Quantum field theoretic origin of structures in the Universe

V.A. Rubakov

Institute for Nuclear Research
of the Russian Academy of Sciences, Moscow

With Big Bang nucleosynthesis theory and observations

we are confident of the theory of the early Universe

at temperatures up to $T \simeq 1$ MeV, age $t \simeq 1$ second

With LHC, we hope to be able to go

up to temperatures $T \sim 100$ GeV, age $t \sim 10^{-10}$ second

Are we going to have a handle on even earlier epoch?

Key: cosmological perturbations

Our Universe is not exactly homogeneous.

Inhomogeneities: $\odot$ density perturbations and associated gravitational potentials (3d scalar), observed;

$\odot$ gravitational waves (3d tensor), not observed (yet?).

Today: inhomogeneities strong and non-linear

In the past: amplitudes small,

$$\frac{\delta\rho}{\rho} = 10^{-4} - 10^{-5}$$

Perturbation theory appropriate.

How are they measured?

- Cosmic microwave background: photographic picture of the Universe at age 380 000 yrs, $T = 3000$ K (transition from plasma to neutral gas, mostly hydrogen and helium)
  - Temperature anisotropy
  - Polarization
- Deep surveys of galaxies and quasars, cover good part of entire visible Universe
- Gravitational lensing, etc.

We have already learned a number of fundamental things

Extrapolation back in time with known laws of physics and known elementary particles and fields $\Longrightarrow$ hot Universe, starts from Big Bang singularity (infinite temperature, infinite expansion rate)

We now know that this is not the whole story:

Properties of perturbations in conventional ("hot") Universe.

Friedmann–Lemaître–Robertson–Walker metric:

$$ds^2 = dt^2 - a^2(t) d\vec{x}^{\,2}$$

Expanding Universe:

$a(t) \propto t^{1/2}$ at radiation domination stage (before $T \simeq 1$ eV,
$t \simeq 60$ thousand years)
$a(t) \propto t^{2/3}$ at matter domination stage (until recently).

Cosmological horizon (assuming that nothing preceeded hot epoch): length that light travels from Big Bang moment,

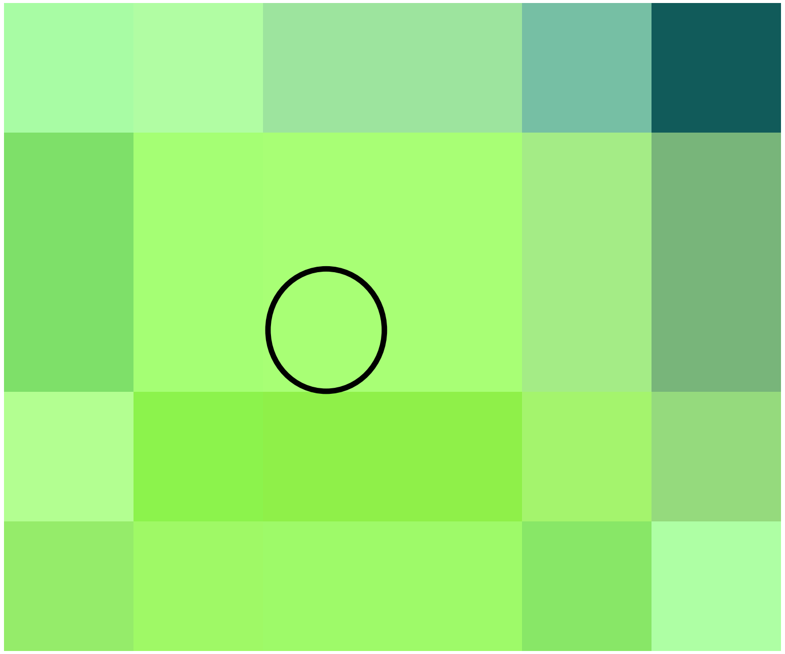
$$l_H(t) = (2-3)t$$

Wavelength of perturbation grows as $a(t)$.
E.g., at radiation domination

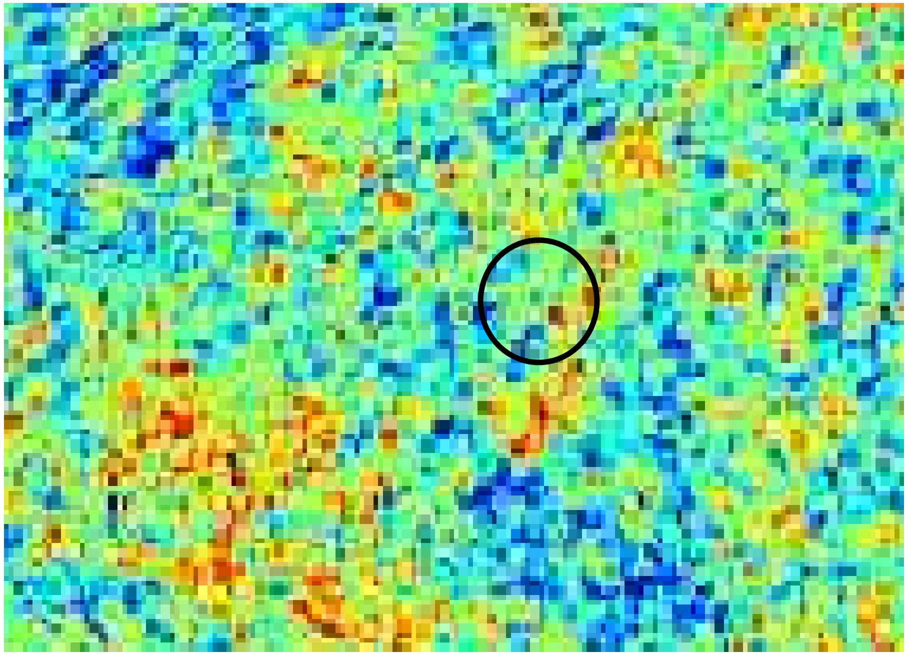
$$\lambda(t) \propto t^{1/2} \quad \text{while} \quad l_H \propto t$$

Today $\lambda < l_H$, subhorizon regime

Early on $\lambda(t) > l_H$, superhorizon regime.



superhorizon mode



subhorizon mode

Major issue: origin of perturbations

Causality $\Longrightarrow$ perturbations can be generated only when they are subhorizon.

Off-hand possibilities:

- Perturbations were never superhorizon, they were generated at the hot cosmological epoch by some causal mechanism.

  E.g., seeded by topological defects (cosmic strings, etc.)

  N. Turok et.al.' 90s

  The only possibility, if expansion started from hot Big Bang.

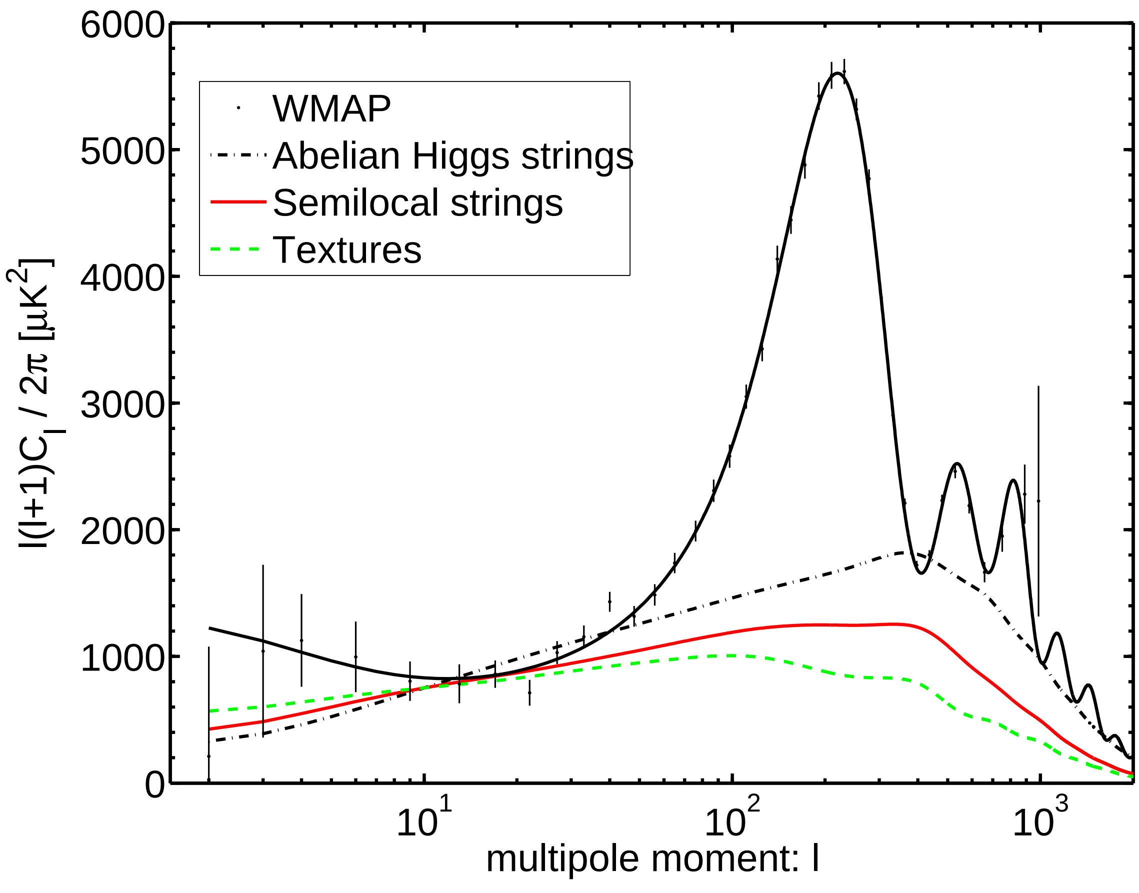
  No longer an option!

  Known properties of density perturbations show that

- Hot epoch was preceeded by some other epoch. Perturbations were generated then.

Only perturbations that were supehorizon at early times can yield oscillatory angular spectrum of CMB.
Furthermore, there are perturbations which were superhorizon at the time of photon last scattering

These properties would not be present if perturbations were generated at hot epoch in causal manner.



Primordial perturbations were generated at some yet unknown epoch before the hot expansion stage.

That epoch must have been long and unusual:
perturbations were subhorizon early at that epoch,
our visible part of the Universe was in a causally connected region.

- Excellent guess: inflation

Starobinsky'79; Guth'81; Linde'82; Albrecht and Steinhardt'82

Exponential expansion with almost constant Hubble rate,

$$a(t) = \mathrm{e}^{\int H dt}\,, \qquad H \approx \text{const}$$

Perturbations subhorizon early at inflation:

$$\lambda(t) \propto a(t) \ll H^{-1} \qquad \text{early times, small } a(t)$$

Alternatives to inflation:

- Contraction — Bounce — Expansion
- Start up from static state

Creminelli et.al.'06; '10

Difficult, but not impossible. Einstein equations (neglecting spatial curvature)

$$H^2 = \frac{8\pi}{3} G\rho$$

$$\frac{dH}{dt} = -4\pi(\rho + p)$$

$\rho =$ energy density, $p =$ pressure, $H = \dot{a}/a$.

Bounce, start up scenarios $\Longrightarrow \frac{dH}{dt} > 0 \Longrightarrow \rho > 0$ and $p < -\rho$

Very exotic matter. Potential problems with instabilities, superluminal propagation/causality. Solvable, if one gives up Lorentz-invariance (or, possibly, General Relativity).

Other suggestive observational facts about density perturbations
(valid within certain error bars!)

- Perturbations in overall density, not in composition (jargon: “adiabatic”)

$$\frac{\text{baryon density}}{\text{entropy density}} = \frac{\text{dark matter density}}{\text{entropy density}} = \text{const in space}$$

Consistent with generation of baryon asymmetry and dark matter at hot stage.

NB: even weak variation of composition over space would mean exotic mechanism of baryon asymmetry and/or dark matter generation $\Longrightarrow$ watch out Planck!

- Primordial perturbations are Gaussian.

  Gaussianity = Wick theorem for correlation functions

  This suggests the origin: enhanced vacuum fluctuations of weakly coupled quatum field(s)

  NB: Linear evolution does not spoil Gaussianity.

- Primordial power spectrum is flat (or almost flat).

  Homogeneity and anisotropy of Gaussian random field:

$$\langle \frac{\delta\rho}{\rho}(\vec{k}) \frac{\delta\rho}{\rho}(\vec{k}') \rangle = \frac{1}{4\pi} \frac{\mathscr{P}(k)}{k^3} \delta(\vec{k}+\vec{k}')$$

  - Free massless quantum field (e.g., scalar) in Minkowski space: equal-time correlation function in momentum representation

$$\langle \varphi(\vec{k}) \varphi(\vec{k}') \rangle = \delta(\vec{k}+\vec{k}') \frac{1}{2(2\pi)^3} \frac{1}{k} \quad \Longrightarrow \quad \mathscr{P}(k) \propto k^2$$

  - Primordial density perturbations just before the hot Big Bang epoch: flat or nearly flat power spectrum,

$$\mathscr{P}(k) \approx \text{ independent of } k$$

$\mathscr{P}(k)$ gives fluctuation in logarithmic interval of momenta,

$$\langle \left(\frac{\delta\rho}{\rho}(\vec{x}) \right)^2 \rangle = \int_0^\infty \frac{dk}{k} \, \mathscr{P}(k)$$

Flat spectrum: all scales give equal contributions

Harrison' 70; Zeldovich' 72

NB: Parametrization

$$\mathscr{P}(k) = A \left(\frac{k}{k_*} \right)^{n_s - 1}$$

$A =$ amplitude, $(n_s - 1) =$ tilt, $k_* =$ fiducial momentum (matter of convention). Flat spectrum $\Longleftrightarrow n_s = 1$.

Observationally $n_s = 0.96 \pm 0.015$.

There must be some symmetry behind flatness of spectrum

- Inflation: symmetry of de Sitter space-time $O(4,1)$

$$ds^2 = dt^2 - \mathrm{e}^{2Ht} d\vec{x}^{\,2}$$

Relevant symmetry: spatial dilatations supplemented by time translations

$$\vec{x} \to \lambda \vec{x}\,, \quad t \to t - \frac{1}{2H} \log \lambda$$

Inflation automatically generates nearly flat spectrum.

- Alternative: conformal symmetry O(4,2)

Conformal group includes dilatations, $x^\mu \to \lambda x^\mu$.
$\Longrightarrow$ No scale, good chance for flatness of spectrum

First mentioned by Antoniadis, Mazur, Mottola' 97
Concrete models: V.R.' 09;
Creminelli, Nicolis, Trincherini' 10.

General setting:

Hinterbichler, Khouri' 11

- Effectively Minkowski space-time
- Conformally invariant theory
- Field ρ of conformal weight $\Delta \neq 0$
- Instability
  of conformally invariant background $\rho = 0$

  Homogeneous classical solution

$$\rho_c(t) = \frac{\text{const}}{(t_* - t)^{\Delta}}$$

by conformal invariance.

NB: Spontaneous breaking of conformal symmetry:
$O(4,2) \to O(4,1)$

- Another scalar field θ of conformal weight 0.
- Kinetic term dictated by conformal invariance (modulo field rescaling)

$$L_\theta = \rho^{2/\Delta}(\partial_\mu\theta)^2$$

- Assume potential terms negligible $\Longrightarrow$ Lagrangian in rolling background

$$L_\theta = \frac{1}{(t_* - t)^2} \cdot (\partial_\mu\theta)^2$$

Exactly like scalar field minimally coupled to gravity in de Sitter space, with $t =$ conformal time, $a(t) = \text{const}/(t_* - t)$.

θ develops perturbations with flat power spectrum.

There are various ways to reprocess perturbations of field θ into density perturbations, e.g., at hot epoch. Density perturbations inherit shape of power spectrum and correlation properties from $\delta\theta$, plus possible additional non-Gaussianity.

Non-linear level

Let the field ρ have conformal weight 1 for definiteness.
Under dilatations $\rho \to \lambda \rho(\lambda x)$.

Consider perturbations $\delta\rho$ about background

$$\rho_c = \frac{1}{t_* - t}$$

Action for quadratic perturbations

$$S_{\delta\rho} = \int d^4x \; \delta\rho \mathscr{L} \delta\rho$$

where $\mathscr{L}$ is second order differential operator.
Invariance under spatial translations and dilatations $\Longrightarrow$

$$\mathscr{L} = -\partial_t^2 + v_s^2 \Delta + \frac{c}{(t_* - t)^2}$$

with time-independent v_s and c.

Time-shift of background is again a solution to full eqn. $\Longrightarrow$
$\delta\rho = t^{-2}$ must be a solution to eqn. for small perturbations $\Longrightarrow$
$c = 6$.

Power spectrum of $\delta\rho$ unique, up to overall normalization,
$\mathscr{P}_{\delta\rho} \propto k^{-2}$.

Libanov, V.R.' 10;

Libanov, S. Mironov, V.R.' 11

Same for arbitrary conformal weight of ρ, $\Delta \neq 0$

Hinterbichler, Khoury' 11

NB: Invariance under special conformal transformations $\Longrightarrow$ $v_s = 1$.
Irrelevant for spectral index

Lagrangian of field θ of conformal weight 0:

$$L_\theta = (\rho + \delta\rho)^2 (\partial_\mu \theta)^2 \, .$$

Leading order non-linear effects: interaction between θ and $\delta\rho$.

- Statistical anisotropy

  Perturbations $\delta\rho$ make the Universe slightly anisotropic at the time when perturbations $\delta\theta$ are generated $\Longrightarrow$ statistical anisotropy in resulting power spectrum

$$\mathscr{P}(\mathbf{k}) = \mathscr{P}_0(k)\left(1 + w_{ij}\frac{k_i k_j}{k^2} + \dots\right)$$

Libanov, V.R.' 10;

Libanov, Ramazanov, V.R.' 11

- Non-Gaussianity
  Invariance $\theta \to -\theta \Longrightarrow$ 3-point function vanishes.

  4-point function fully calculated. Most striking property:
  singularity in "folded" limit:

$$
\langle \frac{\delta\rho}{\rho}(\vec{k}_1) \dots \frac{\delta\rho}{\rho}(\vec{k}_4) \rangle
$$

$$
= c \cdot \delta\left(\sum_{i=1}^{n} \vec{k}_i\right) \cdot \frac{1}{k_{12} k_1^4 k_3^4} \left[1 - 3\left(\frac{\vec{k}_{12} \cdot \vec{k}_1}{k_{12} k_1}\right)^2\right] \left[1 - 3\left(\frac{\vec{k}_{12} \cdot \vec{k}_3}{k_{12} k_3}\right)^2\right]
$$

$$
\vec{k}_{12} = \vec{k}_1 + \vec{k}_2 \to 0
$$

Libanov, V.R.' 10;
Libanov, S. Mironov, V.R.' 11

This is in sharp contrast to single field inflationary models.

Origin: infrared enhancement of radial perturbations $\delta\chi_1$

Can one tell?

Not yet.

But there are promising signatures.

- Primordial gravitational waves

Sizeable amplitude, (almost) flat power spectrum predicted by simplest (and hence most plausible) inflationary models

Starobinsky' 79

but not alternatives to inflation

May make detectable imprint on CMB temperature anisotropy

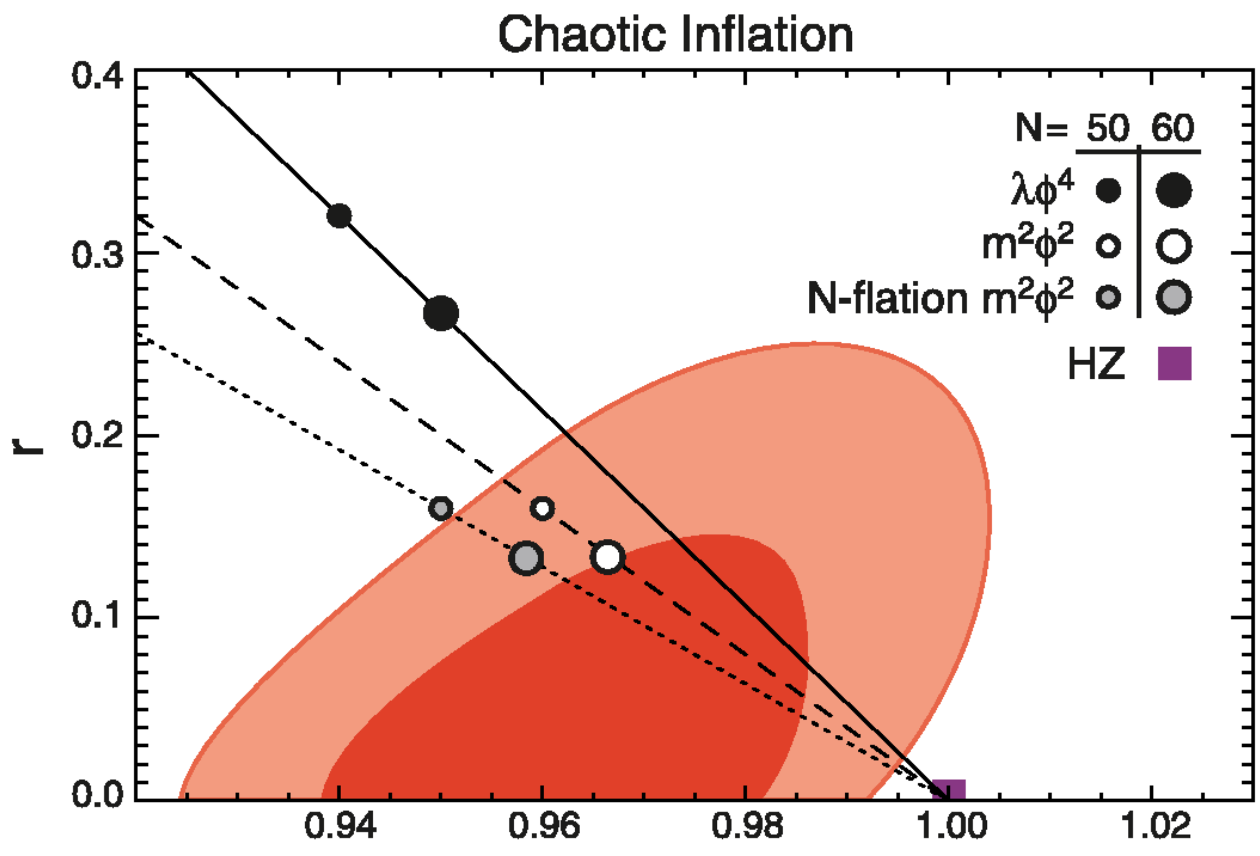
V.R., Sazhin, Veryaskin' 82;
Fabbri, Pollock' 83; ...

and especially on CMB polarization

Kamionkowski, Kosowsky, Stebbins' 96;
Seljak, Zaldarriaga' 96; ...

Smoking gun for inflation

Scalar tilt vs tensor power



NB:

$$r = \left(\frac{\text{amplitude of gravity waves}}{\text{amplitude of density perturbations}} \right)^2$$

- Non-Gaussianity
  - Very small in the simplest inflationary theories

    Maldacena' 03

  - Sizeable in more contrived inflationary models and in alternatives to inflation. Often begins with 3-point function

$$\langle \frac{\delta\rho}{\rho}(\mathbf{k_1})\frac{\delta\rho}{\rho}(\mathbf{k_2})\frac{\delta\rho}{\rho}(\mathbf{k_3})\rangle = \delta(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})G(k_i^2,\mathbf{k_1}\mathbf{k_2},\mathbf{k_1}\mathbf{k_3})$$

    Shape of $G(k_i^2,\mathbf{k_1}\mathbf{k_2},\mathbf{k_1}\mathbf{k_3})$ different in different models $\Longrightarrow$ potential discriminator.

  - Very special shapes of 4-point function in conformal models.

- Statistical anisotropy

$$\mathscr{P}(\mathbf{k}) = \mathscr{P}_0(k)\left(1 + w_{ij}(k)\frac{k_i k_j}{k^2} + \dots\right)$$

  - Anisotropy of the Universe at pre-hot stage
  - Possible in inflation with strong vector fields (rather contrived). Quadrupole only.

    Ackerman, Carroll, Wise' 07; Pullen, Kamionkowski' 07;
    Watanabe, Kanno, Soda' 09
  - Natural in conformal models. May contain all multipoles.
  - Would show up in correlators of CMB anisotropy

    Controversy at the moment

To summarize:

- Available data on cosmological perturbations (notably, CMB anisotropies) give confidence that the hot stage of the cosmological evolution was preceeded by some other epoch, at which these perturbations were generated.

- Inflation is consistent with all data. But there are competitors: the data may rather be viewed as pointing towards early conformal epoch of the cosmological evolution.

More options:

Matter bounce, Finelli, Brandenberger' 01.

Negative exponential potential, Lehners et. al.' 07;
Buchbinder, Khouri, Ovrut' 07; Creminelli, Senatore' 07.

Lifshitz scalar, Mukohyama' 09

- Only very basic things are known for the time being.

At the eve of new physics

LHC $\Longleftrightarrow$ Planck,
dedicated CMB polarization experiments,
data and theoretical understanding
of structure formation ...

Good chance to learn
what preceeded the hot Big Bang epoch

Barring the possibility that Nature is dull

Backup slides

Physical wavenumber (momentum) gets redshifted,

$$q(t) = \frac{2\pi}{\lambda(t)} = \frac{k}{a(t)} \,, \qquad k = \text{const} = \text{coordinate momentum}$$

Today

$$q > H \equiv \frac{\dot{a}}{a}$$

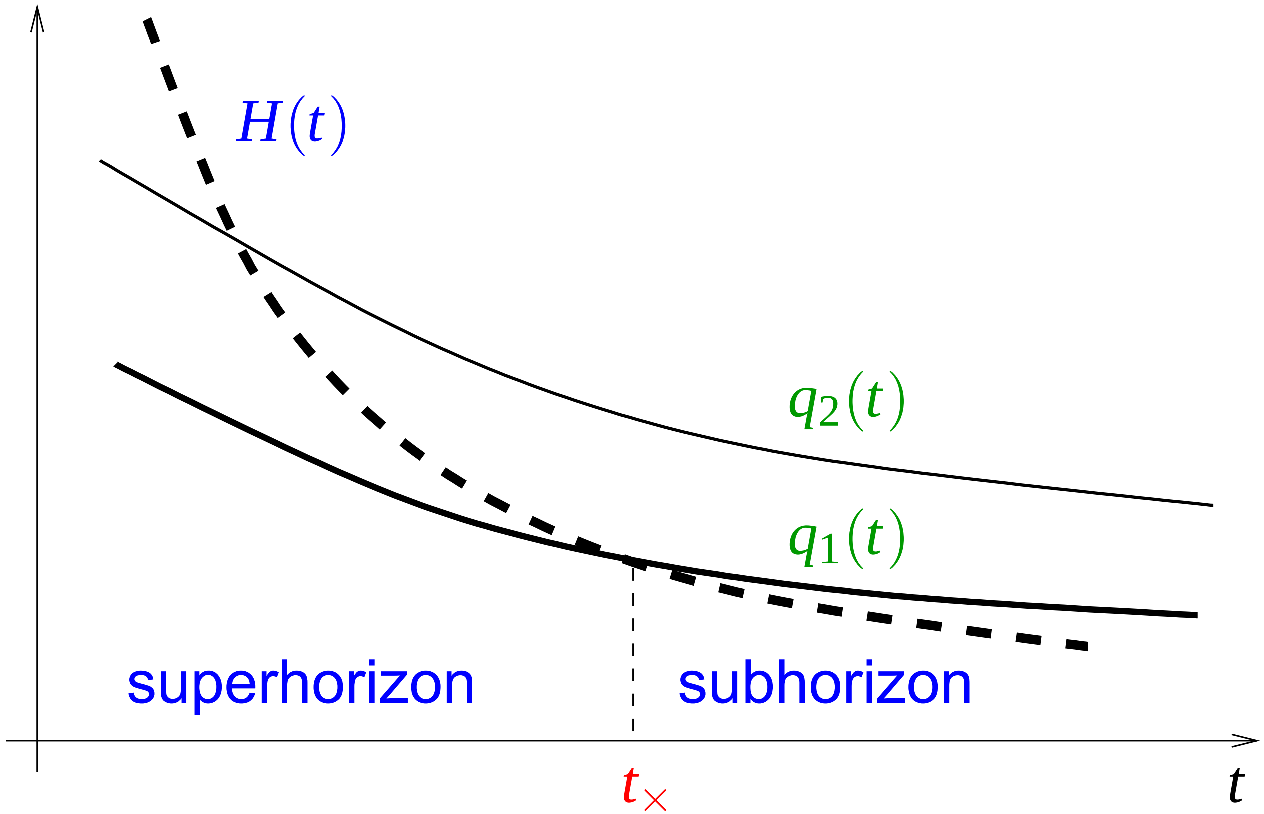
Early on

$$q(t) < H(t)$$

Very different regimes of evolution.

NB: Horizon entry occured after Big Bang Nucleosynthesis epoch for modes of all relevant wavelengths $\Longleftrightarrow$ no guesswork at this point.

Regimes at radiation (and matter) domination



$q_2 > q_1$

Perturbations in baryon-photon plasma = sound waves.

If they were superhorizon, they started off with one and the same phase.

Reason: solutions to wave equation in superhorizon regime in expanding Universe

$$\frac{\delta\rho}{\rho} = \text{const} \quad \text{and} \quad \frac{\delta\rho}{\rho} = \frac{\text{const}}{t^{3/2}}$$

Assume that modes were superhorizon. If the Universe was not very inhomogeneous at early times, the initial condition is unique (up to amplitude),

$$\frac{\delta\rho}{\rho} = \text{const} \implies \frac{d}{dt}\frac{\delta\rho}{\rho} = 0$$

Acoustic oscillations start after entering the horizon at zero velocity of medium $\implies$ phase of oscillations uniquely defined.

Perturbations develop different phases by the time of photon last scattering (= recombination), depending on wave vector:

$$\frac{\delta\rho}{\rho}(t_r) \propto \cos\left(\int_0^{t_r} dt\; v_s\; q(t)\right)$$

(v_s = sound speed in baryon-photon plasma) $\Longrightarrow$

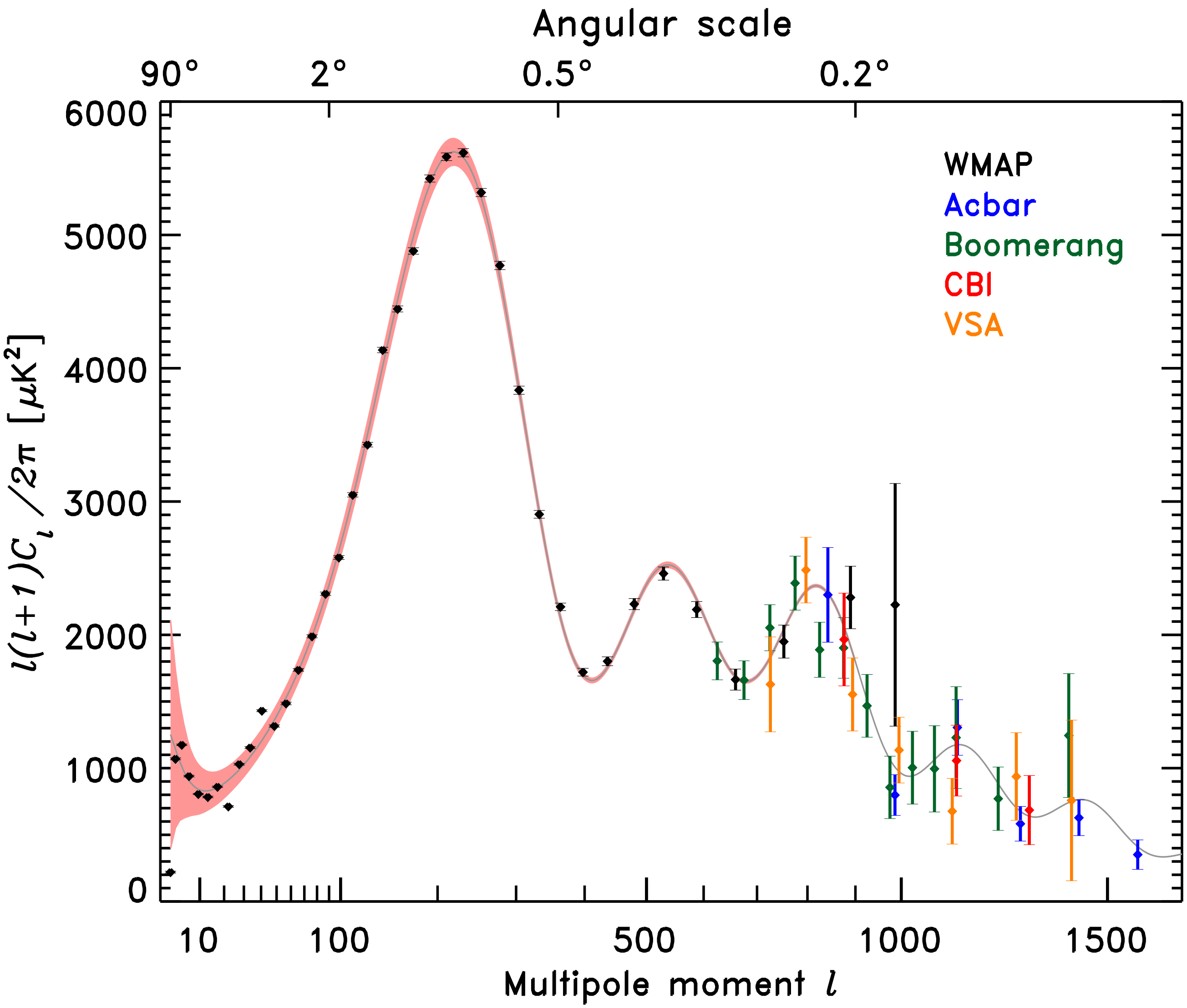
Oscillations in CMB temperature angular spectrum

Fourier decomposition of temperatue fluctuations:

$$\delta T(\theta,\varphi) = \sum_{l,m} a_{lm} Y_{lm}(\theta,\varphi)$$

$\langle a_{lm}^* a_{lm}\rangle = C_l$, temperature angular spectrum;

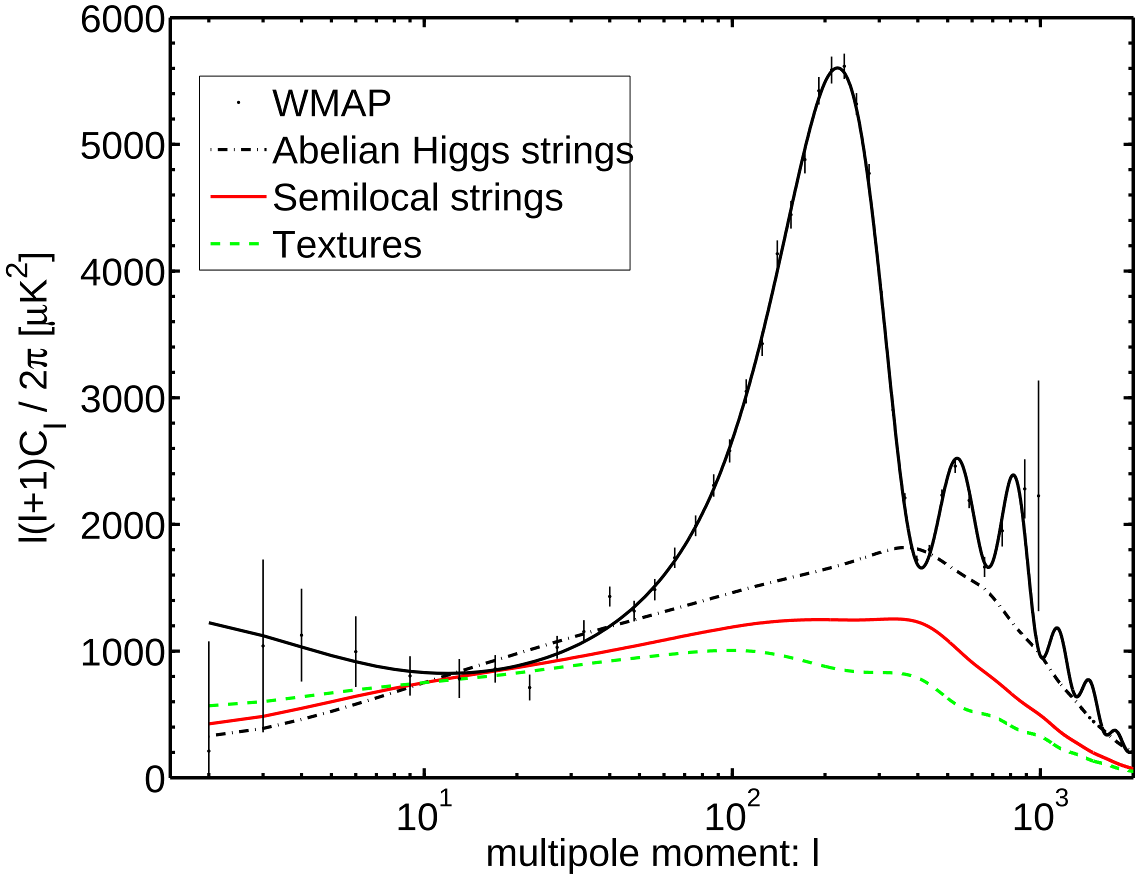
larger $l \Longleftrightarrow$ smaller angular scales, shorter wavelengths



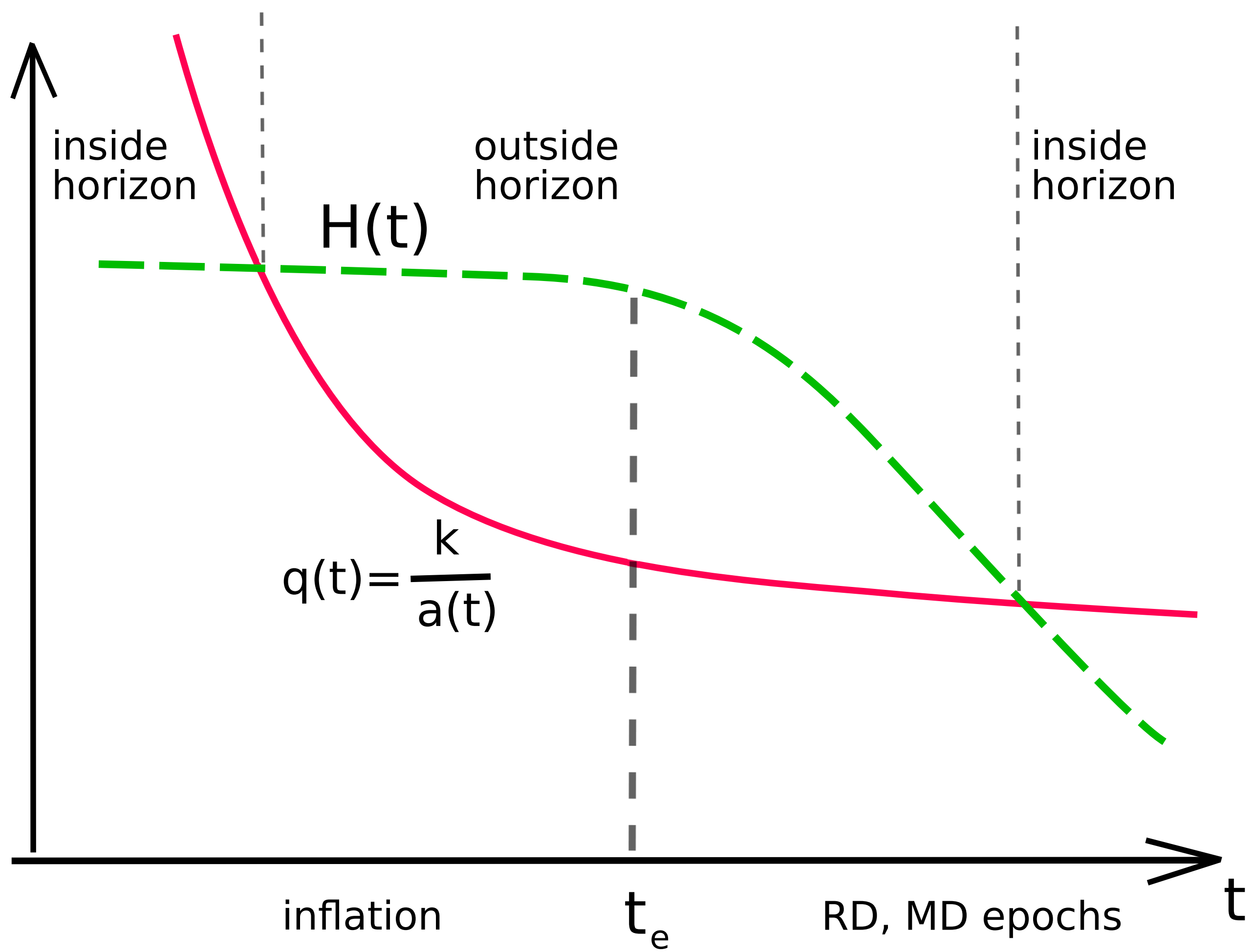
Angular scale
90°
2°
0.5°
0.2°
6000
5000
4000
3000
2000
1000
0
$l(l+1)C_l/2\pi$ [μK^2]
WMAP
Acbar
Boomerang
CBI
VSA
10
100
500
1000
1500
Multipole moment l

Furthermore, there are perturbations which were superhorizon at the time of photon last scattering

These properties would not be present if perturbations were generated at hot epoch in causal manner.



Physical wave number and Hubble parameter at inflation and later:



Alternatives to inflation:

- Contraction — Bounce — Expansion
- Start up from static state

Creminelli et.al.'06; '10

Difficult, but not impossible. Einstein equations (neglecting spatial curvature)

$$H^2 = \frac{8\pi}{3} G\rho$$

$$\frac{dH}{dt} = -4\pi(\rho + p)$$

$\rho =$ energy density, $p =$ pressure, $H = \dot{a}/a$.

Bounce, start up scenarios $\Longrightarrow \frac{dH}{dt} > 0 \Longrightarrow \rho > 0$ and $p < -\rho$

Very exotic matter. Potential problems with instabilities, superluminal propagation/causality. Solvable, if one gives up Lorentz-invariance (or, possibly, General Relativity).

Other suggestive observational facts about density perturbations

(valid within certain error bars!)

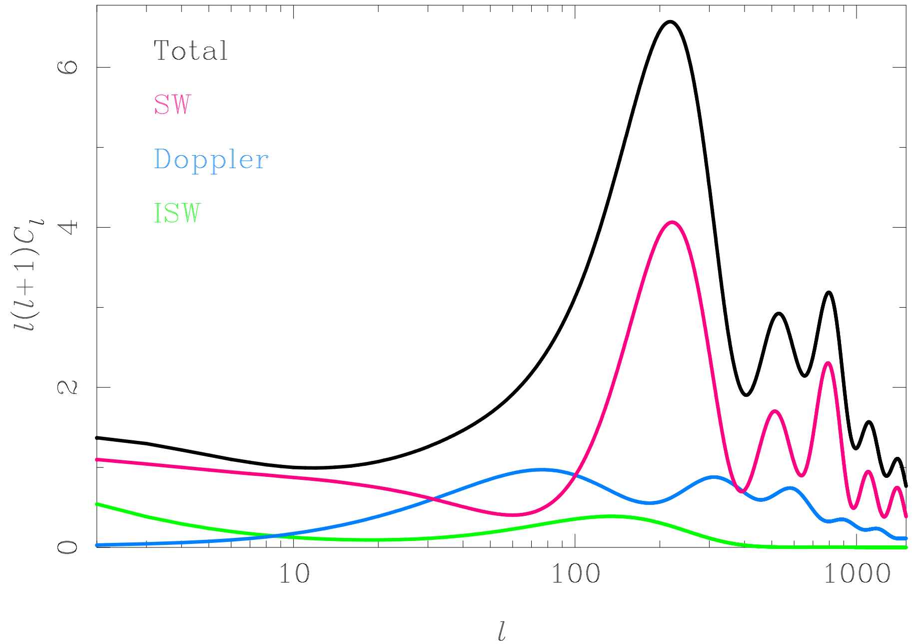
- Perturbations in overall density, not in composition (jargon: “adiabatic”)

$$\frac{\text{baryon density}}{\text{entropy density}} = \frac{\text{dark matter density}}{\text{entropy density}} = \text{const in space}$$

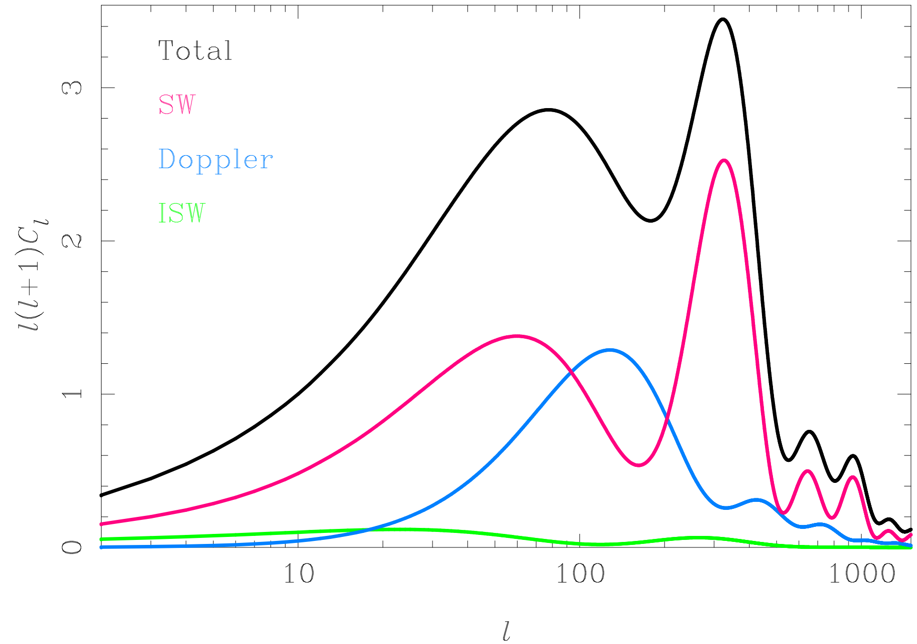
Consistent with generation of baryon asymmetry and dark matter at hot stage.

Perturbation in chemical composition (jargon: “isocurvature” or “entropy”) $\Longrightarrow$ wrong initial condition for acoustic oscillations $\Longrightarrow$ wrong prediction for CMB angular spectrum.

CMB angular spectra



homogeneous chemical composition



inhomogeneous chemical composition

NB: even weak variation of composition over space would mean exotic mechanism of baryon asymmetry and/or dark matter generation $\Longrightarrow$ watch out Planck!

- Primordial perturbations are Gaussian.

  Gaussianity = Wick theorem for correlation functions

  This suggests the origin: enhanced vacuum fluctuations of weakly coupled quatum field(s)

  NB: Linear evolution does not spoil Gaussianity.

  - Inflation does the job very well: fluctuations of all light fields get enhanced greatly due to fast expansion of the Universe.

    Including the field that dominates energy density (inflaton) $\Longrightarrow$ perturbations in energy density.

    Mukhanov, Chibisov'81; Hawking'82; Starobinsky'82; Guth, Pi'82; Bardeen et.al.'83

  - Enhancement of vacuum fluctuations is less automatic in alternative scenarios

NB: Conformal symmetry has long been discussed in the context of Quantum Field Theory and particle physics.

Particularly important in the context of supersymmetry: many interesting superconformal theories.

Large and powerful symmetry behind, e.g., adS/CFT correspondence and a number of other QFT phenomena

Maldacena' 97

It may well be that ultimate theory of Nature is (super)conformal

What if our Universe started off from a conformal state
and then evolved to much less symmetric state we see today?

Exploratory stage: toy models so far.

A toy model:

V.R.' 09;

Libanov, V.R.' 10

Conformal complex scalar field ϕ with negative quartic potential (to mimic instability of conformally invariant state)

$$S = \int \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi^* \partial_\nu \phi + \frac{R}{6} |\phi|^2 - (-h^2 |\phi|^4) \right]$$

Conformal symmetry in 4 dimensions. Global symmetry $U(1)$ (to mimic other symmetries of conformally invariant theory).

Homogeneous and isotropic cosmological background

$$ds^2 = a^2(\eta)[d\eta^2 - d\vec{x}^2]$$

Evolution of the scalar field is basically independent of $a(\eta)$, because of conformal symmetry.
NB: behaviour of scale factor may be arbitrary.
E.g., contraction or start-up.

Homogeneous isotropic evolution:

$$\phi_c(\eta) = \frac{1}{ha(\eta)(\eta_* - \eta)}$$

(in conformal time). Dictated by conformal invariance.
$\eta_* =$ integration constant, end of roll time.

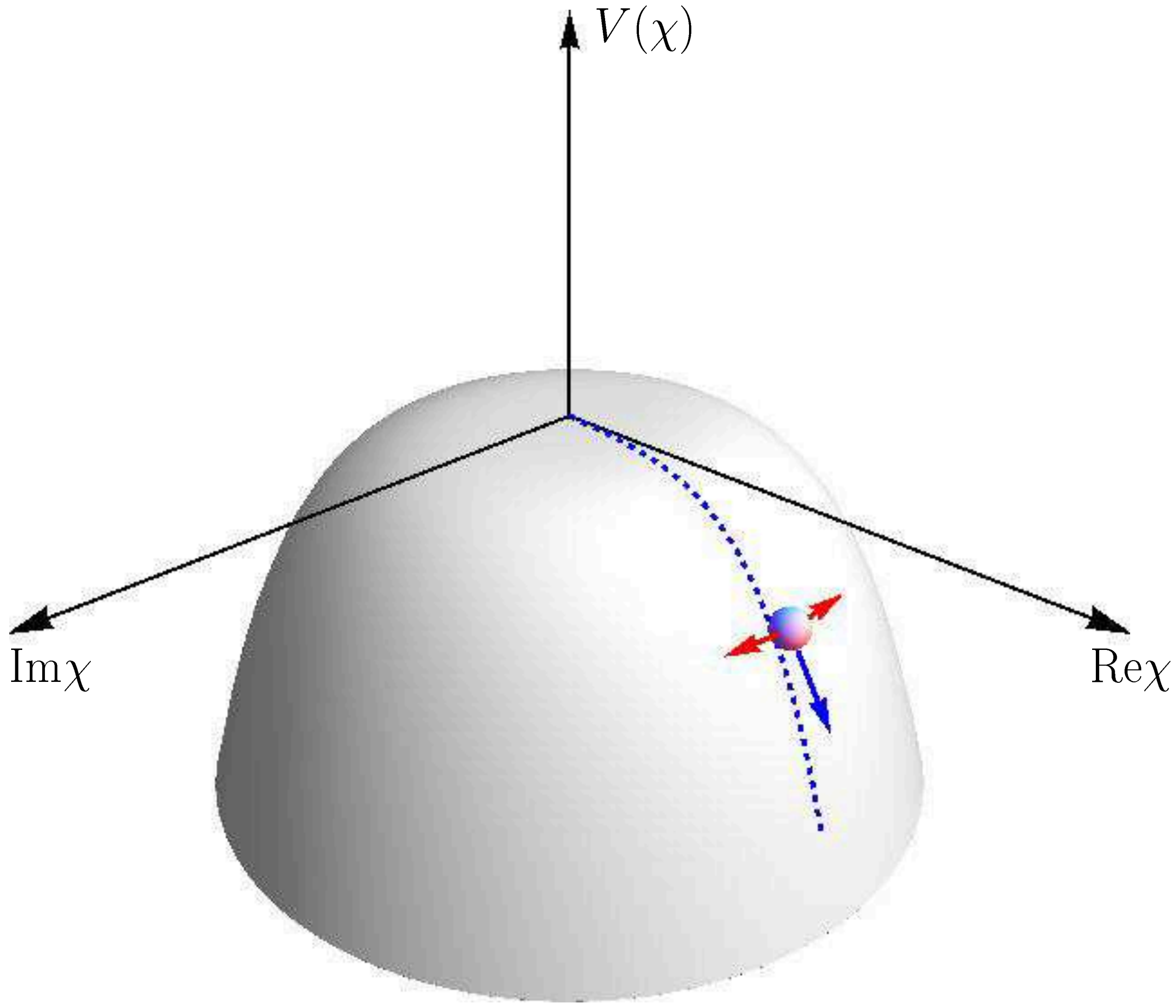
Vacuum fluctuations of the phase Arg ϕ get enhanced, and freeze out at late times.

They become Gaussian random field with flat spectrum,

$$\langle \delta\theta^2 \rangle = \frac{h^2}{2(2\pi)^3} \int \frac{d^3k}{k^3}$$

This is automatic consequence of global $U(1)$
and conformal symmetry

Conformal evolution

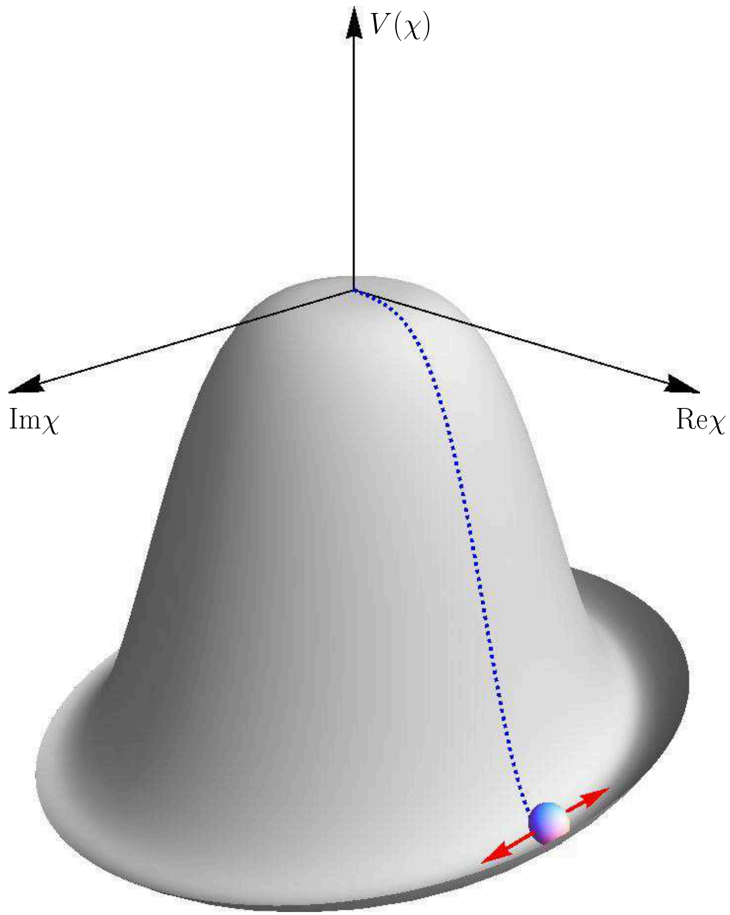


Later on, conformal invariance is broken, and perturbations of the phase get reprocessed into density perturbations.

This can happen in a number of ways

Reprocessing in inflationary context: Linde, Mukhanov' 97;
Enqvist, Sloth' 01; Moroi, Takahasi' 01; Lyth, Wands' 01;
Dvali, Gruzinov, Zaldarriaga' 03; Kofman' 03

One way: θ = pseudo-Nambu-Goldstone field. Generically, it ends up at a slope of its potential



Anisotropy in conformal model

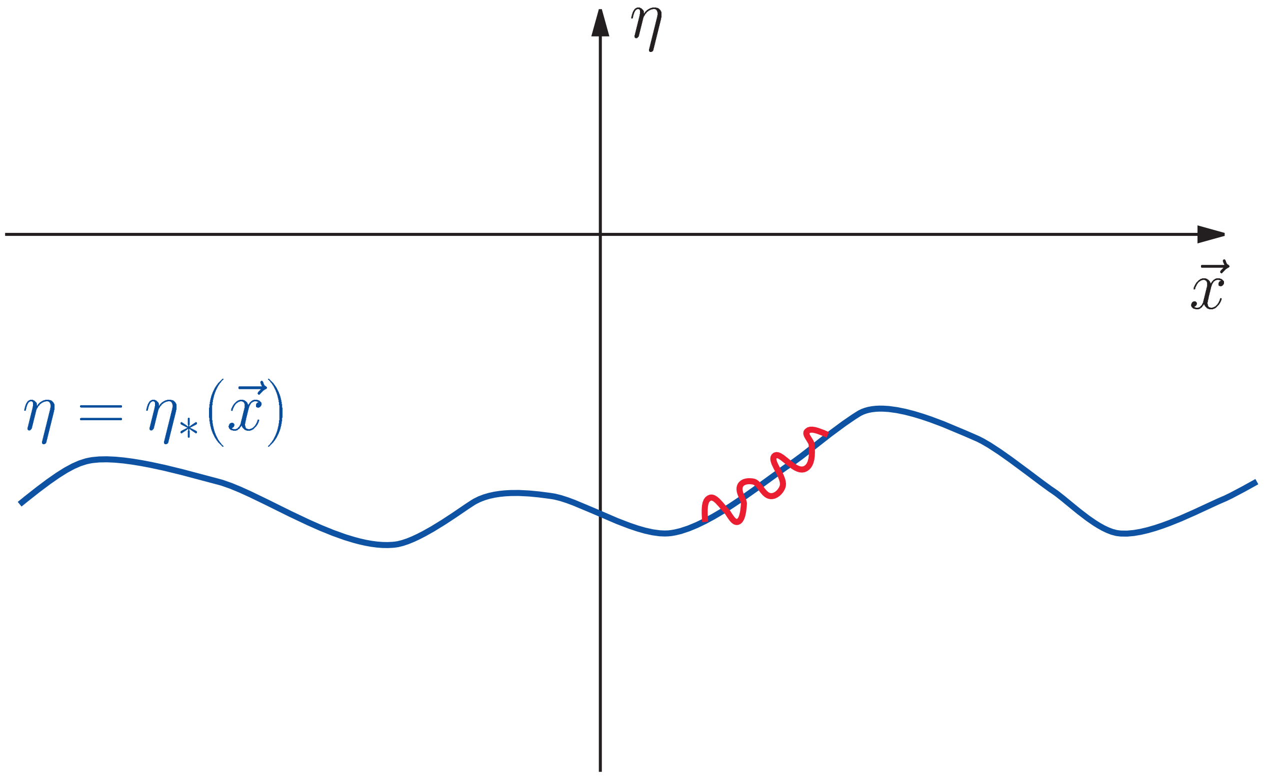
Perturbations of radial field $|\phi| \Longrightarrow$ inhomogeneous background ϕ_c $\Longrightarrow$ inhomogeneous end-of-roll time:

$$\phi_c(\eta, \mathbf{x}) = \frac{1}{ha(\eta)(\eta_*(\mathbf{x}) - \eta)}$$

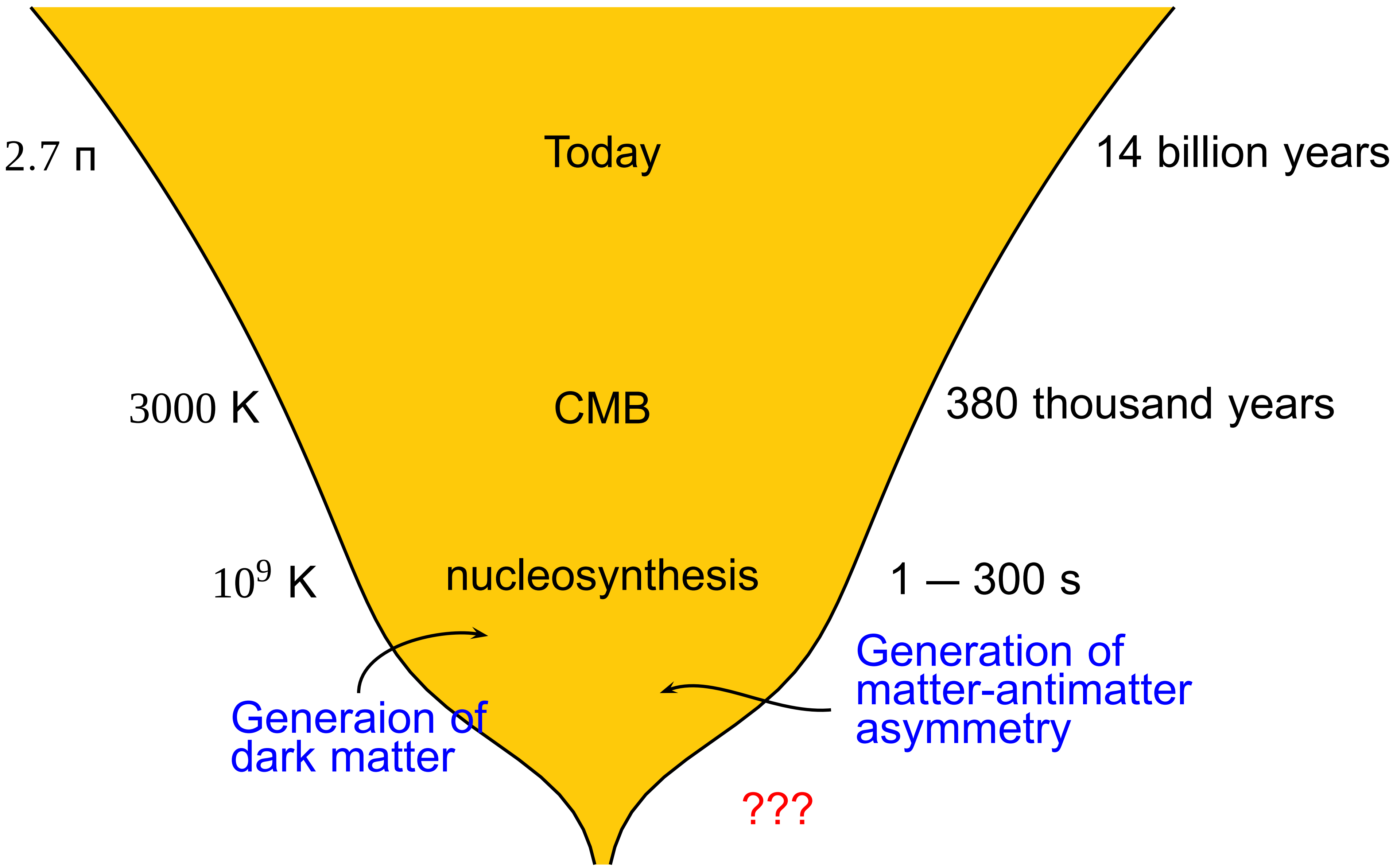
Large wavelength perturbations of $\eta_*(\mathbf{x}) \Longrightarrow$ keep gradient only,

$$\eta_*(\mathbf{x}) = \text{const} + \mathbf{vx}$$

$\Longrightarrow$ frame of homogeneous $\phi_c \neq$ cosmic frame $\Longrightarrow$ anisotropy.

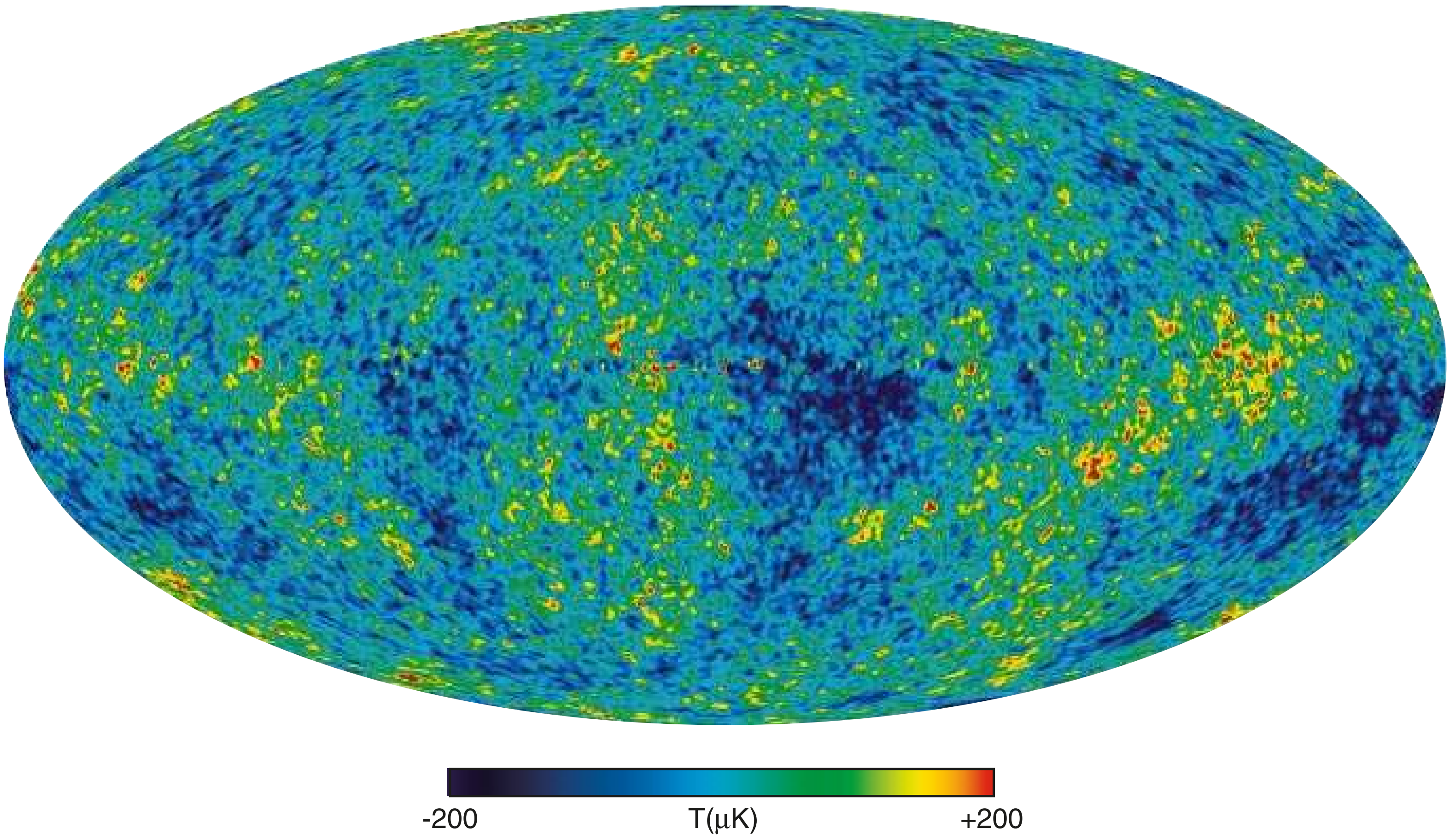


Reference frame of conformal rolling is boosted with respect to cosmic frame $\Longrightarrow$ anisotropy due to relative velocity of the two frames


2.7 п
Today
14 billion years
3000 K
CMB
380 thousand years
10^9 K
nucleosynthesis
1 — 300 s
Generation of
matter-antimatter
asymmetry
Generaion of
dark matter
???

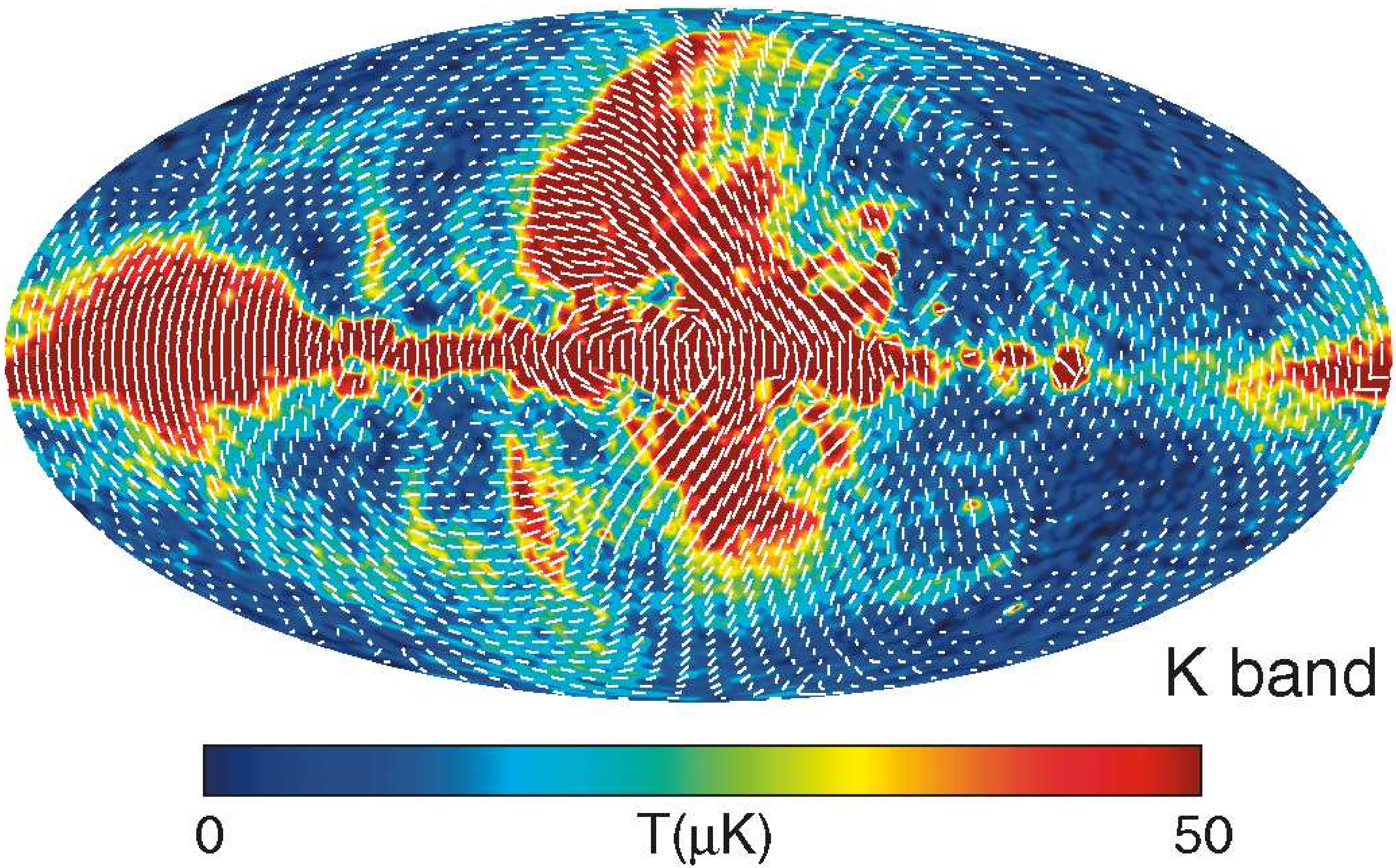
CMB temperature anisotropy

$$T = 2.725^\circ K, \quad \frac{\delta T}{T} \sim 10^{-4} - 10^{-5}$$

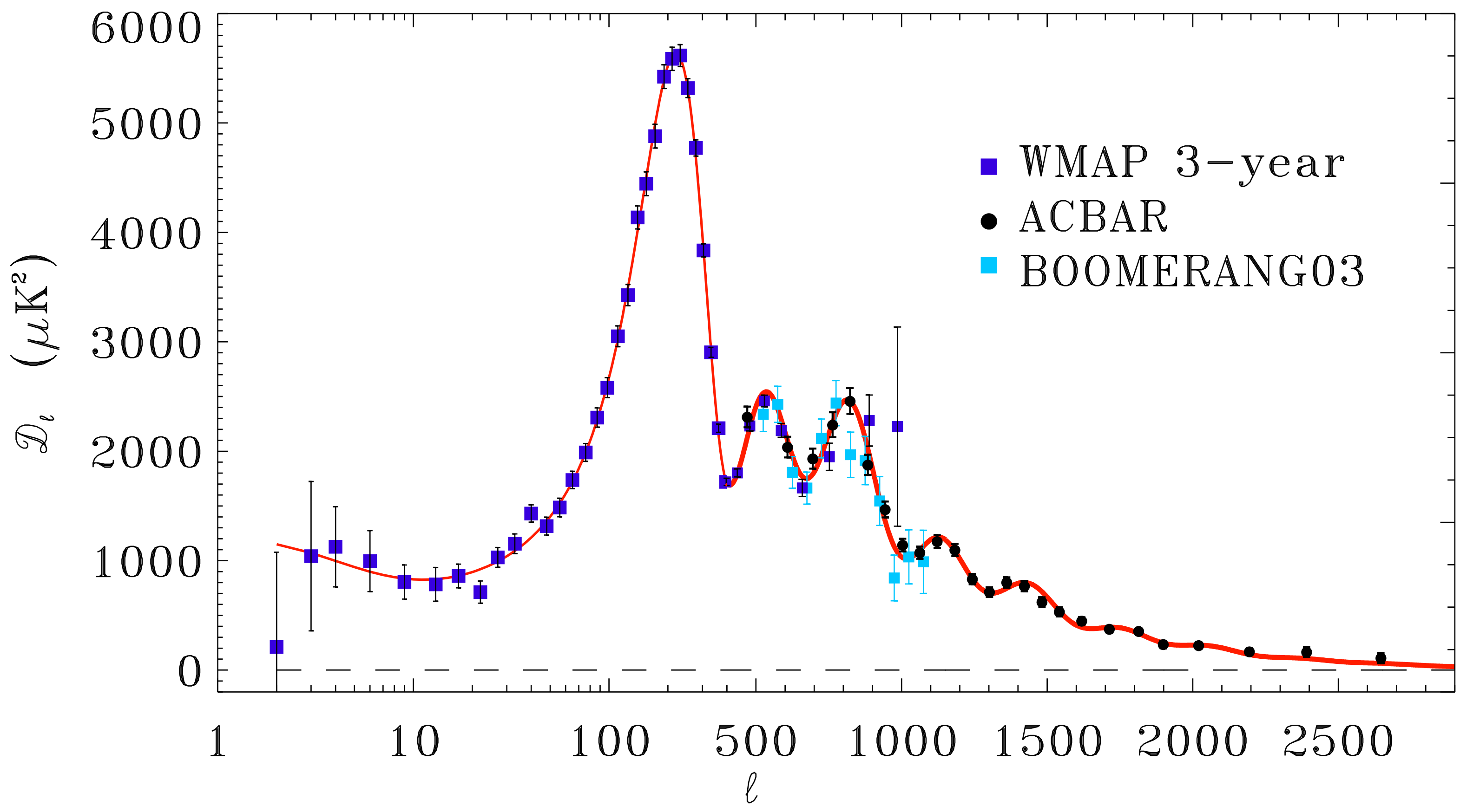


WMAP

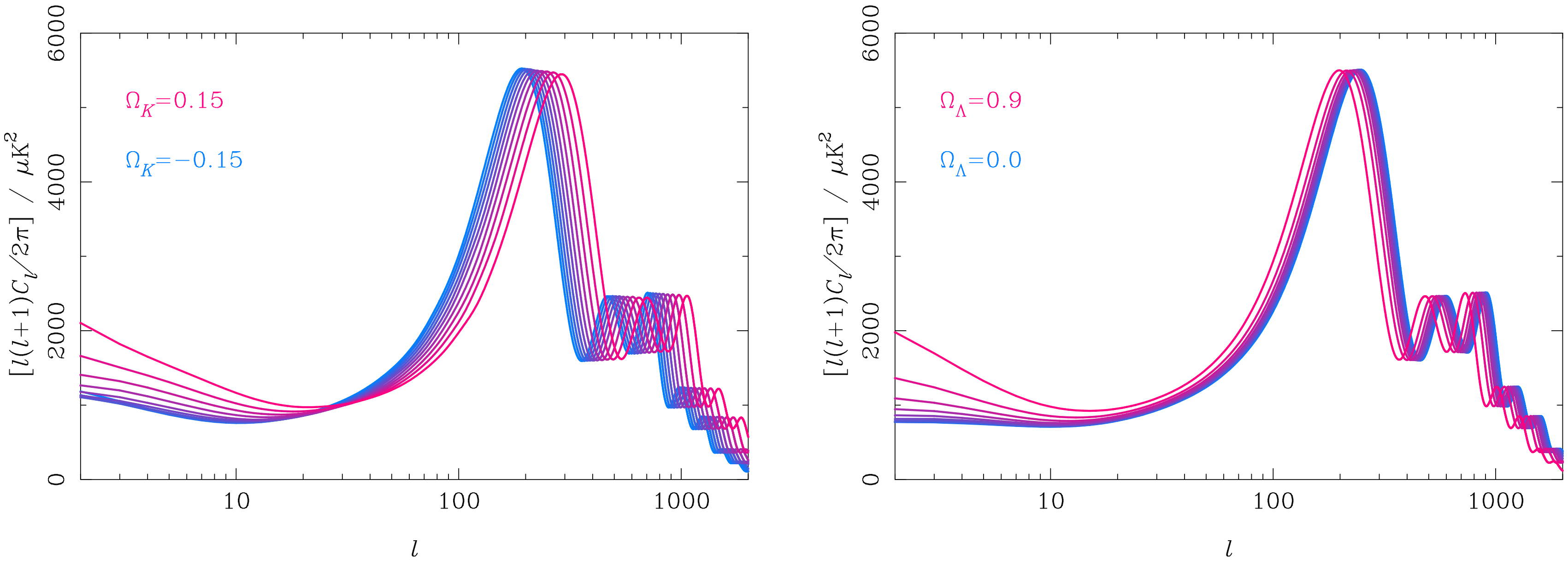
CMB polarization map



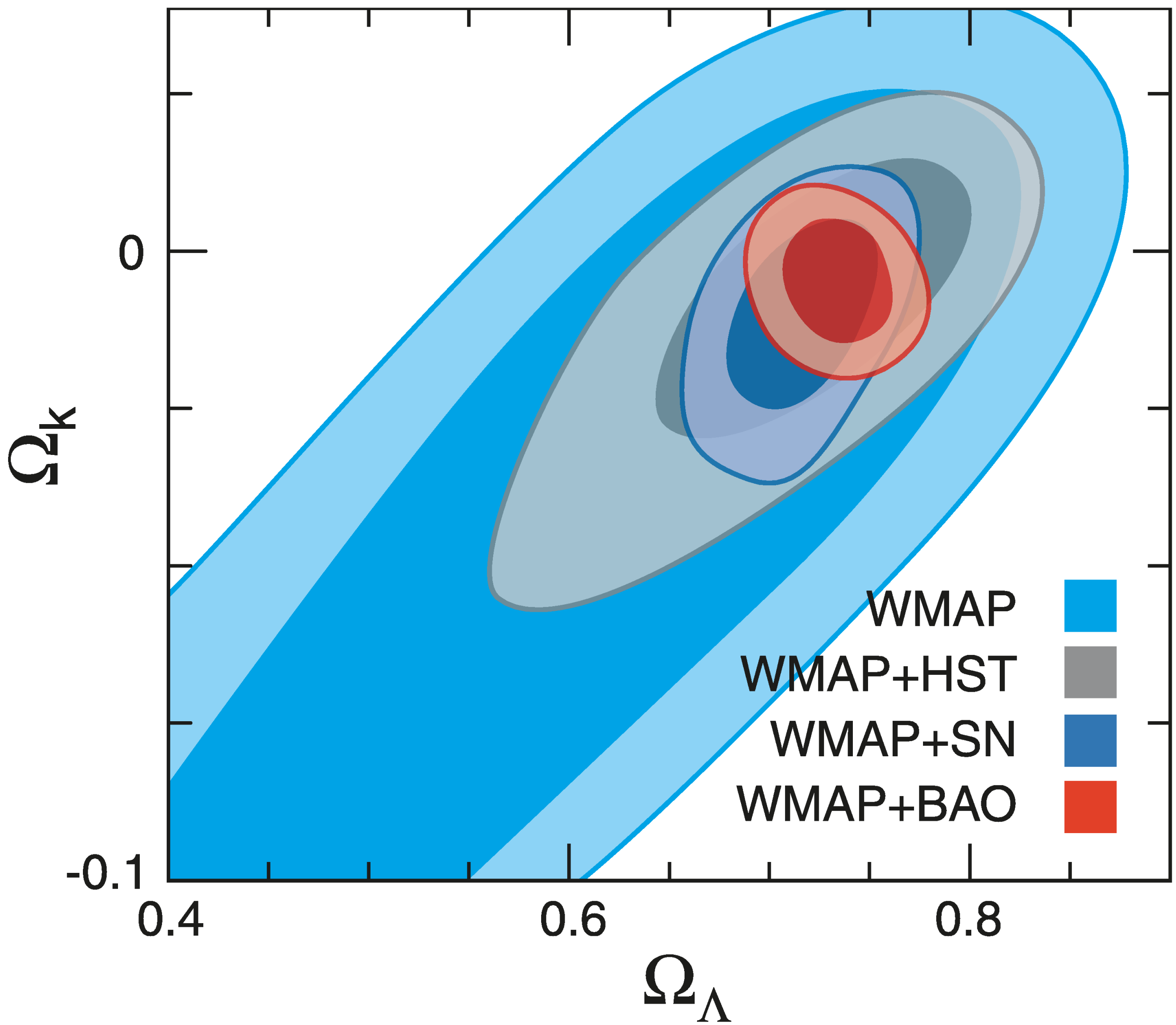
CMB anisotropy spectrum



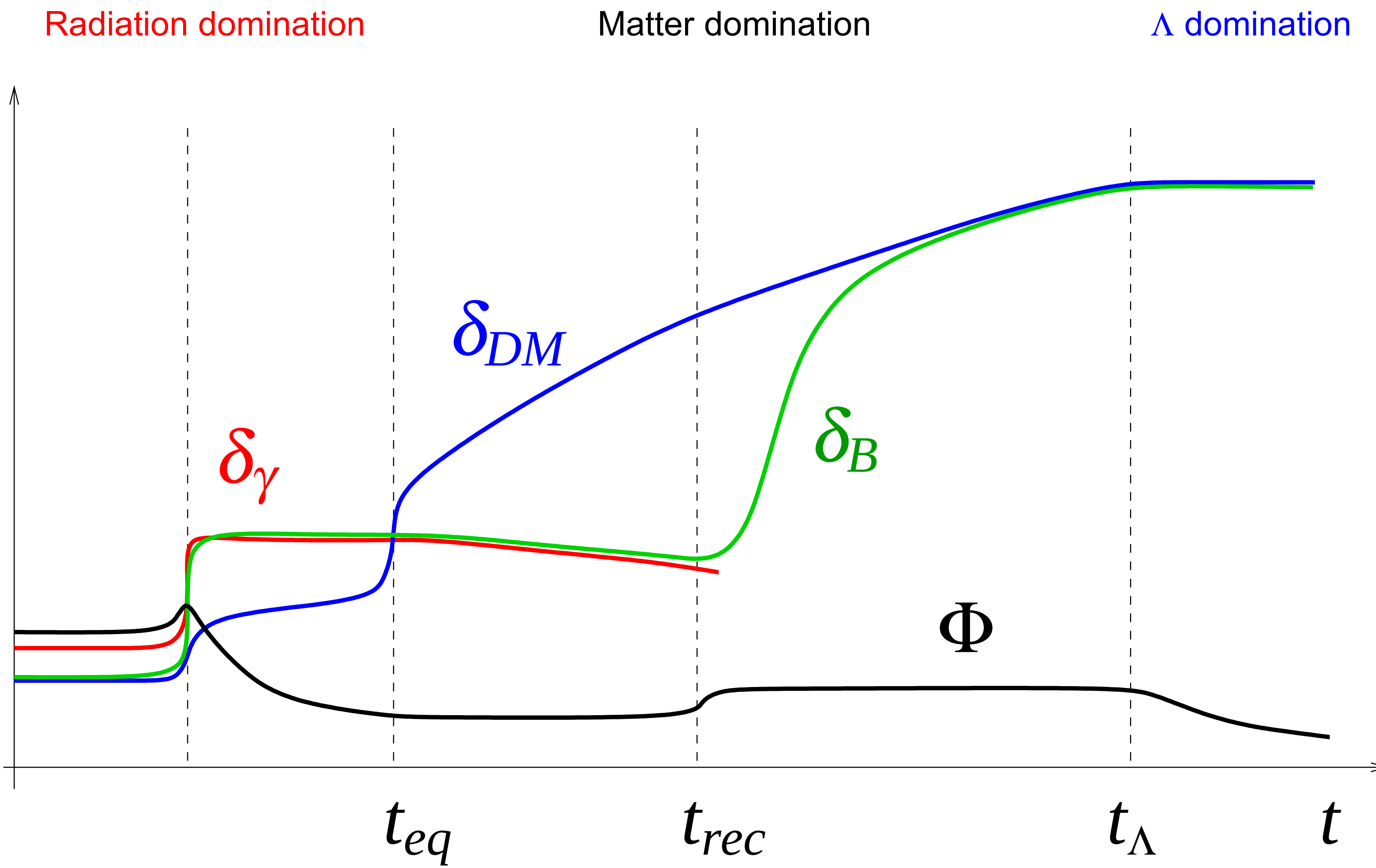
Effect of curvature (left) and Λ



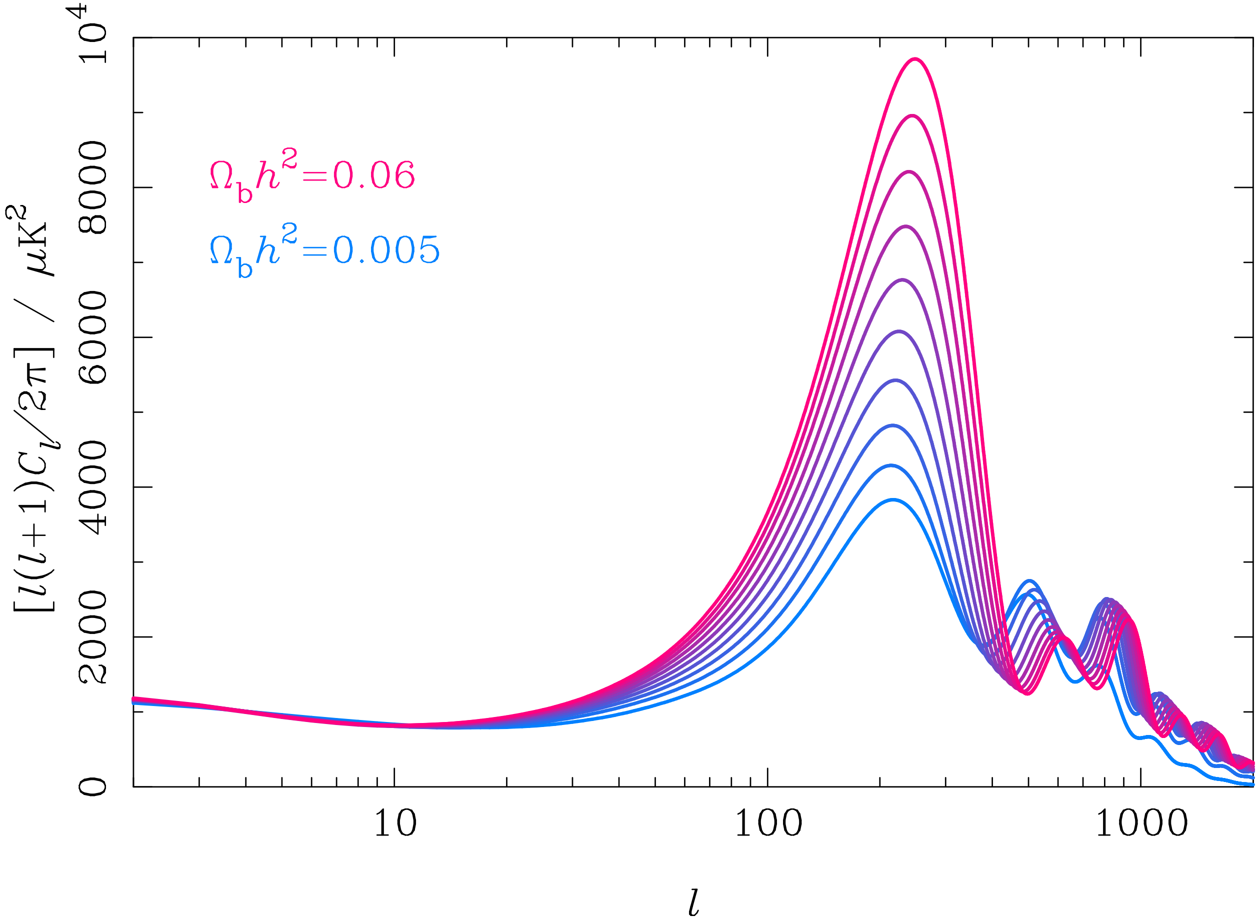
Allowed curvature and Λ



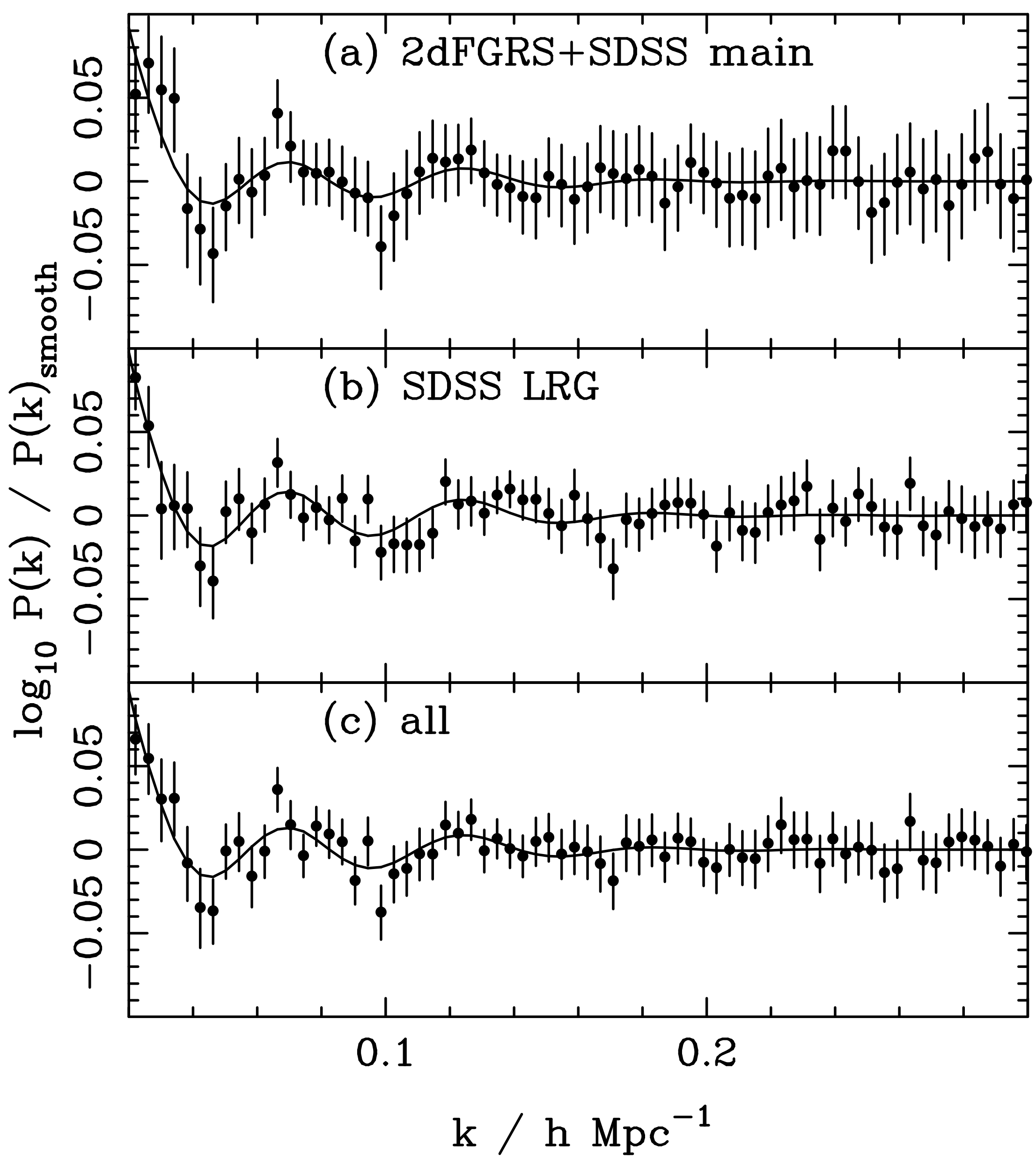
Growth of perturbations (linear regime)



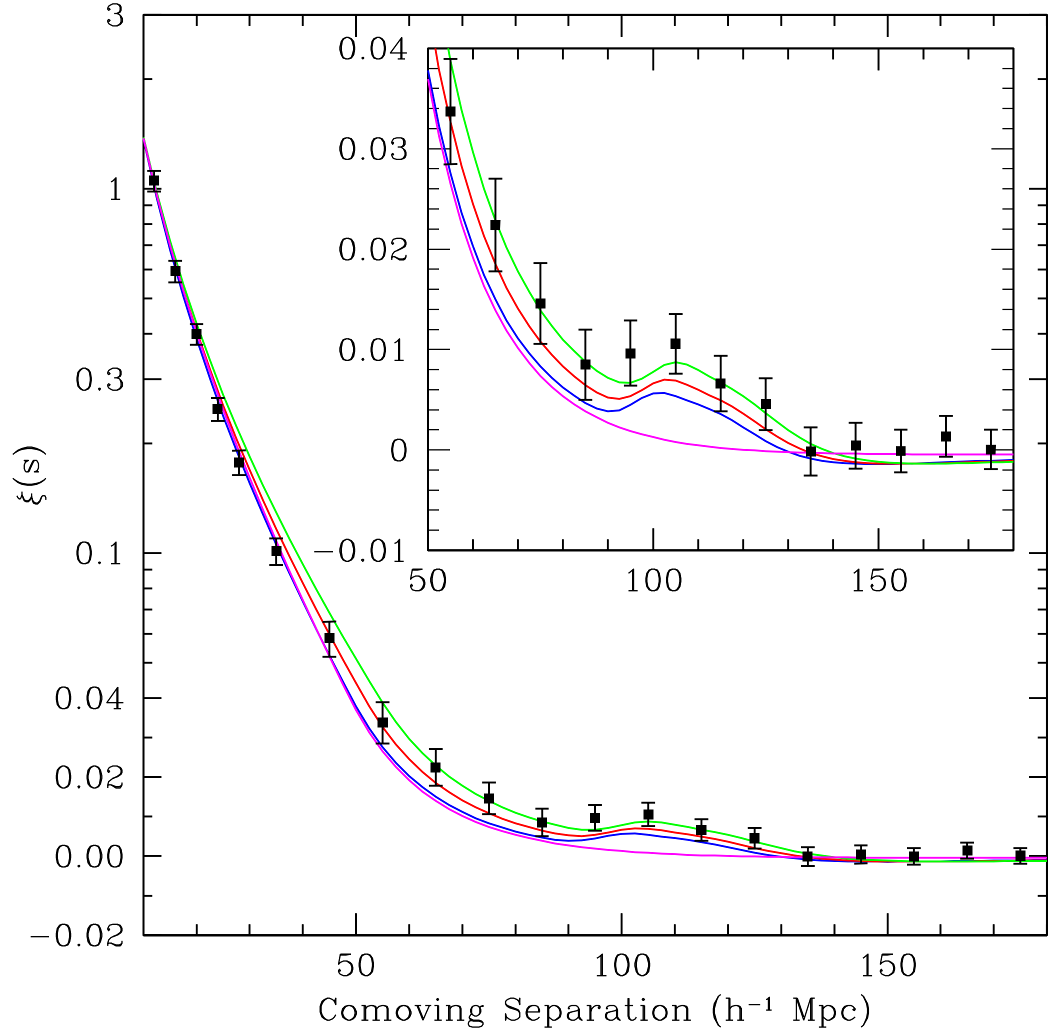
Effect of baryons



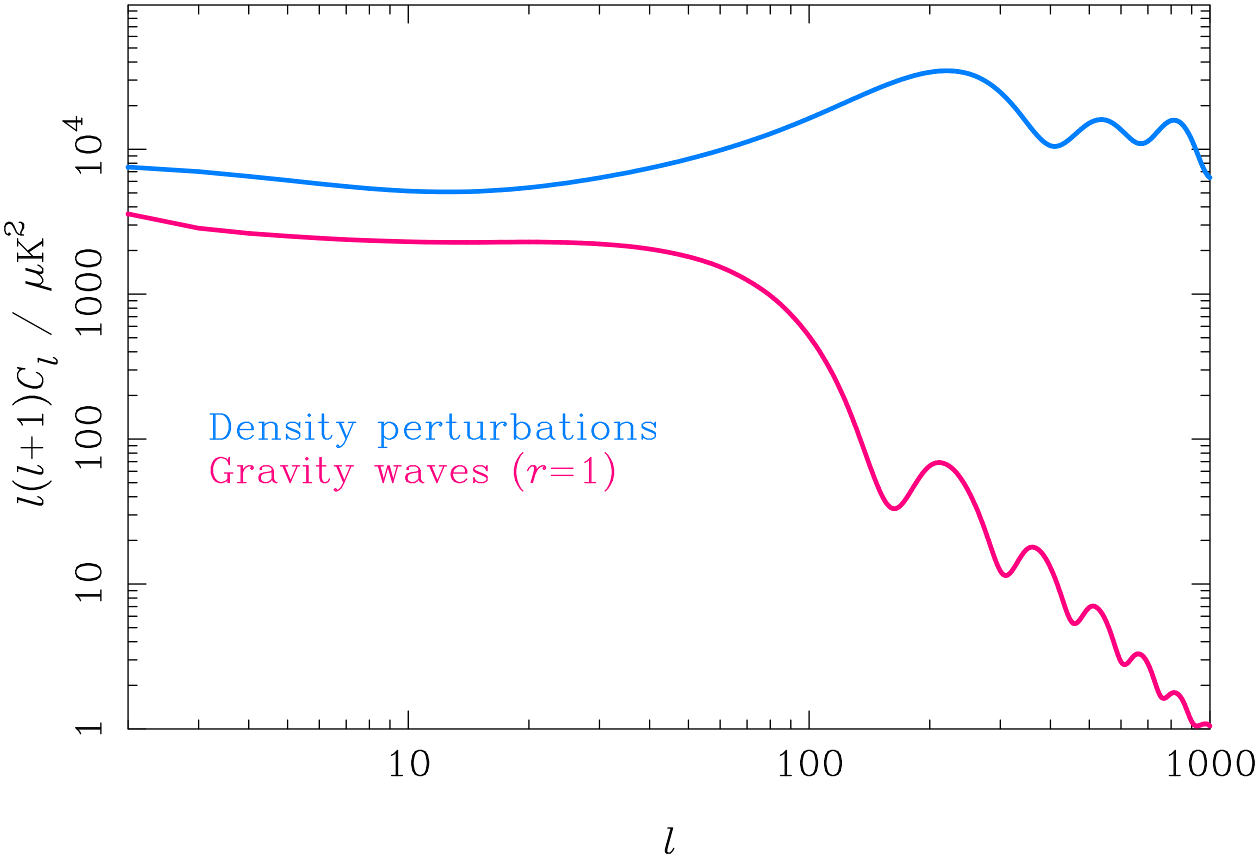
BAO in power spectrum



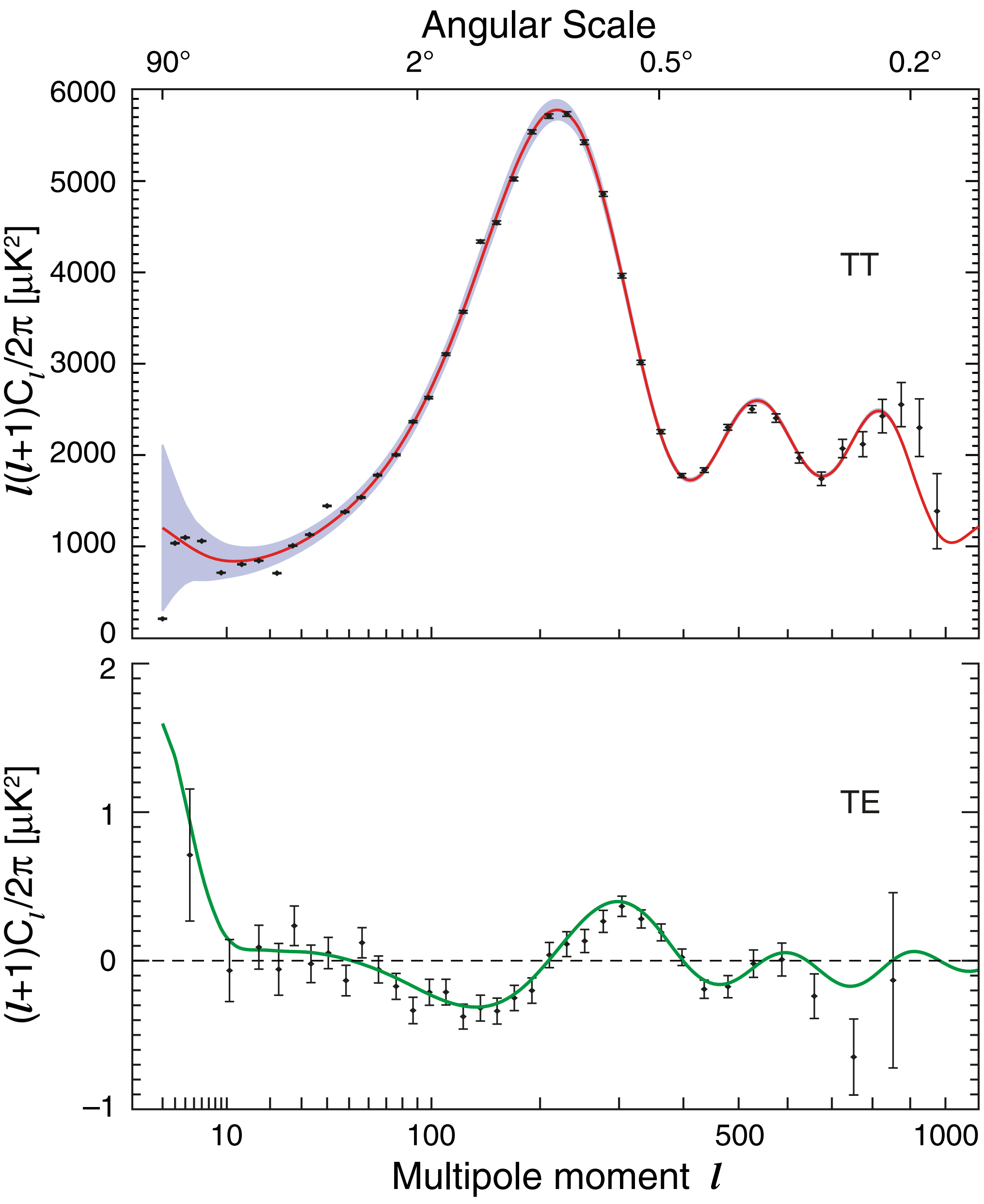
BAO in correlation function



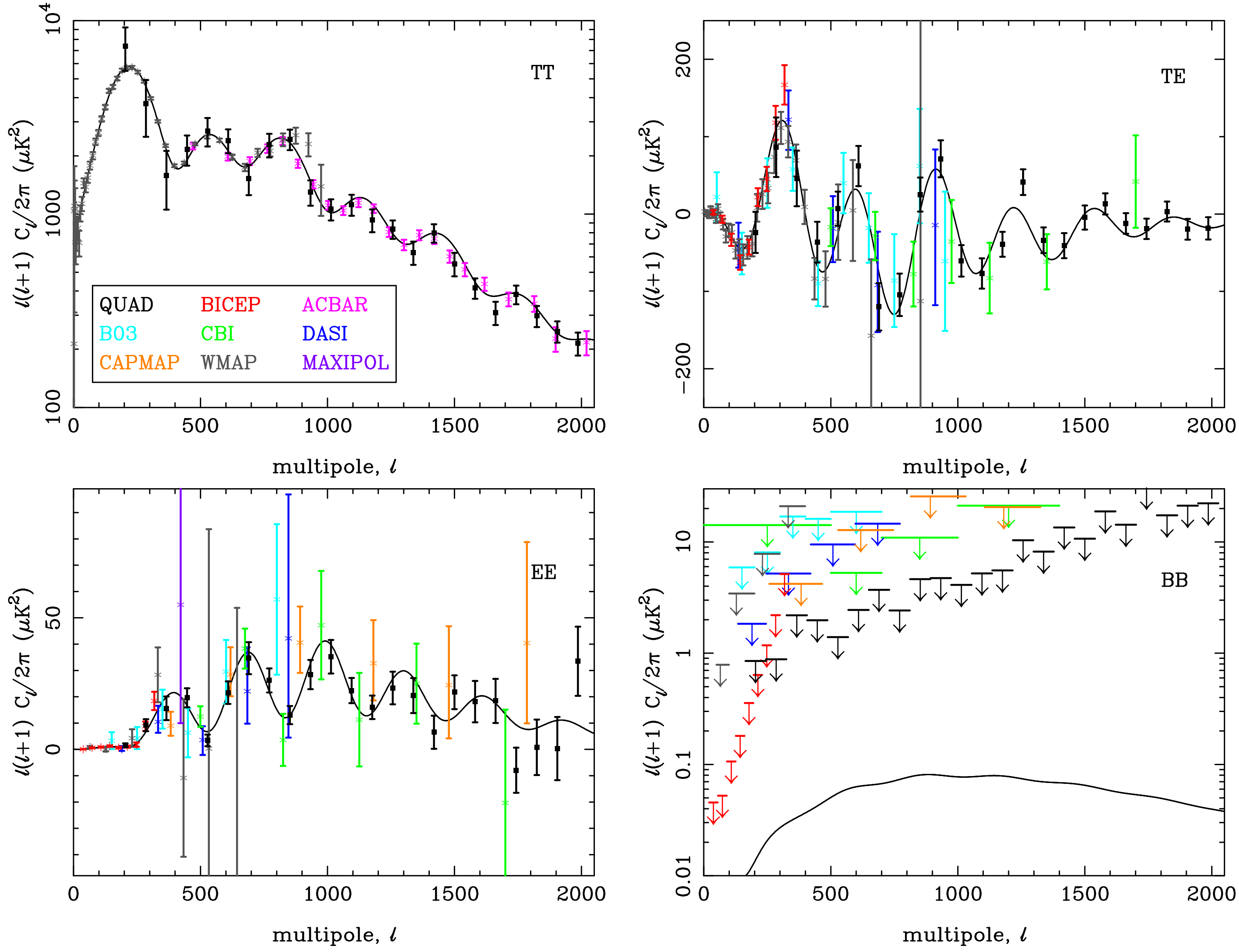
Effect of gravity waves



CMB temperature and polarization



CMB temperature and polarization



Effect of gravity waves on polarization (right)