Form factors in N=4 SYM

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N=4 SYM theory.

- N=4 SYM is perfect theoretical laboratory for studying properties of D=4 gauge theories.
- The correlation functions $G_n = \langle 0 | \mathcal{O}(x_1) \dots \mathcal{O}(x_n) | 0 \rangle$ in this theory can be studied in the weak and strong regimes (via AdS/CFT).
- The computation of anomalous dimensions of local operators in N=4 SYM in planar limit can be reduced to the problem of solving some integrable system.
- The regularized amplitudes $A_n = \langle p_1^{\lambda_1} \dots p_n^{\lambda_n} | 0 \rangle$ (S-matrix) can be also studied in weak/strong coupling regimes. It was realized that likely planar Smatrix in N=4 SYM possesses new type of symmetries - (super)conformal symmetry in momentum space. Algebra of this symmetry together with conventional (super)conformal symmetry can be fused together to infinite dimensional Yangian algebra. It is believed that this symmetries will completely define planar S-matrix of N=4 SYM (*Beisert et al 10*).

Form factors in N=4 SYM.

 There is another class of objects in N=4 SYM which is intermediate between completely off-shell correlation functions and completely on-shell amplitudes – form factors (*van Neerven 85, Selivanov 98*):

$$F_n = \langle p_1^{\lambda_1} \dots p_n^{\lambda_n} | \mathcal{O}(x) | 0 \rangle$$

Some n-particle external state

Local gauge invariant operator

- What is the structure of such objects in general ? Can symmetries of N = 4 SYM fix the structure of such objects ? Is there a dual description in terms of Wilson loops ? e c.t.
- To answer this questions computations in first several orders of PT are required (*Kazakov et al 10, Brandhuber et al 10-11, Kazakov 11 et al*).

On shell computational methods in N=4 SYM.

- The ordinary textbook Feynman diagram based machinery for amplitudes and form factors in D=4 gauge theories quickly becomes bulky when the number of external particles\loops are sufficiently large (5 legs\1 loop, actually).
- Up to now there are several methods beyond ordinary textbook staff that can help at tree and loop level. Some of them are specific to N=4 SYM but most of them are not.
- They are colour and helicity decomposition, helicity spinor notations, MHV vertex expansion (Witten ~05), BCFW recurrence relations (Britto, Cachzo, Feng, Witten 05), unitarity cut constructability technics (Bern, Dixon ~90) and their N=4 on-shell superspace generalizations (Korchemsky et all 08), momentum twistor space formulation of N=4 SYM (Masson Skiner 10).

Colour decomposition, helicity spinors and MHV amplitudes.

It is convenient to decompose polarized amplitudes in the following way:

$$A_n(p_1^{\lambda_1}, \dots, p_n^{\lambda_n}) = \sum_{perm} Tr(T^{a_1} \dots T^{a_n}) \mathcal{A}_n(p_1^{\lambda_1} \dots p_n^{\lambda_n}) + O(1/N_c)$$
Physical amplitude
Colour SU(N_c) factor
Colour ordered
partial amplitude

Introducing new variables (helicity spinors) for massless momenta of external particles:

$$p_{\mu} \to p_{\mu}^{(i)}(\sigma^{\mu})_{\alpha\dot{\alpha}} = \lambda_{\alpha}^{i}\tilde{\lambda}_{\dot{\alpha}}^{i}, \ \langle ij \rangle = \epsilon^{\alpha\beta}\lambda_{\alpha}^{i}\lambda_{\beta}^{j}, \ (\langle ij \rangle)^{*} = [ij]$$

and using BCFW or CSW recurrence relations one can obtain extremely compact expressions for colour ordered polarized amplitudes at tree level. For example for MHV gluon amplitude one have :

$$\mathcal{A}_n^{tree}(g_1^-g_2^-g_3^+\dots g_n^+) = \delta^4(\sum_{k=1}^n \lambda_k^{\alpha} \tilde{\lambda}_k^{\dot{\alpha}}) \frac{\langle 12 \rangle^4}{\langle 12 \rangle \dots \langle n1 \rangle}$$

n

 $\lambda_{s}=n-4$

MHV amplitudes in on-shell momentum superspace.

It is convenient to introduce <u>on-shell momentum superspace</u> parameterized by the following set of coordinates (*Nair 88, Korchemsky 08*): $\{\lambda_{\alpha}, \tilde{\lambda}_{\dot{\alpha}}, \eta^A\}$ SL(2,C) commuting spinors **Grassmann scalars** $\{q^-, \Gamma^A, \phi^{AB}, \overline{\Gamma}^A, q^+\}$ of N=4 Then one can combine all physical on-shell states SYM into single "super-state" : $|\Omega_i\rangle = \left(g_i^+ + \eta^A \Gamma_{i,A} + \frac{1}{2!}\eta^A \eta^B \phi_{i,AB} + \frac{1}{3!}\eta^A \eta^B \eta^C \varepsilon_{ABCD} \bar{\Gamma}_i^D + \frac{1}{4!}\eta^A \eta^B \eta^C \eta^D \varepsilon_{ABCD} g_i^-\right)|0\rangle$ In such notations all MHV component amplitudes can be combined into one N=4 covariant expression: $\mathcal{A}_{n}^{tree,MHV}(\Omega_{1},\ldots,\Omega_{n}) = \delta^{4}(\sum_{k=1}^{n}\lambda_{k}^{\alpha}\tilde{\lambda}_{k}^{\dot{\alpha}})\delta^{8}(\sum_{k=1}^{n}\lambda_{k}^{\alpha}\eta_{A}^{k})\frac{1}{\langle 12\rangle\ldots\langle n1\rangle}$ Similar (but more complicated) expressions can be obtained for all other tree helicity amplitudes (<u>Brandhuber et al ~08</u>).

Example of the conventional Feynman diagram based result.





Example of the conventional Feynman diagram based result.

Result of evaluation (actually only a small part of it):



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$k_1\cdot k_4\,\varepsilon_2\cdot k_1\,\varepsilon_1\cdot \varepsilon_3\,\varepsilon_4\cdot \varepsilon_5$

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Unitarity based methods. Fusing trees into loops.

One can try to reconstruct the basis of scalar integrals for the amplitude using the knowledge of the explicit form of different cuts of amplitude. The scalar integrals themselves should be calculated separately.



It is often enough to consider iterated cuts up to tree level MHV amplitudes, which is reminiscent to MHV vertex expansion (CSW).

In N=4 SYM such approach for the computation of amplitudes is essentially efficient (no problems with rational parts, simpler basis of scalar integrals). One can apply this methods to form factors as well (*Brandhuber et al 10-11, Kazakov et al 11*).

Off-shell N=4 supermultiplets.

In the case of the form factors one would like to construct analog of super MHV amplitude at tree level (super form factor), and then use in in unitarity based computations.

Which parameterization (superspace) one should use to describe and what supermultiplet of operators one can chose ?

$$F_n = \langle p_1^{\lambda_1} \dots p_n^{\lambda_n} | \mathcal{O}(x) | 0 \rangle$$
$$\mathcal{F}_n = \langle \Omega_1 \dots \Omega_n | ??? | 0 \rangle$$

Usually N=4 supermultiplets in N=4 coordinate superspace are on-shell, and in addition the coordinate N=4 superspace is none-chiral in contrast to on-shell one.

One can sensible set of supermultiplets if one will consider chiral truncation of N=4 supermultiplet itself.

Off-shell N=4 supermultiplets. Harmonic superspace.

N=4 supermultiplet of fields can be described superfield in coordinate superspace

$$\begin{split} W^{AB}(x,\theta^{A},\bar{\theta}_{A}) &= \phi^{AB}(x) + \dots \\ \text{This superfield satisfies the following constraints:} \quad \overline{W}^{AB}(x,\theta,\bar{\theta}) = \frac{1}{2} \epsilon^{ABCD} W_{CD}(x,\theta,\bar{\theta}) \\ \text{and most importantly:} \quad \begin{aligned} D^{\alpha}_{C} W^{AB}(x,\theta,\bar{\theta}) &= -\frac{2}{3} \delta^{[A}_{C} D^{\alpha}_{L} W^{B]L}(x,\theta,\bar{\theta}) \\ \bar{D}^{\dot{\alpha}(C} W^{A)B}(x,\theta,\bar{\theta}) &= 0, \end{aligned}$$

and most impo

This constraints can be "solved" in terms of harmonic projected superfield:

$$\begin{split} W^{AB} &\to W^{AB} u_A^{+a} u_B^{+b} = \epsilon^{ab} W^{++} \quad W^{++}(x,0,0,u) = \phi^{++}, \ \phi^{++} = \frac{1}{2} \epsilon_{ab} u_A^{+a} u_B^{+b} \phi^{AB} \\ & \text{Where} \quad \left(u_A^{+a}, \ u_A^{-\dot{a}} \right) \text{ are } \quad \frac{SU(4)}{SU(2) \times SU(2) \times U(1)} \quad \text{harmonics } (\textit{Ogievetsky et al in 80's}). \\ & \text{Coordinate and oh-shell momentum harmonic superspaces are:} \end{split}$$

$$\{x^{\alpha\dot{\alpha}}, \ \theta^{+a}_{\alpha}, \theta^{-\dot{a}}_{\alpha}, \ \bar{\theta}^{+a}_{\dot{\alpha}}, \bar{\theta}^{-\dot{a}}_{\dot{\alpha}}, u\} \ \{\lambda_{\alpha}, \tilde{\lambda}_{\dot{\alpha}}, \ \eta^{+a}, \eta^{-\dot{a}}, \ u\}$$

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Off-shell N=4 supermultiplets. Harmonic superspace.

Then one can observe that chiral truncation of harmonic projected superfield is off-shell

$$\mathcal{W}^{++} = W^{++}(x, \theta^+, 0, u)$$

Perfect building block for 1/2-BPS :

$$\mathcal{O}^{(k)} = Tr([\mathcal{W}^{++}]^k), \ \mathcal{O}^{(k)} \sim Tr(\phi^k), \dots$$

and stress tensor supermutiplet of operators

$$\mathcal{O}^{(2)} = \mathcal{T} = Tr(\mathcal{W}^{++}\mathcal{W}^{++}), \ \mathcal{T} \sim Tr(\phi^2), Tr(\psi\psi), Tr(FF) + O(g), \dots$$

So it is natural as the beginning to consider form factors with operators of such type:

$$\mathcal{F}_n = \langle \Omega_1 \dots \Omega_n | \mathcal{O}^{(k)}(x, \theta^+, u) | 0 \rangle$$

Arbitrary combination of N=4 on-shell states (particles)

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Operators from1\2 BPS or stress tensor chiral truncated supermultiplets

MHV form factor of stress tensor supermultiplet at tree level.

Using BCFW recurrence relations and symmetry arguments one can obtain the expression for MHV (n-2) super form factor of stress tensor supermultiplet (*Kazakov et al 11, Brandhuber et al*):

$$\hat{T}[\mathcal{F}_{n}^{tree,MHV}] \sim \delta^{+4}(\sum_{k=1}^{n} \lambda_{\alpha}^{k} \eta_{k}^{+a} + \lambda_{\alpha}^{\prime} \eta^{\prime + a} + \lambda_{\alpha}^{\prime \prime} \eta^{\prime \prime + a}) \delta^{-4}(\sum_{k=1}^{n} \lambda_{\alpha}^{k} \eta_{k}^{-\dot{a}}) \frac{1}{\langle 12 \rangle \dots \langle n1 \rangle}$$

$$\mathcal{A}_{n}^{tree,MHV} \sim \delta^{+4} (\sum_{k=1}^{n} \lambda_{\alpha}^{k} \eta_{k}^{+a}) \delta^{-4} (\sum_{k=1}^{n} \lambda_{\alpha}^{k} \eta_{k}^{-\dot{a}}) \frac{1}{\langle 12 \rangle \dots \langle n1 \rangle}$$

Both this MHV form factor and amplitude can be used in unitarity based computations to obtain loop corrections for MHV and in principle <u>none-MHV</u> form factors.

Also we define:
$$\hat{T}[\ldots] = \int d^4 \theta_{\alpha}^{+a/-\dot{a}} \exp(\theta_{+a/-\dot{a}}^{\alpha} \sum_{i=1}^2 \lambda_{\alpha}^i \eta_i^{+a/-\dot{a}})[\ldots]$$

After introduction of $M_n^{(l)} = \frac{\hat{T}[\mathcal{F}_n^{(l),MHV}]}{\hat{T}[\mathcal{F}_n^{tree,MHV}]}$ we are ready to discuss loop corrections in planar limit.

MHV form factors of stress tensor super multiplet and 1/2-BPS super multiplets at 1 and 2 loops. Scalar integrals.



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MHV form factors of stress tensor super multiplet and 1/2-BPS super multiplets at 1 and 2 loops. General IR structure.

The discussed above form factors have the following structure in general identical to planar amplitudes : n = n

$$\log(M_n) = \frac{1}{2} \sum_{i=1}^n \left(\hat{M}(s_{i,i+1}/\mu^2) \right) + \text{fin.part} + O(\epsilon) \quad \text{(Kazakov et al 11)}$$
$$\hat{M}(s_{i,i+1}/\mu^2) = -\frac{1}{2} \sum_l \left(\frac{\lambda}{16\pi^2} \right)^l \left(\frac{\Gamma_{cusp}^{(l)}}{(l\epsilon)^2} + \frac{G^{(l)}}{l\epsilon} \right) \left(\frac{s_{i,i+1}}{\mu^2} \right)^{l\epsilon}$$

As a by product one can get the answer for Sudakov-like form factor up to 2 loops in N=4 SYM:

$$\log(M_2) = -\frac{1}{4} \sum_{i=1}^2 \sum_l \left(\frac{\lambda}{16\pi^2}\right)^l \left(\frac{\Gamma_{cusp}^{(l)}}{(l\epsilon)^2} + \frac{G^{(l)}}{l\epsilon} + C^{(l)}\right) \left(\frac{s_{i,i+1}}{\mu^2}\right)^{l\epsilon}$$

$$\Gamma_{cusp}^{(1)} = 4, \ \Gamma_{cusp}^{(2)} = -8\zeta_2,$$

$$G^{(1)} = 0, \ G^{(2)} = -\zeta_3,$$

$$C^{(1)} = -\zeta_2, \ C^{(2)} = 0.$$

While in general:

$$\Gamma_{cusp}^{(1)} = 4, \ \Gamma_{cusp}^{(2)} = -8\zeta_2,$$

$$G^{(1)} = 0, \ G^{(2)} = -7\zeta_3.$$

For
$$\mathcal{O}^{(n)},\;n,\;\lambda_{tot}=0$$
 only.

Note also that transcendentality principle holds.

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MHV form factors of stress tensor super multiplet and 1\2-BPS super multiplets at 1 and 2 loops. Answers.

In general fin.part is complicated function of log's , polylogarithms and Goncharov polylogarithms . It is not clear whether one can combine them in some simpler expressions (symbol based machinery (*Volovich 09*)).

Some finite parts are still simple to be presented. The first case is:

$$\mathcal{T}, MHV : (M_n^{(1)})^{fin} = \sum_{perm. i,j} (Box_{2me}^{fin}(i,j))$$

Also for in some kinematical limits (collinear) one can also obtain simplifications :

$$\log(M_n) \to^{\parallel} \frac{1}{2} \hat{M}_{n-1} + R(\epsilon) + \log(M_{n-1}^{\ coll, fin}) + O(\epsilon)$$

For $\mathcal{O}^{(n)}, n, \lambda_{tot} = 0$ (n=3) one gets:

(Kazakov et al 10)

$$\log(M_2^{coll,fin}) = \left(\frac{\lambda}{16\pi^2}\right)\frac{\zeta_2}{2} - \left(\frac{\lambda}{16\pi^2}\right)^2 \frac{1}{2880} \left(75\log^4\frac{s_{12}}{s_{13}} + 120\pi^2\log^2\frac{s_{12}}{s_{13}} - 317\pi^4\right)$$

 $\log(M_{\rm el}, \ldots)$

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Limiting procedure for scalar integrals. Dual conformal invariance.

The answers for finite parts of form factors are not very interesting themselves. *The really* relevant question is the following - are there some structure (symmetry) behind them ?

On the level of scalar integrals one can observe the following property. ALL scalar integral for MHV stress tensor form factors and ½-BPS form factors with external state with there total helicity 0 can be obtained from *dual conformal once* encountered in amplitude computations by limiting procedure (*Kazakov et al 10*):



It is believed that amplitudes (S-matrix) in N=4 SYM can be completely fixed by "dual conformal based" Yangian symmetry. Form factors are also affected by dual conformal symmetry somehow ?

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Limiting procedure for scalar integrals. Dual conformal invariance. Once again scalar integrals.



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Conclusions.

- The powerful on-shell methods in N=4 SYM can be applied to partially offshell objects as well.
- The form factors in N=4 SYM share lots of similarities with amplitudes factorization of IR divergences, similar super space formulations, dual description in terms of Wilson line-like objects, maximal trancendentality, ec.t.
- There are some hints that rich symmetries of N=4 SYM amplitudes (Smatrix) may (partially) survive for form factors as well.
- There is hope that N=4 SYM is integral theory and its S-matrix can be completely fixed by the symmetry arguments. If this is the case, then one may expect even more rich structure for form factors (like in 2D sin-gordon models).



Happy birthday Dmitry I gorevich!