziger,... ('97 - '09)]

$$Z(q^2) \propto (q^2)^{\,\kappa_D}\,, \quad J(q^2) \propto (q^2)^{-\kappa_G} \quad \text{for} \quad q^2 \to 0$$

with

$$\kappa_D = 2\,\kappa_G + (4-d)/2 \quad \Longrightarrow \quad \kappa_D = 2\,\kappa_G, \quad \kappa_G \simeq 0.59 \quad \text{for} \quad d=4$$

is claimed

- to be consistent with BRST quantization,
- to hold without any truncation of the tower of DS or FRG eqs.,
- to be independent of the number of colors N_c,
- to look qualitatively the same in any dimension $d = 2, 3, 4$.

Running coupling:

$$\alpha_s(q^2) \to \text{const.} \quad \text{for} \quad q^2 \to 0$$

i.e. infrared fixed point as in analytic perturbation theory

[D.V. Shirkov, I.L. Solovtsov ('97 - '02)].

Alternative "decoupling" IR solution

[Boucaud et al. ('06 -'08), Aguilar et al. ('07-'08), Dudal et al. ('05-'08)]

$$\kappa_D = 1\,,\ \kappa_G = 0$$

i.e.

$$D(q^2) = Z(q^2)/q^2 \to \text{const.}\,, \qquad J(q^2) \to \text{const.}$$

such that

$$\alpha_s(q^2) = \frac{g^2}{4\pi}\, Z(q^2) \cdot [J(q^2)]^2 \quad \to 0 \ \text{ for } \ q^2 \to 0\,.$$

Existence has been confirmed.

[Fischer, Maas, Pawlowski, Annals Phys. '09, arXiv:0810-1987 [hep-ph]]

No debate any more on who is right, but about criteria what is the physically correct solution (BRST).

Claim: $J(0)$ might be chosen as an IR boundary condition.

Expect: close relation to the notorious Gribov problem.

Question: Relevance for phenomenology ?

IR “scaling” solution for Z, J at a first view in agreement with confinement scenarios:

- Kugo-Ojima confinement criterion [Ojima, Kugo ('78 - '79)]:
  absence of colored physical states $\Longleftrightarrow$ ghost (gluon) propagator more (less) singular than simple pole for $q^2 \to 0$.

- Gribov-Zwanziger confinement scenario
  [Gribov ('78), Zwanziger ('89 - ...)]:
  gauge fields restricted the Gribov region

$$\Omega = \left\{ A_\mu(x) : \ \partial_\mu A_\mu = 0, \ M_{FP} \equiv -\partial D(A) \geq 0 \right\}$$

  are accumulated at the Gribov horizon $\partial\Omega$:

  non-trivial eigenvalues of M_{FP}: $\lambda_0 \to 0$.

$$\Longrightarrow \quad \begin{array}{ll} \text{Ghost:} & J(q^2) \ \to \ \infty \\ \text{Gluon:} & D(q^2) \ \to \ 0 \end{array} \qquad \text{for} \ \ q^2 \to 0.$$

There are attempts to modify scenarios such, that IR “decoupling” solution can be accomodated, too. [Dudal et al. ('08 - '09), Kondo ('09)].

The Gribov problem:

- Existence of many gauge copies inside Ω.

- What are the right copies?

  Restriction inside Ω to fundamental modular region (FMR) required

$$\Lambda = \left\{ A_\mu(x) : \; F(A^g) > F(A) \;\; \text{for all} \;\; g \neq \mathbf{1} \right\},$$

  i.e. to global extremum of the Landau gauge functional $F(A^g)$?

Answer in the limit of infinite volume [Zwanziger ('04)]:

Non-perturbative quantization requires only restriction to Ω,

$$\text{i.e.} \quad \delta_\Omega(\partial_\mu A_\mu) \det(-\partial_\mu D^{ab}_\mu) e^{-S_{YM}[A]} \,.$$

Expectation values taken on Ω or Λ should be equal in the thermodynamic limit.

- What happens on a (finite) torus?

- How Gribov copies influence finite-size effects?

Questions to Yang-Mills theory on the lattice:

- What kind of infrared DS and FRG solutions are supported ?
- What is the influence of Gribov copies on gluon and ghost propagators ?
- Finite-volume effects ?
- Continuum limit, scaling, non-perturbative multiplicative renormalization at finite volume ?

Lattice investigations of gluon and ghost propagators over many years in

Adelaide: Bonnet, Leinweber, von Smekal, Williams, et al.;

Berlin: Burgio, Ilgenfritz, M.-P., Sternbeck, et al.;

Dubna/Protvino: Bakeev, Bogolubsky, Bornyakov, Mitrjushkin;

San Carlos: Cucchieri, Maas, Mendes;

Paris: Boucaud, Leroy, Pene, et al.;

Coimbra: Oliveira, Silva;

Tübingen: Bloch, Langfeld, Reinhardt, Watson et al.;

Utsunomiya: Furui, Nakajima.

2. How to compute Landau gauge gluon and ghost propagators on the lattice

A few technicalities:

i) Generate lattice discretized gauge fields $U = \{U_{x,\mu} \equiv e^{iag_0 A_\mu(x)} \in SU(N_c)\}$ by MC simulation from path integral

$$Z_{\text{Latt}} = \int \prod_{x,\mu} [dU_{x,\mu}] \ (\det Q(\kappa, U))^{N_f} \ \exp(-S_G(U))$$

– standard Wilson plaquette action

$$S_G(U) = \beta \sum_x \sum_{\mu<\nu} \left(1 - \frac{1}{N_c} \, \mathfrak{Re} \, \mathrm{tr} \, U_{x,\mu\nu}\right),$$

$$U_{x,\mu\nu} \equiv U_{x,\mu} U_{x+\hat{\mu},\nu} U^\dagger_{x+\hat{\nu},\mu} U^\dagger_{x,\nu}, \qquad \beta \equiv 2N_c/g_0^2$$

– (clover-improved) Dirac-Wilson fermion operator $Q(\kappa, U)$:

$N_f = 0$ – pure gauge case,

$N_f = 2$ – full QCD with equal bare quark masses $ma = 1/2\kappa - 1/2\kappa_c$,

$a(\beta)$ – lattice spacing.

ii) Z_{Latt} is simulated with (Hybrid) MC method without gauge fixing.

iii) Gauge fix each lattice field U:

$$U^g_{x\mu} = g_x \cdot U_{x\mu} \cdot g^\dagger_{x+\hat{\mu}}$$

standard orbits: $\{g_x\}$ periodic on the lattice;

extended orbits: $\{g_x\}$ periodic up to global $Z(N)$ transformations;

Landau gauge: linear definition $\mathcal{A}_{x+\hat{\mu}/2,\mu} = \frac{1}{2iag_0} \left(U_{x\mu} - U^\dagger_{x\mu}\right)\Big|_{\text{traceless}}$

$$(\partial\mathcal{A})_x = \sum_{\mu=1}^{4} \left(\mathcal{A}_{x+\hat{\mu}/2;\mu} - \mathcal{A}_{x-\hat{\mu}/2;\mu}\right) = 0$$

equivalent to minimizing the gauge functional

$$F_U(g) = \sum_{x,\mu} \left(1 - \frac{1}{N_c}\, \mathfrak{Re}\ \mathrm{tr}\, U^g_{x\mu}\right) = \text{Min.}.$$

For uniqueness (FMR) one requires to find the global minimum

[Parrinello, Jona-Lasinio ('90), Zwanziger ('90)].

Well understood in compact $U(1)$ theory in order to get

e.g. massless photon propagator

[Bogolubsky, Bornyakov, Mitrjushkin, M.-P., Peters, Zverev ('93 - '99)].

Optimized minimization in (our) practice: simulated annealing (SA) + overrelaxation (OR)

Gribov problem: global minimum of $F_U(g)$ very hard or impossible to find.

"Best copy strategy": repeated initial random gauges

$\Longrightarrow$ best copies (bc) from subsequent SA + OR minimizations,

$\Longrightarrow$ compared with first (random) copies (fc)).

iv) Compute propagators

- Gluon propagator:

$$D^{ab}_{\mu\nu}(q) = \left\langle \widetilde{A}^a_\mu(k) \widetilde{A}^b_\nu(-k) \right\rangle \equiv \delta^{ab} \left(\delta_{\mu\nu} - \frac{q_\mu \; q_\nu}{q^2} \right) D(q^2)$$

for lattice momenta

$$q_\mu(k_\mu) = \frac{2}{a} \sin\left(\frac{\pi k_\mu}{L_\mu}\right), \qquad k_\mu \in \left(-L_\mu/2, L_\mu/2\right]$$

with certain cuts in order to suppress artifacts of lattice discretization.

- Ghost propagator:

$$G^{ab}(q) = \frac{1}{V^{(4)}} \sum_{x,y} \left\langle \mathrm{e}^{-2\pi i\, k\cdot(x-y)} [M^{-1}]^{ab}_{xy} \right\rangle \equiv \delta^{ab} G(q)\,.$$

$M \sim \partial_\mu D_\mu$ - Landau gauge Faddeev-Popov operator

$$M^{ab}_{xy}(U) = \sum_\mu A^{ab}_{x,\mu}(U)\,\delta_{x,y} - B^{ab}_{x,\mu}(U)\,\delta_{x+\hat{\mu},y} - C^{ab}_{x,\mu}(U)\,\delta_{x-\hat{\mu},y}$$

$$\begin{aligned} A^{ab}_{x,\mu} &= \quad \mathfrak{Re}\,\mathrm{tr}\left[\{T^a, T^b\}(U_{x,\mu} + U_{x-\hat{\mu},\mu})\right], \\ B^{ab}_{x,\mu} &= 2\cdot\mathfrak{Re}\,\mathrm{tr}\left[T^b T^a\, U_{x,\mu}\right], \\ C^{ab}_{x,\mu} &= 2\cdot\mathfrak{Re}\,\mathrm{tr}\left[T^a T^b\, U_{x-\hat{\mu},\mu}\right], \qquad \mathrm{tr}[T^a T^b] = \delta^{ab}/2\,. \end{aligned}$$

M^{-1} from solving

$$M^{ab}_{xy}\phi^b(y) = \psi^a_c(x) \equiv \delta^{ac} \exp(2\pi i\, k\cdot x)$$

with (preconditioned) conjugate gradient algorithm.

3. Results for gluon, ghost propagators and the running coupling and the running coupling in lattice quenched and full QCD

- Pure gauge case $N_f = 0$:
  $\beta = 5.7, 5.8, 6.0, 6.2$; $12^4, \ldots, 56^4$, $aL_{max} \simeq 9.5$fm;
  $\beta = 5.7$; $64^4, \ldots, 96^4$, $aL_{max} \simeq 16.3$fm.

- Full QCD case $N_f = 2$:
  thanks: configurations provided by QCDSF - collaboration,
  $\beta = 5.29, 5.25$; mass parameter $\kappa = 0.135, ..., 0.13575$;
  $16^3 \times 32$, $24^3 \times 48$.

- Results for propagators / dressing functions and α_s

$$\text{Gluon} \quad Z(q^2) \equiv q^2 D(q^2), \qquad \text{Ghost} \quad J(q^2) \equiv q^2 G(q^2)$$

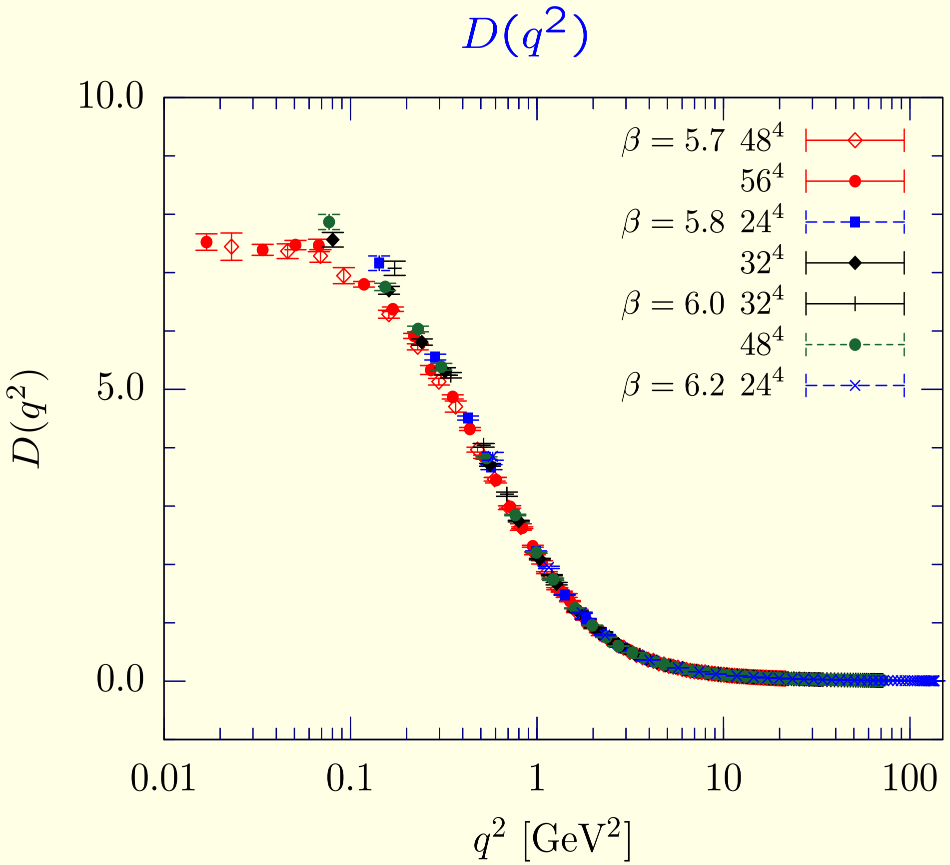
  as well as ghost-ghost-gluon vertex and Kugo-Ojima parameter.

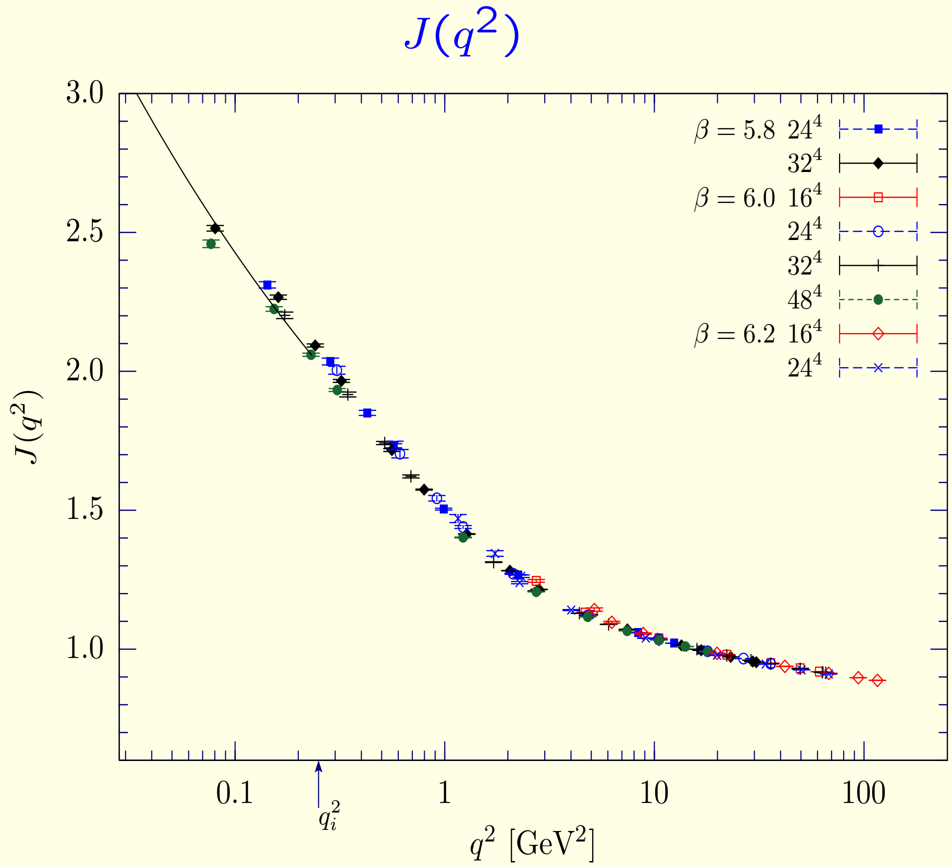
- Propagators at $T > 0$ studied, too $\Longrightarrow$ V.K. Mitrjushkin's talk.

First results: Gluon propagator and ghost dressing function

quenched QCD ($N_f = 0$), renorm. pt.: $q = \mu = 4\text{GeV}$, first OR copies

[Sternbeck, Ilgenfritz, M.-P., Schiller, PRD 72 (2006), Proc. IRQCD '06]





$\Longrightarrow$ Gluon prop. $D(q^2)$ shows plateau and not $D(q^2) \to 0$ for $q^2 \to 0$,

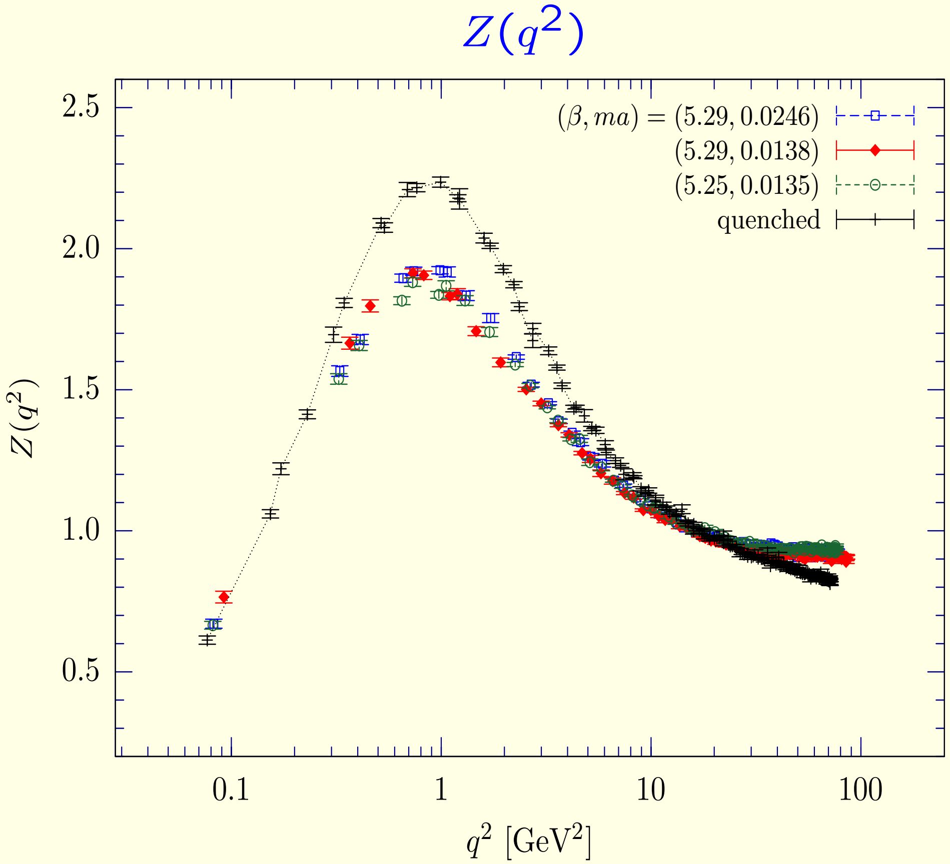
$\Longrightarrow$ corresponds to an effective gluon mass behaviour.

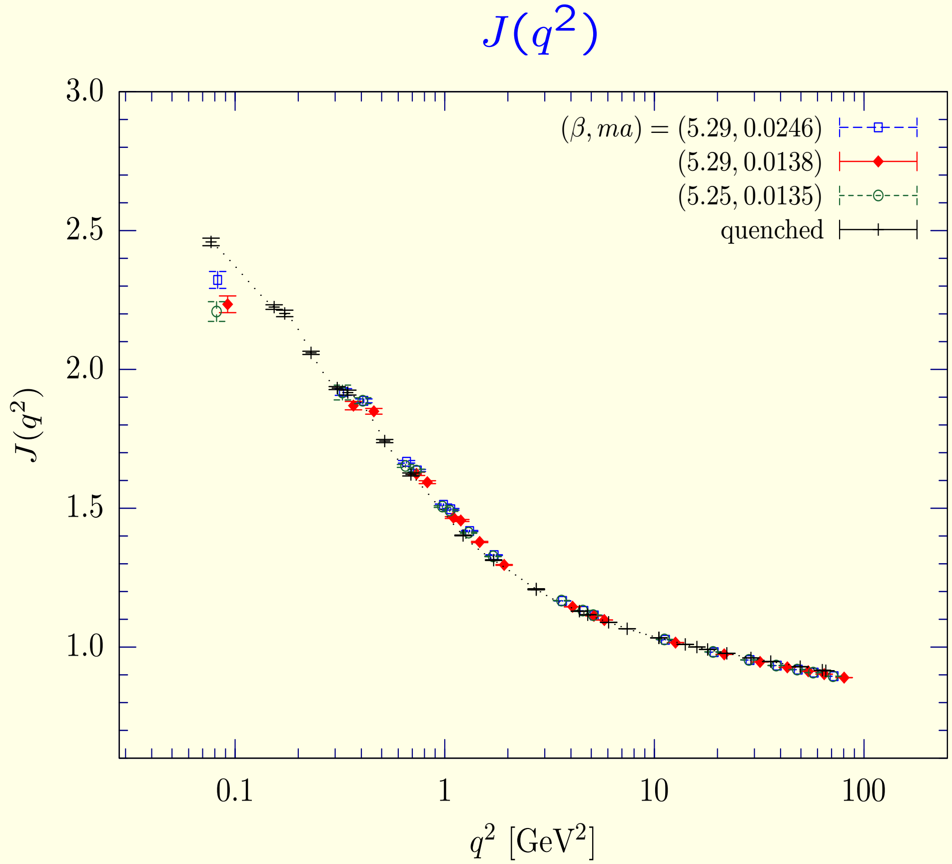
$\Longrightarrow$ Ghost dress. fct. $J(q^2)$ power-like, expon. too small for scaling solution.

Gluon and ghost dressing functions

full QCD ($N_f = 2$) versus quenched QCD ($N_f = 0$), renorm. point: $q = \mu = 4$GeV

[Ilgenfritz, M.-P., Schiller, Sternbeck (A. DiGiacomo 70, '06)]





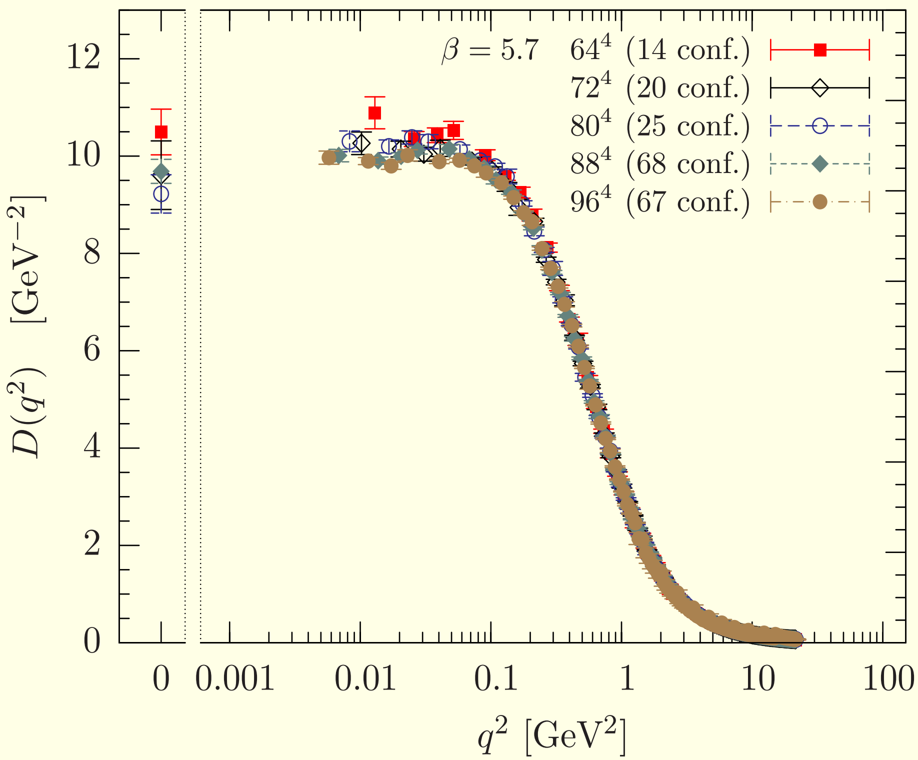
$\Longrightarrow$ Influence of virtual quark loops in $Z(q^2)$ clearly visible.

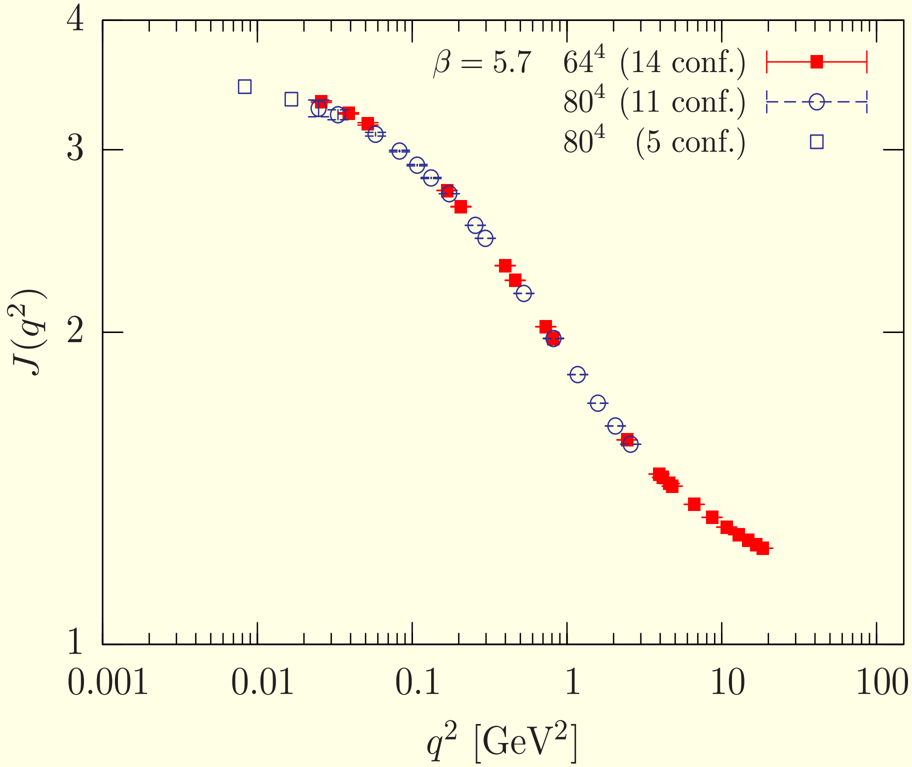
$\Longrightarrow$ No quenching effect in $J(q^2)$, as ghosts do not directly couple to quarks.

Gluon propagator and ghost dressing function on huge volumes

quenched QCD, first but long run SA + OR copies, unrenormalized

[Bogolubsky, Ilgenfritz, M.-P., Sternbeck, PLB 676 (2009)]





⟹ Both $D(q^2)$ and $J(q^2)$ seem to tend to const..

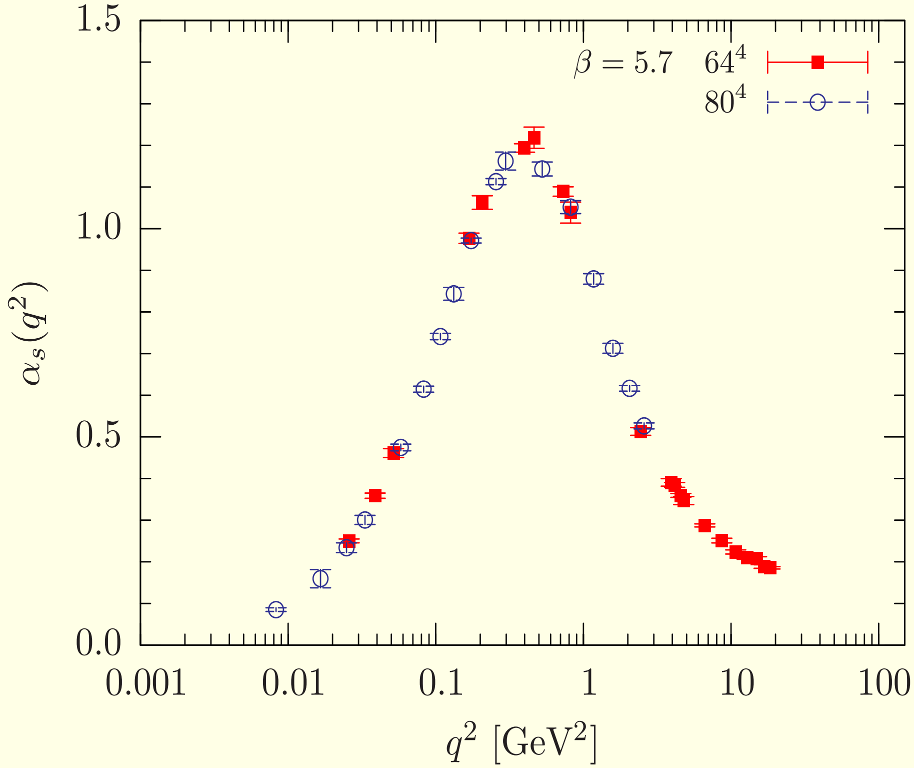
⟹ Clear indication for "decoupling" solution.

⟹ Here coarse lattices used. Question: continuum limit ?

Result for the running coupling on large volumes

quenched QCD, first but long run SA + OR copies, coarse lattices

[Bogolubsky, Ilgenfritz, M.-P., Sternbeck, PLB 676 (2009),]



- Running coupling not monotonous, $\alpha_s \rightarrow 0$ for $q \rightarrow 0$,
  $\Longrightarrow$ "decoupling behaviour".
- Agrees with other lattice studies, in particular for the three-gluon vertex.
- At large q^2 allows to fix $\Lambda_{\overline{MS}}$.

4. Gribov copies, finite-volume effects, multiplicative renormalization, continuum limit

Systematic effects somewhat easier to study in pure $SU(2)$ gauge theory.

[Bakeev, Bogolubsky, Bornyakov, Burgio, Ilgenfritz, Mitrjushkin, M.-P. ('04 - '09)]

Improved gauge fixing $\Longrightarrow$ getting 'close' to the FMR:

- Simulated annealing (SA):

  Find g's randomly with statistical weight:

$$W \;\propto\; \exp\left(-\frac{F_U(g)}{T_{SA}}\right) \,.$$

  Let "temperature" T_{SA} slowly decrease. Infinitely slow cooling ends at the global extremum. In practice SA clearly wins for large lattice sizes. (Over)relaxation (OR) has to be applied subsequently in order to reach

$$(\partial\mathcal{A})_x = \sum_{\mu=1}^{4} \left(\mathcal{A}_{x+\hat{\mu}/2;\mu} - \mathcal{A}_{x-\hat{\mu}/2;\mu}\right) < \epsilon \quad \text{for all} \quad x \,.$$

- $\mathbb{Z}(N_c)$ flips:

  Gauge functional $F_U(g)$ minimized by enlarging the gauge orbit with respect to $\mathbb{Z}(N_c)$ non-periodic gauge transformations:

$$g(x + L\hat{\nu}) = z_\nu g(x)\,, \quad z_\nu \in \mathbb{Z}(N_c)\,.$$

  For $SU(N_c)$ the N_c^4 different sectors of Polyakov loop averages are combined.

In order to view Gribov copy effects we compare:

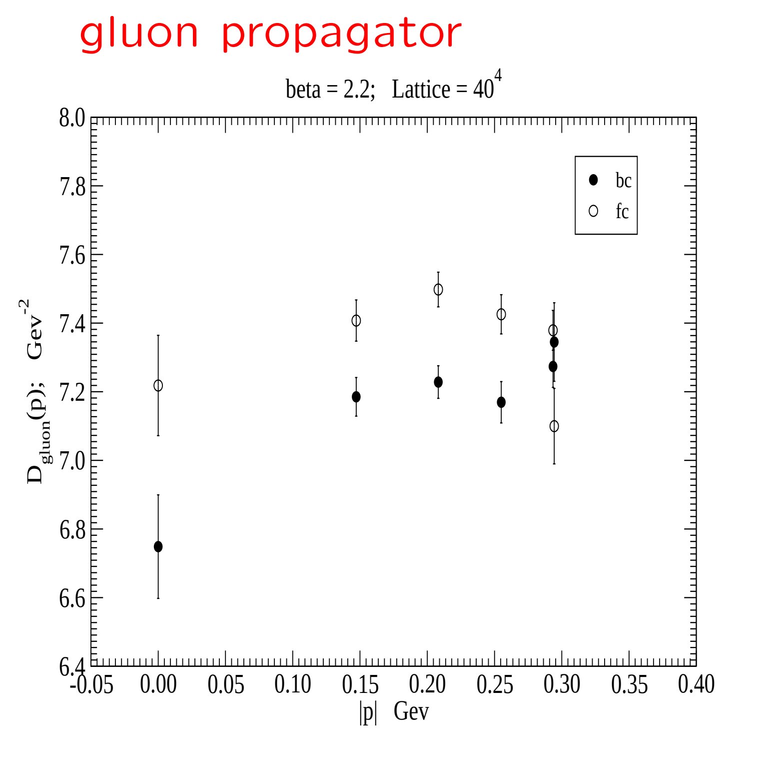
i) standard method: first (i.e. random) copy overrelaxation = "fc OR",

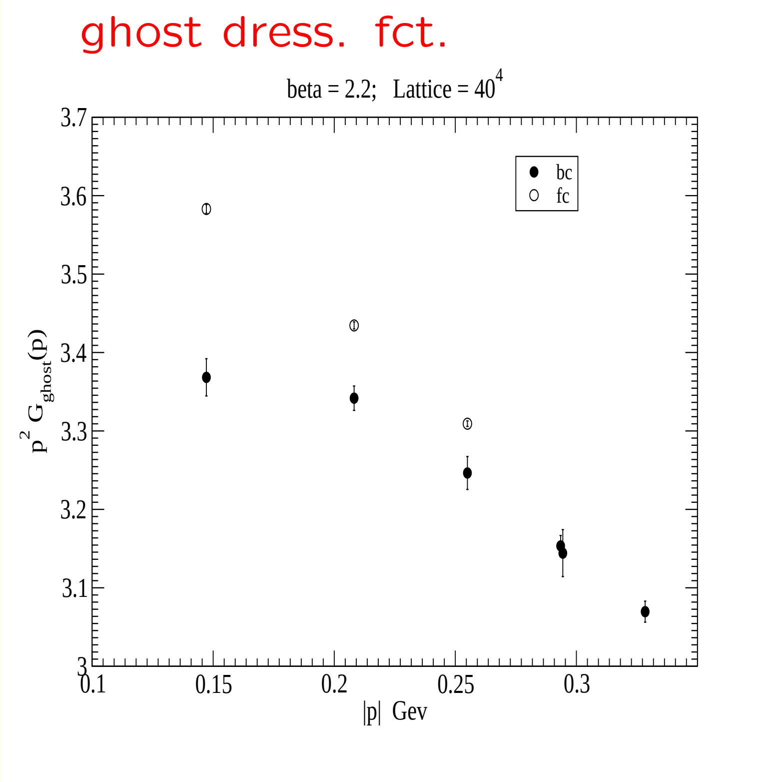
ii) first copy simulated annealing (incl. finalizing overrelaxation) = "fc SA",

iii) best copy $\mathbb{Z}(2)$ flips + simulated annealing (+OR) = "bc FSA",
compare typically 5 copies in each of the 16 Polyakov loop sectors
(= 80 copies).

Gluon propagator and ghost dressing fct.: fc SA versus bc FSA

[Bornyakov, Mitrjushkin, M.-P., PRD 79 (2009), arXiv:0812.2761 [hep-lat]]





$\Longrightarrow$ Gribov copies important for both gluon and ghost!

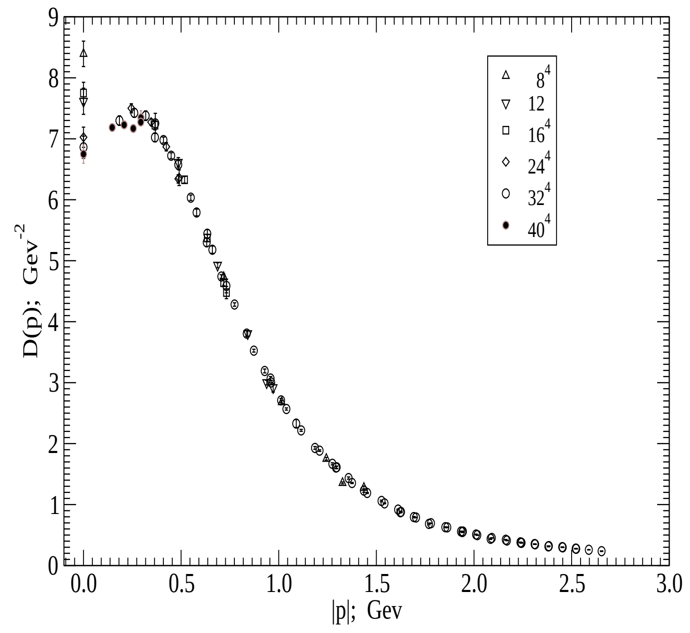
$\Longrightarrow$ The closer to the global minimum (FMR), the weaker the ‘singularity’ of the ghost dressing fct., the lower the IR values of the gluon propagator.

$\Longrightarrow$ $D(q^2 \to 0) = 0$? Together with a non-singular ghost dress. fct. this would completely contradict DS and FRG eqs. and (modified) Zwanziger approach.

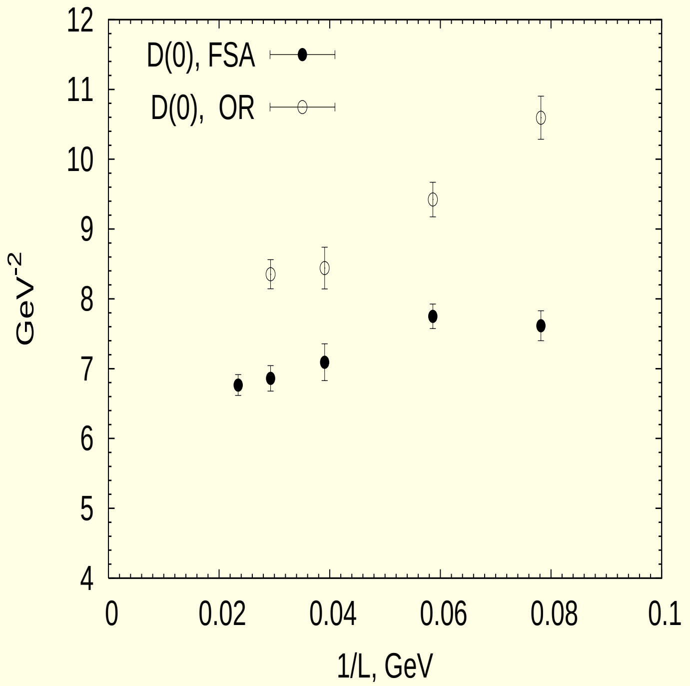
Gluon propagator $\beta = 2.2$: bc FSA versus fc OR

[Bornyakov, Mitrjushkin, M.-P., PRD 79 (2009), arXiv:0812.2761 [hep-lat]]

bc FSA $D(q^2)$ for diff. L^4



$D(0)$ versus $1/L$



$\Longrightarrow$ Finite-size effects weaker for FSA, i.e. when approaching the FMR Λ.

$\Longrightarrow$ Extrapolation $D(0) \neq 0$ for $V \to \infty$.

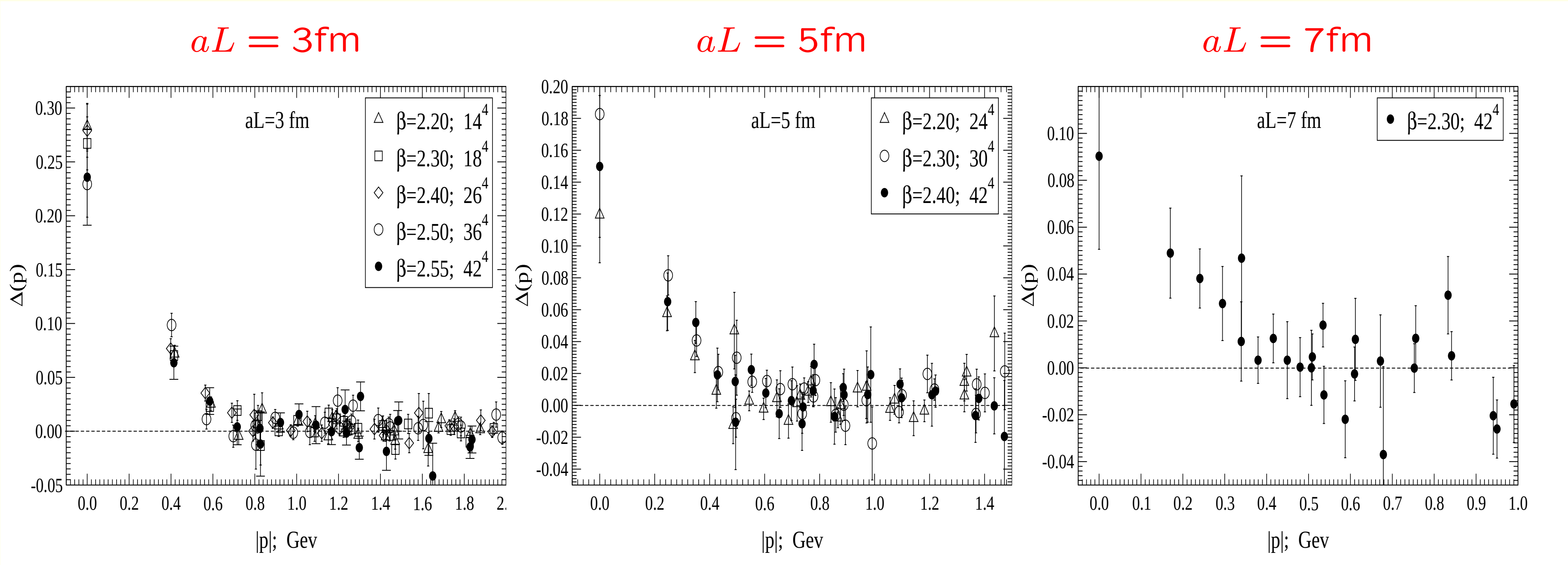
$\Longrightarrow$ Again support for "decoupling solution".

But still we have strong coupling, i.e. coarse lattices.

Gribov copy sensitivity for the gluon propagator bc FSA versus fc SA

$$\Delta(p) = \frac{D^{fc}(p) - D^{bc}(p)}{D^{bc}(p)}$$

[Bornyakov, Mitrjushkin, M.-P., PRD 81 (2010), arXiv:0912.4475 [hep-lat]].



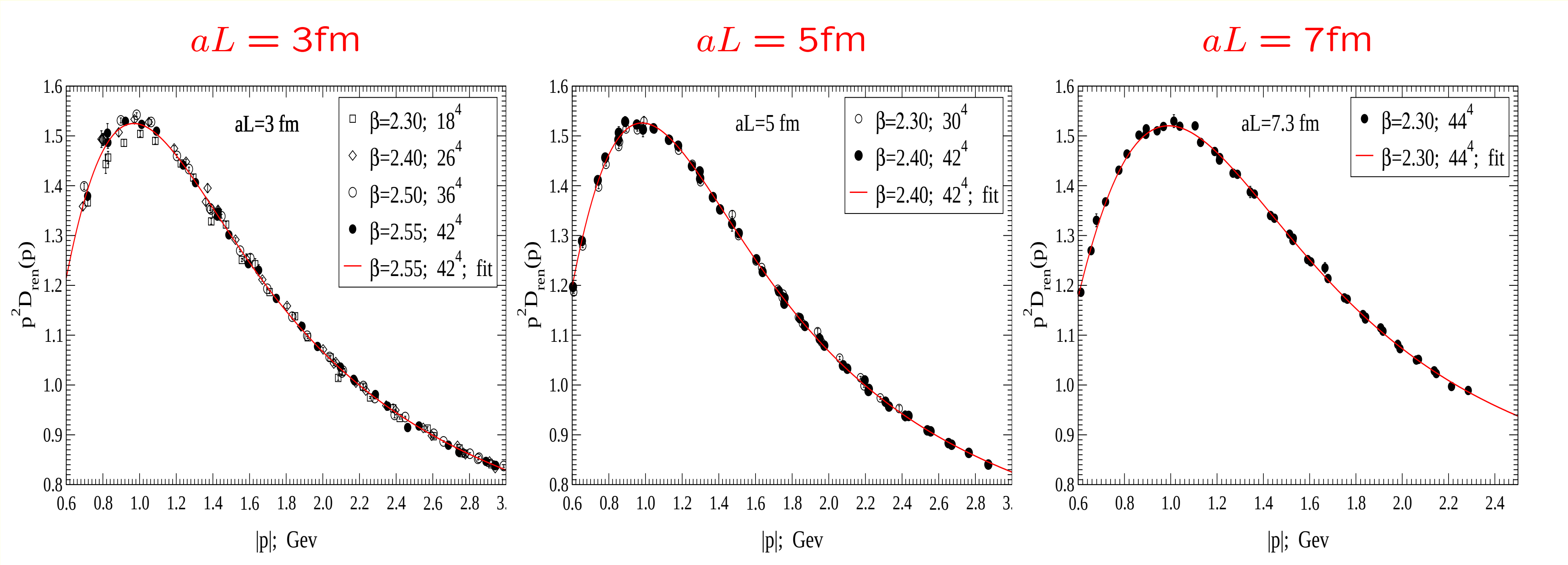
Gribov copy effect: ⇒ important at low momenta,

⇒ almost independent of lattice spacing,

⇒ weakens with increasing physical volume [Zwanziger ('04)].

Finite-volume and cont. limit results for renormalized gluon dressing fct. bc FSA

[Bornyakov, Mitrjushkin, M.-P., PRD 81 (2010), arXiv:0912.4475 [hep-lat]].



$\Longrightarrow$ For momenta $p > 0.4$MeV, $\beta \geq 2.40$ the renormalized data fall on top of each other,

$\Longrightarrow$ contin. result reached, good fits available $\longrightarrow$ IR effect. gluon mass,

$\Longrightarrow$ Curves for different linear sizes $3, 5, 7$ fm nicely agree.

Gluon and ghost propagators with alternative definition(s) of $A_\mu(x)$

Use logarithmic definition for the lattice gluon field

$$A^{(\log)}_{x+\frac{\hat{\mu}}{2},\mu} = \frac{1}{i\ a\ g_0} \log\left(U_{x,\mu}\right) ,$$

minimize lattice gauge functional directly translated from continuum

$$F_U^{(\log)}[g] = \sum_{x,\mu} \frac{1}{N_c} \mathrm{tr}\left[{}^g A^{(\log)}_{x+\frac{\hat{\mu}}{2},\mu} \, {}^g A^{(\log)}_{x+\frac{\hat{\mu}}{2},\mu} \right] .$$

Faddeev-Popov determinant derived accordingly.

Numerical treatment differs: accelerated multigrid algorithm + preconditioning.

$\Longrightarrow$ Compare results for linear and logarithmic definition.

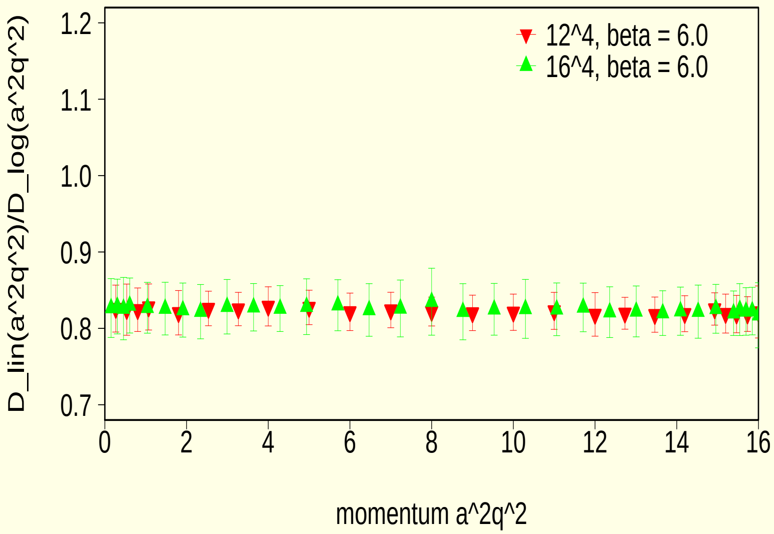
$\Longrightarrow$ Check independence of the running coupling.

$\Longrightarrow$ Compare with stochastic perturbation theory.

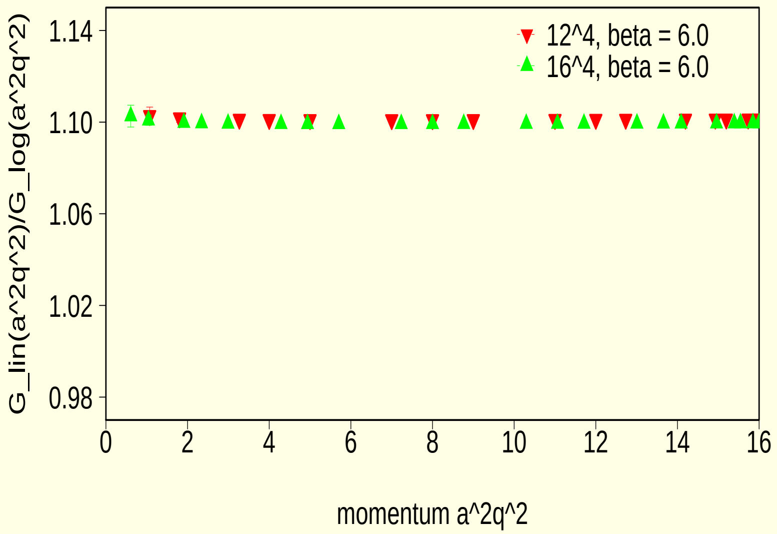
Related work: [Petrarca et al., '99; Cuchieri, Karsch, '99; Bogolubsky, Mitrjushkin, '02;...]

Linear definition results vs. logarithmic definition, $\beta = 6.0$

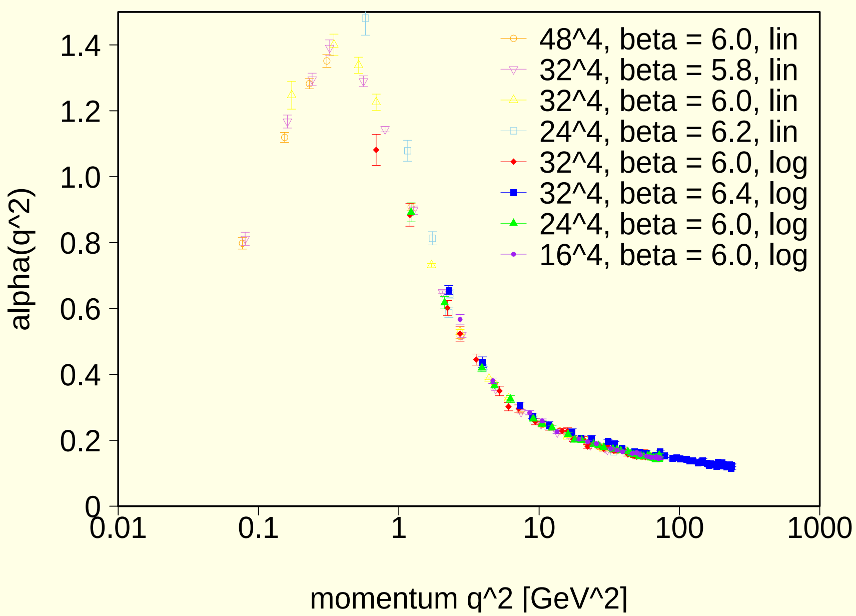
Gluon propagator ratio



Ghost propagator ratio



Running coupling



$\Rightarrow$ multipl. renormalizability confirmed.

$\Rightarrow$ $\alpha_s(q^2)$ in given MOM scheme approx. renorm. independent.

[Ilgenfritz, Menz, M.-P., to be published]

Monte Carlo vs. numerical stochastic perturbation theory (NSPT)

NSPT with Langevin technique allows for higher loop perturbation theory.

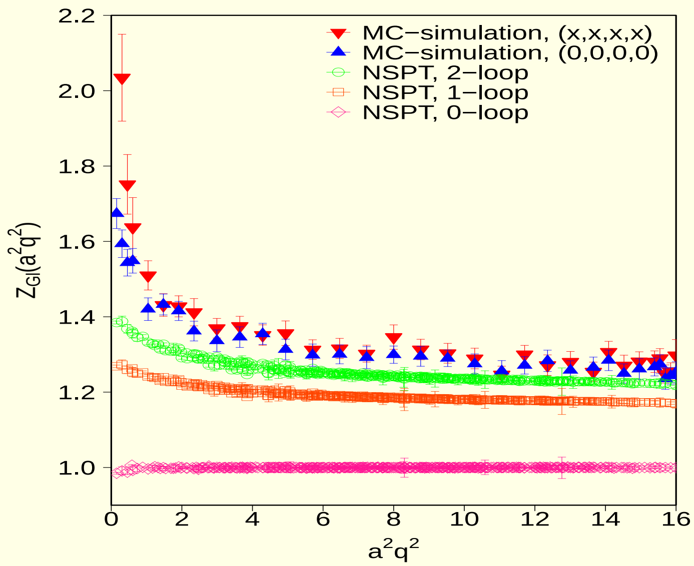
Logarithmic definition for A_μ is natural.

See [di Renzo, Ilgenfritz, Perlt, Schiller, Torrero, '09 - '10]

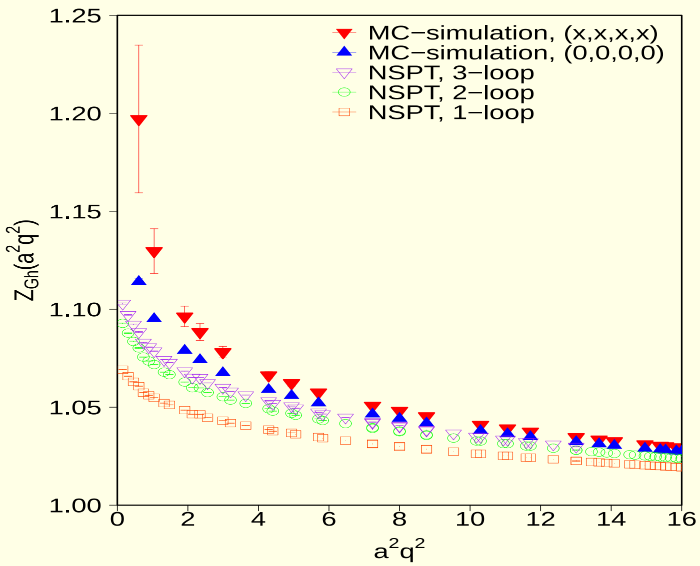
Compare arbitrary Polyakov loop sectors (x, x, x, x) with real sector $(0, 0, 0, 0)$.

Here: 16^4, large $\beta = 9.0$ for both approaches.

Gluon dressing fct.



Ghost dressing fct.



$\Rightarrow$ Nice consistency, approach to full result can be checked !

5. Conclusion and outlook

- Lattice results support "decoupling solution" as long as we assume approach

$$F_U(g) \rightarrow \text{Global Min.}$$

- Gribov effects turn out to be important. For pure LGT simulated annealing $+ \mathbb{Z}(N)$ flips ("bc FSA") provides a solution with weak finite-size effects.
- Continuum limit can be consistently reached within the non-perturbatively and phenomenologically important range around 1GeV.
- Analogous results, when available for full QCD will allow to tune DS and FRG truncations and provide immediate input into Bethe-Salpeter or Faddeev Eqs.
- Debate "scaling" versus "decoupling" solution continues.
  On the lattice it could mean to give up the condition $F_U(g) \rightarrow$ Global Min.
  [Maas et al. ('09)]
- Continuum alternative ?
  $\Longrightarrow$ A.A. Slavnov – Y.-M. theory without Gribov ambiguity

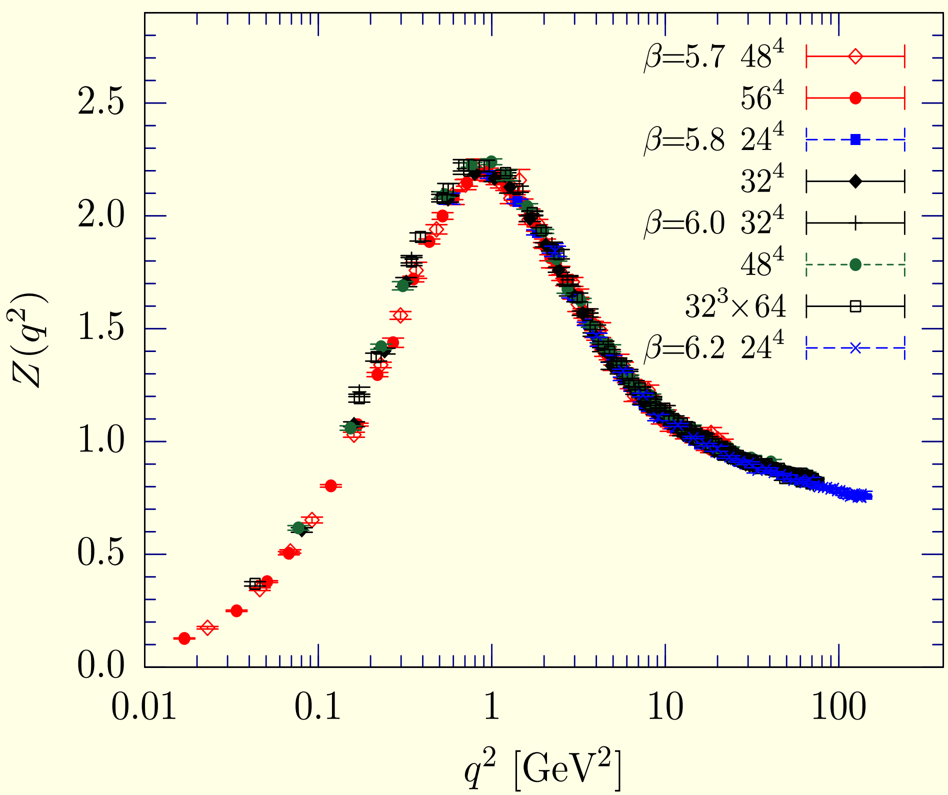
Thank you for your attention.

Gluon dressing function and propagator from first OR copies

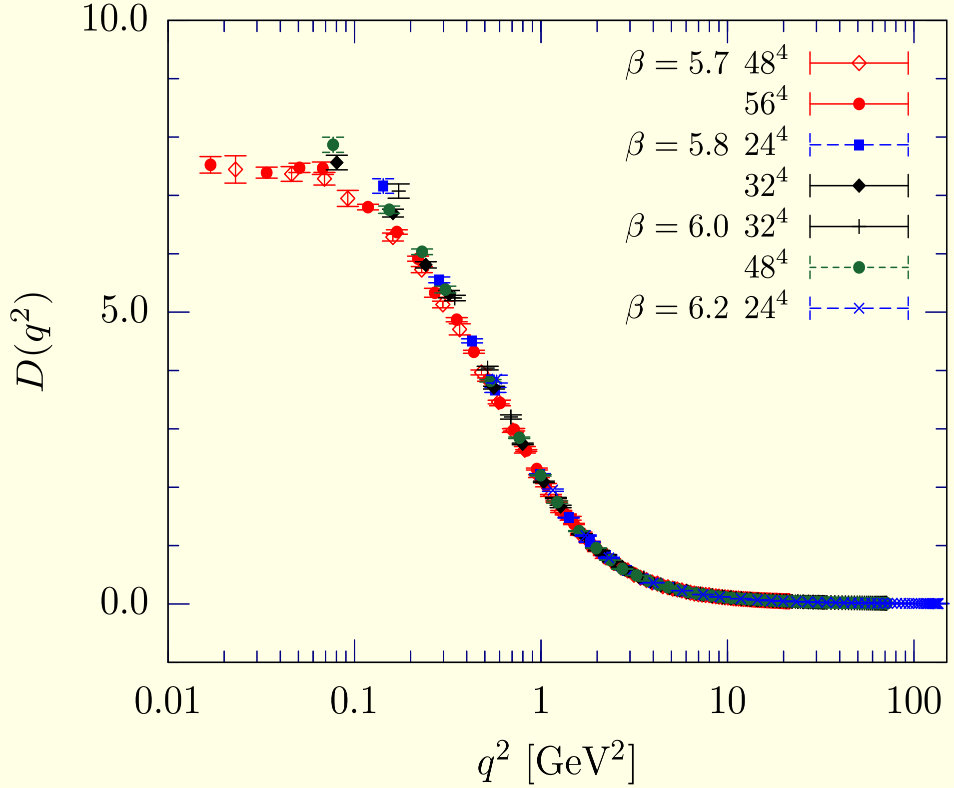
quenched QCD ($N_f = 0$), renorm. point: $q = \mu = 4$GeV

[Sternbeck, Ilgenfritz, M.-P., Schiller, PRD 72 (2006), Proc. IRQCD '06]

$Z(q^2)$



$D(q^2) = Z(q^2)/q^2$



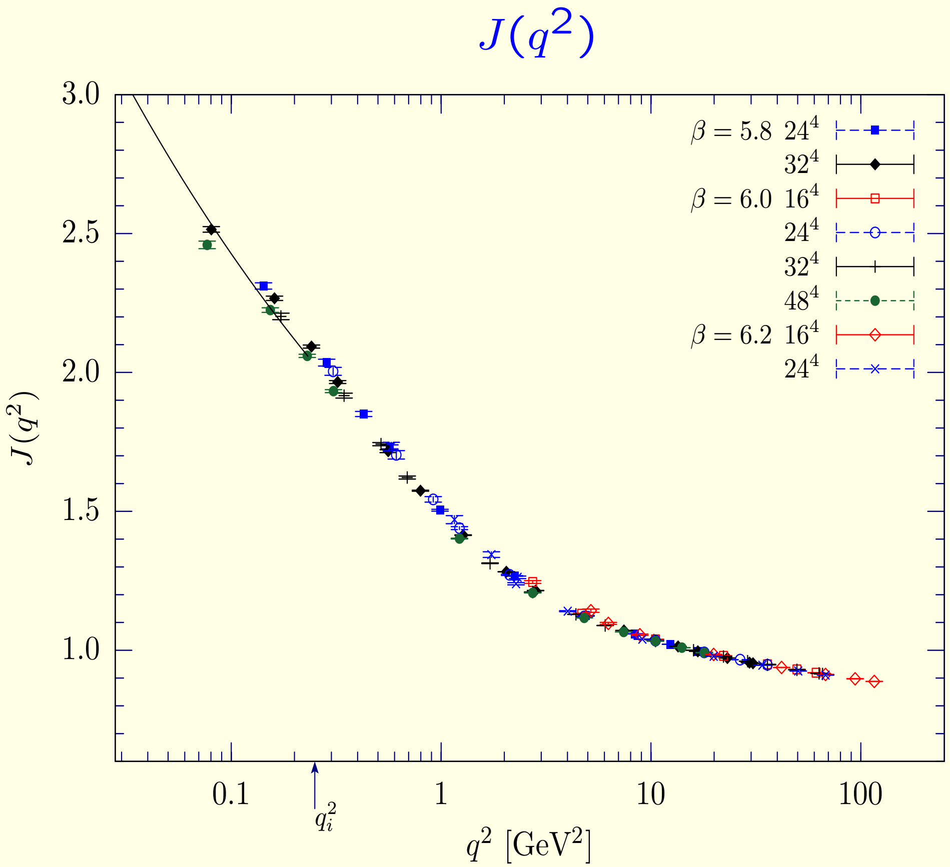
$\Longrightarrow$ $D(q^2) = Z(q^2)/q^2$ shows plateau and not $D(q^2) \to 0$ for $q^2 \to 0$,

$\Longrightarrow$ corresponds to an effective gluon mass.

Ghost dressing function from first OR copies

quenched QCD ($N_f = 0$), renorm. point: $q = \mu = 4$GeV

[Sternbeck, Ilgenfritz, M.-P., Schiller, PRD 72 (2006), Proc. IRQCD '06]



$\Longrightarrow$ looks still singular but with too small exponent

in comparison with "scaling" solution;

$\Longrightarrow$ no serious finite-volume effects (??).

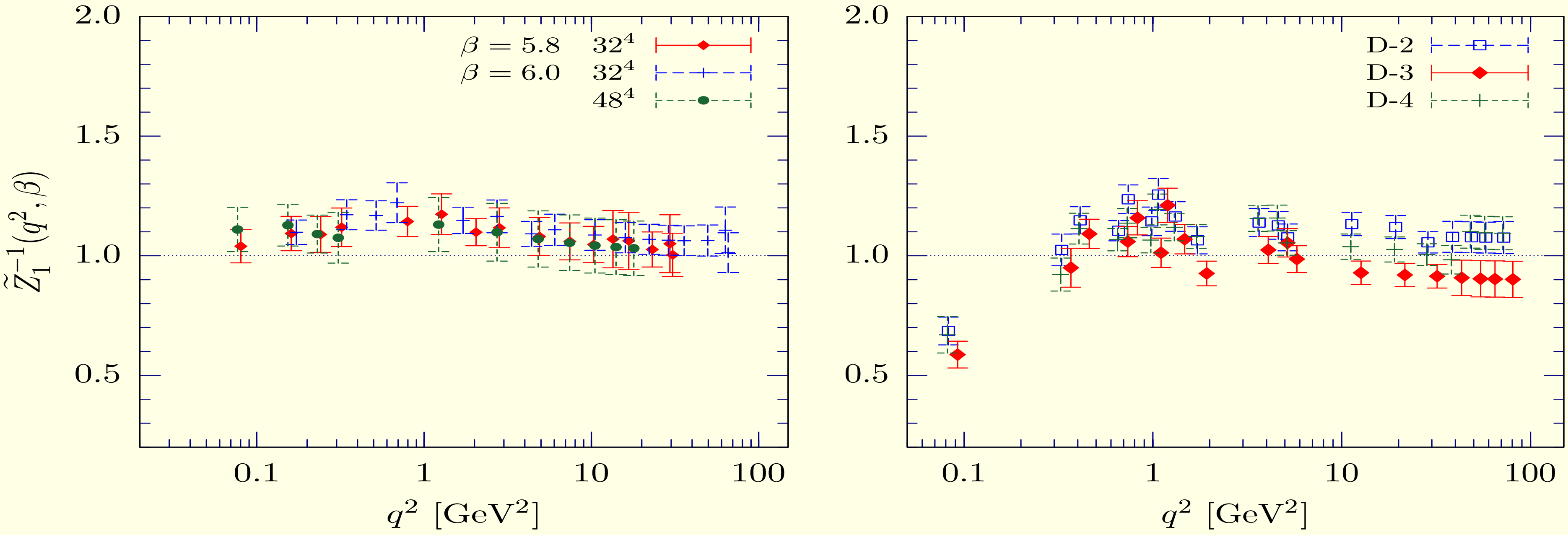
The running coupling from ghost-ghost-gluon vertex

$$\alpha_s(q^2) = \frac{g_0^2}{4\pi} Z(q^2)\,(J(q^2))^2 \quad \text{assuming} \quad \tilde{Z}_1 = 1$$

[perturbation theory: Taylor ('71) / LGT $SU(2)$: Cucchieri et al. ('04)]

The $SU(3)$ vertex renormalization function $\tilde{Z}_1$, gluon momentum $k = 0$.

$N_f = 0$ Ilgenfritz et al. '05 $N_f = 2$ Sternbeck thesis '06



Renormalized gluon propagators at $T > 0$

Temperature dependence from $T \equiv 1/aL_\tau, \quad L_\tau \ll L_\sigma$.

Separate time and space components, Matsubara frequency $\omega \sim q_4$.

Transverse propagator:

$$D_T \sim \langle \sum_{i=1}^{3} A_i^a(q)\; A_i^a(-q) - \frac{q_4^2}{\vec{q}^2}\; A_4^a(q)\; A_4^a(-q)\; \rangle$$

Longitudinal propagator:

$$D_L \sim (1 + \frac{q_4^2}{\vec{q}^2})\; \langle\; A_4^a(q)\; A_4^a(-q)\; \rangle$$

$T > T_c \implies$ spontaneous $Z(3)$ symmetry breaking.

Polyakov loop average $\langle L \rangle$ takes values in 3 sectors.

Real sector = "physical" sector.

[See Cucchieri, Karsch, '00; Bogolubsky, Mitrjushkin, '02; Fischer, Maas, Mueller, '10; ....]

Here: quenched QCD, gauge fixing – first copies with SA + OR,

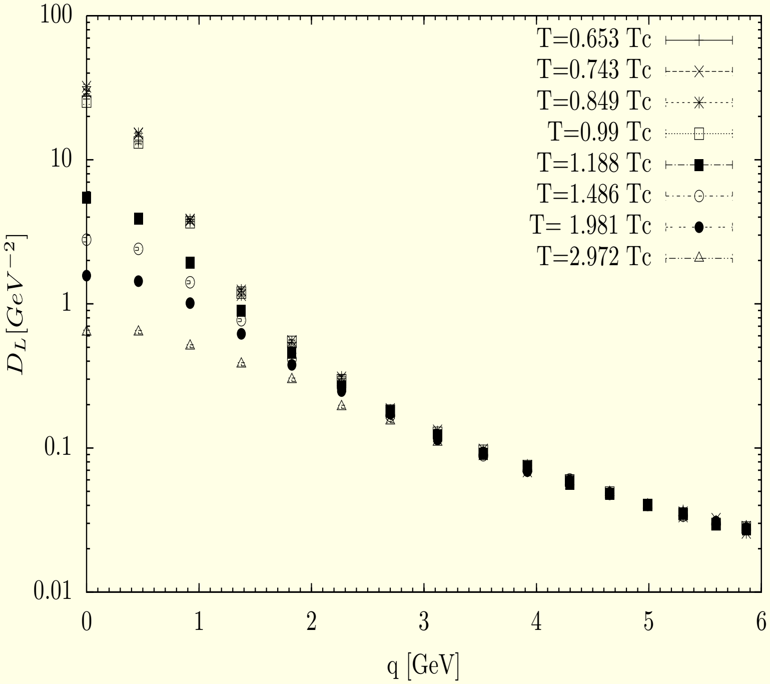
$48^3 \times L_\tau, \quad L_\tau = 4, 6, \ldots, 18$ varies, spacing $a = a(\beta = 6.337)$ fixed,

$T_c \longleftrightarrow L_\tau = 12$

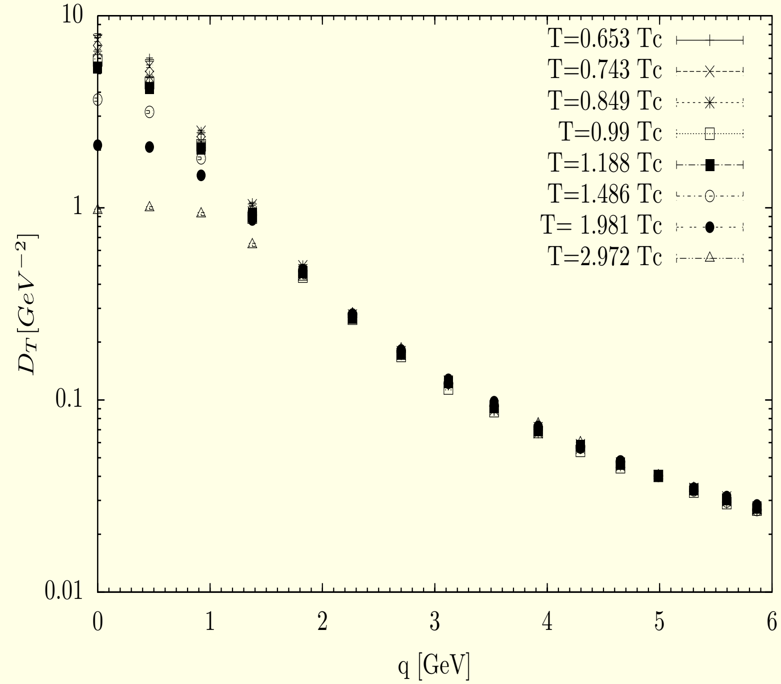
[Aouane, Ilgenfritz, M.-P., Sternbeck, preliminary result]

Renormalized gluon propagators at $T > 0,\ q_4 = 0$

longitudinal gluon propagator



transversal gluon propagator



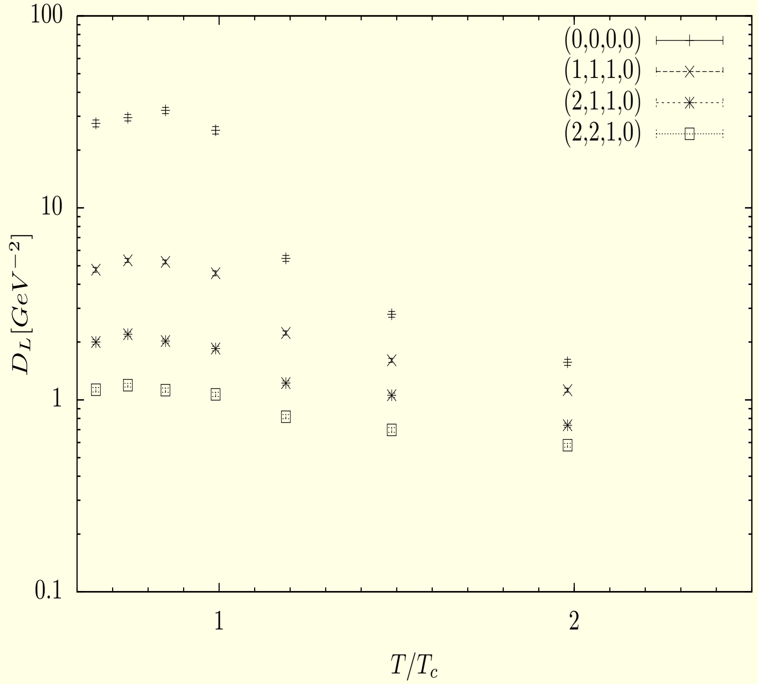
$\Longrightarrow$ Gluon propagators depend on T at low momenta.

$\Longrightarrow$ Not shown: Ghost propagator T-independent.

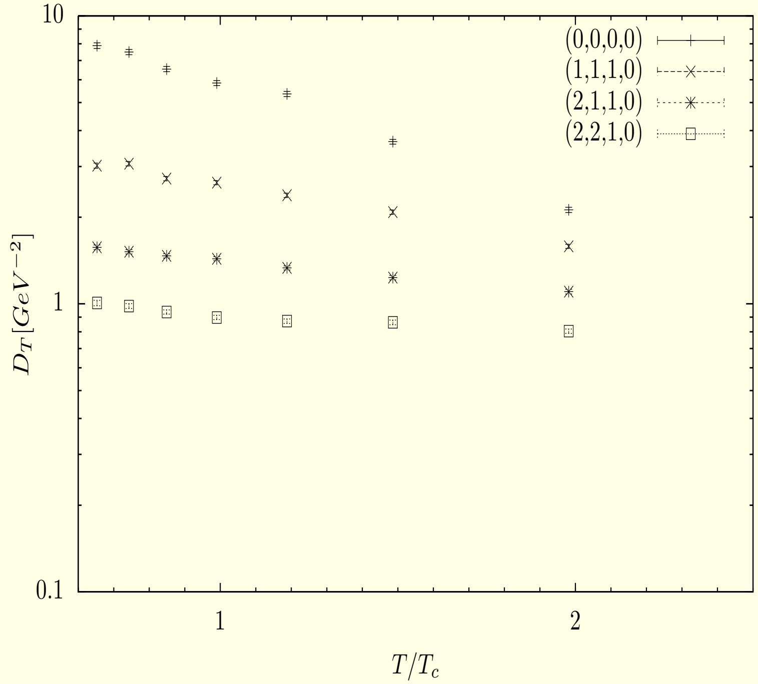
$\Longrightarrow$ Longitudinal gluon propagator most sensitive.

Renormalized gluon propagators vs. T at lowest momenta

longitudinal gluon propagator



transversal gluon propagator



$\Longrightarrow$ Longitudinal propagator at low momenta can serve as "order parameter".

$\Longrightarrow$ cf. talk by V. Mitrjushkin.