Finite temperature phase transition in Lattice QCD with two flavors

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DIK and QCDSF collaborations

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Outline

1. Introduction

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Introduction

Goals:

- precise value of T_c
- nature of the phase transition

Some notations:

Temperature definition
$T = \frac{1}{aN_t}$, a - lattice spacing

$\beta = 6/g^2$

$N_f = 2$ QCD – $m_u = m_d$, $m_s = \infty$, ...

$N_f = 2 + 1$ QCD – $m_u = m_d$, $m_s > m_u$, $m_c = \infty$, ...

Present situation is rather controversial

- RBC/Bielefeld collaboration, improved staggered fermions, $N_f = 2 + 1$, N_t up to 8

$$T_c(\text{deconf}) = T_c(\text{CSB}) = 196(3)\ \text{MeV}$$

- Wuppertal group, improved staggered fermions, $N_f = 2 + 1$, N_t up to 16

$$T_c(\text{deconf}) = 151(6)\text{MeV}, \quad T_c(\text{CSB}) = 176(7)\ \text{MeV}$$

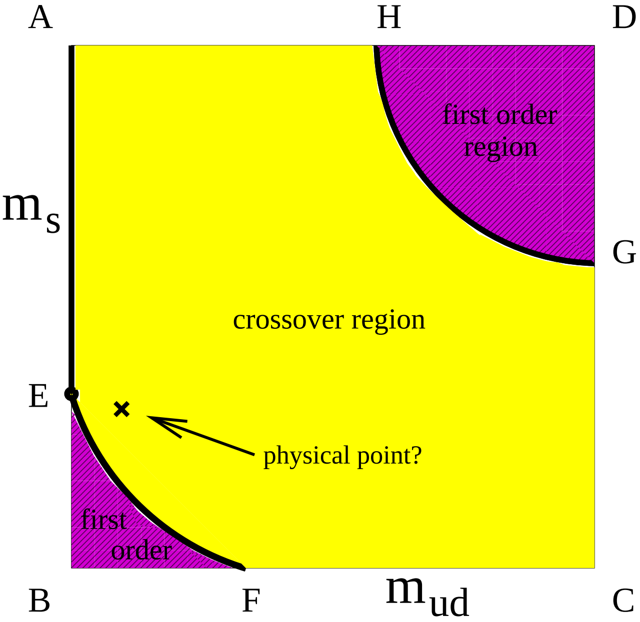
- WHOT-QCD collaboration, improved Wilson fermions, $N_f = 2$, $N_t = 6$

$$T_c(\text{deconf}) = 150 \text{ to } 180\ \text{MeV},$$

- DIK and QCDSF collaborations, improved Wilson fermions, $N_f = 2$, N_t up to 12

$$T_c(\text{deconf}) = T_c(\text{CSB}) = 174(3)(6)\ \text{MeV}$$

- Preliminary results:
  tmfT collaboration, twisted mass fermion action, $N_f = 2$
  Munster University group, Improved Wilson fermions, $N_f = 2$


A
H
D
first order region
m_s
G
crossover region
E
physical point?
first order
B
F
m_{ud}
C

Order of transition in the limit of massless quarks

Pisarski, Wilczek, 1984
$N_f = 2$ – second order in 3d $O(4)$ class of universality
$N_f = 3$ – first order

$O(4)$ scaling was observed for Wilson (Iwasaki et al, 1997) and improved Wilson (Ali Khan et al, 2001)
only recently for improved staggered fermions (Ejiri et al, 2009)
there are claims about signals of 1st order transition (Cossu et al, 2007)

Simulation details and results

- $N_f = 2$ lattice QCD
- Wilson gauge field action
- Improved Wilson fermionic action

$$S_F = S_F^{(0)} - \frac{\mathrm{i}}{2}\kappa\, g\, c_{sw} a^5 \sum_s \bar{\psi}(s)\sigma_{\mu\nu}F_{\mu\nu}(s)\psi(s)$$

- $N_t \times N_s^3 = 8 \times 16^3, 10 \times 24^3, 12 \times 24^3, 12 \times 32^3, 14 \times 40^3$
- $0.6 < r_0 m_\pi < 2.9$
- $r_0 m_\pi$ and r_0/a obtained by interpolation/extrapolation of results by QCDSF-UKQCD

New way to compute chiral condensate susceptibility- Maxwell relation

$$\frac{1}{V}\,\frac{\partial}{\partial\beta}\ln Z\Big|_{\hat{m}} = -6\,P + 2\,\frac{\partial\,\hat{m}_c}{\partial\,\beta}\,\hat{\sigma} - 2\,\frac{\partial\,c_{SW}}{\partial\,\beta}\,\hat{\delta}\,, \tag{1}$$

$$\frac{1}{V}\,\frac{\partial}{\partial\,\hat{m}}\ln Z\Big|_{\beta} = 2\,\hat{\sigma}\,, \tag{2}$$

$$\frac{1}{V}\frac{\partial^2}{\partial\,\beta\,\partial\,\hat{m}}\ln Z = 2\,\frac{\partial\,\hat{\sigma}}{\partial\,\beta}\Big|_{\hat{m}} = -6\,\frac{\partial\,P}{\partial\,\hat{m}}\Big|_{\beta} + 2\,\frac{\partial\,\hat{m}_c}{\partial\,\beta}\,\frac{\partial\,\hat{\sigma}}{\partial\,\hat{m}}\Big|_{\beta} - 2\,\frac{\partial\,c_{SW}}{\partial\,\beta}\,\frac{\partial\,\hat{\delta}}{\partial\,\hat{m}}\Big|_{\beta} \tag{3}$$

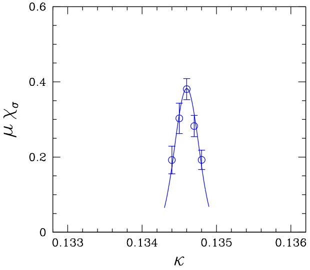
chiral condensate susceptibility

$$\chi_\sigma = \frac{1}{\mu}\frac{\partial P}{\partial \hat{m}}\,,$$

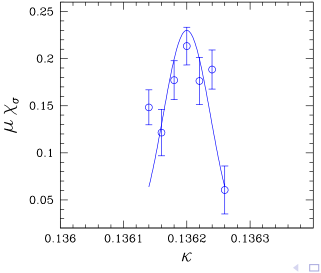
where

$$\mu^{-1} = 3\left(\frac{\partial\,\hat{m}_c}{\partial\,\beta} + \left.\frac{\partial\,\hat{m}}{\partial\,\beta}\right|_{\hat{\sigma}}\right)^{-1}$$

$16^3 8$


0.6
0.4
0.2
0
μ χσ
0.133
0.134
0.135
0.136
κ

$40^3 14$


0.25
0.2
0.15
0.1
0.05
μ χσ
0.136
0.1361
0.1362
0.1363
κ

Polyakov loop

$$L = \frac{1}{N_s^3} \sum_{\vec{x}} \mathrm{Re}\, L(\vec{x}) \,, \; L(\vec{x}) = \frac{1}{3} \mathrm{Tr} \prod_{x_4=1}^{N_t} U_4(x) \,. \tag{4}$$

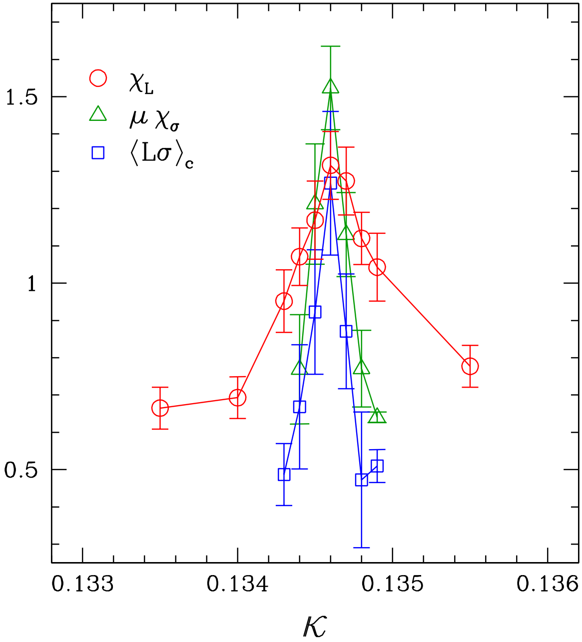
and its susceptibility

$$\chi_L \equiv N_s^3 \langle L^2 \rangle_c \,, \; \langle L^2 \rangle_c = \left(\langle L^2 \rangle - \langle L \rangle^2 \right) \tag{5}$$

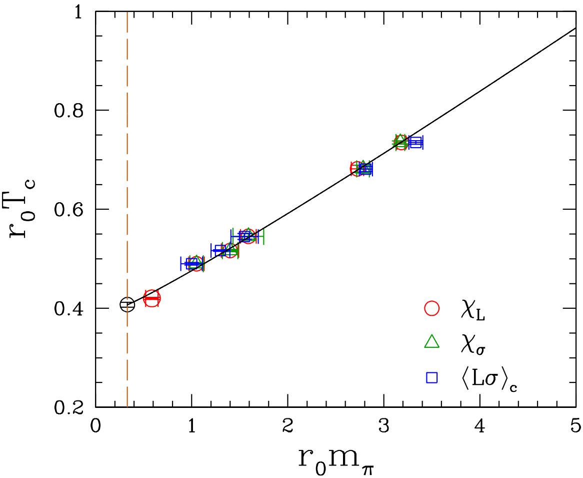
Correlator

$$\langle L\sigma \rangle_c = \langle L\sigma \rangle - \langle L \rangle \langle \sigma \rangle \,, \tag{6}$$

β	V	$r_0\, T_c(m)$	$r_0\, m_\pi^{T_c}$		
			χ_L	χ_σ	$\langle L\sigma\rangle_c$
5.25	$24^3\, 8$	0.735(3)	3.18(4)	3.17(4)	3.33(7)
5.20	$16^3\, 8$	0.682(7)	2.73(6)	2.78(6)	2.81(7)
5.20	$24^3\, 10$	0.545(6)	1.59(8)	1.59(16)	1.55(14)
5.29	$24^3\, 12$	0.517(2)	1.49(8)	1.40(9)	1.3(1)
5.25	$32^3\, 12$	0.490(2)	1.00(11)	1.05(8)	1.05(7)
5.25	$40^3\, 14$	0.420(2)		0.59(6)	



χ_L
$\mu\,\chi_\sigma$
$\langle L\sigma \rangle_c$
1.5
1
0.5
0.133
0.134
0.135
0.136
κ



$$r_0 T_c(r_0 m_\pi) = r_0 T_c(0) + c_m \cdot (r_0 m_\pi)^d \quad (7)$$

with d =1.07 predicted by $O(4)$ scaling at the physical pion mass

$$r_0\, T_c = 0.408(5) \;\longrightarrow T_c = 172(3)(6)\ \text{MeV}$$

r_0=0.467 fm

Conclusions and outlook

- – New method to compute χ_σ has been used
- – Numerical value for T_c at the physical point is in agreement with staggered fermions result for $T_c(deconf)$
- – Peaks in χ_σ, χ_L, $\langle L_\sigma \rangle_c$ coinside, implying $T_c(deconf) = T_c(CSB)$
- – Agreement with $O(4)$ scaling in $T_c(m)$
- – Direct computation of χ_σ is desirable
- – $2+1$ QCD simulations are planned