$SU(2)$ lattice gluon propagators at finite temperatures in the deep infrared region

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Why do we need gauge fixed propagators ?

1) Testground for analytical methods ($T = 0$; $T \neq 0$).

Example : Dyson-Schwinger Equations (DSE)

Solutions depend on approximations.

Lattice gauge theories $\Longrightarrow$ first-principle approach.

2) To check confinement scenarios ($T < T_c$).

Example 1 : Gribov-Zwanziger confinement scenario.

(Gribov '78, Zwanziger '91)

Example 2 : Kugo-Ojima confinement scenario :

(Kugo, Ojima, '78)

3) Phenomenological analysis of experimental data

Especially, in the deep infrared.

'QCD–inspired' models.

Examples.

- Ratios of widths of $J/\psi \to e^+e^-$ and $J/\psi \to \text{hadrons}$.
  Inclusive photon spectrum in decays $J/\psi \to \gamma X$.
- Non-leptonic B meson decays.
- Jet quenching (the radiative energy loss in dense nuclear matter which results from the energy loss of high energy partons moving through the plasma).
- ...

Linde (1980) has discovered that the thermal perturbation expansion in non-Abelian gauge theories breaks down because the magnetic sector is not amenable to perturbative studies.

The idea of the appearence of the (dynamically generated) 'magnetic' screening mass $m_M(T)$ at non-zero T has been proposed.

This mass is entirely non-perturbative.

Zahed, Zwanziger (1999) suggested that the proximity of the Gribov horizon at finite T forces the spatial gluon propagator $D_T(0;\vec{p})$ to vanish at $p=0$. If so, then the finite T analog of Gribov formula

$$|\vec{p}|^2/(|\vec{p}|^4+M_M^4) \; \equiv \; 1/(|\vec{p}|^2+m_{eff}^2(\vec{p}))$$

suggests that the effective magnetic screening mass $m_{eff}(\vec{p})$ becomes infinite in the infrared.

Interpretation : magnetic gluons 'confinement'.

Main goals.

- The momentum dependence of the gluon propagators $D_T(p)$ and $D_L(p)$ in the infrared region.

  Screening masses m_M and m_E ?

- Gribov copy effects.

- Temperature dependence of the propagators ($T < T_c$ and $T > T_c$).

  Propagators as 'order parameters' ?

- Finite volume effects and scaling violation.

Wilson action

$$S = \beta \sum_x \sum_{\mu>\nu} \left[1 - \frac{1}{2}\, \mathrm{Tr}\Big(U_{x\mu} U_{x+\mu;\nu} U^\dagger_{x+\nu;\mu} U^\dagger_{x\nu} \Big) \right] .$$

$$\beta = 4/g^2_{bare}(a)\,; \qquad a \text{ – spacing.}$$

The standard definition for the dimensionless lattice gauge vector potential $\mathcal{A}_{x+\hat{\mu}/2,\mu}$ is

$$\mathcal{A}_{x+\hat{\mu}/2,\mu} = \frac{1}{2i}\, \Big(U_{x\mu} - U^\dagger_{x\mu} \Big) \equiv \mathcal{A}^a_{x+\hat{\mu}/2,\mu} \frac{\sigma_a}{2} \,.$$

Gauge transformations : $U_{x\mu} \longrightarrow U^g_{x\mu} \equiv g_x U_{x\mu} g^\dagger_{x+\mu}$.

The usual choice of the Landau (Lorenz) gauge condition is

$$(\partial\mathcal{A})_x = \sum_{\mu=1}^{4} \left(\mathcal{A}_{x+\hat{\mu}/2;\mu} - \mathcal{A}_{x-\hat{\mu}/2;\mu}\right) = 0 \ .$$

Equivalent to finding a local extremum of the gauge functional

$$F_U(g) = \frac{1}{4L^4} \sum_{x\mu} \frac{1}{2} \operatorname{Tr} U_{x\mu}^g$$

with respect to transformations g_x .

The problem : Gribov ambiguity

Functional $F_U(g)$ has many local extrema $\longrightarrow$ Gribov copies.

Our recipy :

- To choose efficient gauge fixing method;
- To generate many gauge copies;
- To choose copy with $F_U(g)$ as close as possible to the global extremum (best copy or bc) ('absolute' LG).

Indeed,

(i) a consistent non-perturbative gauge fixing procedure proposed by Parrinello-Jona-Lasinio and Zwanziger (PJLZ-approach) presumes that the choice of a unique representative of the gauge orbit should be through the global extremum of the chosen gauge fixing functional;

Parrinello, Jona-Lasinio '90
Zwanziger '90

(ii) in the case of lattice compact $U(1)$ gauge theory in the weak coupling (Coulomb) phase some of the gauge copies produce a photon propagator with a decay behavior inconsistent with the expected zero-mass behavior. The choice of the global extremum permits to avoid such copies and to obtain the physical - massless - photon propagator.

Nakamura '92
Bornyakov, Mitrjushkin, Müller-Preussker and Pahl '93

Bare gluon propagator

$$D^{ab}_{\mu\nu}(p) = \frac{a^2}{g_0^2}\langle \mathcal{A}^a_\mu(p)\mathcal{A}^b_\nu(-p)\rangle$$
$$= \delta_{ab}\left(P^T_{\mu\nu}(p)D_T(p) + P^L_{\mu\nu}(p)D_L(p)\right) ,$$

$P^{T;L}_{\mu\nu}(p)$ – orthogonal projectors

$$P^T_{ij}(p) = \left(\delta_{ij} - \frac{p_i p_j}{\vec{p}^2}\right), \quad P^T_{\mu 4}(p) = 0 ;$$
$$P^L_{44}(p) = 1 ; \quad P^L_{\mu i}(p) = 0 .$$

at $p = (p_4 = 0;\ \vec{p} \neq 0)$.

$D_T(p)$ – magnetic propagator.

$D_L(p)$ – electric propagator.

Lattice size : $L_4 \cdot L_s^3$

The temperature T is

$$T = \frac{1}{aL_4} \,, \qquad a \text{ – lattice spacing.}$$

T/T_c	β	L_4
0.9	2.260	4
~ 1.0	2.300	4
1.1	2.350	4
2.0	2.512	4
2.0	2.635	6

L_s between 16 and 48.

Gribov copy effects

We use FSA (Flip Simulating Annealing).

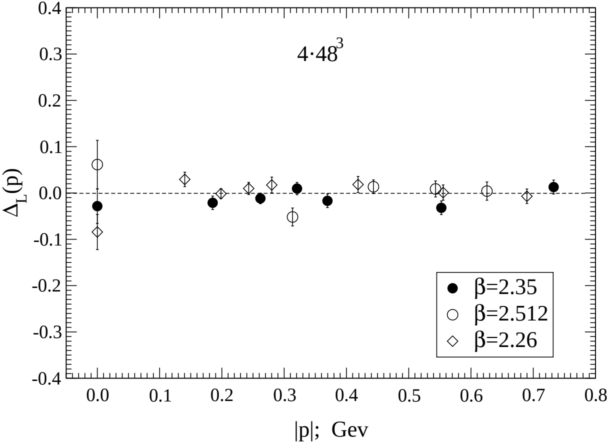
Much more efficient than the standard OR–method.

For every thermalized configuration the number of gauge copies $n_{copy} \leq 40$.

Define Gribov copy sensivity parameters $\Delta_T(p)$ and $\Delta_L(p)$:

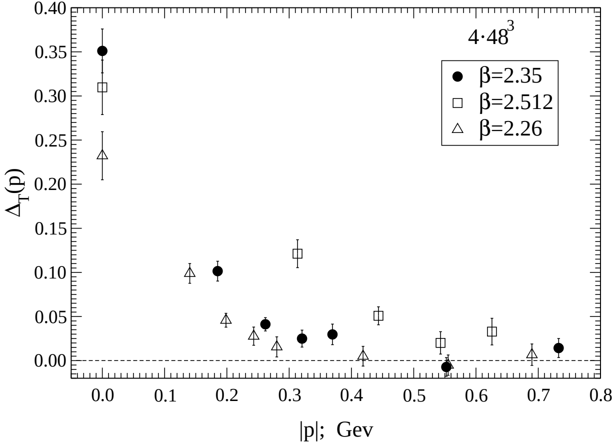
$$\Delta_L(p) = \frac{D_L^{fc}(p) - D_L^{bc}(p)}{D_L^{bc}(p)} \; ; \qquad \Delta_T(p) = \frac{D_T^{fc}(p) - D_T^{bc}(p)}{D_T^{bc}(p)} \; ,$$

Longitudinal propagator



Consistent with zero.
No Gribov copy effects.

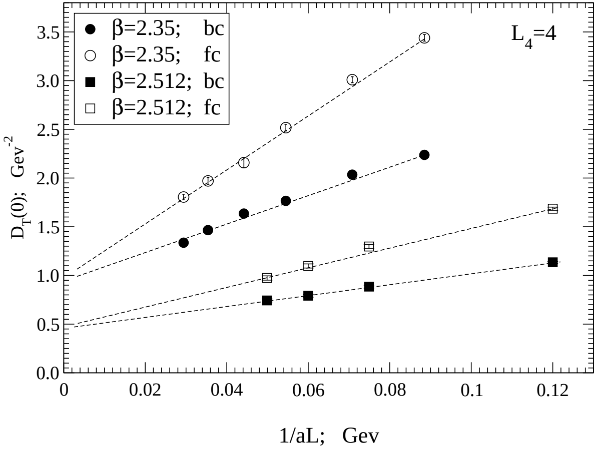
Transverse propagator



Gribov copy effects are very strong.

Efficient gauge fixing method is of crucial importance !

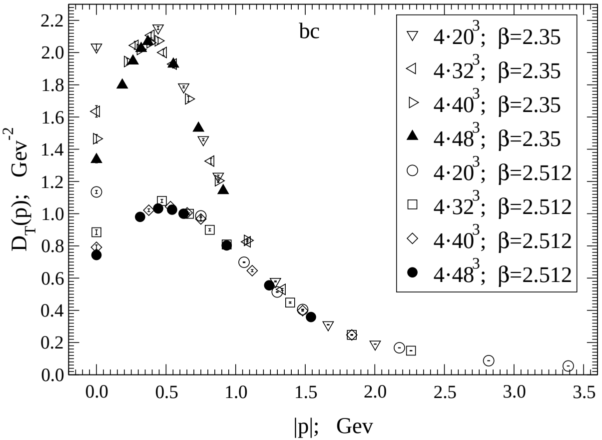
$D_T(0)$ asymptotics in the thermodynamic limit.



- Gribov copy effects are strong !
- At $T = 2T_c$ fc and bc values of $D_T(0)$ converge.

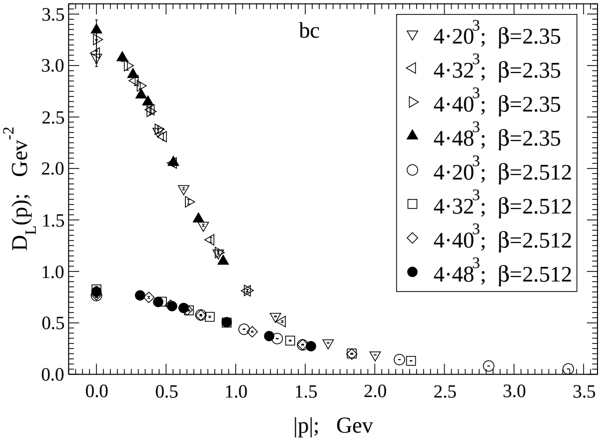
  At $T = 1.1T_c$ the convergence is under question.
- The asymptotic value of $D_T(0)$ is non-zero
  (in contradiction to Zahed&Zwanziger suggestion).

Transverse propagator



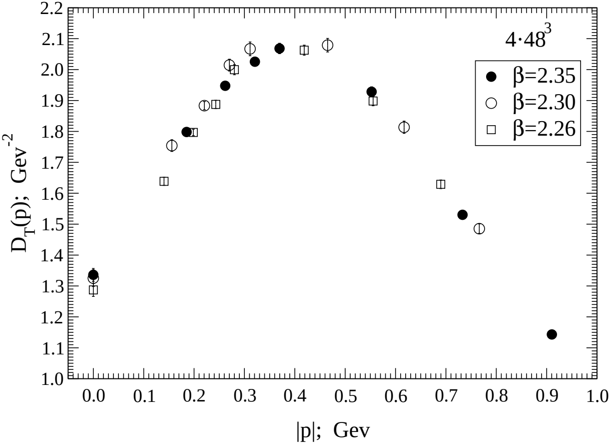
Maximum at non-zero momentum.
No 'Linde magnetic mass'.

Longitudinal propagator



Maximum at $p = 0$.

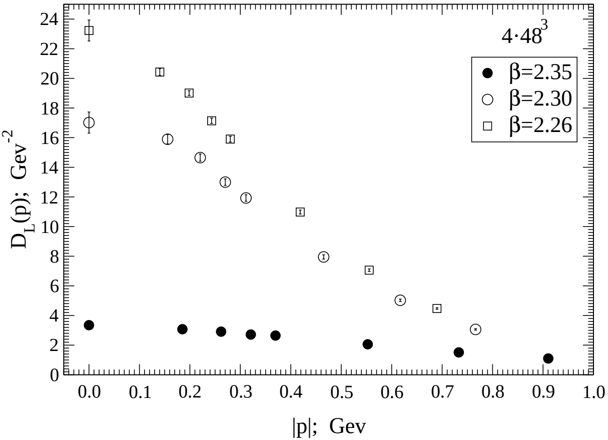
Transverse propagator



No temperature dependence at $T \sim T_c$.

Magnetic gluons 'confinement'.

Longitudinal propagator



Very strong temperature dependence at $T \sim T_c$.

D_L as an 'order parameter' !

Screening masses.

At finite T – two infrared mass scale parameters : m_E (electric) and m_M (magnetic), or 'screening masses'.
Define screeening of electric and magnetic fields on large distances.
Control the infrared behavior of $D_L(p)$ and $D_T(p)$.

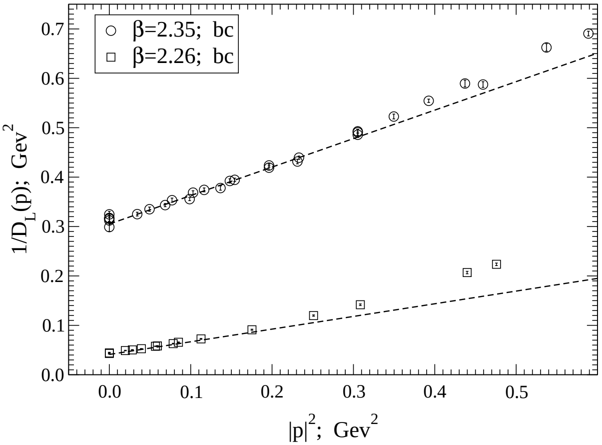
Perturbation theory fails.

$m_E^2 = \frac{2}{3}g^2T^2$ in the leading order of PT. At higher orders – the problem of infrared divergencies.

m_M is entirely nonperturbative in nature.

First principle nonperturbative calculations can resolve this problem.

Inverse longitudinal propagator



1) Electric mass m_E

$$D_L^{-1}(p) = A \cdot (p^2 + m_E^2).$$

More reliable way to determine m_E than propagators $D_E(z)$ in the coordinate space.

β	T/T_c	m_E[Gev]	m_E/T
2.260	0.9	0.41(1)	1.53(4)
2.300	1.0	0.46(1)	1.54(4)
2.350	1.1	0.73(2)	2.06(6)
2.512	2.0	1.21(2)	2.02(4)
2.635	2.0	1.15(3)	1.92(6)

- At $T = 2T_c$ coincides with results Heller&Karsch&Rank (1998).
- $m_E(T) \sim T$ at $T > T_c$ – in accordance with the lowest order PT.

1) Magnetic mass m_M

$D_T(p)$ is not compatible with the simple pole-type behavior.

No 'Linde mass' both at $T > T_c$ and $T < T_c$

Cucchieri&and Karsch&Petreczky (2001)($T > T_c$

Consistent with the $T = 0$ case.

Bornyakov&Mitrjushkin&Mueller-Preussker (2009).

Therefore, for m_M another, different from pole mass, definition is necessary.

We applied two fitting functions to our data for $D_T(p)$.

$$f_G(p) = Ce^{-(p-p_0)^2/m_M^2} \;; \qquad \text{and} \qquad f_P(p) = \frac{C}{(m_M^2 + (p-p_0)^2)}$$

Both work well in the infrared ($|p| \leq 0.8$ Gev) : $\chi^2/ndf < 1$. Consistent values of parameters.

Conclusions.

- The Gribov copy dependence of $D_T(p)$ is very strong in the infrared.

  For fixed p (at least, at $p \neq 0$) the effect of Gribov copies decreases with increasing volume.

  No dependence of the Gribov copy effects on T or spacing.

- The Gribov copy dependence of $D_L(p)$ is very weak, at least, at $p \neq 0$, and is comparable with the statistical errors ('Gribov noise').

- With increasing size L_s the bc-values and fc-values of $D_T(0)$ decrease.

  $D_T^{bc}(0)$ and $D_T^{fc}(0)$ seem to (slowly) converge when $L_s \to \infty$ in accordance with Zwanziger (2003) and with the $T = 0$ case Bornyakov&Mitrjushkin&Mueller-Preussker (2009).

  However, $D_T(0)$ is non-zero in $L_s \to \infty$ limit, in disagreement with Zahed, Zwanziger (1999)

- D_L changes drastically at $T \sim T_c$ $\rightarrow$ 'order parameter'.

  In contrast, D_T practically does not feel T_c ('magnetic gluon confinement').

- The maximum of the $D_T(p)$ at $|p| \sim 0.4 \div 0.5$ Gev not only in the deconfinement phase but also for $T < T_c$. Thus, no pole-type behavior $\sim 1/(p^2 + m^2)$ at $p \sim 0$.

  No 'Linde mass' both at $T > T_c$ and $T < T_c$.

  Another definition(s) of mass scale parameter $m_M(T)$ proposed.

- For $D_L(p)$ – good agreement with pole-type behavior at $p \sim 0$.
  Our method to compute m_E in the momentum space rather than in the coordinate space gives rise to higher precision.