On quantum gravity problem within geometric approach

D.G. Pak
JINR, LTP

Int. Workshop “Bogolyubov Readings

Sept. 2010

Plan of talk

* Preliminaries: geometric approach to quantum gravity problem

I. Vacuum tunneling in Einstein gravity.

II. Quantum gravity models with torsion

- Main principles, ideas
- Yang-Mills type gravity model
- Induced effective Einstein gravity

III. Minimal model of Lorentz gauge gravity with torsion

- Basic idea, the model
- Dynamical content in Lagrange formalism
- Covariant quantization

* Conclusions

Preliminaries

Quantum Gravity – Great Puzzle – numerous approaches:

Quantum Einstein gravity may exist non-perturbatively

Classical geometric generalizations including torsion, Weyl fields, non-metriicty

Canonical gravity approach

Loop gravity

Spin foam models

Super-gravity, -strings, M-theory

Extra-dimensions, brane worlds

Non-commutative geometry

Analogous models in condensed matter physics

………………………

Gravitation is manifestation of geometry and it does not exist as a fundamental force at all ----- an extreme viewpoint

I. Vacuum tunneling in Einstein gravity

Definitions of vacuum:

1. $\Lambda = 0: \quad R_{ijkl} = 0 \rightarrow g_{ij} = \eta_{ij}$ flat space-time

2. $\Lambda \neq 0: \quad \mathrm{R}_{\mathrm{ij}} - \frac{1}{2} R g_{ij} + \Lambda g_{ij} = 0 \rightarrow g_{ij} = 0$

$\downarrow$

(ii) Any static vacuum solution

(i) an absolute vacuum

Fubini-Study instanton CP^2

describes vacuum-vacuum transition

$g_{mn} \rightarrow 0$ when $t \rightarrow \pm\infty$

$\chi = 3, \quad \tau = 1$

3. Λ □ $0:$ Asymptotically flat metric with space topology S^3 or RP^3

this type of vacuum metrics corresponds to asymptotically locally Euclidean instantons (ALE)

On Fubini-Study instanton on CP^2

$$g_{mn} = \frac{4a^2}{a^2+\rho^2}\left(\delta_{mn} - \frac{x_m x_n + \tilde{x}_m \tilde{x}_n}{a^2+\rho^2}\right),$$

$$\tilde{x}_m = C_{mn} x^n,$$

$$C_{mn} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix},$$

$$\chi = 3, \tau = 1, \quad \Lambda = \frac{3}{2a^2}$$

Axial ABJ anomaly

$$\partial_\mu j^{5\mu} = \partial_\mu(\sqrt{g}\, e^{\mu a} \bar{\psi}\gamma_a\gamma_5\psi) = \frac{1}{4} RR^*$$

/Egichi&Freund, PRL’1976/

An analog to BPST instanton

The anti-instanton is given by the same metric
but with opposite orientation of the space-time, i.e. with inversed vielbein

Topological structure of the vacuum: the main assumption:

* The vielbein (tetrad) e^{am} is a fundamental variable of quantum gravity, not a metric tensor.

The non-trivial topology is provided by $\pi_3(SO(1,3)) = \pi_3(SO(3)) = \mathbb{Z}$

In Euclidean gravity the vacuum is classified by two integers (m, n) since SO(4)~SU(2)xSU(2)

$$\pi_3(SO(4)) = \pi_3(SU(2) \times SU(2)) = \mathbb{Z} \otimes \mathbb{Z}$$

S. Hawking (1978): There is no vacuum tunneling due to infinite

/Cargese lectures procs./ barrier between vacuums.

In search of instantons: a simple ansatz

- A pure gauge vielbein and pure gauge spin connection are:

$$e_0^{am}(x) = L^{ab}(x)\delta^{bm} = L^{am}(x)$$

$$\varphi_0^{mcd} = L^{ce}\partial^m \tilde{L}^{ed}$$

In temporal gauge the topological vacuums are classified by Chern-Simons number

$$N_{CS} = \frac{1}{16\pi^2} Tr\int d^3x \omega_{CS}(\varphi_0^{mcd})$$

A simple hedgehog ansatz with an arbitrary trial function "g" produces a class of conformally flat metrics

$$e_{ma} = g(\rho) e^{\omega\eta^i \hat{x}^i}\delta_{ma} = \Theta_{ma}^n \frac{x^n}{\rho}$$

$$\hat{x} = x/r, \quad \tan\omega = r/t, \; \rho^2 = r^2 + t^2$$

Θ_{ma}^n is a generalization of 't Hooft matrices η_{ma}^i

For the vanishing Ricci scalar

$$R = 2g(g'' + \frac{3g'}{\rho}) = 0$$

one obtains a simple regular solution -
- Hawking wormhole :

$$g(\rho) = 1 + \frac{\lambda^2}{\rho^2}$$

Ricci and Weyl tensors are:

$$R_{mn} = (\delta_{mn}\rho^2 - 4x_m x_n)\frac{4\lambda^2}{\rho^2(\rho^2+\lambda^2)^2}$$

$$C_{ijkl} = 0 \quad \rightarrow \quad \tau = 0$$

The solution can be interpreted as instanton-antiinstanton pair in conformal gravity

No other appropriate instanton solutions
(asymptotically locally flat and with non-zero Hirzebruch signature)
were found

In search of instantons: 1-1 correspondence between topological vacuums in Yang-Mills theory and gravity

- The spin connection φ_{mcd} can be decomposed into rotation and boost parts

$$\varphi_{mcd} = \left(\Omega_m, B_m\right)$$

Since

$$SU(2) \approx SO(3) \subset SO(1,3)$$
$$\pi_3(SO(1,3)) = \pi_3(SO(3)) = \pi_3(SU(2))$$

one can construct vacuum spin connection in terms of SU(2) gauge potential

$$\varphi_{mcd}^{vac}(e) = \left(\Omega_m, 0\right)$$

By this way it is not easy to retrieve the vacuum vielbein from the spin connection.

Explicit construction of non-trivial topological vacuums in SU(2) Yang-Mills theory /Baal&Wipf-2001, Cho-2006/

Introduce orthonormal basis of SU(2) triplets

$$\hat{n}_i : \quad D_m \hat{n}_i^{\alpha} = 0, \quad \alpha = 1,2,3 \in SU(2)$$

One has the integrability condition

$$[D_m, D_n]\hat{n}_i = \vec{F}_{mn} \times \hat{n}_i = 0$$

Solution to these conditions gives a pure gauge vacuum potential /Cho-2006/

$$\vec{\Omega}_m = -C_m^k \hat{n}_k,$$

$$C_m^k = -\frac{1}{2}\varepsilon_{ij}{}^{k}(\hat{n}_i \cdot \hat{n}_j)$$

Parameterizing the triplet $\hat{n}_i$ by angles of $S^3 \sim SU(2)$ one obtains explicitly:

$$C_m^1 = \sin\gamma\partial_m\alpha - \sin\alpha\cos\gamma\partial_m\beta,$$

$$C_m^2 = \cos\gamma\partial_m\alpha + \sin\alpha\sin\gamma\partial_m\beta,$$

$$C_m^3 = \cos\alpha\partial_m\beta + \partial_m\gamma$$

In 4d spherical coordinate system the radial coordinate hypersurfaces are given by S^3. So one can define the basis triple of left invariant 1-forms on S^3

$$\sigma^i = \frac{1}{2}dx^m C_m^i,$$

$$d\sigma^i = 2\varepsilon^{ijk}\sigma^j\sigma^k$$

Maurer-Cartan eqn.

Finally, the basis of pure gauge vielbein 1-forms in polar c.s. (ρ,θ,ϕ,ψ) is defined as follows:

$$e_0^a = (d\rho, \rho\sigma^i) \qquad \sigma^i = \frac{1}{2} dx^m C_m^i(\alpha,\beta,\gamma)$$

where the angle functions $\alpha(\theta,\phi,\psi), \beta(\theta,\phi,\psi), \gamma(\theta,\phi,\psi)$

define the homotopy classes $\pi_3(SU(2))$

To find instanton solutions one can apply a simple ansatz:

$$e^a = (g_0(\rho) d\rho, g_i(\rho)\rho\sigma^i)$$

We will consider an ansatz corresponding to topological class with winding number $\tau = 1$, i.e., we put $\alpha=\theta, \beta=\phi, \gamma=\psi$

The ansatz with two functions g_0 , g_3 (g_1=g_2=1) applied to Einstein eqn. produces the well-known Eguchi-Hanson instanton $g_3^2 = 1/g_0^2 = 1 - a^2/\rho^4$

Explicit proof of vacuum tunneling

The space-like vielbein of E-H instanton defines SU(2) gauge potential A^i_m:

$$e^{i=1,2,3} = g_i(\rho)\rho\sigma^i = dx^m A^i_m(x)$$

Passing to temporal gauge

$$\vec{A}_t \to U\vec{A}_t U^{-1} + U\partial_t U^{-1} = 0$$

in Cartesian coords. gives a system of eqs. for gauge parameters $\omega, \hat{f}$

$$U = \exp[i\omega(r,t)\vec{\tau}^i \hat{f}^i(r,t)], \qquad \hat{f}^2 = 1$$

In asymptotic region $t \square \pm\infty$ one has the solution $\hat{f} \square 1, \quad \omega \square \int dt \dfrac{rg_3}{\rho^2} + c_1(r)$

where c_1 is determined by initial condition $\omega(t=-\infty)=0$

This implies transition from the trivial vacuum defined by $\hat{n}(t=-\infty) = (0,0,1)$

to non-trivial vacuum with $N_{CS}=1$ at $t=+\infty$ defined by

$$\hat{n}_{t=+\infty} = -U_{t=+\infty}\hat{n}_{t=-\infty} = \begin{pmatrix} \sin\alpha(r)\cos\beta(r) \\ \sin\alpha(r)\sin\beta(r) \\ \cos\alpha(r) \end{pmatrix}$$

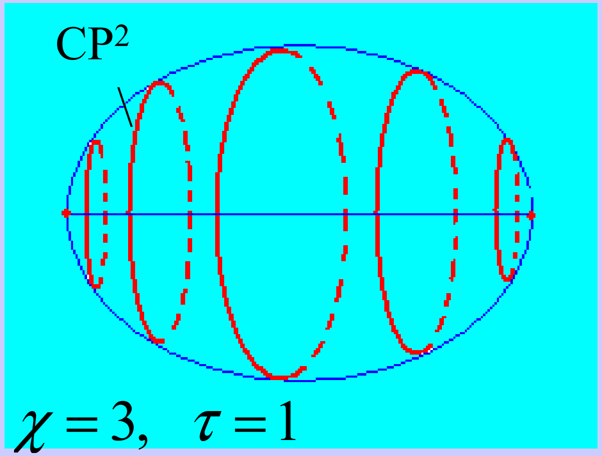
where the functions α, β are defined by

$$U_{t=+\infty}(\omega, \hat{f}) = \exp[\frac{i}{2}\alpha(r)\vec{\tau}^i\hat{\beta}^i(r)],$$

$$\hat{\beta}^i(r) = (\sin\beta(r), \cos\beta(r), 0).$$

$$< N_{CS}=1 | N_{CS}=0 >_{vac} \square\ e^{-S_{inst}}$$

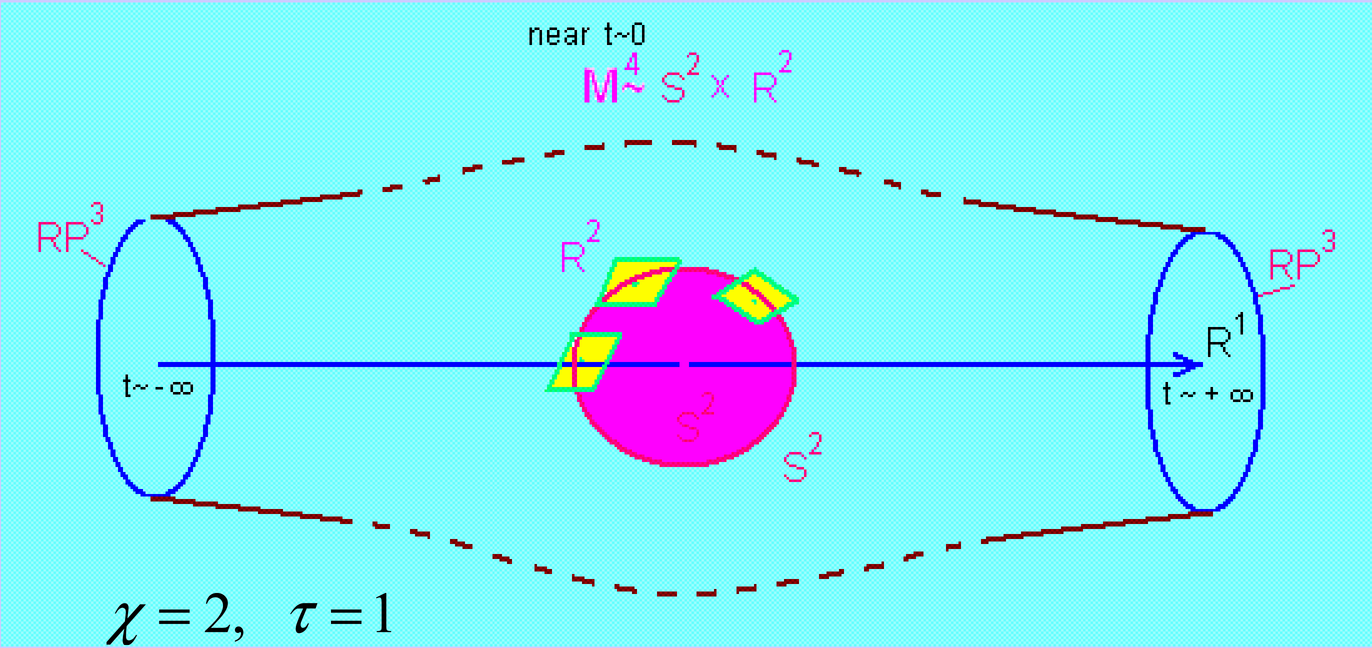
Vacuum tunneling



via Fubini-Studi instanton $\Lambda \neq 0$

$$g_{mn}(t = \pm\infty) = 0$$

via Eguchi-Hanson instanton



Λ [illegible] 0

What is strange in this vacuum tunneling?

* 1979: Hawking's claim was rather limited to asymptotic euclidean instanton.
* 1979: Why others did not claim the vacuum tunneling?
* 2008: Vacuum tunneling revisited /Y.M. Cho, **Prog.Th.Phys.Suppl.,2008/**
* ~1920s: Schwarzschild prefers RP^3 as more simple than S^3
* Indications to RP^3 topology of our space:

--non-zero index $I_{3/2}$ of Dirac operator for E-H instanton;

-- existence of the electrons.

The more principal question is:

Whether vielbein really represents a variable of quantum gravity?

* Is the vielbein like a kinematic variable locally introduced on water surface?
  If this is so, what describes the microscopic structure of the space?

* testing the quantum nature via gravitational Aharonov-Bohm effect:
  calculation of holonomy operator and experimental verification.

II. Quantum gravity models with torsion

Why torsion (contortion, Lorentz connection)?

* Equivalence principle, local Lorentz symmetry, gauge principle.

If the vielbein is classic then the quantum fluctuation of spin connection will create general Lorentz connection

$$\varphi_{mcd}(e) \xrightarrow{\text{fluctn}} A_{\text{mcd}}$$

* Einstein gravity as effective theory induced by quantum dynamics.

Contortion (torsion) may provide the microscopic structure of the space

* Existence of spin particles should imply torsion.

A problem of non-existency of solution for the electron in Einstein gravity

* Ideas from QCD: confinement, quantum condensate

Torsion might be unobservable as a classic object like gluon in QCD--Quantum chromodynamics, there is no classical chromodynamics.

* Contortion should possess properties of connection.

Contortion as a part of Lorentz connection, not a tensor.

Yang-Mills type Lorentz gauge gravity

Utiyama gauge approach to gravity

$$A_{\mu cd} = \varphi_{\mu cd}(e) + K_{\mu cd}$$

Riemann-Cartan curvature :

$$R_{abcd} = \hat{R}_{abcd} + \widetilde{R}_{abcd}(e; K_{mcd})$$

$$\widetilde{R}_{abcd}(e; K_{mcd}) = \hat{D}_{[a} K^{d}_{b]c} + K^{e}_{[a|c} K^{d}_{b]e}$$

The Lagrangian is :

$$L = -\frac{1}{4} e R^{abcd} R_{abcd}$$

This Lagrangian admits

an additional local symmetry:

$$\delta e^{\mu}_{a} = \delta \varphi_{\mu cd} = 0;$$

$$\delta K_{\mu} = \hat{D}_{\mu} \Lambda + [K_{\mu}, \Lambda],$$

One-loop effective action in constant curvature space-time background

- Splitting the contortion into classical and quantum parts

$$K_{\mu cd} = K_{\mu cd}^{class} + Q_{\mu cd}$$

and assuming the vacuum averaged value for Riemann-Cartan curvature to be positively constant

$$< \tilde{R}_{abcd} >= M^2 (\eta_{ac}\eta_{bd} - \eta_{ad}\eta_{bc})$$

one can calculate a one-loop effective potential which has a non-trivial minimum leading to torsion condensate:

$$V_{eff} = \frac{\tilde{R}^2}{3} + \frac{11 g^2 \tilde{R}^2}{48\pi^2} (\ln \frac{g\tilde{R}}{\mu^2} - c)$$

/Class. Quant. Grav.,2008/

Induced Einstein gravity as an effective theory

Expanding the initial Lagrangian
around vacuum one obtains
the effective Lagrangian with Einstein
Hilbert term and cosmological constant

$$L = -\frac{1}{4} e R^{abcd} R_{abcd} = -\frac{1}{4} e(\hat{R}_{abcd} + < \tilde{R}_{abcd} >)^2 =$$
$$= -\frac{1}{4} \hat{R}\hat{}2 - \frac{1}{2} \hat{R} M^2 - \frac{3}{2} M^4$$

The value of M is related with the
minimum of the effective potential
and vacuum energy density.

Weak coupling phase:

$$\rho_{vac} \approx 2 \times 10^{-47} Gev^4$$
$$M^2 = 3.6 \times 10^{-24} Gev^2$$
$$\alpha = \frac{g^2}{4\pi} \approx 0.012$$

Strong coupling phase:

$$\Lambda \approx 1,$$
$$\alpha \approx 1.5$$

this value is close to
coupling constant in SO(10) SS GUT
at unification scale 10^{17} Gev

Problems, drawbacks of the Yang-Mills type gravity model

* The Hamiltonian is not positively defined;
* Two independent variables, vielbein and contortion on equal footing. Formally the vielbein should be quantized also. But the fundamental quantum variable should be rather only one.
* The content of spin states of contortion is too big to compare with vielbein: two spin 2 states, four spin 1 states implies 24 degrees of freedom;

Possible interrelation of QCD and Gravity

QCD

(i) Dynamical symmetry breaking, Meissner effect, confinement, gluon condensate

(ii) A vacuum pure gauge potential is described by SU(2) gauge potential

$$A^i_\mu = C^k_\mu \hat{n}^i_k ,$$

$$C^k_\mu = \frac{1}{2}\varepsilon^{ijk}(\hat{n}_i \cdot \partial_\mu \hat{n}_j)$$

Gravity

* Meissner effect with forming torsion condensate (Regge&Hanson'80s), torsion as a genuine connection can be confined

* Flat vielbein is represented by the same pure gauge SU(2) potential (in spherical coordinates)

$$e^a = (d\rho, \rho\sigma^i), where$$

$$\sigma^i = \frac{1}{2} dx^\mu C^i_\mu$$

(iii) Abelian decomposition of the gauge potential

$$\vec{A}_\mu = \hat{A}_\mu + \vec{X}_\mu$$

$$\hat{A}_\mu = A_\mu{}^r \hat{n}_r - \frac{1}{g} \hat{n}_r \times \partial_\mu \hat{n}_r$$

$$\hat{D}_\mu \hat{n}_r = 0$$

** $\hat{n}_i^a \in \pi_3(SU(2)/U(1))$

is unobservable topological degree which becomes dynamical in effective Faddeev-Niemi-Skyrme model /1998

* Decomposition of Lorentz connection

$$A_{\mu cd} = \varphi_{\mu cd}(e) + K_{\mu cd}$$

$$\hat{D}_\mu g_{\nu\rho} = 0$$

$\rightarrow$ e_{am} ??

III. Minimal model of Lorentz gauge gravity with torsion

The main idea: Existence of the topological phase in the absence of torsion

The most general P, CP invariant Lagrangian quadratic in curvature:

$$L = -\frac{1}{4}\left[(\alpha+\gamma)R_{abcd}^2 - (\alpha-\gamma)R_{abcd}R^{cdab} - 4\beta(R_{bd}^2 - R_{bd}R^{db}) + 4\gamma R_{abcd}R^{acdb}\right]$$

$$R_{abcd} = \hat{R}_{abcd} + \widetilde{R}_{abcd}(e;K_{mcd})$$

$$\widetilde{R}_{abcd}(e;K_{mcd}) = \hat{D}_{[a}K_{b]c}^{d} + K_{[a|c}^{e}K_{b]e}^{d}$$

In constant curvature space-time,

$$\hat{R}_{abcd} = \frac{\hat{R}}{12}(\eta_{ac}\eta_{bd} - \eta_{ad}\eta_{bc})$$

we consider linear in K_{mcd} equations of motion:

$$\frac{\delta L^{(2)}}{\delta K_{bcd}} = 0$$

Decomposition into irreducible field components

- Pure gauge degrees of freedom (6 dof): $\mathbf{K_{0cd}}$

Space components of contortion:

$$K_{\mu\gamma\delta} = \varepsilon_{\gamma\delta\rho}\tilde{K}_{\mu\rho},$$

$$\tilde{K}_{\mu\rho} = S^{tt}_{\mu\rho} + \frac{1}{2}(\delta_{\mu\rho}\Delta - \partial_\mu\partial_\rho)S^t + (\partial_\mu S_\rho + \partial_\rho S_\mu) + \varepsilon_{\mu\rho\sigma}A_\sigma,$$

$$K_{\mu 0\rho} = R^{tt}_{\mu\rho} + \frac{1}{2}(\delta_{\mu\rho}\Delta - \partial_\mu\partial_\rho)R^t + (\partial_\mu R_\rho + \partial_\rho R_\mu) + \varepsilon_{\mu\rho\sigma}Q_\sigma$$

two spin 2 states (4 dof): $S^{tt}_{\mu\rho}, R^{tt}_{\mu\rho}$

four vector spin 1 states (12 dof): $S_\mu, A_\mu, R_\mu, Q_\mu$

two spin 0 states (2 dof): S^t, R^t

total number of physical dof: 18

There are constraints in the eqns. of motion $\frac{\delta L^{(2)}}{\delta K_{bcd}} = 0$

Additional local symmetries of the equations of motion:

$$\delta_\lambda K_{bcd} = \frac{1}{3}\eta_{bc}\hat{D}_d\lambda - \eta_{bd}\hat{D}_c\lambda,$$

$$\delta_\chi K_{bcd} = \hat{D}_c\chi_{db} - \hat{D}_d\chi_{dc}$$

$$\chi_{bc} = \chi_{cb},$$

$$\chi_{cc} = 0,$$

$$\hat{D}_c\chi_{cb} = 0$$

We impose 6 gauge fixing conditions

to fix the local Lorentz symmetry:

(the gauge is consistent with

equations of motion)

$$\partial^i(K_{i0\delta} - K_{\delta 0i}) = 0,$$

$$(\alpha+\gamma)\partial^i K_{i\gamma\delta} - \gamma(\partial^i K_{\gamma i\delta} - \partial^i K_{\delta i\gamma}) = 0,$$

$$\partial^i\partial^j K_{i0j} = 0$$

Technical details

we split the Lorentz connection into background and quantum parts.

We consider linearized in K_{mcd} Euler-Lagrange equations of motion

in the constant curvature background Riemannian space.

$$A_{\mu cd} = \varphi_{\mu cd}(e) + K_{\mu cd}$$

Normal gauge decomposition of geometric quantities is used in sufficient order of R.

$$e_m^a = \delta_m^a + \frac{\hat{R}}{36}(\delta_m^a x^k x_k - x^a x_m) + \ldots$$

the theory is highly degenerated, there appear constraints which strongly suppress the dynamics of torsion. We solve all constraints while keeping the consistence with dynamical eqns. of motion .

The final result of calculation is the following:

Special case α=1, β=0, γ= -3: torsion obtains dynamical degrees

All irreducible fields in the decomposition of contortion K_{mcd} are expressed in terms of four fields:

A_μ^{tr}	massless vector,	2 d.o.f.
$S_{\mu\nu}^{tt}$	spin 2,	2 d.o.f.
$\varphi_1 = \partial_\mu S_\mu^{long}$		
$\varphi_2 = \partial_\mu Q_\mu^{long}$	two spin 0 fields,	2 d.o.f.

The number of torsion dynamic d.o.f. equals the number of d.o.f. for the metric tensor!

Classical effective Lagrangian is:

$$\rho = \frac{\hat{R}}{24}$$

$$L_{eff} = A_\alpha^{tr}\left(-\partial_t^2 + \vec{\partial}^2 - 2\rho\right)A_\alpha^{tr} + \frac{3\rho}{8} S_{\alpha\beta}^{tt}\frac{1}{\vec{\partial}^2}\left(-\partial_t^2 + \vec{\partial}^2\right)S_{\alpha\beta}^{tt}$$

$$-\varphi_1\frac{1}{\vec{\partial}^2}\left(-\partial_t^2 + \vec{\partial}^2 - 6\rho\right)\varphi_1 - \varphi_2\left(-\partial_t^2 + \vec{\partial}^2 - 6\rho\right)\varphi_2 - \left(\varphi_1 + \partial_t\varphi_2\right)^2$$

Covariant quantization in one-loop approximation

- Effective Lagrangian with gauge fixing terms and Faddeev-Popov ghosts has a simple form:

$$L_{tot} = L^{(2)}_{class}(e,K) + \sum_{i=1,2,3} (L^{(i)}_{gf} + L^{(i)}_{FP})$$

$$L^{(1)}_{gf} = -\frac{1}{2\xi_1}(\hat{D}^b \widetilde{K}_{bcd})^2$$

$$L^{(2)}_{gf} = -\frac{1}{2\xi_2}(\hat{D}^d \widetilde{K}_d)^2$$

$$L^{(3)}_{gf} = -\frac{1}{2\xi_3}(\hat{D}^c \widetilde{K}_{bcd} + \hat{D}^c \widetilde{K}_{dcb})^2$$

$$L^{(1)}_{FP} = \bar{c}_{1cd}(\hat{D}\hat{D} + \frac{\hat{R}}{6})c_{1cd}$$

$$L^{(2)}_{FP} = \bar{c}_2 \hat{D}\hat{D} c_2$$

$$L^{(3)}_{FP} = \bar{\psi}_{cd}(\hat{D}\hat{D} - \frac{\hat{R}}{3})\psi_{cd}$$

Conclusions

Drawbacks of Yang-Mills type Lorentz gauge gravity:

* The Hamiltonian is not positively defined;

* Two independent variables, vielbein and contortion, on equal footing.
  Vielbein should be quantized also.
  Which variable is fundamental?

* The content of spin states of contortion is too big to compare with vielbein, 24 degrees of freedom;

How are they resolved in the minimal gravity with torsion:

* The kinetic terms for spin 1,2 are positive. There is a hope that the Hamiltonian of whole non-linear theory is positive.

* Torsion can be treated as a unique dynamic degree of quantum gravity.

* The number of physical d.o.f. for torsion and for the metric tensor are the same.

Metric becomes dynamical in the effective Einstein gravity. Topological d.o.f. turn into dynamical ones.

Open questions

* Implications in standard cosmology:
  Dark matter as a classical or quantum condensate of torsion.
* Idea from analogous gravity models in condensed matter: the torsion condensate provides microscopic structure of space as a superfluid.
* If torsion does not exist as a classical object then the space before Big Bang is topological, non-metric one.
* If there is no torsion at all then we'll have a chance to invent more sophisticated and beautiful theory.