Joint Institut for Nuclear Research

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"Bogoliubov Readings"

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2010

Extra dimensions and Gravitation Potential with taking into account the hadron form-factor

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- Introduction
- Coulomb-hadron interference
- Extra-dimensional gravity
- (ADD-scenario)
- Born amplitude and
- eikonalization
- Properties of the “oscillation”
- GPDs and Hadron gravtation form-factor
- Gravitational potential
- Summary

Elastic scattering amplitude

$pp \rightarrow pp$ $\qquad$ $p\bar{p} \rightarrow p\bar{p}$

$$\frac{d\sigma}{dt} = 2\pi[|\Phi|_1^2 + |\Phi|_2^2 + |\Phi|_3^2 + |\Phi|_4^2 + 4|\Phi|_5^2]$$

$$\Phi_i(s,t) = \Phi_i^h(s,t) + \Phi_i^e(t)\, e^{i\alpha\varphi}$$

$$\varphi(s,t) = \mp [\, \gamma + \log(\mathrm{B}(s,t)\, |t| / 2) + \nu_1 + \nu_2 \,]$$

$\gamma = 0{,}577...$ (*the Euler constant*) $\qquad$ ν_1 *and* ν_2 *are small correction terms*

$$\varphi(s,t_i) = [log(\frac{B(s,t)}{|t|}) + \gamma + \nu_1 + \nu_2]$$

ν_1 – 2 second Born approximation (2 photon diagram)

O.V. Selyugin ***Mod.Phys.Lett. A11, 2317 (1996)***

ν_2 – 2 second Born approximation (photon-Pomeron interference with taking into account dipole form-factor of the nucleons)

O.V. Selyugin, ***Mod.Phys.Lett., A12, 1379, (1997);***

Phys.Rev. D60, 074028 (1999)

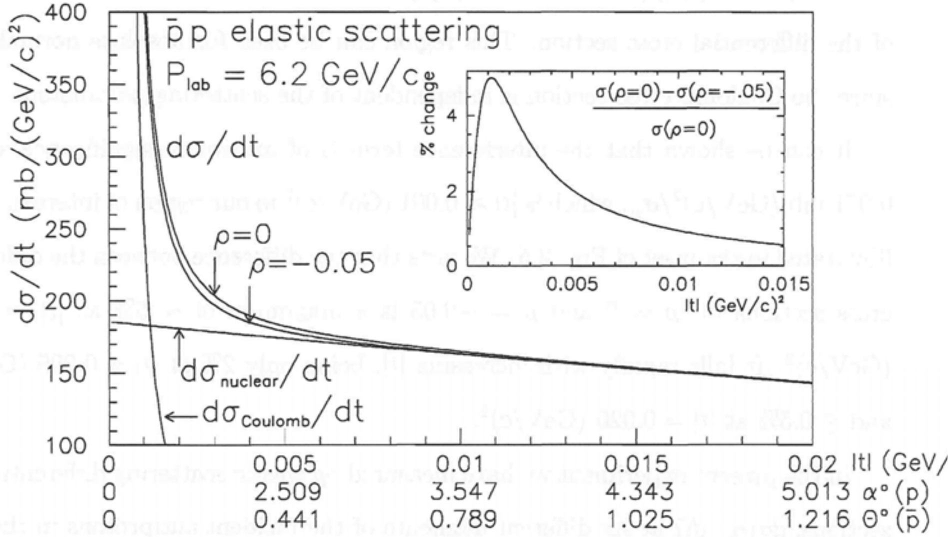
The ρ parameter

$$\rho(s,t) = \frac{\mathrm{Re}\, F_N(s,t)}{\mathrm{Im}\, F_N(s,t)};$$

linked to σ_{tot} via dispersion relations

sensitive to σ_{tot} beyond the energy

at which is measured



$$\frac{d\sigma}{dt}\Big|_{Interference} \propto F_C \,\mathrm{Im}\, F_h \,(\rho(s,t) + \alpha\varphi)$$

$$\rho_{\pm}(E)\,\sigma_{\pm}(E) = \frac{C}{P} + \frac{E}{\pi P}\int_m^{\infty} dE'\, P'\left[\frac{\sigma_{\pm}(E')}{E'(E'-E)} - \frac{\sigma_{\mp}(E')}{E'(E'+E)}\right].$$

predictions of σ_{tot} beyond LHC energies

Or, are dispersion relations still valid at LHC energies?

J.-R. Cudell, O.Selyugin
Phys.Rev.Lett. 102, 032003, (2009)

If the additional amplitude is almost REAL

$$(d\sigma / dt)(s,t) \Box [\operatorname{Re} F_C(t) + \operatorname{Re} F_N(s,t) + \operatorname{Re} F_{osc}(s,t)]^2$$

$$[\operatorname{Im} F_C(t) + \operatorname{Im} F_N(s,t) + \operatorname{Im} F_{osc}(s,t)]^2 \;;$$

$$\Delta(d\sigma / dt)_{ad}(s,t) \approx$$

$$\{2 \operatorname{Re} F_C \operatorname{Im} F_N (\rho(s,t) + \sin[\alpha_{em}(\varphi_c(t) + \varphi_{CN}(s,t))]\}$$

$$2 \operatorname{Im} F_{osc}(s,t)[\operatorname{Re} F_C(t) \sin[\alpha_{em}(\varphi_c(t) + \varphi_{CN}(s,t))] + \operatorname{Im} F_N(s,t)] +$$

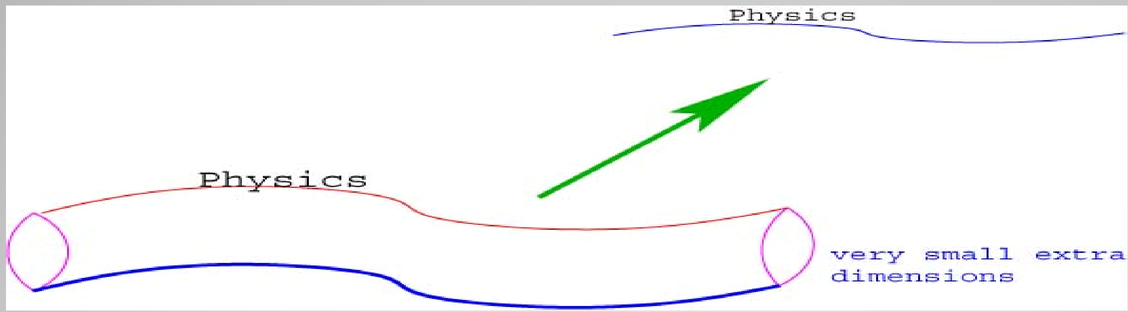
$$2 \operatorname{Re} F_{osc}(s,t)[[\operatorname{Re} F_C(t) \cos[\alpha_{em}(\varphi_c(t) + \varphi_{CN}(s,t))] + \rho(s,t) \operatorname{Im} F_N(s,t)] \;;$$

$$\Delta(d\sigma / dt)_{ad}(s,t) \Box$$

$$\{2 \operatorname{Re} F_C \operatorname{Im} F_N (\rho(s,t) + \sin[\alpha_{em}(\varphi_c(t) + \varphi_{CN}(s,t))]\} +$$

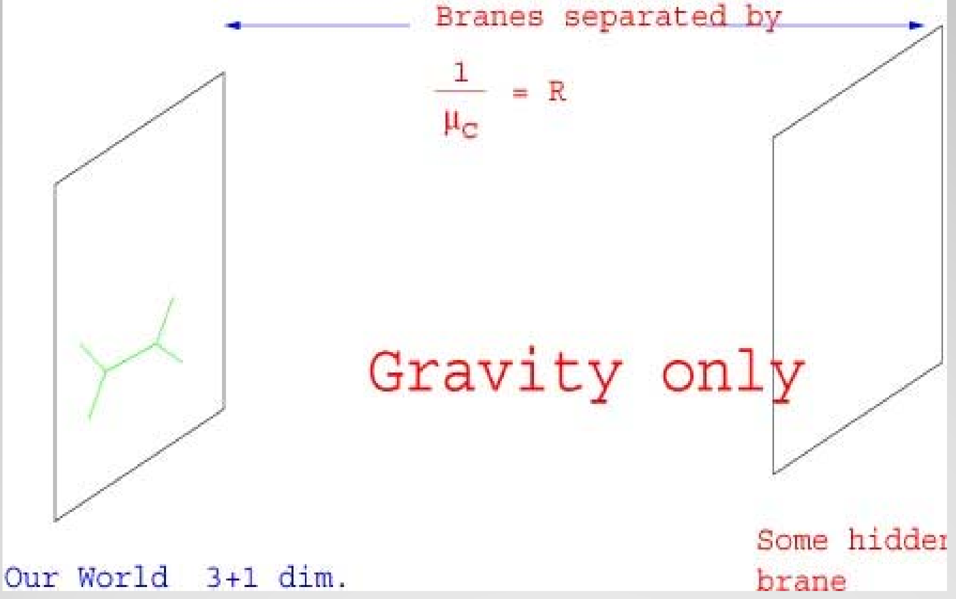
$$2 \operatorname{Re} F_{ad}(s,t)[\operatorname{Re} F_C(t) + \rho(s,t) \operatorname{Im} F_N(s,t)] \;;$$

Extra dimensions

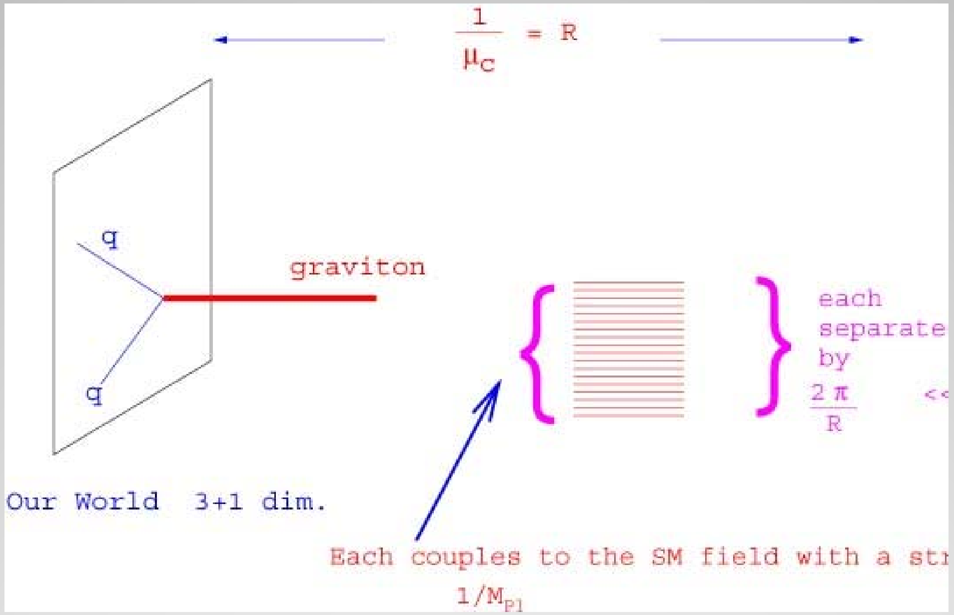


$\{x^1, x^2, x^3\}$+1(circle with small radius R)

Large extra dimensions



Kaluza-Klein tower



Arkani-Hamed, Dimopoulos, Dvali

ADD scenario

[Cylinder homogeneous and metric flat
neglect tension of brane]

N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, **Phys.Lett., B 429 (998) 263;**

Phys.Rev., 59 (1999) 0860004.

- G.F. Giudice, R.Rattazzi and J.D.Wells **Nucl.Phys., B 544 (1999) ;**
- **Nucl.Phys., B 630 (2002) ;**
- T. Han, J.Lykken, and R.-J. Zhng, **Phys.Rev 59 (1999) 105006.**
- R. Emparan, **Phys.Rev. D64 (2001);**
- R. Emparan, M. Masir, R. Ratttazzi , **Phys.Rev. D65 (2002);**

I. Ya, Aref'eva, arXiv: 1007.4777.

$$M_{(4)}^{2} = r_d^{d} \, M_{(4+d)}^{d+2}$$

$$r_d = M_{(4+d)}^{-1} \left(\frac{M_{Pl}}{M_{(4+d)}} \right)^{2/d} \sim 10^{32/d} \times 10^{-17} cm$$

$d = 2$ $\quad$ $M_{4+2} = 1 \, TeV$

$r_2 = 0.\ 1\mathrm{mm}$

Virtual graviton exchange

d *continuous*

$$A_{grav.} \square \int_{o}^{\infty} \frac{q_T^{d-1}\, dq_T}{q^2 + q_T^2} = \frac{\pi}{2}\left(\frac{1}{q^2}\right)^{1-d/2} \csc\left(d\,\frac{\pi}{2}\right)$$

$$A_{grav.} \square \int_{0}^{q_{max}} \frac{q_T^{d-1} dq_T}{q^2 + q_T^2} = \frac{M_s^d}{d}\,\frac{1}{q^2}\;{}_2F_1\left[1, \frac{d}{2}, 1+\frac{d}{2}, -\frac{M_s^2}{q^2}\right]$$

Born amplitude

O.V. Selyugin, O.V. Teryaev, **Phys.Rev. D 85, (2009)**

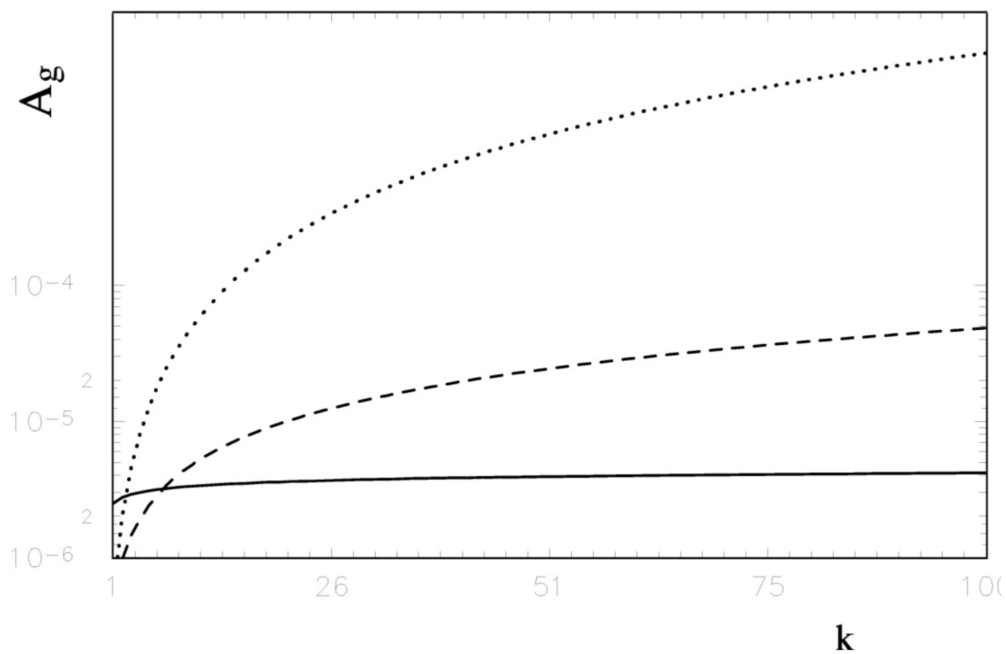
d = 2 $$A^{\mathrm{Born}}_{\mathrm{grav.}} = \frac{\pi s^2}{M^4_{4+2}} \ln(1 + \frac{M_s^2}{q^2})$$

d = 3 $$A^{\mathrm{Born}}_{\mathrm{grav.}} = \frac{S_d s^2}{M^4_{4+d}} (\frac{M_s}{M_{4+d}})^{d-2} [1 - \frac{q}{M_s} ArcTan(\frac{M}{q})]$$

d = 4 $$A^{\mathrm{Born}}_{\mathrm{grav.}} = \frac{S_d s^2}{M^4_{4+d}} (\frac{M_s}{M_{4+d}})^{d-2} [1 - \frac{q^2}{M_s^2} \ln(1 + \frac{M_s^2}{q^2})]$$

$$S_d = \frac{2\pi^{d/2}}{\Gamma(d/2)}$$

A_g^{Born} as a function of the upper limit – k M_s



Extra dimensions d = 2, 3, 4

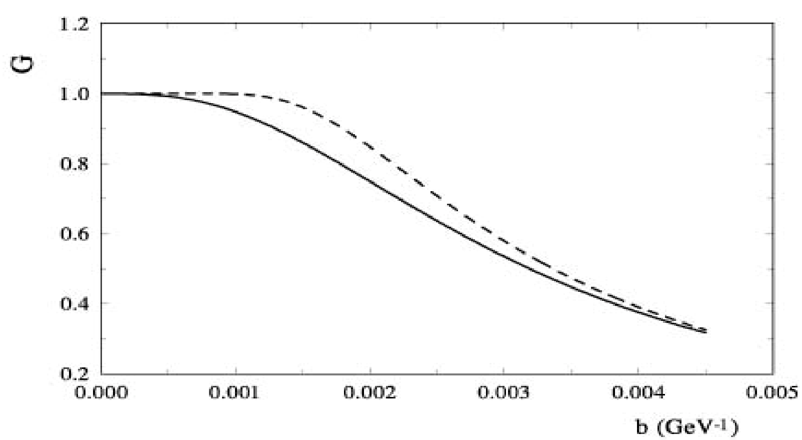
Impact parameter representation (d=2)

$$T(s,t) = \frac{1}{2\pi}\int_0^\infty b\ J_0(bq)\,[1-e^{-\chi(s,b)}]\,db$$

$$\chi(s,b) = 2\pi\int_0^\infty q\ J_0(bq)\,T_B(s,q)\,dq$$

(d=2)

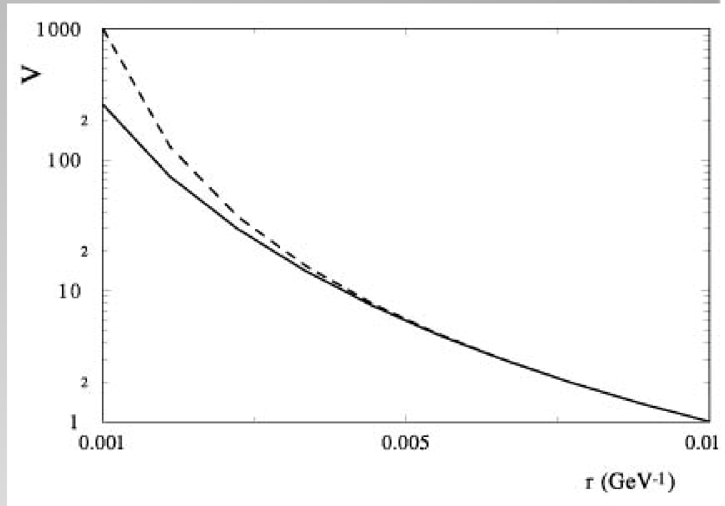
$$\chi(s,b) \sim \frac{S}{M_d^4}(1 - b M_s K_1(b M_s))]/b^2$$

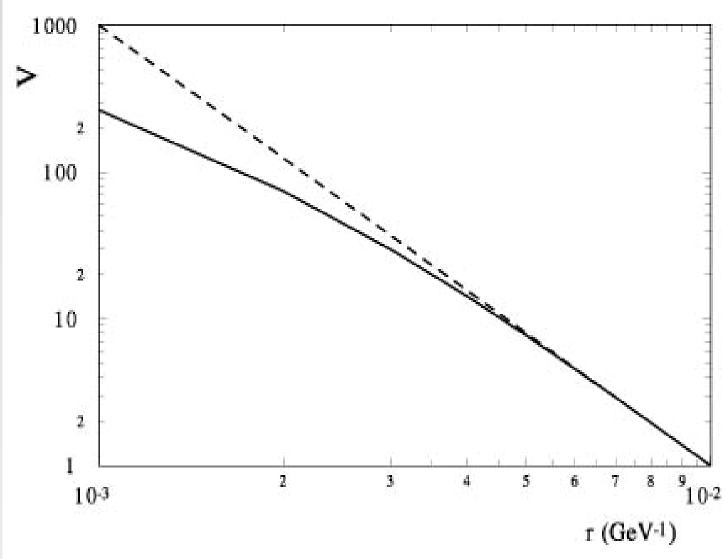


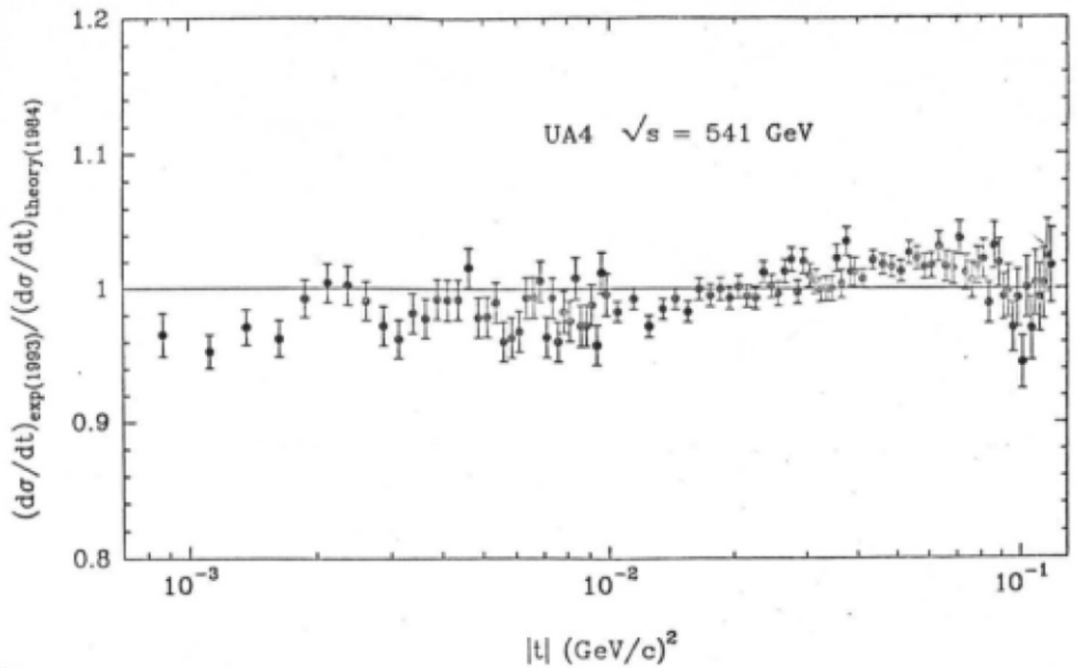
Gravi-potential (d=2)

$$V(r) = -\frac{2}{\pi r}\frac{d}{dr}\int_r^\infty \frac{b\,\chi(s,b)}{\sqrt{b^2 - r^2}}\,db$$

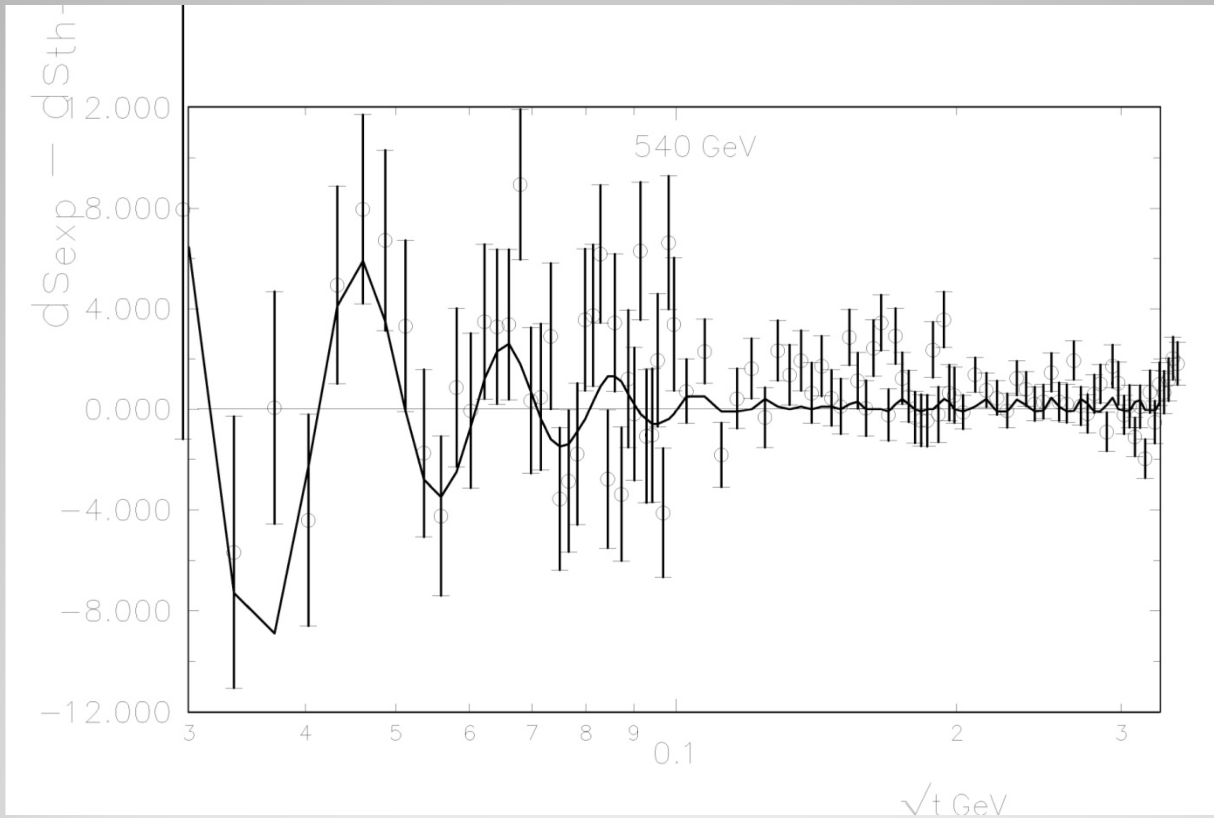
$$V(r) \sim \frac{1}{r^3}\left(1 - e^{M_s r} - M_s\, r\, e^{-M_s r}\right)$$






UA4 √s = 541 GeV
1.2
1.1
1
0.9
0.8
(dσ/dt)exp(1993)/(dσ/dt)theory(1984)
10^-3
10^-2
10^-1
|t| (GeV/c)^2
Fig. 2

Experimental data UA4/2

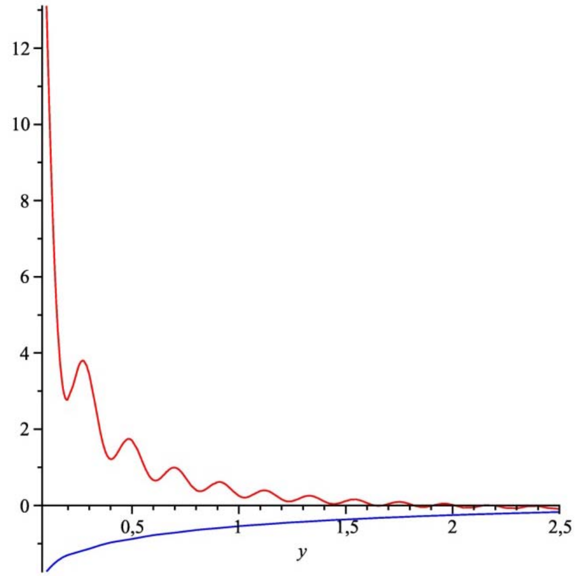


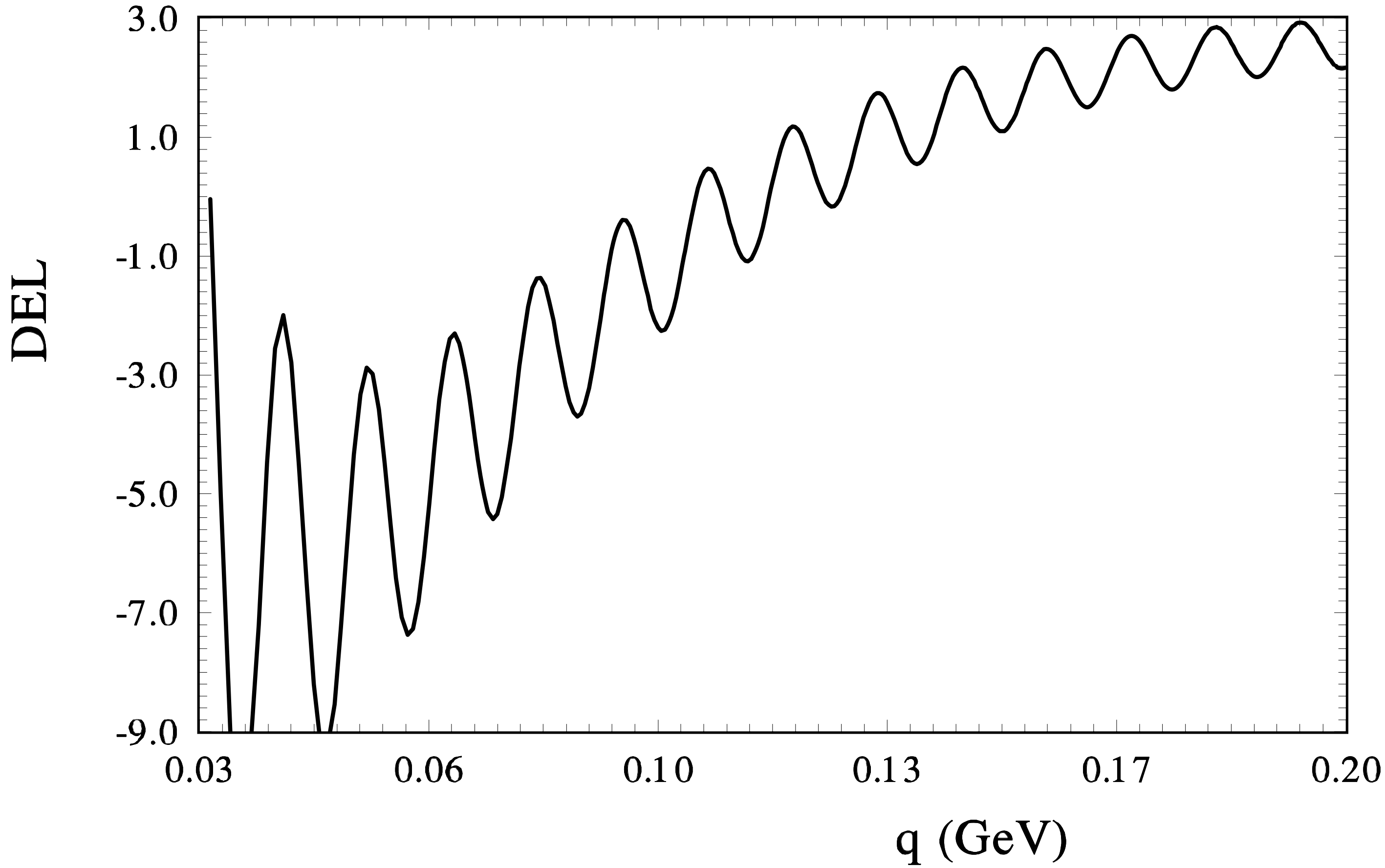
I.Ya. Aref'eva - arXiv:1007.4777

$$F_n(y) = -i\int_0^\infty x\ J_0(xy)\ [e^{ix^{-n}} - 1]\,dx$$

$$y = b_c q; \qquad x = b/b_c;$$

$$b_c = \left[\frac{(4\pi)^{n/2-1} s\Gamma(n/2)}{2M_d^{n+2}}\right]^{1/n}$$




3.0
1.0
-1.0
-3.0
-5.0
-7.0
-9.0
DEL
0.03
0.06
0.10
0.13
0.17
0.20
q (GeV)

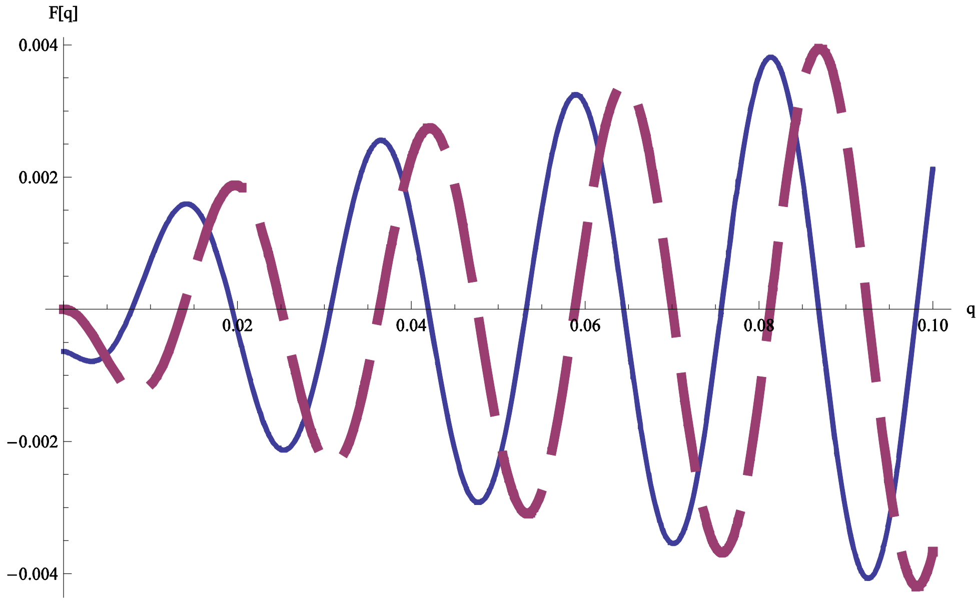
Screening long range potential – (rigid string)

$$T_{osc}(s,t) = is\int_0^\infty b\,db\,J_0(bq)\;\chi_{osc}(s,b)\exp[-\chi(s,b)])$$

$$\chi_{osc}(s,b)=\frac{h_{osc}(s)}{(r_{scr}^2-b^2)^2};$$

$$\chi_{osc}(s,b)=[1-e^{-h/(b^2-r_{scr}^2)}];$$

$$T_{osc}(s,t)=\frac{1}{2}\frac{iq}{r_{scr}}K_1(i\,r_{scr}q);$$



Nucleon-Gravitation Interaction
(Gravitation form-factors)

O.V. Selyugin, O.V. Teryaev, **Phys.Rev. D 85, (2009)**

$$\mathrm{H}^{q}(x,t) \sim q(x)\exp[a_{+}\frac{(1-x)^{2}}{x^{0.4}}t];$$

$$E^{q}(x,t) \sim \varepsilon^{q}(x)\exp[a_{-}\frac{(1-x)^{2}}{x^{0.4}}t];$$

$$F_{1}^{q}(t)=\int_{-1}^{1} dx\ H^{q}(x,\xi,t); \qquad F_{2}^{q}(t)=\int_{-1}^{1} dx\ E^{q}(x,\xi,t);$$

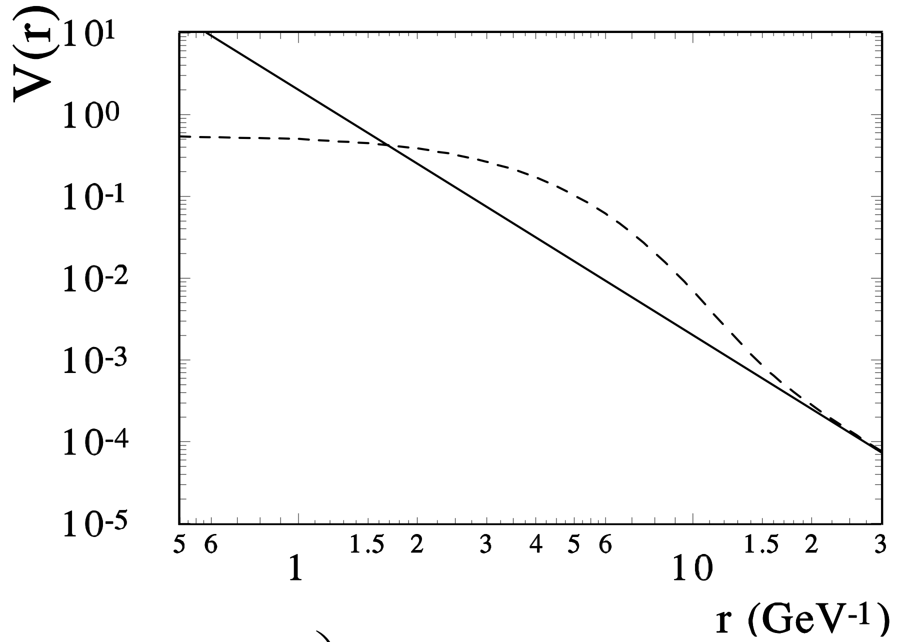
$$\int_{-1}^{1} dx\ x[H^{q}(x,\xi,t)+E^{q}(x,\xi,t)] = A_{q}(\Delta^{2})+B_{q}(\Delta^{2});$$

Nucleon-Gravitation Interaction

O.V. Selyugin, O.V. Teryaev, **Found.Phys. V.40(7),(2010)**

$$G(t) = 1/[1 - t/\Lambda^2]^2; \quad \Lambda^2 = 1.8 \ GeV^2.$$

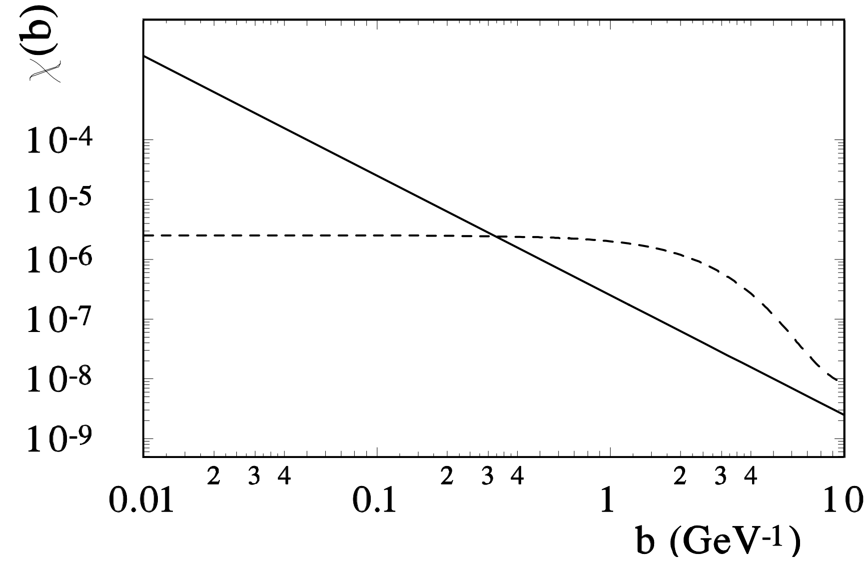
$$\tilde{A}^{Born}_{grav.} = \frac{\pi s^2}{M^4_{4+2}} \ln(1 + \frac{M_s^2}{q^2}) \, G^2(t)$$



$$\tilde{V}^{Born}_{grav.}(r) = \frac{1}{r^3}\left(1 - (1 + \frac{\Lambda r}{18}(18 + \Lambda r(9 + \Lambda r)))e^{-\Lambda r}\right).$$

$$\chi_{grav.}(b) \sim \int_0^\infty q\, J_0(qb)\log(\frac{M_s^2}{q^2})\, G^2(t) = \frac{\Lambda^2}{48}\Lambda^3 b^3\, K_3(\Lambda b)\log(M_s^2);$$

$$\tilde{\chi}_{grav.}(b) = \frac{s}{M_s^4}\left[\frac{\log(M_s^2)\,\Lambda^5 b^3}{1.4\cdot 48} K_3(\Lambda b)\,(1+\frac{1+5b^4}{2.2+10b^4}) + \frac{1}{2(1+b^{1.5})}\right].$$



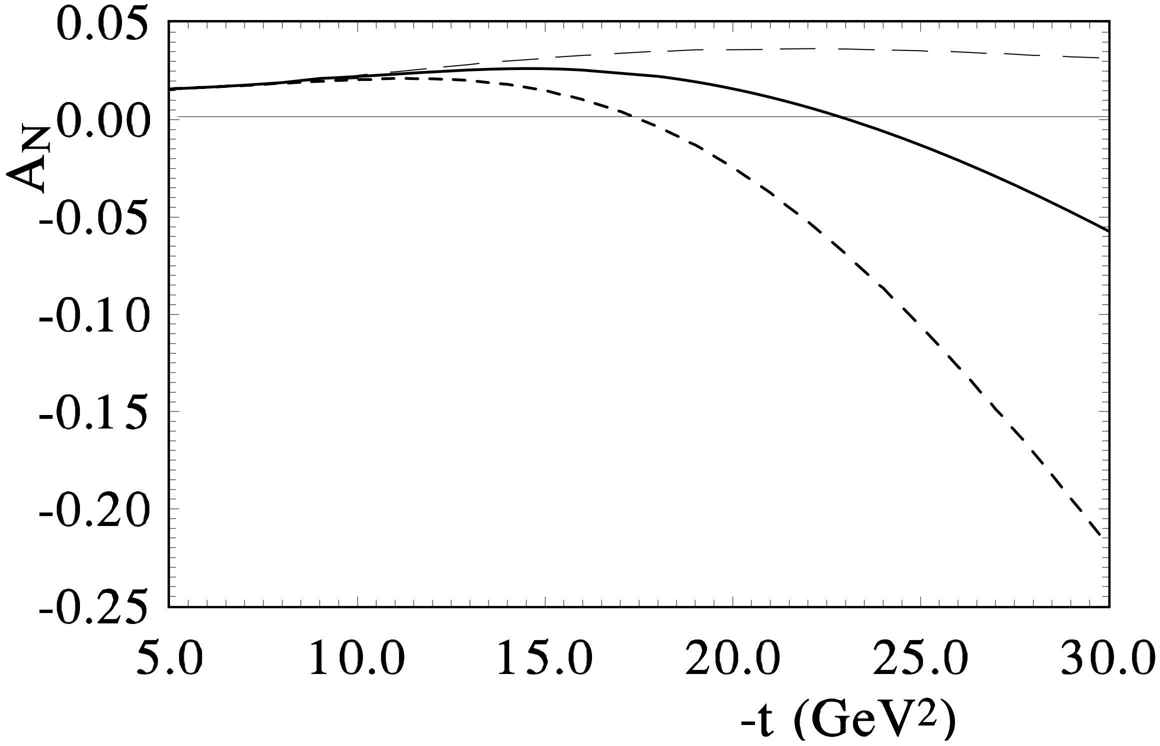
Impact gravitation contribution (d=2)
on spin correlation parameter

$$A_N = \frac{d\sigma\uparrow - d\sigma\downarrow}{d\sigma\uparrow + d\sigma\downarrow}$$

$$A_N \frac{d\sigma}{dt} = \frac{4\pi}{s^2} \mathrm{Im}[(\Phi_1(s,t) + \Phi_2(s,t) + \Phi_3(s,t) - \Phi_4(s,t))\Phi_5^*(s,t)]$$

$$A_N \frac{d\sigma}{dt} = \frac{4\pi}{s^2} \mathrm{Im}[F_{nfl} F_{fl}^*]$$

$$A_N \frac{d\sigma}{dt} = \frac{4\pi}{s^2} |F_{nfl}||F_{fl}^*| \sin(\varphi_1 - \varphi_2)$$


0.05
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
A_N
5.0
10.0
15.0
20.0
25.0
30.0
-t (GeV2)

SUMMARY

* The additional dimension d=2 do not contradict the existence experimental data.

* The gravitatin hadron form-factor can be obtained from GPDs of hadron. It leads to changing of gravitation potential on the distances order the hadron size. It is need take into account when the Black Hole production is examined.

SUMMARY

* The long range potential can be leads to the some periodic structure in the hadron differential cross sections.

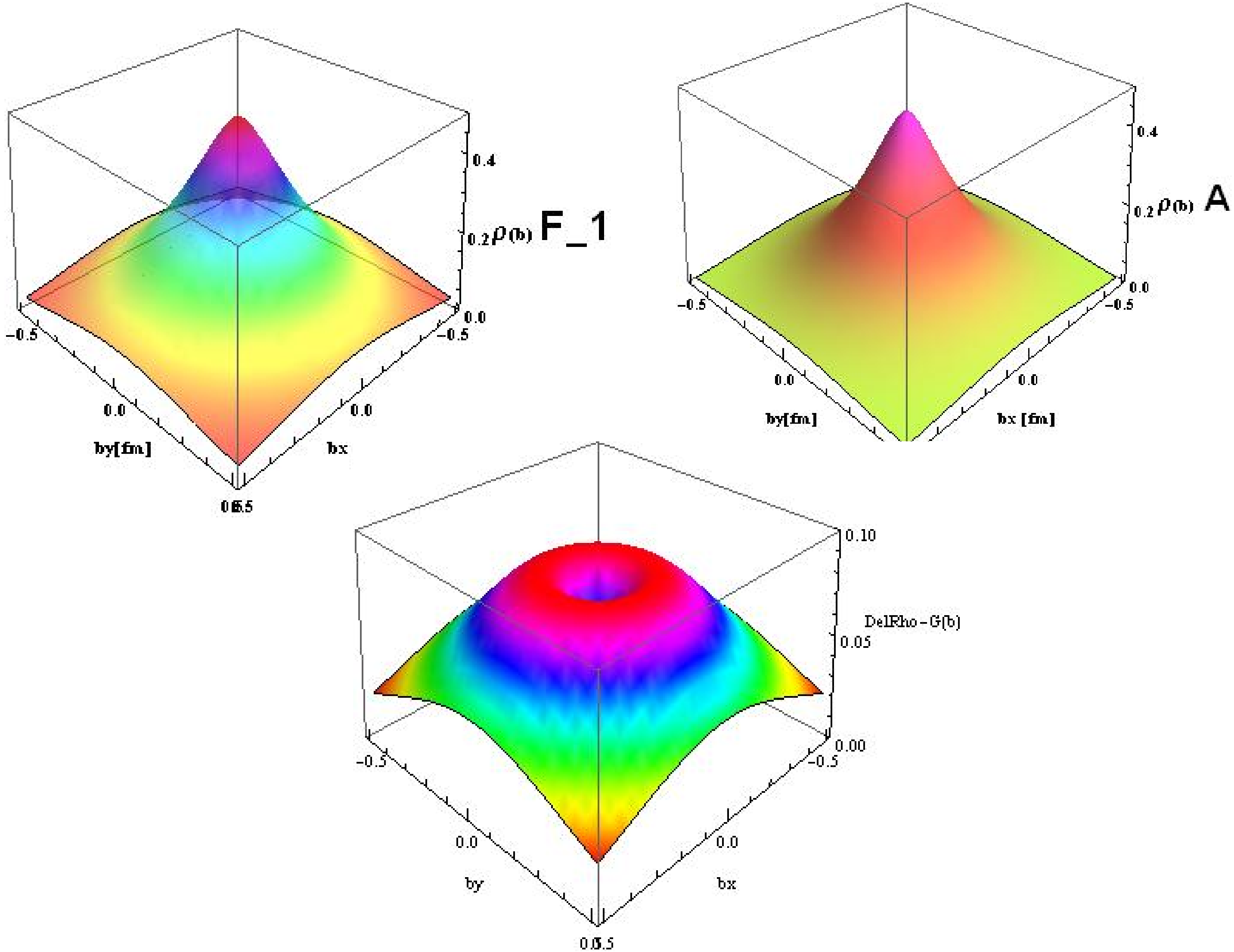
* We find that with d=2 in the framework of ADD scenario there is the manifestation of the additional dimensions as the specific behaviour of the analyzing power of the hadron-hadron scattering.

* It is need research these and other effects in the universal scenario where the all filds can be live in the extra dimentions.

THE END

Thanks for your attention

Proton - Del = (F1_em – A_gr)



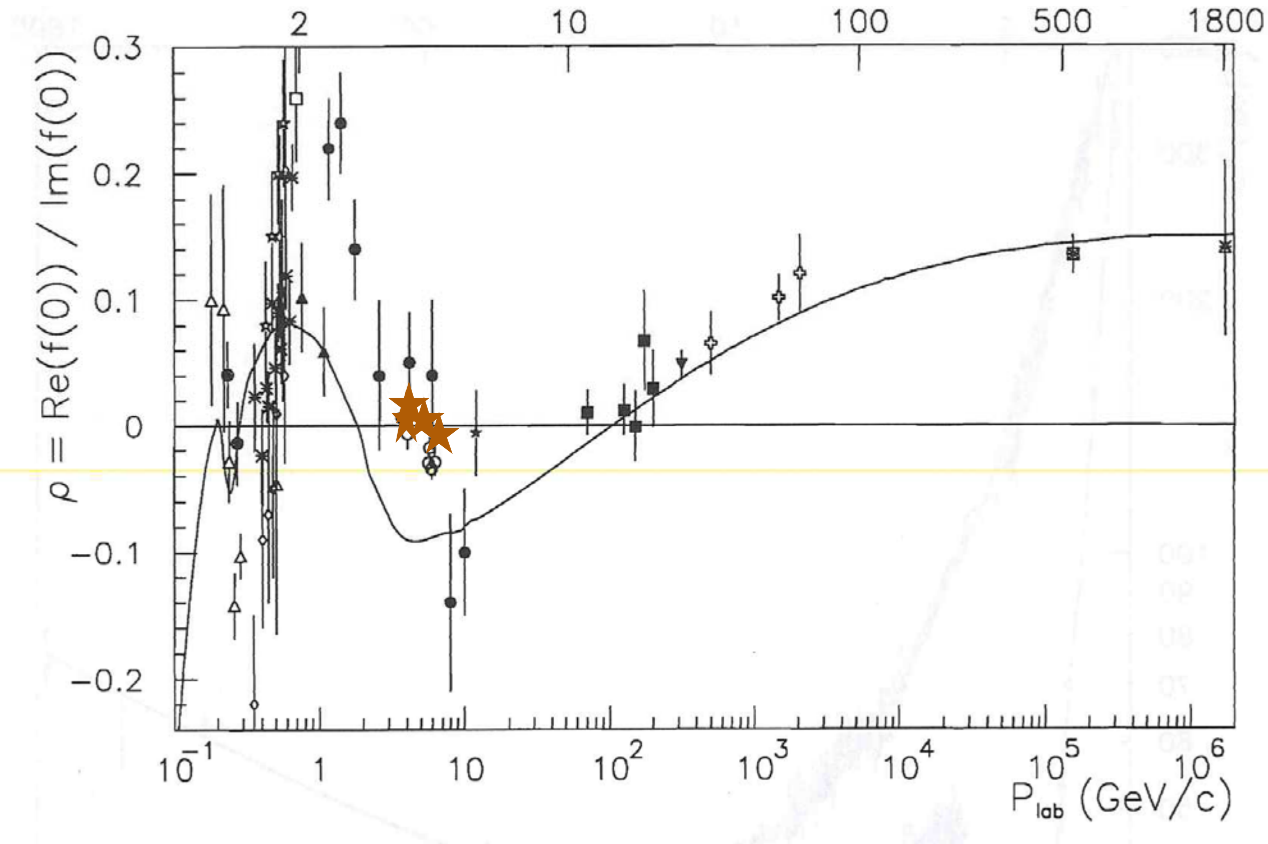
Newton case (N=4)

$$F(r) = G_N \frac{m_1 m_2}{r^2} = \frac{1}{M_{Pl}^2} \frac{m_1 m_2}{r^2}.$$

G_N ´= $10^{´-39}$ GeV

$$M_{Pl} = \sqrt{\frac{hc}{G_N}} = 1.22 \times 10^{19}\ GeV - \ Planck\ mass$$

$$l_{Pl} = [\frac{\hbar c^5}{8\pi G_N}]^{1/2} \approx 10^{-33}\, cm - \ Planck\ length$$



2
10
100
500
1800
0.3
0.2
0.1
0
−0.1
−0.2
ρ = Re(f(0)) / Im(f(0))
10−1
1
10
10²
10³
10⁴
10⁵
10⁶
P_lab (GeV/c)

K. Chadan, A. Martin: "Scattering theory and dispersion relations for a class of long-range oscillating potentials", CERN (1979)

$$V(r) \sim \sin[\exp(\mu r)]/(1+r^2)^2;$$

2. a) Van-der-Waals potential $V_{ad} \sim h/r^4$

b) F. Ferrer, M. Nowakowski (1998)
(Golstoun boson – long range forces) $V_{ad} \sim h/r^3$

3. S-L interaction

$$F_C(s,t) + F_{ad}(s,t) = is\int_0^\infty b\,db\,J_0(bq)[(1-e^{\chi_c(s,b)}) + \chi_{LS}^2(s,b)]$$

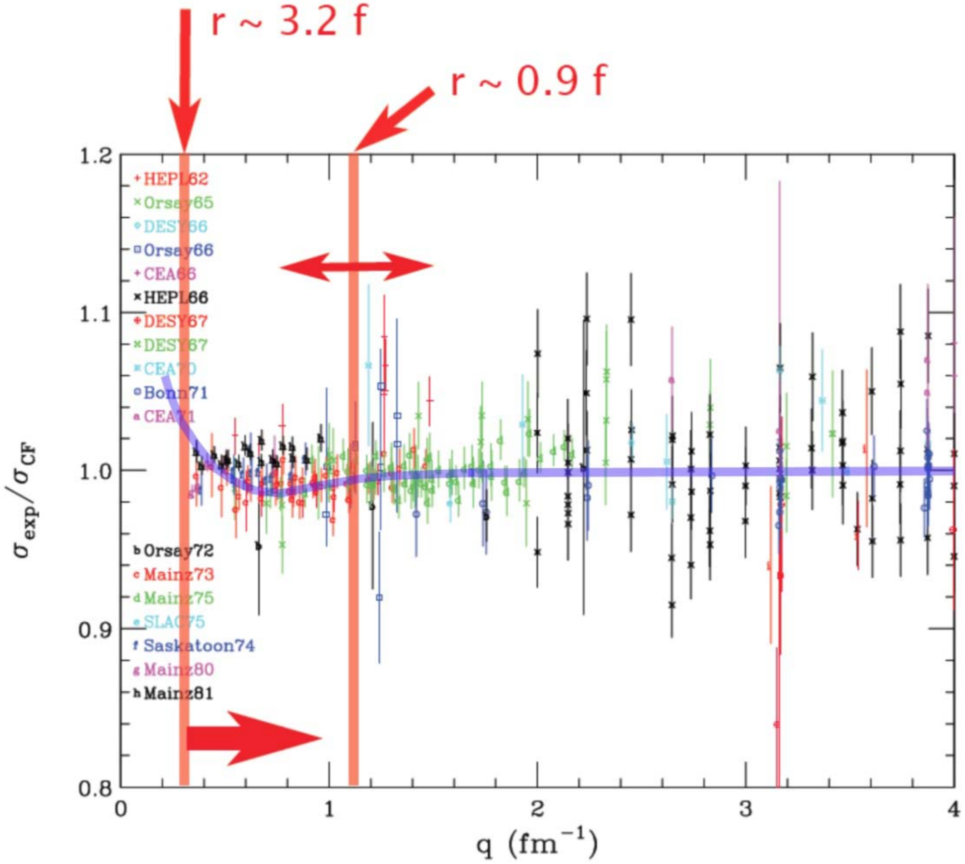
4. N-dimensional gravipotential (ADD-model)
Oscillations"- I. Aref'eva [1007.4777:arXiv-hep-ph]

$$F_{ad}(s,t) \sim \frac{s}{M_d^2};$$

Universal scenario?

A. De Rujula , arXiv:1008.3861

$$[\langle r_p^3 \rangle_{(2)}]^{1/3} = 3.32 \pm 0.22 \; fm.;$$



critic: I. Cloet, G.A. Miller
arXiv: 1008.4345

Literature

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• . D59 (1999) 105002

• T. Han, J.Lykken, and R.-J. Zhng, Phys.Rev 59 (1999) 105006.

1. The N-dimensional word manifests itself as a difference in the inverts power of the radius 1/r^d

$$F_{(4+d)}(r) = G_{N(4+d)} \frac{m_1 m_2}{r^{2+d}}.$$

V_d is the volume of compactified dimensions

$$G_{N(4+d)} = \frac{4\pi V_d}{S_{3+d}} G_N$$

$$M^2_{(4+d)} = M^2_{(4)} / V_d$$

$$G_{N(4+d)} = \frac{1}{S_{3+d} M^{(2+d)}_{(4+d)}}$$

Kaluza - Klein picture

From 4-n point o view - (4+d) g (q_1, q_2, …q_d) looks like a massive particle of mass |q|

\+ Yukawa potentials mediated by all the massive modes

for r >> L

$$\frac{V(r)}{m_1 m_2} = \; = \frac{S_d \Gamma(d)}{(2\pi)^d} \frac{G_{N(4)} V_d}{r^{d+1}} \qquad N_{KK} \sim (\sqrt{s}\, r_d)^d$$

$$\frac{1}{s} A_{grav.} \sim G_d \sim M_{Pl}^{-2} N_{KK} = \frac{(\sqrt{s})^d M_{Pl}^2}{M_{4+d}^{2+d} M_{Pl}^2} = \frac{(\sqrt{s})^d}{M_{4+d}^{2+d}}$$