# Learning on dynamical scalar fields from confinement and Yang-Mills plasma

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# Motivation

Scalar fields are introduced in

- theory of electroweak interactions
- cosmology
- theory of confinement

However, field theory seems to fail
on the scalar particles ( divergences)

Also, no Higgs observed so far

Pressure to appreciate lessons for Yang-Mills scalars
If not from theory, then from the lattice

# Pre-conclusions on the Higgs

- No immediate suggestions for the missing Higgs
- the Yang-Mills dynamics in the scalar sector
  is highly non-trivial
  might be potentially important

# Why Yang-Mills scalars at all?

Confinement via Abrikosov string:

$$\lim_{R\to\infty} V_{Q\bar{Q}}(R) \;=\; \sigma \cdot R$$

$$\sigma \;\sim < \Phi_M >^2$$

where $\Phi_M$ is magnetically charged 4d scalar
with no a priori understanding, what it means

In fact, somewhat misleading, as see later

Quark-gluon plasma might be a cleaner case

# Quark-gluon plasma (QGP) and superfluidity

"Modern superfluidity": From uncertainty principle

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

where $\eta$ is viscosity, $s$ is entropy density

For the plasma:

$$\left(\frac{\eta}{s}\right)_{QGP} \approx \frac{1}{4\pi}$$

Superfluidity scenario is encouraged (not granted of course)

# Relativistic superfluidity and scalars

Non-relativistic wave function:

$$\Psi_{superfluid} \sim \sqrt{N} \exp\ i\phi(x,t)$$

$$\partial_t \phi(x,t) = \mu, \quad (\partial_i)^2 \phi(x,t) = 0$$

($\mu$ is the chemical potential)

Relativistically, no wave function.
Instead, postulate

$$< \phi_{3d} > \neq 0 \ , \quad \phi^*_{3d} \neq \phi_{3d}$$

Spontaneous breaking implies 3d Goldstone
Apparently, no such a symmetry in Yang-Mills case

# Thermal scalar (1986)

Within effective (infrared) stringy theories
novel U(1) symmetries arise

Stringy topological quantum number:
wrapping around the compact Euclidean time
Symmetry could be spontaneously broken

At the Hagendorn phase transition, $T < T_H$ the whole partition function reduces to that of a static scalar:

$$m_\beta^2 = \frac{(\beta - \beta_H)\beta_H}{(2\pi^2)(\alpha')^2}$$

$\beta_H = 1/T_c$
Hits the Higgs region at $T = T_H$.

# Conclusions to introductory remarks

- Spontaneously broken, non-pert U(1) symmetries,
  both 4d and 3d are welcome,
  to explain confinement and low viscosity, resp.
- The only example relevant seems to be thermal scalar,
  or stringy topological quantum numbers

# Confinement-related geometry (Witten)

Deforming the N=4 SUSY YM one gets theory in the same universality class as YM (large $N_c$)

Geometry: compact Euclidean time +3d + 5th dimension with properties:

1. conjugated to inverse momentum.
2. The 4d world is the UV limit, ($z = 0$)
3. there is horizon, $z < z_H$, confinement-related

# Extra compact dimension

Central point: there is extra compact 6th dim responsible for the $\theta$-dependence

survives in the ultraviolet does not fit reality, limits the applicability of the model to distances

$$R \gg \Lambda_{QCD}^{-1}$$

Only confinement and hydrodynamics and massless particles could be captured

# Cigar-shape geometries, phase transition

At $T < T_c$ cigar-shape in the $(z, \theta)$ coordinates:

$$R_\theta(z = z_H) \ = 0$$

$$R_\tau(z) = const = 1/T$$

At $T = T_c$ the geometry is changed:

$$R_\tau(z = z_H) \ = 0 \qquad\qquad R_\theta(z) = const$$

Cigar-shape geometry in $(\tau, z)$ coordinates $(z_H \ \equiv \ z_T)$

Implication of vanishing radius is similar to
the thermal scalar: vanishing in infrared mass/tension

# Geometry and vanishing actions/tensions

The nonperturbative physics is physics of defects

Above $T_c$ the time direction is free
Similar to the thermal scalar (static)

Non-perturbative physics becomes 3d at $T = T_c$
This is dramatically confirmed on the lattice:
defects become time oriented (Gorsky et al 0902.1842)

Below $T_c$ the (gluonic) topological charge is free
(like instantons)
The object which can condense is magnetic strings

# Conclusions to holographic models

- possible condensation of topologically charged magnetic strings below $T_c$
- thus, physics of confinement is rather physics of strings, not field theory
- Euclidean-time oriented defects above $T_c$
- possible condensation of a scalar above $T_c$. (probably) back to field theory

# Condensation of strings. Hints from the lattice

In the absence of theory, the only source is lattice.
Remarkable results (pure numerical, though):

Condensation of strings has been observed on the lattice

Turns to be equivalent to a single scalar living on a string:

$$(condensation\ of\ strings) \equiv < |\phi_M|^2 > \sim \Lambda_{QCD}^2$$

$$< \phi_M >^2 \sim \Lambda_{QCD}^3 \cdot a,\ < \phi_M >_{continuum} = 0$$

although at any finite $a$ suffices to produce confinement (!)

# Condensation of magnetic strings, cn'd

In field theory one would have instead

$$< \phi^2 > \sim \ a^{-2} \qquad < \phi >^2 \sim \ \Lambda^2_{QCD}$$

That is, strings are 'milder' by two dimensions

Vanishing of $< \phi_M >$, $< \phi_M > \sim \ (a\Lambda_{QCD})^{1/2}$
opens the possibility

$$\frac{< \phi_M >}{\Lambda_{QCD}} \ll \ 1$$

upon inclusion of (say, e-m) corrections. Very interesting.

At $T > T_c$ strings become time-oriented and condensation of strings becomes condensation of a 3d scalar (observed)

# Effective 3d scalars at $T > T_c$

At $T > T_c$ we can try theory as well
(not only measurements)

Imagine that there is condensation of a Euclidean scalar
Then there is pole in static correlator:

$$< T_{0i}, T_{0k} >_{\mathbf{q}\to 0,\omega\equiv 0} \sim \frac{q_i q_k}{q^2}(T^3 < \phi_{3d} >^2)$$

(standard signature of superfluidity). In Minkowski:

$$\rho_s \;=\; 0 \quad since \;\; \mu = 0$$

(very specific feature)

Try to guess scalar theory reproducing these conditions

# An exotic liquid

The answer is provided by

$$S = (const) \int d^4x \sqrt{\gamma} \sqrt{(\partial_t \phi)^2 - (\partial_i \phi)^2}$$

where $\gamma_{ab} \equiv r_c d\tau^2 - dx_i^2$ ($r_c$ is a parameter)

$$\phi_{equilibrium} = \tau$$

$$\left(T_{ab}\right)_{equilibrium} = (0, p, p, p)$$

$(p = 1/\sqrt{r_c})$

# Exotic liquid (cn'd)

- Liquid is holographic dual to vacuum Einstein space, Rindler space (Compere et al 1103.3022)
- Upon inclusion dissipation satisfies $\eta/s = 1/4\pi$
- Action $\int d^3x \sqrt{1 + (\partial_i \phi)^2}$ is straightforward to derive. Specific for duality is large pressure $p$.
- similar to the liquid on the stretched horizon of black hole

# Partial check on the lattice

The only relevant measurement on the lattice

$$(\epsilon - 3p)_{non-pert} \; < \; 0$$

and large,

$$|(\epsilon - 3p)_{non-pert}| \; \approx \; 4 \cdot (\epsilon - 3p)_{total}$$

although occupies a tiny fraction of the whole volume

That is, rather in agreement with theory

# Conclusions on condensates properties

- Both 4d and 3d condensate seem to exist and relevant to the confinement and low viscosity of YM plasma, respectively
- theoretical guesses tend to agree with the lattice measurements

# Absence of massless excitations

A generic manifestation of extra dimensions is massless excitations living on the brane (the Luscher term is a well known example)

Could expect massless 3d scalar at $T > T_c$
manifested in the correlator of the momentum components and present in the action derived from duality

By similarity, at $T = 0$ could suspect massless 4d scalar coupled to topological $K_\mu$ current

None is there

# Nonperturbative component and unitarity

At closer look, the 3d scalar turns superluminal
with respect to the perturbative vacuum
and is not carried on to the full theory

The 4d scalar would be ghost-like
(Since non-pert effects lower the energy of the vacuum)

In other words the non-pert component is non-unitary by itself and is incomplete. (recall the Kogut-Susskind ghost)
Some correlators have wrong sign non-pert
In full theory reduce to local terms:

$< G\tilde{G}, G\tilde{G} >$ at $T = 0$ (well known)
and in $< T_{0i}, T_{0k} >$ at $T > T_c$ (new observation)

# Overall conclusions

- Condensates associated with nonperturbative scalars, 4d and 3d, seem to exist and exhibit exciting properties
- Associated with the condensation massless excitations are not observed. Non-perturbative physics cannot be treated as an independent component in this respect