Electric and magnetic screening in plasma with charged Bose condensate

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Based on:

A.D. Dolgov, A. Lepidi, G. Piccinelli,
JCAP 0902 (2009) 027;
Phys. Rev D, 80 (2009) 125009;
A.D. Dolgov, A. Lepidi,
Phys. Lett. A375 (2011) 3188.

Some similar results but another method:

G. Gabadadze, R.A. Rosen,
Phys. Lett. B 658 (2008) 266;
JCAP 0810 (2008) 030;
JCAP 1004 (2010) 028.

Textbook formula for screening:

$$U(r) = \frac{Q}{4\pi r} \to \frac{Q\ \exp(-m_D r)}{4\pi r},$$

because the time-time component of the photon propagator acquires "mass":

$$k^2 \to k^2 + \Pi_{00}(k) = k^2 + m_D^2\,,$$

where e.g. for relativistic fermions

$$m_D^2 = e^2\left(T^2/3 + \mu^2/\pi^2\right).$$

In presence of condensate the screened potential drops as a power of r and oscillates.

Strangely until recently effects on screening from condensate of a charged Bose field were not studied (in relativistic theory), though it is a simple textbook problem.

Consider electrically neutral plasma with large electric charge density of fermions compensated by charged bosons.

Bosons condense when their chemical potential reaches maximum value:

$$\mu_B = m_B \, .$$

Equilibrium distribution of condensed bosons:

$$f_B^C = C\delta^{(3)}(\mathrm{q}) + \frac{1}{\exp\left[(E - m_B)/T\right] \pm 1}$$

annihilates collision integral for an arbitrary constant C.

f_{eq} **is always determined by two parameters, either** T **and** μ**, or** T **and** C**, iff** $\mu = m_B$**.**

Collision integral:

$I_{coll} \sim |A_{fi}|^2 \Pi f_f \Pi(1 \pm f_i) - \text{(inverse)}$

If T-invariance holds, i.e. $|A_{if}| = |A'_{fi}|$:

$$I_{coll} \sim \left[\Pi f_i (1 \pm f_f) - (i \leftrightarrow f)\right] d\tau \, .$$

$I_{coll} = 0$ **for arbitrary** T **and** C
iff $\mu = m$.

If T-invariance is broken and $|A_{if}| \neq |A'_{fi}|$, :

$$I_{coll}[f_{eq}] \sim \Pi f_i (1 \pm f_f) \left[|A_{fi}|^2 - |A_{if}|^2 \right]$$

This term is surely non-vanishing!
Do equilibrium distibutions remain the same in T-broken theory?

Breaking of T-invariance is unobservs-ble if only one reaction channel is open.

In this case $T_{if} = T^*_{fi}$ with time reflected momenta.

f^C_B annihilates collision integral after summation over all relevant processes, due to S-matrix unitarity or CPT and conservation of probability.

Instead of the detailed balance condition there operates "the cyclic balance" condition.

If CPT and unitarity are broken equilibrium distributions change.

Screening propereties of medium are expressed through f which is not necessarily equiibrium one. In calculations neither imaginary time method which may be inconvenient in presence of condensate or out of equiibroum, nor Matsubara-Keldysh technique are used. We started from the quantum equations of motion, solved them up to e^2 order, and averaged the corresponding operators not only over vacuum but also over "non-empty" medium.

Operator Maxwell equations:

$$\partial_\nu F^{\mu\nu}(x) = \mathcal{J}_B^\mu(x) + \mathcal{J}_F^\mu(x)\,,$$

where bosonic current is

$$\mathcal{J}_B^\mu(x) = -i\,e[(\phi^\dagger(x)\partial^\mu\phi(x)) - (\partial^\mu\phi^\dagger(x))\phi(x)] + 2e^2 A^\mu(x)|\phi(x)|^2\,,$$

plus fermionic current:

$$\mathcal{J}_F^\mu(x) = e\bar{\psi}\gamma_\mu\psi\,.$$

Equation for quantum operator ϕ:

$$(\partial^2 + m^2)\phi(x) = \mathcal{J}_\phi(x)\,,$$

where

$$\mathcal{J}_\phi = -i\,e\Big[\partial_\mu A^\mu(x) + 2A_\mu(x)\partial^\mu\Big]\phi(x) + e^2 A^\mu(x)A_\mu(x)\phi(x)\,,$$

can be formally solved as (next page):

$$\phi(x) = \phi_0(x) + \int d^4y \, G_B(x-y) \mathcal{J}_\phi(y) \,,$$

where ϕ_0 is the free field operator.
In the lowest order in e, i.e. up to e^2 terms, take $\phi = \phi_0$ in electromagnetic current: $\mathcal{J}^\mu_B(x)$ and in $\mathcal{J}_\phi(y)$.
Next: insert expressions for e.m. currents into Maxwell equations for classical field A_μ and take average of operators ϕ and ψ over medium.

The r.h.s. of the Maxwell equations in e^2 order is linear (but non-local) in A_μ and bilinear in ϕ_0 and ψ_0.

Expand free fields as usually:

$$\phi_0(x) = \int d\tilde{q} \left[a(q) e^{-iqx} + b^\dagger(q) e^{iqx} \right] .$$

Average over medium as:

$$\langle a^\dagger(\mathrm{q}) a(\mathrm{q}') \rangle = f_B(E_q) \delta^{(3)}(\mathrm{q} - \mathrm{q}'),$$
$$\langle a(\mathrm{q}) a^\dagger(\mathrm{q}') \rangle = [1 + f_B(E_p)] \delta^{(3)}(\mathrm{q} - \mathrm{q}') .$$

Solving Fourier transformed linear Maxwell equation for A_t we find:

$$\Pi_{tt}(0,k) = \frac{e^2}{2\pi^2}\int_0^\infty \frac{dq\, q^2}{E_B}[f_B(E_B,\mu_B)$$

$$+\bar{f}_B(E_B,\bar{\mu}_B)][1+\frac{E_B^2}{kq}\ln|\frac{2q+k}{2q-k}|]\,,$$

plus similar contribution from fermions which neutralize the plasma.

This is the well known result for Π_{tt} in order e^2.

The screened Coulomb potential is the Fourier transform of tt-component of the photon Green's function in medium:

$$U(r) = e^2 \int \frac{d^3k}{(2\pi)^3} \frac{e^{ikr}}{k^2 + \Pi_{tt}(k)} = \frac{e^2}{2\pi^2 r} \int_0^\infty \frac{dkk \, \sin kr}{k^2 + \Pi_{tt}}.$$

Asymptotics of the potential for large r, created by charged impurities is determined by the singularities of the integrand in complex k-plane.

Comment.

Singularities of $f(z)$:

$$f(z) = \int_a^b dy g(z, y)$$

in complex z-plane appear at such z for which singularites of $g(z, y)$, i.e. $y_c(z)$, in complex y-plane coinsides with the bounds of integration, a or b, or $y_c(z)$ pinches the contour of integration.

Two types of singularities:

1. Poles of $[k^2 + \Pi_{tt}(k)]^{-1}$.

E.g. Debye pole. Necessary to check that the position of the poles are at small k, such that the infrared asymptotics of Π_{tt} is valid.

2. Singularities of $\Pi_{tt}(k)$, originating from the pinch of the integration contour in q-plane by poles of f and by branch points of log.

Without condensate one obtains the usual k-independent Debye screening:

$$\Pi_{tt}(0,k) = m_D^2$$

originating from a pole at imaginary axis of k.

With condensate the corrections to Π_{tt} at low k are infrared singular:

$$\frac{\Delta\Pi_{tt}}{e^2} = \frac{m_B^2 T}{2k} + \frac{C}{(2\pi)^3 m_B}\left(1 + \frac{4m_B^2}{k^2}\right)$$

Both terms in the r.h.s. appear only if $\mu = m_B$.

Instead of exponential the screening becomes power law and oscillating, depending upon parameters, m_j:

$$\Pi_{tt} = m_0^2 + m_1^3/k + m_2^4/k^2.$$

May this infrared singularity have something to do with confinement?
See: P. Gaete, E. Spalucci, 0902.00905 – confinement in the Higgs phase.

Contribution from poles in the limit of large $m_2 r$ but when power law terms are subdominant:

$$U(r)_{pole} = \frac{Q}{4\pi r} \exp\left(-\sqrt{e/2}\, m_2 r\right) \times \cos\left(\sqrt{e/2}\, m_2 r\right).$$

Oscillating screening is known for degenerate fermions, Friedel oscillations. Observed in experiment.

Comment.

Friedel oscillations are usually considered at $T = 0$. In this case the integral over q is in finite interval and the singularity in k appears when log branch point coincides with the upper limit of the integration.

$T = 0$ limit can be obtained in the "pinch" method by summing all the singularities.

Contribution from the integral along imaginary axis, which is nonzero because Π_{tt} contains an odd in k term. If $m_2 \neq 0$, the dominant term is

$$U(r) = -\frac{12Qm_1^3}{\pi^2 e^2 r^6 m_2^8}.$$

If $T \neq 0$, $\mu = m_B$, but the condensate is not yet formed, the asymptotic decrease of the potential becomes:

$$U(r) = -\frac{Q}{\pi^2 e^2 r^4 m_1^3} = -\frac{2Q}{\pi^2 e^2 r^4 m_B^2 T}.$$

Contribution from logarithmic cuts (analogous to Friedel oscillations for fermions).

If the first "pinch" (between the poles of $f(q)$ and logarithmic branch point) dominates:

$$U_1(r) = -\frac{32\pi Q}{e^2 m_B r^2} \frac{e^{-z}}{\ln^2(2\sqrt{2}z)} \sin z \, ,$$

where $z = 2r\sqrt{2\pi T m_B}$.

NB: $U_1(r)$ is inversely proportional to e^2 and formally vanishes at $T \to 0$, but remains finite if $\sqrt{T m_B} r \neq 0$.

All pinches are comparable:

$$U(r) \approx -\frac{3Q}{2e^2T^2m_B^3r^6\ln^3(\sqrt{8m_BTr})}.$$

$U \sim T^{-2}$ **valid if** $r \ll 1/\sqrt{16\pi T m_B}$, **i.e. if** $T = 0.1$**K and** $m_B = 1GeV$ **the distance should be bounded from above as** $r \ll 3 \cdot 10^{-8}$ **cm.**

Condensation of vector bosons.

$W^{\pm}$ would condense in the early universe if lepton asymmetry was sufficiently high. It leads to large electric asymmetry of W, such that
$\mu_W = m_W$. Plasma neutrality was maintained by quarks and leptons.

Vector bosons have additional degrees of freedom: their spin states, and their condensation demonstrates richer possibilities.
Depending on the sign of the pairwise spin-spin coupling, W's would condense either in $S = 0$ (scalar) state or in $S = 2$ (ferromagnetic) state.

Magnetic spin-spin interaction through one photon exchange (similar to Breit equation):

$$U_{em}^{spin}(r) = \frac{e^2\rho^2}{4\pi m_W^2}\Big[\frac{(S_1 \cdot S_2)}{r^3} - 3\,\frac{(S_1 \cdot r)(S_2 \cdot r)}{r^5} - \frac{8\pi}{3}(S_1 \cdot S_2)\delta^{(3)}(r)\Big].$$

Here ρ is the ratio of magnetic moment of W to the standard one.

For S-wave the energy is shifted by the last term only.

Local quartic self-coupling of W:

$$U_{4W}^{(spin)} = \frac{e^2}{8m_W^2 \sin^2\theta_W}(S_1 S_2)\delta^{(3)}(r).$$

The net result $U_{em} + U_{4W}$ is negative in the standard model, so $S = 2$ state is energetically favorable and spontaneous magnetization in the early universe may be possible.

Suppression due to screening.

The ij component of W propagator probably remains massless: $\Pi_{ij} \sim 1/q^2$. In Abelian QED it is true in perturbation theory, while in non-Abelian theories the screening may occur in higher orders of perturbation theory due to infrared singularities. The screening would diminish the long-range ferromagnetic spin-spin coupling while the local W^4 coupling is not screened.

If the propagator is modified, and the wave function of W-bosons is constant in space, the spin-spin energy shift is:

$$\delta E \sim \int \frac{d^3q\,\delta(q)}{(2\pi)^3} \frac{q^2(S_1 S_2) - (\mathrm{q}\mathrm{S}_1)(\mathrm{q}\mathrm{S}_2)}{q^2 + \Pi_{ss}(\mathrm{q})}$$

$\delta E = 0$, if $\Pi_{ss} \neq 0$ at $q = 0$.

However, the space integration should be done with an upper limit, l, which by an order of magnitude is equal to the average distance between W bosons, so instead of $\delta^{(3)}(q)$, we obtain:

$$\int_0^l d^3r e^{iqr} = \frac{4\pi}{q^3}\left[\sin(ql) - ql\cos(ql)\right].$$

and the energy shift is non-zero:

$$\delta E = -\frac{(S_1 S_2)e^2}{l^3 m_W^2} F(l),$$

$$F(l) = \int_0^\infty \frac{dx\left[x\ \sin x + l^2\Pi_{ss}\cos x\right]}{x^2 + l^2\Pi_{ss}(x/l)}.$$

If $l^2\Pi_{ss}$ is nonnegligible the e.m. part of the spin-spin interaction would be suppressed and the ferromagnet may turn into an antiferromagnet. This might happen at T above the EW phase transition when the Higgs condensate is destroyed and $m_{W,Z}$ appear as a result of temperature and density corrections and are relatively small.

The quantitative statement depends upon the modification of the space-space part of the photon propagator in presence of the Bose condensate of charged W.

The screening of magnetic fields is determined by Π_{ij}, which, in the homogeneous and isotropic case, can be written as:

$$\Pi_{ij} = a(k)\left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2}\right) + b(k)\frac{k_i k_j}{\mathbf{k}^2}\,.$$

Multiplying Π_{ij} by δ_{ij} and by $k_i k_j$ we obtain:

$$b = 0\,,$$

$$a = \int \frac{e^2 d^3 q}{16\pi^3 E}\left(f + \bar{f}\right)\left[1 + \frac{k^2(4q^2 - k^2)}{4(\mathbf{kq})^2 - k^4}\right]$$

If only the condensate term (delta-function) is retained in distribution function, we obtain:

$$a^{(C)} = \frac{e^2 C}{8\pi^3 m_B}.$$

Since $a \neq 0$ at $k = 0$ magnetic fields at large distances from the source would exponentially decrease.
BEC is superconducting, high T superconductivity in contrast to BCS.

If $\mu < m_B$, so the condensate is not formed, $\Pi_{ij}(k)$ vanishes in the limit $k^2 \to 0$, as expected:

$$a(k) \approx \frac{e^2 k^2}{24\pi^2} \int \frac{dq}{E} \left(f + \bar{f}\right) .$$

However, if $\mu = m$ the integral more infrared singular and

$$a(k) = \frac{e^2}{32\pi^3} \int \frac{d^3q}{E} \, (f + \bar{f}) \left[2 + \frac{2k^2(4q^2 - k^2)}{4(kq)^2 - k^4} \right] .$$

It leads to:

$$a^{(s)}(k) = \frac{e^2 T}{16}\, k\,.$$

If $m_C > e^2 T/32$, the resulting screened potential would be exponentially cut with superimposed oscillations. For $e^2 T \ll 32\, m_C$, the Green function takes the form:

$$G(r) \sim \exp\left(-\, e m_C r\right)\, \cos\left(e^2 r T/32\right).$$

In this case the spatial damping scale is much shorter than the oscillation scale. However, if $eT \sim m_C$ the scales are comparable.

The screening is effective if the screening length $\lambda \equiv 1/em_C$ is larger than the inter-particle distance $d \equiv C^{-1/3}$. Thus the screening is not effective if $m_B/C^{1/3} \ll 10^{-4}$. Thus it is improbable that the ferromagnetic state could be realized in the broken phase, where m_B, is essentially determined by the Higgs VEV. The screening may be ineffective in the unbroken phase, where m_B is determined by radiative corrections and small.

Problem of large scale magnetic fields:
$B \sim \mu G$ at several kpc. In the intergalactic space the fields are probably 2-3 orders of magnitude weaker, but still non-vanishing.
Dynamo operates only in galaxies.
Maybe ferromagnetism of W might create seeds for large scale magnetic fields.

DIMA,
MY BEST WISHES TO YOU,
GOOD HEALTH,
AND NON-SCREENED SUCCESS WITH YOUR WORK.