Black stars induced by matter on a brane: exact solutions

Maxim Kurkov
in collaboration with Prof. A.A. Andrianov
based on: arXiv:1008.2705

V.A.Fock Department of Theoretical Physics
Saint-Petersburg State University

International Workshop Bogoliubov Readings
Russia, Dubna, 2010
What is the problem?

- Black hole creation may be a consequence of strong gravity at short distances attainable in high energy experiments if our space is realized on a hypersurface – three-brane in a multidimensional space-time.
- Correct (or better exact) description of black hole geometry when the matter universe is strictly situated on the three-dimensional brane but gravity propagates into extra space dimensions is needed.
- Appearance of delta-like singularities in matter distribution hidden under horizon for static locally stable black holes is a problem: in fact the matter (quarks and gluons) must be smoothly distributed.
- Therefore one expects that rather black stars are created with matter both inside and outside an event horizon in a finite brane-surface volume.
Techniques

- **Stress-energy tensor structure.**
  - The Einstein equations in the bulk read,
    \[
    (5) G_{AB} = \kappa_5 T_{AB}, \quad T_{AB} = \delta^\mu_A \delta^\nu_B T_{\mu\nu}\delta(z)
    \]
    with \( \kappa_5 = 1/M^3_* \) and \( M_* \) is a Planck scale in five dimensions.
  - In order to define \( \tau_{\mu\nu} \) let us introduce extrinsic curvature tensor \( K_{\mu\nu} \).
    \[
    K_{\mu\nu} = -\frac{1}{2} \frac{\partial g_{\mu\nu}}{\partial z} \text{ valid in the Gaussian normal coordinates (i.e.} g_{zz} = -1, g_{\mu z} = 0) \text{ only!}
    \]
  - \( \tau_{\mu\nu} \) is defined by the Israel-Lanczos junction conditions,
    \[
    [g_{\mu\nu} K - K_{\mu\nu}]^+_{-0} = \kappa_5 \tau_{\mu\nu}.
    \]
  - \( K^+_{\mu\nu} \), and \( K_{\mu\nu-0} \) are the extrinsic curvature tensors of hypersurfaces \( z = +0 \) and \( z = -0 \) correspondingly.
Techniques

- **General construction.**
  - To build a brane we search for a metric $g_{AB}(x, y)$ which is a bulk vacuum solution of the Einstein equations with event horizon.
  - Suppose that:
    - a) the induced metric $g_{\mu\nu}(x, y)$ is asymptotically flat for any hypersurface $y = \text{const}$ and inherits the horizon;
    - b) in the chosen coordinate systems $g_{5B}(x, y) = 0$ and the remaining metric components provide orbifold geometry $g_{AB}(x, y) = g_{AB}(x, -y);$  
    - c) Coordinate $y$ is spacelike i.e. $g_{yy} \equiv g_{55} < 0.$
  - In order to generate a brane filled by matter we proceed the following transformation,

$$g_{AB}(x, y) \Longrightarrow g_{AB}(x, |z| + a).$$

- Brane: $z = 0.$
Construction of the solution.

- Preparing of the suitable coordinate system.
  - We start from the metric describing a five-dimensional static neutral black hole in Schwarzschild coordinates \( \{t, r, \theta_1, \theta_2, \theta\} \),

\[
g_{AB} = \text{diag} \left[ U(r), -\frac{1}{U(r)}, -r^2 \cos^2 \theta, -r^2 \cos^2 \theta \cos^2 \theta_1, -r^2 \right],
\]

where \( U(r) = 1 - \frac{M}{r^2} \), \( M \) is related to the Schwarzschild-Tangherlini radius \( M \equiv r_{Sch-T}^2 \).

- Let's define the Gaussian normal coordinates in respect to hypersurface with space-like normal vector \( \theta = 0 \).
- The vector orthonormal to this hypersurface \( n^A = [0, 0, 0, 0, 1/r] \).
- The required change of coordinates acts on two variables \( r = r(\rho, y) \), \( \theta = \theta(\rho, y) \).
Construction of the solution.

- Our coordinate transformation has the following form.

\[
|y| = \int_{\rho}^{r} \frac{\text{sign}((r - \rho)) x^2}{\sqrt{(x^2 - M)(x^2 - \rho^2)}} \, dx, \quad \theta = \int_{\rho}^{r} \frac{\text{sign}((r - \rho)y)}{\sqrt{(x^2 - M)(x^2 - \rho^2)}} \, dx.
\]

- We have: inside the horizon \( r < \rho < \sqrt{M} \) and outside the horizon \( \sqrt{M} < \rho < r \).

- The metric in new coordinates \( \{t, \rho, \theta_1, \theta_2, y\} \), reads,

\[
g_{AB}(x, y) = \text{diag} \left[ U(r), -\frac{r^2 \rho^2}{\rho^2 U(r)}, -r^2 \cos^2 \theta, -r^2 \cos^2 \theta \cos^2 \theta_1, -1 \right],
\]

where \( r = r(\rho, y), \theta = \theta(\rho, y) \).

- The final answer for black star metric and \( \tau_{\mu\nu} \) has the following form:

\[
g_{AB}^{\text{final}}(x, z) = g_{AB}(x, y)|_{y=|z|+a}, \quad \kappa_5 \tau_{\mu\nu}(x, a) = \left( \frac{\partial g_{\mu\nu}}{\partial y} - g_{\mu\nu} \frac{g^{\lambda\delta} \partial g_{\lambda\delta}}{\partial y} \right)|_{y=a}
\]
Construction of the solution.

Figure 1: Pairs of hypersurfaces symmetric in respect to the horizontal axis to be glued into a brane are shown by red curves. The circle of horizon in dim = 5 is depicted by green line. 

\[ g_{AB}(x, y) \rightarrow g_{AB}(x, |z| + a), \quad a = 0.69868\sqrt{M} \div \pi \sqrt{M}/2 \]
Construction of the solution.

- Some technical remarks.
  - Note, that situation on the horizon is O.K. All quantities that must be continues are continues. for example for scalar curvature on the brane $(4)R$ we have the following limit:

$$
(4)R(a) = -2 \frac{B + 1 - \cos^2 a - 4|\sin a|\sqrt{1 + B}\sqrt{B}\cos a}{(1 + B)\cos^2 a},
$$

$$
B(a) \equiv \lim_{\rho \to \sqrt{M}} \frac{r(\rho, a) - \rho}{\rho - \sqrt{M}} = \frac{1}{2} \left( \cosh \left( \frac{2a}{\sqrt{M}} \right) - 1 \right).
$$

- In this construction space-time is asymptotically flat and the following asymptotic takes place:

$$
(4)R = \frac{4M^2a^2}{\rho^8} \left( 1 + O \left( \frac{a^2}{\rho^2} \right) \right).
$$
Matter distribution.

- Effective 4-D stress-energy tensor $S_{\mu \nu}$.
  - projection of Einstein equations onto the brane: SMS equations
    \[ (4) G_{\mu \nu} \equiv G_{\mu \nu} = \kappa_5^2 \Sigma_{\mu \nu} - E_{\mu \nu} \equiv \kappa_4 S_{\mu \nu}, \quad \kappa_4 \equiv \frac{1}{M_{Pl}^2}, \]
  - where
    \[ \Sigma_{\mu \nu} = \frac{1}{24} \left( -2 \tau \tau_{\mu \nu} + 6 \tau_\sigma \tau_{\sigma \nu} + g_{\mu \nu} \left( -3 \tau_\rho \tau_{\sigma \rho} + \tau^2 \right) \right), \]
  - and
    \[ E_{\mu \nu} = (5) C_{BCD}^A n_A n_C q_\mu^B q_\nu^D. \]
  - Compare with 5-D Einstein equations:
    \[ (5) G_{AB} = \kappa_5 \delta_A^\mu \delta_B^\nu \tau_{\mu \nu} \delta(z). \]
Matter distribution.

- Here and below we use new radial coordinate $R(\rho, a) \equiv r(\rho, a) \cos \theta(\rho, a)$ on the brane.

- The total mass in 4+1 dimension is given by

$$M = \frac{3}{16\pi\kappa_5} \int_{t=\text{const}} d^4V R_{AB} \xi^A m^B \equiv \int_0^\infty dR f_5(R).$$

- $M$ does not depend on the value of parameter $a$!

- The exact calculations show that the 3-dim Komar integral, $^{(4)}M_{\text{eff}} = 0$.

- compare with $g_{00} - 1 = O(1/R^2)$ but not $O(1/R) \ll$ infinite size of an extra dimension.
Figure 2: The matter-density radial distributions $f_5(R, a)$ on the brane with $M = 1$ are presented by a magenta colored line. The corresponding horizons are indicated by green lines. $a_1 > a_2 > a_3 > a_4 > a_5$. 
Figure 3: The matter-density radial distribution $f_5(R)$ on the brane with $a = 1.1$, $M = 1$ is presented for $\kappa_5 = 1$ by a magenta colored line. The effective matter-density $f_4(R)$ is shown by blue line for the value $\kappa_4 = 50$ to compare with $f_5(R)$. The horizon is indicated by green line.
Conclusions

- **Results.**
  - We have shown that by cut-and-paste method in special Gaussian normal coordinates one can build the exact geometry of multidimensional black star with horizon, generated by a smooth matter distribution in our universe.
  - In our approach, for a given total mass, the profiles of available configurations for matter distribution are governed by the parameter $a$ which is presumably related to the collision kinematics when a black object ("black hole") is created by partons on colliders.

- **Generalizations.**
  - charged and rotated black stars as well as black rings.
  - compact extra dimensions and warped geometries.
Thanks for attention!!!!!!