Dynamics of Dirac particle spins in arbitrary stationary gravitational fields

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International Workshop “Bogoliubov readings”

Dubna 2010

OUTLINE

- Dirac particles in static gravitational fields and uniformly accelerated frames
- Dirac particles in stationary gravitational fields and rotating frames
- Comparison between classical and quantum equations of spin motion
- Dirac particles in arbitrary strong stationary gravitational fields
- Summary

Dirac particles in static gravitational fields and uniformly accelerated frames

This problem has been solved in the work:

- A.J. Silenko and O.V. Teryaev, Phys. Rev. D **71**, 064016 (2005).

The quantum theory is based on the Dirac equation:

$$(ig^{m}D_{m}-m)y=0,\qquad m=0,1,2,3$$

The exact transformation of the Dirac equation for the metric

$$ds^2=V^2(\mathbf{r})(dx^0)^2-W^2(\mathbf{r})(d\mathbf{r}\cdot d\mathbf{r})$$

to the Hamilton form was carried out by Obukhov:

$$i\frac{\partial y}{\partial t}=Hy\,,\quad H=b\,mV+\frac{1}{2}\{F,\boldsymbol{\alpha}\cdot\mathbf{p}\},\quad F=\frac{V}{W}$$

- Yu. N. Obukhov, *Phys. Rev. Lett.* **86**, 192 (2001); *Fortsch. Phys.* **50**, 711 (2002).

This Hamiltonian covers the cases of a weak Schwarzschild field and a uniformly accelerated frame

- **Silenko and Teryaev used the Foldy-Wouthuysen transformation for relativistic particles in external fields and derived the relativistic Foldy-Wouthuysen Hamiltonian:**

$$H_{FW}=H_{FW}^{(1)}=\beta\epsilon+\frac{\beta}{2}\left\{\frac{m^2}{\epsilon},V-1\right\}+\frac{\beta}{2}\left\{\frac{p^2}{\epsilon},F-1\right\}$$

$$-\frac{\beta m}{4\epsilon(\epsilon+m)}[\boldsymbol{\Sigma}\cdot(\boldsymbol{\varphi}\times\mathbf{p})-\boldsymbol{\Sigma}\cdot(\mathbf{p}\times\boldsymbol{\varphi})+\nabla\cdot\boldsymbol{\varphi}]$$

$$+\frac{\beta m(2\epsilon^3+2\epsilon^2 m+2\epsilon m^2+m^3)}{8\epsilon^5(\epsilon+m)^2}(\mathbf{p}\cdot\nabla)(\mathbf{p}\cdot\boldsymbol{\varphi})$$

$$+\frac{\beta}{4\epsilon}[\boldsymbol{\Sigma}\cdot(\mathbf{f}\times\mathbf{p})-\boldsymbol{\Sigma}\cdot(\mathbf{p}\times\mathbf{f})+\nabla\cdot\mathbf{f}]-\frac{\beta(\epsilon^2+m^2)}{4\epsilon^5}(\mathbf{p}\cdot\nabla)(\mathbf{p}\cdot\mathbf{f}).$$

$$\boldsymbol{\varphi}=\nabla V,\quad \mathbf{f}=\nabla F$$

$$\epsilon=\sqrt{m^2+p^2}$$

- Quantum mechanical equations of momentum and spin motion

$$\frac{d\mathbf{p}}{dt}=i[H_{FW},\mathbf{p}]=-\frac{b}{2}\left\{\frac{m^2}{e},\boldsymbol{\varphi}\right\}-\frac{b}{2}\left\{\frac{p^2}{e},\mathbf{f}\right\}$$

$$+\frac{m}{2e(e+m)}\nabla(\boldsymbol{\Pi}\cdot(\boldsymbol{\varphi}\times\mathbf{p}))-\frac{1}{2e}\nabla(\boldsymbol{\Pi}\cdot(\mathbf{f}\times\mathbf{p}))$$

$$\frac{d\boldsymbol{\Pi}}{dt}=i[H_{FW},\boldsymbol{\Pi}]=\frac{m}{e(e+m)}\boldsymbol{\Sigma}\times(\boldsymbol{\varphi}\times\mathbf{p})-\frac{1}{e}\boldsymbol{\Sigma}\times(\mathbf{f}\times\mathbf{p})$$

Semiclassical equations of momentum and spin motion

$$\frac{d\mathbf{p}}{dt}=-\frac{m^2}{e}\boldsymbol{\varphi}-\frac{p^2}{e}\mathbf{f}+\frac{m}{2e(e+m)}\nabla(\mathbf{P}\cdot(\boldsymbol{\varphi}\times\mathbf{p}))$$

$$-\frac{1}{2e}\nabla(\mathbf{P}\times(\mathbf{f}\times\mathbf{p})), \qquad \mathbf{P}=\frac{\mathbf{S}}{S}$$

$$\frac{d\mathbf{S}}{dt}=\frac{m}{e(e+m)}\mathbf{S}\times(\boldsymbol{\varphi}\times\mathbf{p})-\frac{1}{e}\mathbf{S}\times(\mathbf{f}\times\mathbf{p})$$

When the Foldy-Wouthuysen representation is used, the derivation of semiclassical equations consists in replacing p, Π, Σ operators with corresponding classical quantities

Uniformly accelerated frame

$$H_{FW} = b\left(e + \frac{1}{2}\{e, \mathbf{a}\cdot\mathbf{r}\}\right) + \frac{\mathbf{\Pi}\cdot(\mathbf{a}\times\mathbf{p})}{2(e+m)}, \quad e = \sqrt{m^2 + \mathbf{p}^2}$$

$$\frac{d\mathbf{p}}{dt} = -be\,\mathbf{a}, \qquad \frac{d\mathbf{\Pi}}{dt} = -\frac{\mathbf{\Sigma}\times(\mathbf{a}\times\mathbf{p})}{e+m}$$

An observer can distinguish between a gravitational field (g = – a) and a uniformly accelerated frame

Helicity evolution is the same

$$\mathbf{o} = \boldsymbol{\omega} - \mathbf{\Omega} = -\frac{m}{\mathbf{p}^2}(\mathbf{a}\times\mathbf{p})$$

Angular velocities of precession of spin and unit momentum vector

Dirac particles in stationary gravitational fields and rotating frames

Spin motion in the Lense-Thirring metric

Albert Einstein's theory of general relativity predicts that rotating bodies drag spacetime around themselves (**frame dragging or** the **Lense-Thirring effect**)

Lense-Thirring metric

(an example of a stationary spacetime):

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right)(cdt)^2 + \frac{1}{1 - \dfrac{2GM}{c^2 r}} dr^2$$

$$+ r^2 \left(dq^2 + \sin^2 q \, df^2\right) - \frac{4GMa}{c^2 r} \sin^2 q \, df \, dt$$

Initial Dirac equation for the Lense-Thirring metric can be transformed to the Hamilton form by the Obukhov's method:

- Yu. N. Obukhov, *Phys. Rev. Lett.* **86**, 192 (2001);
  Fortsch. Phys. **50**, 711 (2002).

$$i\mathbf{h}\frac{\partial y}{\partial t} = Hy\,, \qquad F = \frac{V}{W},$$

$$H = b\,mc^2 V + \frac{c}{2}\{F, \boldsymbol{\alpha}\cdot\mathbf{p}\} + \frac{2G}{c^2 r^3}\mathbf{l}\cdot\mathbf{J}$$

$$+\frac{\mathbf{h}G}{2c^2 r^5}\left[3(\mathbf{r}\cdot\boldsymbol{\Sigma})(\mathbf{r}\cdot\mathbf{J}) - r^2\boldsymbol{\Sigma}\cdot\mathbf{J}\right]$$

After the Foldy-Wouthuysen transformation, the Hamiltonian takes the form

$$H_{FW} = H_{FW}^{(1)} + H_{FW}^{(2)},$$

$$H_{FW}^{(2)} = \frac{2G}{c^2 r^3}\mathbf{l}\cdot\mathbf{J} + \frac{\mathbf{h}G}{2c^2 r^5}\left[3(\mathbf{r}\cdot\mathbf{\Sigma})(\mathbf{r}\cdot\mathbf{J}) - r^2\mathbf{\Sigma}\cdot\mathbf{J}\right]$$

$$-\frac{3\mathbf{h}G}{8c^2}\left\{\frac{1}{e(e+m)},\left(\frac{2\{(\mathbf{\Sigma}\cdot\mathbf{l}),(\mathbf{l}\cdot\mathbf{J})\}}{r^5}\right.\right.$$

$$\left.\left.+\frac{1}{2}\left\{(\mathbf{\Sigma}\cdot[\mathbf{p}\times\mathbf{l}] - \mathbf{\Sigma}\cdot[\mathbf{l}\times\mathbf{p}]),\frac{\mathbf{r}\cdot\mathbf{J}}{r^5}\right\} + \left\{\mathbf{\Sigma}\cdot[\mathbf{p}\times[\mathbf{p}\times\mathbf{J}]],\frac{1}{r^3}\right\}\right)\right\}$$

$$-\frac{3\mathbf{h}^2}{8c^2}\left\{(5p_r^2 - \mathbf{p}^2)\frac{2e^2 + em + m^2}{e^4(e+m)^2},\frac{\mathbf{l}\cdot\mathbf{J}}{r^5}\right\}.$$

Quantum mechanical equations of momentum and spin motion

Force operator:

$$F^i = \frac{dp^i}{dt} = -\frac{dp_i}{dt} + \frac{1}{4}\left\{\left\{v^j, \frac{\partial g^{im}}{\partial x^j}\right\}, p_m\right\},$$

$$p_m = i\hbar\frac{\partial}{\partial x^m} = \left(\frac{H_{FW}}{c}, -\mathbf{p}\right)$$

$$\mathbf{F} = \frac{c}{2}\left(\mathrm{curl}\mathbf{K}\times\mathbf{p} - \mathbf{p}\times\mathrm{curl}\mathbf{K}\right) + \mathbf{F}_s$$

agrees with the known nonrelativistic classical result

$$\mathrm{curl}\mathbf{K} = \frac{2G}{c^2 r^3}\left(\frac{3\mathbf{r}\left(\mathbf{r}\cdot\mathbf{J}\right)}{r^2} - \mathbf{J}\right)$$

Spin-dependent part

$$\mathbf{F}_s = \nabla\Bigg(\frac{\hbar G}{2c^2 r^5}\left[3(\mathbf{r}\cdot\boldsymbol{\Sigma})(\mathbf{r}\cdot\mathbf{J}) - r^2\boldsymbol{\Sigma}\cdot\mathbf{J}\right]$$

$$-\frac{3\hbar G}{8}\left\{\frac{1}{e(e+mc^2)}, \left(\frac{2\{(\boldsymbol{\Sigma}\cdot\mathbf{l}),(\mathbf{l}\cdot\mathbf{J})\}}{r^5}\right.\right.$$

$$\left.\left.+\frac{1}{2}\left\{(\boldsymbol{\Sigma}\cdot[\mathbf{p}\times\mathbf{l}] - \boldsymbol{\Sigma}\cdot[\mathbf{l}\times\mathbf{p}]), \frac{\mathbf{r}\cdot\mathbf{J}}{r^5}\right\} + \left\{\boldsymbol{\Sigma}\cdot[\mathbf{p}\times[\mathbf{p}\times\mathbf{J}]], \frac{1}{r^3}\right\}\right)\right\}$$

$$-\frac{3\hbar^2}{8c^2}\left\{(5p_r^2 - \mathbf{p}^2)\frac{2e^2 + em + m^2}{e^4(e+m)^2}, \frac{\mathbf{l}\cdot\mathbf{J}}{r^5}\right\}\Bigg)$$

agrees with the previously obtained nonrelativistic classical result:

R. Wald, Phys. Rev. D **6**, 406 (1972); B.M. Barker and R. F. O'Connell, Gen. Relativ. Gravit. **11**, 149 (1979).

Operator equation of spin motion

$$\frac{d\mathbf{\Pi}}{dt} = \mathbf{\Omega}^{(1)} \times \mathbf{\Sigma} + \mathbf{\Omega}^{(2)} \times \mathbf{\Pi}$$

Term depending on
the static part of the metric

$$\mathbf{\Omega}^{(2)} = \frac{G}{c^2 r^3}\left[\frac{3\mathbf{r}\left(\mathbf{r}\cdot\mathbf{J}\right)}{r^2} - \mathbf{J}\right]$$

$$-\frac{3G}{4}\left\{\frac{1}{e\left(e+mc^2\right)}, \left(\frac{2\left\{\mathbf{l},\left(\mathbf{l}\cdot\mathbf{J}\right)\right\}}{r^5}\right.\right.$$

$$\left.\left.+\frac{1}{2}\left\{\left(\mathbf{p}\times\mathbf{l}-\mathbf{l}\times\mathbf{p}\right),\frac{\mathbf{r}\cdot\mathbf{J}}{r^5}\right\}+\left\{\mathbf{p}\times\left[\mathbf{p}\times\mathbf{J}\right],\frac{1}{r^3}\right\}\right)\right\}$$

Semiclassical limit of quantum mechanical equations of momentum and spin motion can be found

Relativistic formula for the angular velocity of the Lense-Thirring spin precession:

$$\mathbf{\Omega}_{LT} = G\frac{3\mathbf{r}(\mathbf{r}\cdot\mathbf{J}) - r^2\mathbf{J}}{c^2r^5}$$

$$-\frac{3G}{m^2g\left(g+1\right)c^2r^5}\left[\mathbf{l}\left(\mathbf{l}\cdot\mathbf{J}\right)+\left(\mathbf{r}\cdot\mathbf{p}\right)\left(\mathbf{p}\times\left(\mathbf{r}\times\mathbf{J}\right)\right)\right].$$

Spin motion in the rotating frame

- The simplest example of nonstatic spacetimes
- The exact Dirac Hamiltonian was obtained by Hehl and Ni:

$$H = b\,m + \boldsymbol{\alpha}\cdot\mathbf{p} - \boldsymbol{\omega}\cdot\mathbf{J},$$

$$\mathbf{J} = \mathbf{L} + \mathbf{S}, \quad \mathbf{L} = \mathbf{r}\times\mathbf{p}, \quad \mathbf{S} = \frac{\boldsymbol{\Sigma}}{2}$$

- F. W. Hehl and W. T. Ni, *Phys. Rev. D* **42,** 2045 (1990).

- The result of the **exact** Foldy-Wouthuysen transformation is given by

$$H_{FW} = b\sqrt{m^2 + p^2} - \boldsymbol{\omega}\cdot\mathbf{J}.$$

- A.J. Silenko and O.V. Teryaev, Phys. Rev. D **76**, 061101(R) (2007).
- The equation of spin motion coincides with the Gorbatsevich-Mashhoon equation:

$$\frac{d\mathbf{S}}{dt} = -\boldsymbol{\omega}\times\mathbf{S}$$

- **The particle motion is characterized by the operators of velocity and acceleration:**

$$v^i \equiv \frac{dx^i}{dx^0} = i[H, x^i], \quad x^0 \equiv t,$$

$$w^i \equiv \frac{dv^i}{dx^0} = i[H, v^i] = -\left[H, [H, x^i]\right].$$

- **For the particle in the rotating frame**

$$\mathbf{v} = b\frac{\mathbf{p}}{e} - \boldsymbol{\omega}\times\mathbf{r}, \quad e = \sqrt{m^2 + p^2},$$

$$\mathbf{w} = 2b\frac{\mathbf{p}\times\boldsymbol{\omega}}{e} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) = 2\mathbf{v}\times\boldsymbol{\omega} - \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}).$$

w is the sum of the Coriolis and centrifugal accelerations

Comparison between classical and quantum equations of spin motion

- The Equivalence Principle manifests in the general equations of motion of classical particles

$$\frac{Du^m}{dt} = 0$$

- and their spins:

$$\frac{DS^m}{dt} = 0$$

- A.A. Pomeransky and I.B. Khriplovich, Zh. Eksp. Teor. Fiz. **113**, 1537 (1998) [J. Exp. Theor. Phys. **86**, 839 (1998)].
- When one neglects a non-geodesic motion of spinning particles, Pomeransky-Khriplovich and Mathisson-Papapetrou approaches bring the same results

Pomeransky-Khriplovich equation for the three-component spin

$$\frac{d\mathbf{S}}{dt} = \mathbf{\Omega} \times \mathbf{S}$$

$$\Omega_i = ce_{ikl}\left(\frac{1}{2}g_{klc} + \frac{u^k}{u^0+1}g_{0lc}\right)\frac{u^c}{u^0}.$$

Tetrad variables are blue, $t \equiv x^0$

For nonstatic metric, the Pomeransky-Khriplovich equation of spin motion depends on a choice of a tetrad!

For the rotating frame, only the Schwinger gauge

$$e_i^0 = 0, \quad e_i^0 = 0$$

leads to the exact equation of spin motion

Lense-Thirring effect in general relativity

Lense-Thirring effect for spin:

L.I. Schiff, Am. J. Phys. **28**, 340 1960;

Proc. Nat. Acad. Sci. **46**, 871 (1960);

Phys. Rev. Lett. **4**, 215 (1960).

L. Schiff has described the effect of precession of gyroscope (classical spin) caused by a rotation of a central body which angular momentum is J

$$\frac{d\mathbf{S}}{dt} = \mathbf{\Omega} \times \mathbf{S}, \quad \mathbf{\Omega} = G\frac{3\mathbf{r}(\mathbf{r}\cdot\mathbf{J}) - r^2\mathbf{J}}{c^2 r^5}.$$

Our relativistic formula for the Lense-Thirring spin precession:

$$\mathbf{\Omega}_{LT} = G\frac{3\mathbf{r}(\mathbf{r}\cdot\mathbf{J}) - r^2\mathbf{J}}{c^2 r^5}$$

$$-\frac{3G}{m^2 g\,(g+1)\,c^2 r^5}\left[\mathbf{l}\left(\mathbf{l}\cdot\mathbf{J}\right) + \left(\mathbf{r}\cdot\mathbf{p}\right)\left(\mathbf{p}\times\left(\mathbf{r}\times\mathbf{J}\right)\right)\right].$$

Pomeransky-Khriplovich formula:

$$\mathbf{\Omega}_{LT}^{(\mathbf{PK})} = G\frac{3\mathbf{r}(\mathbf{r}\cdot\mathbf{J}) - r^2\mathbf{J}}{c^2 r^5}$$

$$-\frac{G}{m^2 g\,(g+1)\,c^2 r^5}\left[3\mathbf{l}\left(\mathbf{l}\cdot\mathbf{J}\right) + r^2\left(\mathbf{p}\times\left(\mathbf{p}\times\mathbf{J}\right)\right)\right].$$

A. A. Pomeransky and I. B. Khriplovich, Zh. Eksp. Teor. Fiz. **113**, 1537 (1998) [Sov. Phys. JETP **86**, 839 (1998)].

Semiclassical equation of spin motion does not agree with the Pomeransky-Khriplovich result

- **When the symmetric tetrad is used, the Pomeransky-Khriplovich equation should be added by the correction for the gravitoelectric field including the correction for the Thomas precession:**

$$d\mathbf{\Omega} = \frac{G}{m^2 g\,(g+1)\,c^2 r^5}\Big[3(\mathbf{r}\cdot\mathbf{p})\big(\mathbf{p}\times(\mathbf{r}\times\mathbf{J})\big) - r^2\big(\mathbf{p}\times(\mathbf{p}\times\mathbf{J})\big)\Big].$$

Relativistic formula for the Lense-Thirring spin precession is our new result

The classical and quantum equations of spin motion agree

The classical and quantum approaches are in the best agreement

Dirac particles in arbitrary strong stationary gravitational fields

Any stationary metric can be brought to the form diagonal in spatial coordinates and then to the isotropic form by an appropriate transformation of spatial coordinates:

$$ds^2 = V^2 c^2 dt^2 - W^2 d_{ij}\left(dx^i - K^i c dt\right)\left(dx^j - K^j c dt\right)$$

Schwinger gauge:

$$e_m^0 = V d_m^0, \quad e_m^i = W\left(d_m^i - K^i d_m^0\right)$$

Dirac Hamiltonian:

$$H_D = b\, mc^2 V + \frac{c}{2}\{F, \boldsymbol{\alpha}\cdot\mathbf{p}\} + \frac{2G}{c^2 r^3}\mathbf{l}\cdot\mathbf{J}$$

$$+\frac{\mathbf{h}G}{2c^2 r^5}\left[3(\mathbf{r}\cdot\boldsymbol{\Sigma})(\mathbf{r}\cdot\mathbf{J}) - r^2\boldsymbol{\Sigma}\cdot\mathbf{J}\right], \qquad F = \frac{V}{W}$$

Foldy-Wouthuysen Hamiltonian:

$$H_{FW} = H_{FW}^{(1)} + H_{FW}^{(2)},$$

$$H_{FW}^{(1)} = be' - \frac{b\mathbf{h}m}{4}\left\{\frac{1}{2e'^2 + m\{e',V\}}, [\boldsymbol{\Sigma}\cdot(\boldsymbol{\Phi}\times\mathbf{p}) - \boldsymbol{\Sigma}\cdot(\mathbf{p}\times\boldsymbol{\Phi}) + \mathbf{h}\nabla\cdot\boldsymbol{\Phi}]\right\}$$

$$+b\frac{\mathbf{h}}{16}\left\{\frac{1}{e'}, [\boldsymbol{\Sigma}\cdot(\mathsf{G}\times\mathbf{p}) - \boldsymbol{\Sigma}\cdot(\mathbf{p}\times\mathsf{G}) + \mathbf{h}\nabla\cdot\mathsf{G}]\right\}$$

$$e' = \sqrt{m^2c^4V^2 + \frac{1}{2}c^2\left\{F^2, \mathbf{p}^2\right\}}, \quad \boldsymbol{\Phi} = F^2\nabla V, \quad \mathsf{G} = \nabla\left(F^2\right)$$

$$H_{FW}^{(2)} = \frac{c}{2}\left(\mathbf{K}\cdot\mathbf{p} + \mathbf{p}\cdot\mathbf{K}\right) + \frac{\mathbf{h}c}{4}\boldsymbol{\Sigma}\cdot\left(\nabla\times\mathbf{K}\right)$$

$$-\frac{\mathbf{h}c}{16}\left\{\frac{1}{2e'^2 + m\{e',V\}}, \left\{F^2, \boldsymbol{\Sigma}\cdot\mathbf{Q}\right\}\right\}$$

$$\mathbf{Q} = \mathbf{p}\times\nabla\left(\mathbf{K}\cdot\mathbf{p} + \mathbf{p}\cdot\mathbf{K}\right) - \nabla\left(\mathbf{K}\cdot\mathbf{p} + \mathbf{p}\cdot\mathbf{K}\right)\times\mathbf{p}$$

$$-\mathbf{p}\times\left(\mathbf{p}\times\left(\nabla\times\mathbf{K}\right)\right) - \left(\left(\nabla\times\mathbf{K}\right)\times\mathbf{p}\right)\times\mathbf{p}$$

Only ħ is a small parameter. All terms are exact. All terms of order of ħ and leading terms of order of ħ² describing contact interaction are included

Operator equation of spin motion

$$\frac{d\mathbf{\Pi}}{dt} = \mathbf{\Omega}^{(1)} \times \mathbf{\Sigma} + \mathbf{\Omega}^{(2)} \times \mathbf{\Pi}$$

Term depending on the static part of the metric

$$\mathbf{\Omega}^{(1)} = -\frac{m}{2}\left\{\frac{1}{2e'^2 + m\{e',V\}}, (\mathbf{\Phi}\times\mathbf{p} - \mathbf{p}\times\mathbf{\Phi})\right\} + \frac{1}{8}\left\{\frac{1}{e'}, (\mathsf{G}\times\mathbf{p} - \mathbf{p}\times\mathsf{G})\right\}$$

$$\mathbf{\Omega}^{(2)} = \frac{c}{2}(\nabla\times\mathbf{K}) - \frac{c}{8}\left\{\frac{1}{2e'^2 + m\{e',V\}}, \left\{F^2, \mathbf{Q}\right\}\right\}$$

Semiclassical equation of spin motion

$$\frac{d\mathbf{S}}{dt} = \mathbf{\Omega}\times\mathbf{S}$$

$$\mathbf{\Omega} = -\frac{m}{e'^2 + me'V}\mathbf{\Phi}\times\mathbf{p} + \frac{1}{2e'}\mathsf{G}\times\mathbf{p} + \frac{c}{2}(\nabla\times\mathbf{K}) - \frac{c}{4\left(e'^2 + me'V\right)}F^2\mathbf{Q}$$

This is the general solution of the problem

Summary

- Foldy-Wouthuysen Hamiltonians and operator and semiclassical equations of spin motion are derived for Dirac particles in static gravitational fields and uniformly accelerated frames. An observer can distinguish between a static gravitational field and a uniformly accelerated frame
- Foldy-Wouthuysen Hamiltonians and operator and semiclassical equations of spin motion are obtained for Dirac particles in the Lense-Thirring metric and rotating frames
- Behavior of classical and quantum spins in stationary spacetimes is the same and any important quantum effects do not appear. The classical and quantum approaches are in the best agreement
- Foldy-Wouthuysen Hamiltonians and operator and semiclassical equations of spin motion are derived for Dirac particles in arbitrary strong stationary gravitational fields

Thank you for attention