Thermodynamics and Fabric of Spacetime

Dmitri V. Fursaev

Dubna U & JINR

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Gravity as an emergent phenomenon

A. Sakharov's suggestion (1968):

the Einstein theory can be induced at one-loop

$$\ln \det (\nabla_\mu \nabla^\mu + m^2) \sim \frac{1}{16\pi G_{eff}} \int d^4x \sqrt{-g} \, (\Lambda_{eff} + R + a_{eff} \, "R^2" + \ldots)$$

$$\frac{\Lambda_{eff}}{G_{eff}} \sim M^4 \quad , \quad M - \text{UV cutoff}$$

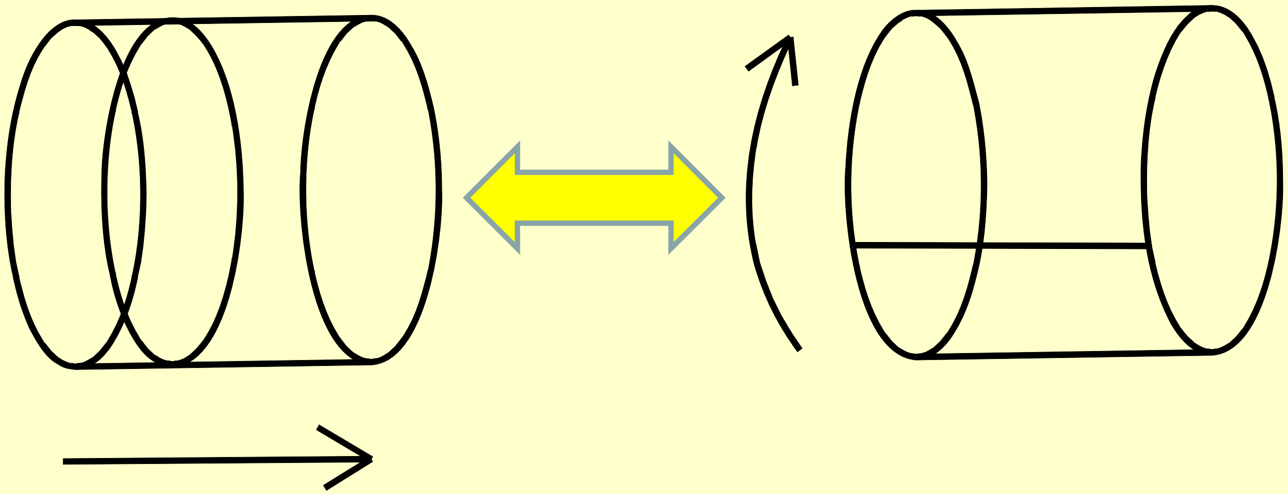
$$\frac{1}{G_{eff}} \sim M^2$$

Gravitons = collective excitations of underlying degrees of freedom

analogy: phonons in solid state physics

Λ_{eff} , G_{eff} - Young's modulus

String theory:



“Tree-level” diagram
(closed strings)

“one-loop” diagram
(open strings)
“Sakharov’s picture”

low-energy limit (10D (super)gravity, …)

Consequences of an emergent nature of gravity?

- entropy of a black hole (since 1970’s) ?

- T.Jacobson: laws of gravity can be inferred from ‘thermodynamical’ properties of event horizons

- E.Verlinde: laws of gravity have a thermodynamical form (horizons are replaced with a more general concept of ‘holographic’ screens)

Entropic origin of gravity (E. Verlinde)

Consider a massive source and a holographic screen around it;

1st postulate the screen is equipotential surface which carries certain entropy:

$$dN = \frac{d\sigma}{G} -$$ number of degrees of freedom on the screen on the area $d\sigma$

2d postulate: $\boxed{\delta S = 2\pi m l}$ - change of the entropy under the movement of a test particle toward the screen;

3d postulate: the energy takes an 'equipartition' form on the screen

$$\boxed{M = \frac{1}{4\pi G}\int d\sigma \partial_n \phi = \frac{1}{2}\int T dN}$$ (T. Padmanabhan)

$$T = \frac{\partial_n \phi}{2\pi} -$$ a local temperature on the screen

E.Verlinde
arXiv:1001.0785 [hep-th]

Consequences

use an analog of the 1st law $\boxed{T\delta S = W}$

$W = Fl -$ a work done by the system,

$F = mw$ - force acting on the test particle;

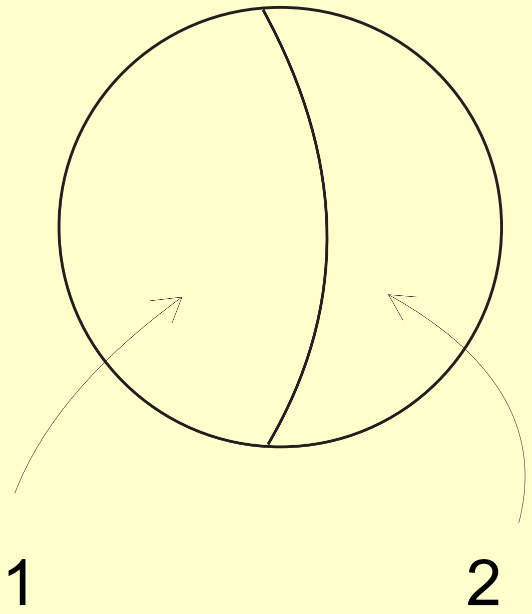
w - acceleration of the particle

$$\boxed{F = mw = m\partial\phi = \frac{mMG}{r^2} \text{ - the Newton law,} \qquad F = T\frac{\partial S}{\partial l}}$$

gravity is an emergent phenomenon;
the force of gravity has an entropic origin
direction of the force – gradients of the entropy

Main problem:

mechanism of generation of the entropy?



<u>quantum entanglement</u>: states of subsystems cannot be described independently

entanglement has to do with quantum gravity:

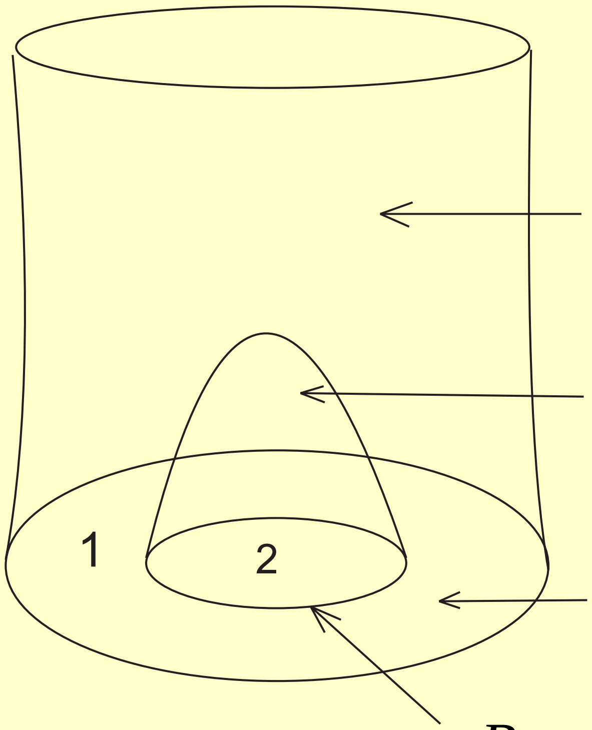
- possible source of the entropy of a black hole (states inside and outside the horizon);

- d=4 supersymmetric BH's are equivalent to 2, 3,… qubit systems

- entanglement entropy allows a *holographic interpretation* for CFT's with AdS duals

Holographic Formula for the Entropy

Ryu and Takayanagi,
hep-th/0603001, 0605073



AdS_5 (bulk space)

$\tilde{B}$ minimal (least area) surface in the bulk

4d space-time manifold (asymptotic boundary of AdS)

B separating surface

entropy of entanglement

$$\boxed{S = \frac{\tilde{A}}{4G^{(d+1)}}}$$

is measured in terms of the area of $\tilde{\mathcal{B}}$

$G^{(d+1)}$ is the gravity coupling in AdS

Holographic formula enables one to compute entanglement entropy in strongly correlated systems with the help of classical methods (the Plateau problem)

What about entanglement in quantum gravity?

Can one define an entanglement entropy, $S(B)$,
of fundamental degrees of freedom spatially separated by a surface B?

How can the fluctuations of the geometry be taken into account?

the hypothesis

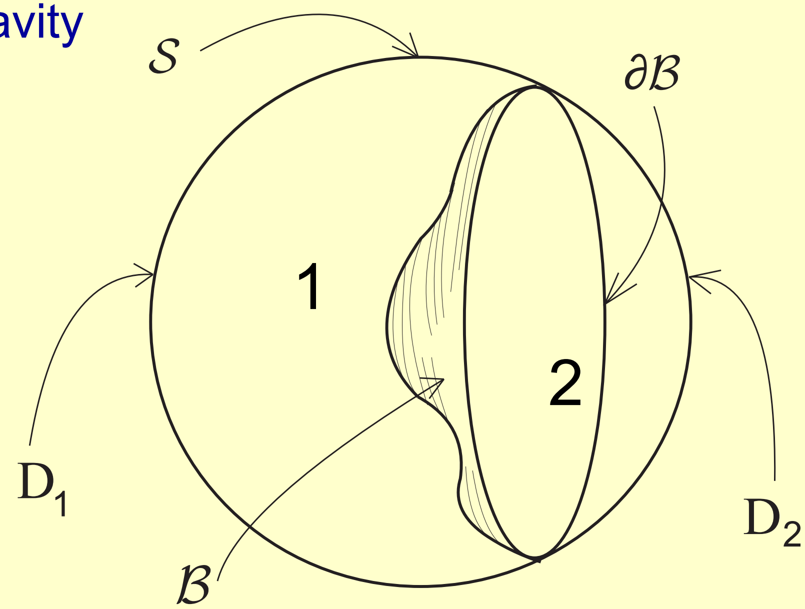
- $S(B)$ is a macroscopical quantity (like thermodynamical entropy);

- $S(B)$ can be computed without knowledge of a microscopical content of the theory (for an ordinary quantum system it can't)

- the definition of the entropy is possible at least for a certain type of boundary conditions

Suggestion (DF, 06,07): EE in quantum gravity between degrees of freedom separated by a surface B is

$$S(B) = \frac{A(B)}{4G}$$



B is a least area minimal hypersurface in a constant-time slice

conditions:

- static space-times

the system is determined by a set of boundary conditions; subsets, “1” and “2” , in the bulk are specified by the division of the boundary

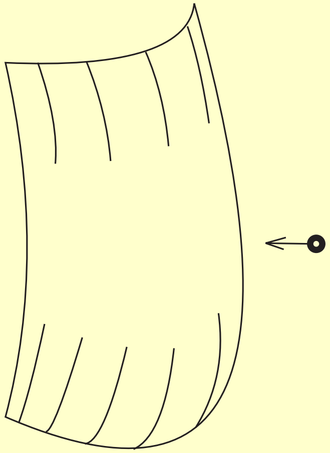
The shape of the separating surface is
formed under fluctuations of the geometry;

As a result the surface is minimal, i.e. has a least area

Details: D.V. Fursaev, Phys. Rev. D77 (2008) 124002,
e-Print: arXiv:0711.1221 [hep-th]

If the entanglement entropy in QG is a macroscopic quantity, does it allows a thermodynamic interpretation

simple variational formulae
(weak field approximation)



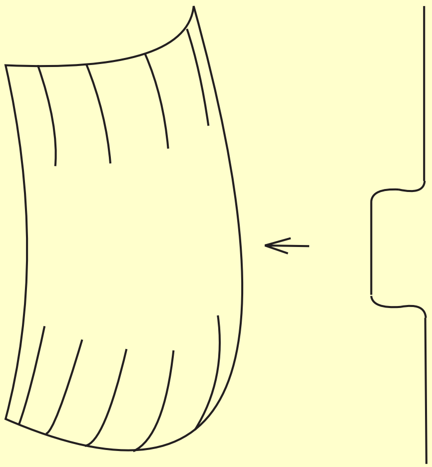
$$\delta S = \pi m l$$

$m -$ mass of a particle

$l -$ shift (toward of the surface)

$$\delta S \sim 10^{37} \quad \text{if} \quad m = 1g, \quad l = 1cm$$

$\delta S = O(1)$ if l is a Compton wavelength



$$\delta S \sim \pi \mu l \delta z$$

$\mu -$ string tension

$\delta z -$ lenght of the segment

aim of the talk

to study simplest dynamics of a minimal surface;

to look for its thermodynamic analogy;

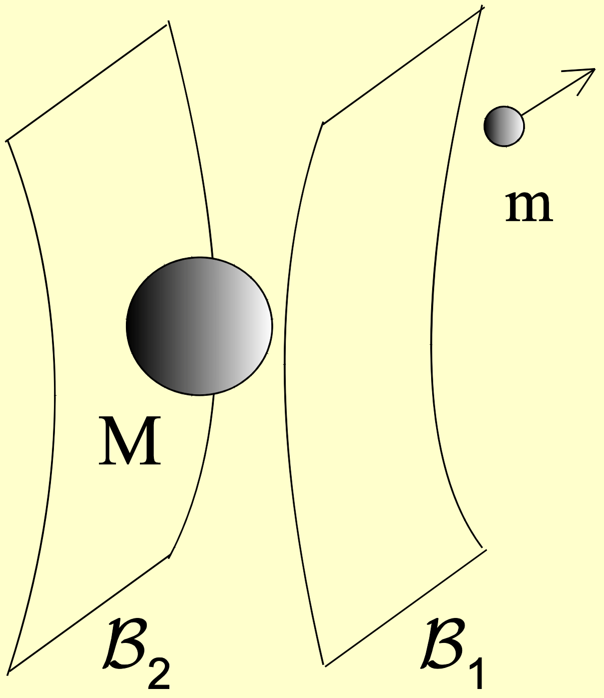
to relate this analysis to a hypothesis about an entropic origin of gravity (as suggested by E.Verlinde)

see D.V. Fursaev, **arXiv:1006.2623** [hep-th]

Minimal surfaces may play
a role of holographic screens

2-component 'screen' around
a massive source

in weak field approximation
screen = 2 parallel planes



$$ds^2 = -(1 + 2\phi)\, dt^2 + (1 - 2\phi)\,(dx^2 + dy^2 + dz^2), \quad \phi = -\frac{2MG}{r}$$

$$E(B_k) = \frac{1}{4\pi MG}\int d\sigma w_n\,, \quad w_n = \partial_n \phi - \text{ acceleration on the screen}$$

$$E = E(B_1) + E(B_2) = M - \text{ the Komar energy of the source}$$

Dynamics in the weak field approximation

$$S_k = \frac{A(B_k)}{4G} = \overline{S_k} - \frac{1}{2}\int \phi(r_k)\, dN; \qquad \overline{S_k} -$$ entropy for a plane

$$\phi(r) = -\frac{MG}{r}$$ - potential of the massive source

$$A'(B_k) = -2\int \phi'(r'_k)\, d\sigma$$ - modification of the area by a test particle

$$\phi'(r'_k) = -\frac{mG}{r'_k}$$ - potential of the test particle on the screen

r'_k - distance from a point on the screen to the particle

$$\boxed{\delta A(B_k) = -2\int \delta\phi'(r'_k)\, d\sigma = -4\pi mGl}$$

Notes on the computation

$$\phi'(r) = 4\pi mG\, D(x_r - x_0)$$

x_0 - position of the particle, x_r - position of a point on a screen

$$\Delta D(x) = \delta(x_0)$$

shift of the particle $(\delta x_0)^k = l^k$ results in the variation

$$\delta_0 D(x_r - x_0) = -l^k \partial_k D(x_r - x_0)$$

$$\int \delta\phi'(r')\, d\sigma \quad \rightarrow$$

$$\int l^k \partial_k D(x_r - x_0) d\sigma = l_\perp \int \Delta D(x) d^3 x = \frac{l}{2}$$

$l \equiv l_\perp$ shift in the direction orthogonal to the screen

shifts along the screen (plane) do not change the area

`Thermodynamics'

$\delta S_k = \delta S(B_k) = -\pi m l$ - for a particle moving out of the surface

$\delta S = 0$ - for a particle moving inside the screen

$\boxed{\delta S = \delta S_1 + \delta S_2 = -2\pi m l}$ - can be derived, is not a postulate!

single surface = half of the screen:

$$E(B) = \frac{1}{4\pi MG}\int_B d\sigma w_n = \frac{M}{2}$$

energy balance:

$$\boxed{T(x)\delta S(B) = -\frac{1}{2}\delta W(x) \;\rightarrow T(x) = \frac{w_n}{2\pi} \;\rightarrow E(B) = \frac{1}{2}\int_B T dN}$$

$\delta W(x) -$ work done by an external force to drag the test particle with coordinates x out of the surface

Static space-time backgrounds
(which are solutions to the Einstein equations)

$$ds^2 = g_{00}(x)dt^2 + g_{ab}(x)dx^a dx^b$$

`holographic screen` is a minimal surface (with a topology of a hyperplane) in a constant-time slice

$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$ - perturbation caused by a test particle

$A(B) \to A(B) + A'(B)$ - perturbation of the area of a minimal surface

$$\boxed{A'(B) = \frac{1}{2}\int_B d\sigma\, \gamma^{ij} X^a{}_{,i} X^b{}_{,j} h_{ab}}$$

$X^a = X^a(y)$- change in position of the surface does not count in the linear approximation

$\gamma^{ij} X^a{}_{,i} X^b{}_{,j} h_{ab}$ - variation of the metric induced on the surface

perturbations:

$$L\,\bar{h}^{\mu}_{\nu} = 16\pi G\, t^{\mu}{}_{\nu} \quad , \quad \bar{h}^{\mu}_{\nu} = h^{\mu}{}_{\nu} - \frac{1}{2} h \delta^{\mu}{}_{\nu}$$

$$\nabla_{\mu} \bar{h}^{\mu}_{\nu} = 0$$

$$L\bar{h}^{\mu}_{\nu} = -\nabla^{2}\bar{h}^{\mu}_{\nu} - 2R_{\nu\rho}{}^{\mu\lambda}\bar{h}^{\rho}_{\lambda} - \frac{8\Lambda}{n-2}\bar{h}^{\mu}_{\nu}$$

Λ - cosmological constant, $n -$ number of dimensions

$$\nabla^{2}\bar{h}^{0}_{0} = \Delta\bar{h}^{0}_{0} - w^{a}\partial_{a}\bar{h}^{0}_{0} + 2w^{2}\bar{h}^{0}_{0} - 2w^{a}w_{b}\bar{h}^{b}_{a}$$

Δ - Laplacian on constant-time slice

$t^{\mu}{}_{\nu}(x) = m u^{\mu} u_{\nu} \delta^{(n-1)}(x, x_{0})$, m - mass of the particle

u^{μ} - velocity of the particle , $w^{\mu} = \nabla u^{\mu}$ - acceleration

approximation:

curvature terms, lambda term, and acceleration terms are "slowly" changing,

perturbations caused by the particle are rapidly changing ;

curvature-, lambda-, and acceleration terms can be neglected

$$A'(B) = \frac{1}{2}\int_B d\sigma\, \bar{h}_0^{\,0} - \text{area perturbation}$$

$$\bar{h}_0^{\,0} = 16\pi G m\, D(x, x_0)$$

$$\Delta_x D(x, x_0) = \delta^{(n-1)}(x, x_0)$$

in a thin layer near a minimal surface the space is “flat”
in the direction orthogonal to the surface (z-direction)

$$dl^2 = dz^2 + \gamma_{ij}(z,y)dy^i dy^j$$

$z = 0 -$ position of the surface

$k_{ij} = 0 -$ extrinsic curvature of the minimal surface

$$\partial_z \gamma_{ij}(0,y) = 0$$

$\boxed{\delta A(B) = \delta A'(B) = -4\pi m G l}$ $-$ for the shift of the particle out of the surface by distance l

a universal formula, does not depend on the background and its dimensionality

`Thermodynamical’ parameters of a minimal surfaces

$$S = \frac{A(B)}{4G} - \text{entropy}$$

$$\boxed{E = \frac{1}{4\pi G} \int_B |g_{00}|^{1/2} w_n d\sigma = \frac{1}{2} \int_B T dN} - \text{energy (Komar mass)}$$

$$dN = \frac{d\sigma}{G} - \text{number of states on the area}$$

$$w_n - \text{acceleration at the surface (part normal to the surface)}$$

$$\boxed{T = \frac{|g_{00}|^{1/2} w_n}{2\pi}} - -\text{local temperature on the surface}$$

temperature coincides with the Hawking temperature for the surface located near a back hole horizon

`Thermodynamics'

the surface is located between a gravitating body
and a test particle

one obtains "1st law" $T(x)\delta S = -\frac{1}{2}\delta W(x)$

$\delta S = -\pi m l -$ entropy change when dragging particle out of the surface

$$\delta W(x) = Fl = m\left|g_{00}\right|^{1/2} w_n$$

$F -$ force applied by an observer at infinity

(for asymptotically flat spacetimes)

Summary

● support to and developing hypothesis about entropic origin of gravity (E.Verlinde):

- minimal surfaces as holographic screens
- reducing the number of postulates
- **interpretation of the entropy: entanglement of fundamental degrees of freedom = fabric of spacetime**

● a universal variational formula for minimal surfaces

● a simple version of `thermodynamics' of minimal surfaces (entropy, temperature, energy)

thank you for attention