Multiparticle production in QCD at strong coupling

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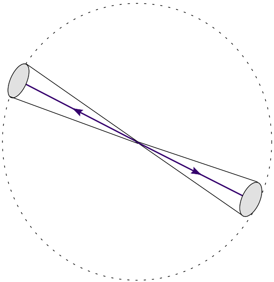
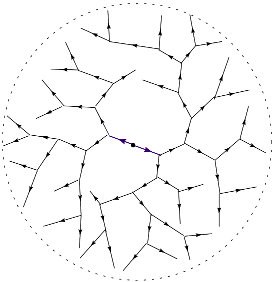
Motivation

- Multiparticle production is a fundamental topic related to the nature of cross sections at asymptotic energies
- Very interesting data on multiparticle production at LHC in pp and AA collisions, more to come.
- Meeting point for ideas from various domains of physics.

Particle production at strong coupling: e^+e^- annihilation





- Left: particle production in the weak coupling regime.
- Right: particle production in the strong coupling regime.

Particle production at strong coupling: e^+e^- annihilation

Y. Hatta, T. Matsuo (2008)

- DGLAP evolution equation:

$$\begin{aligned}\partial_{\ln Q^2} D_{S/T}(j, Q^2) &= \gamma_{S/T}(j) D_{S/T}(j, Q^2) \\ D_{S/T}(j, Q^2) &= \int_0^1 dx\, x^{j-1} D_{S/T}(x, Q^2)\end{aligned}$$

- Relation between spacelike and timelike anomalous dimensions in perturbation theory:

$$\begin{aligned}\gamma_S(j) &= \gamma_T(j - 2\gamma_S(j)) \\ \gamma_T(j) &= \gamma_S(j + 2\gamma_T(j))\end{aligned}$$

- There exist arguments [B. Basso, G. Korchemsky (2007)] that the above relation also holds at strong coupling at $j \gg 1$. Let us assume that it holds for all j.

Particle production at strong coupling: e^+e^- annihilation

Y. Hatta, T. Matsuo (2008)

- In conformal theory multiplicity is controlled by $\gamma_T(1)$:

$$n(Q^2) \propto Q^{2\gamma_T(1)}$$

- Weak coupling:

$$n(Q^2) \propto Q^{\sqrt{\frac{\lambda}{2\pi^2}}}$$

- Strong coupling:

$$2\gamma_T(1) = 1 - \frac{3}{2\sqrt{\lambda}} = J_{\rm Pom} - 1 \quad \Rightarrow \quad n(Q)|_{\lambda\to\infty} \propto Q$$

- Qualitative interpretation: democratic cascading

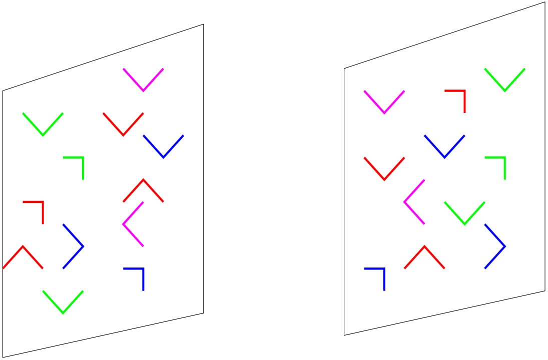
$$Q_i = \frac{Q}{2^i} \quad \Rightarrow \quad Q_N = \frac{Q}{2^N} \sim \Lambda \quad \Rightarrow \quad n(Q^2) \sim 2^N \sim \frac{Q}{\Lambda}$$

Particle production at strong coupling: e^+e^- annihilation

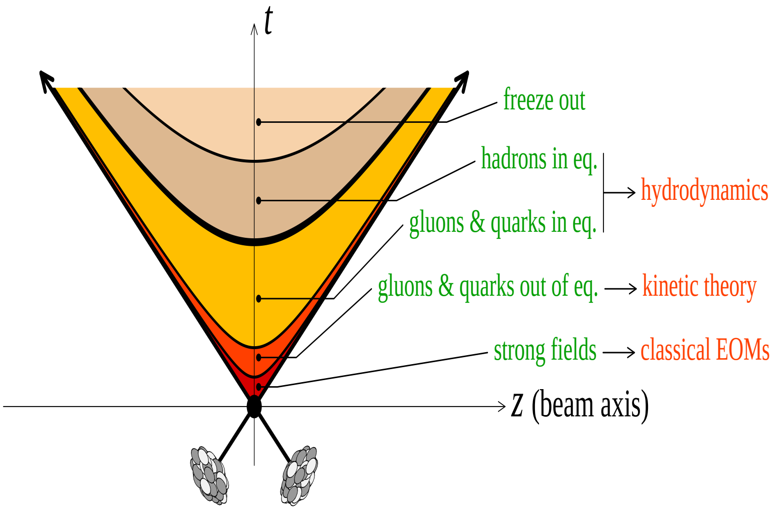
Y. Hatta (2008)

- Weak coupling:
  - Intrajet multiplicity is controlled by collinear logarithms. Multiplicity growth controlled by DGLAP.
  - Interjet multiplicity is controlled by radiation at large angles. Multiplicity growth controlled by BFKL!
- Strong coupling: equivalence between
  - Shock wave description of e^+e^- annihilation
  - Shock wave description of a wave function of fast hadron

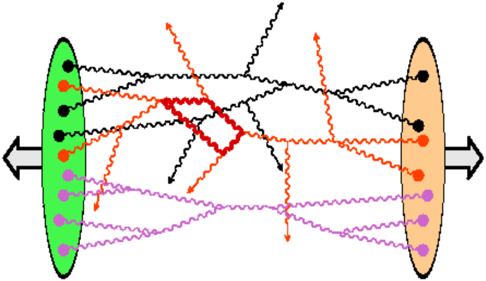
Particle production at weak coupling: nuclear collisions. Initial state.



Particle production at weak coupling: nuclear collisions. Collision stages



Nuclear collisions: classical solution



$$[D_\mu, F^{\mu\nu}] = J^\nu$$

$$J^\mu = \delta^{\mu +}\rho_1(\mathbf{x}_\perp, x^-) + \delta^{\mu +}\rho_2(\mathbf{x}_\perp, x^-)$$

Looking for a solution to all orders in $\rho_{1,2}$

Boost-invariant classical solution

- Coordinates τ, η

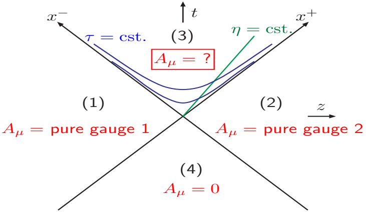
$$x^0 + x^3 = \tau \mathrm{e}^{\eta}, \qquad x^0 - x^3 = \tau \mathrm{e}^{-\eta}$$

- For one source the convenient gauge is $A^{\pm} = 0$
- For two sources the convenient gauge is $A^{\tau} = 0$

$$A_\tau = A^\tau \equiv \frac{1}{\tau}(x^+ A^- + x^- A^+)$$

Boost-invariant solution does not depend on η

Boost invariant classical solution

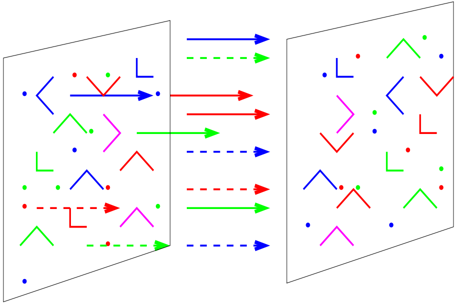


- $$\begin{aligned} A^i &= \theta(-x^+)\theta(x^-)A^i_{(1)} + \theta(x^+)\theta(-x^-)A^i_{(2)} + \theta(x^+)\theta(x^-)A^i_{(3)} \\ A^\eta &= \theta(x^+)\theta(x^-)A^\eta_{(3)} \end{aligned}$$

- consistency condition at $\tau = 0$:

$$\begin{aligned} A^i_{(3)}|_{\tau=0} &= A^i_{(1)} + A^i_{(2)} \\ A^\eta_{(3)}|_{\tau=0} &= \frac{ig}{2}\left[A^i_{(1)}, A^i_{(2)}\right] \end{aligned}$$

Immediately after collision: glasma



$$
\begin{aligned}
E^z &= ig\left[A^i_{(1)}, A^i_{(2)}\right] \\
B^z &= ig\epsilon^{ij}\left[A^i_{(1)}, A^j_{(2)}\right]
\end{aligned}
$$

Initial conditions: hydro

- Equations of motion
$$\partial_\mu T^{\mu\nu} = 0$$
- Equation of state
$$p = f(\epsilon)$$
- Initial conditions are specified at some $\tau = \tau_0$
$$T^{\mu\nu}(\tau = \tau_0, \eta, \mathbf{x}_\perp)$$
- Generic structure of $T^{\mu\nu}$:
$$T^{\mu\nu} = \begin{pmatrix} \epsilon & & & \\ & \frac{\epsilon}{3} & & \\ & & \frac{\epsilon}{3} & \\ & & & \frac{\epsilon}{3} \end{pmatrix}$$

Initial conditions: color glass condensate

For configuration $\mathbf{E}^a_\mu = \lambda \mathbf{B}^a_\mu$

$$\langle T^{\mu\nu}(\tau = 0^+, \eta, \mathbf{x}_\perp) \rangle = \begin{pmatrix} \epsilon & & & \\ & \epsilon & & \\ & & \epsilon & \\ & & & -\epsilon \end{pmatrix}$$

Does not look like hydro nut looks like QCD string model (negative p_z)

glasma tubes	strings
negative p_z	string tension
glasma instabilities	string breaking

Isotropization mechanism?

Initial conditions: color glass condensate

- $$\frac{dN}{d\eta}|_{\eta=0} = c_N \frac{\pi R_A^2 Q_S^2}{\alpha_s}$$
- $$\frac{dE_\perp}{d\eta}|_{\eta=0} = c_E \frac{\pi R_A^2 Q_S^3}{\alpha_s}$$

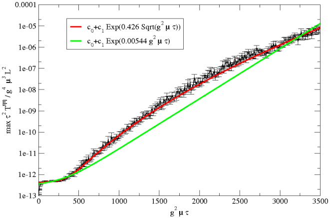
$$\mathrm{HERA} \Rightarrow Q_S^2 \simeq 1.2\,\mathrm{GeV} \Rightarrow \frac{dN}{d\eta}|_{\eta=0} \simeq 1100$$

Glasma instability

- Consider η - dependent perturbations:

$$E_i(0,\eta,\mathbf{x}_\perp) = \delta E_i(\eta,\mathbf{x}_\perp)\ ,$$
$$E_\eta(0,\eta,\mathbf{x}_\perp) = i\,g\,[\alpha_1^i,\alpha_2^i] + \delta E_\eta(\eta,\mathbf{x}_\perp,)\ ,$$

- Maximal Fourier component of longitudinal pressure $P_L = \tau^2 T^{\eta\eta}$



- Deviations from boost invariance generate exponentially growing transverse fields

$$|E_\perp|, |B_\perp| \sim \mathrm{e}^{\sqrt{Q_s\tau}}$$

Initial conditions: quantum evolution. Factorization.

- LO

$$T^{\mu\nu}_{\mathrm{LO}} = \frac{1}{4} g^{\mu\nu} F^{\lambda\sigma} F_{\lambda\sigma} - F^{\mu\lambda} F^{\nu}_{\lambda}$$

$$[F_\mu, F^{\mu\nu}] = J^\nu, \qquad \lim_{t\to-\infty} A^\mu(t, \mathbf{x}) = 0$$

- NLO

$$\delta T^{\mu\nu}_{\mathrm{NLO}} = \left[\ln\left(\frac{\Lambda_0^+}{\Lambda_1^+}\right) \mathcal{H}_1 + \ln\left(\frac{\Lambda_0^-}{\Lambda_1^-}\right) \mathcal{H}_2 \right] T^{\mu\nu}_{\mathrm{LO}}$$

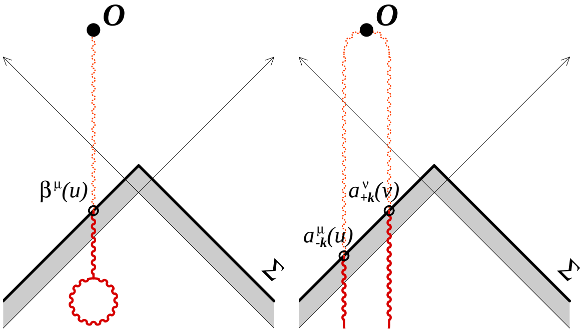
- Quantum corrections can be packed into the evolution of the density matrix

$$\langle T^{\mu\nu}_{\mathrm{LO}} + \delta T^{\mu\nu}_{\mathrm{NLO}} \rangle = \langle T^{\mu\nu}_{\mathrm{LO}} \rangle_{\Lambda_1}$$

- New averaging

$$W_{\Lambda_1^\pm} = \left[1 + \ln\left(\frac{\Lambda_0^\pm}{\Lambda_1^\pm}\right) \mathcal{H}_{1,2} \right] W_{\Lambda_0^\pm}$$

Initial conditions: quantum evolution. Factorization.



- General structure of quantum corrections

$$\langle O\rangle_{\rm NLO} = \left[\frac{1}{2}\int_\Sigma \mathrm{d}^3\mathbf{u}\,\mathrm{d}^3\mathbf{v}\,\mathcal{G}_{\mu\nu}(\mathbf{u},\mathbf{v})\mathbb{T}^\mu_\mathbf{u}\mathbb{T}^\nu_\mathbf{v} + \int_\Sigma \mathrm{d}^3\mathbf{u}\,\beta_\mu(\mathbf{u})\mathbb{T}^\nu_\mathbf{u}\right]\langle O\rangle_{\rm LO}.$$

- $\mathbb{T}_\mathbf{u}$ is a Lie derivative

$$a^\mu(x) = \int_{\vec{\mathbf{u}}\in\Sigma} a(\vec{\mathbf{u}})\cdot\mathbb{T}_\mathbf{u}\mathcal{A}^\mu(x)$$

Glasma instabilities: resummation

- Resummation:

$$\langle O\rangle_{\mathrm{res}} = \exp\left[\frac{1}{2}\int_{\Sigma}\mathrm{d}^3\mathbf{u}\,\mathrm{d}^3\mathbf{v}\,\mathcal{G}_{\mu\nu}(\mathbf{u},\mathbf{v})\mathbb{T}^{\mu}_{\mathbf{u}}\mathbb{T}^{\nu}_{\mathbf{v}} + \int_{\Sigma}\mathrm{d}^3\mathbf{u}\,\beta_{\mu}(\mathbf{u})\mathbb{T}^{\nu}_{\mathbf{u}}\right]\langle O\rangle_{\mathrm{LO}}$$

- Useful identity:

$$\exp\left[\frac{1}{2}\int_{\Sigma}\alpha(\mathbf{u})\mathbb{T}_{\mathbf{u}}\right]\langle O\rangle_{\mathrm{LO}}[\phi_0] = O[\phi_0+\alpha]$$

- Proposed resummation is equivalent to Gaussian averaging over fluctuations of initial field configuration:

$$O_{\mathrm{res}} = \exp\left[\frac{1}{2}\int_{\Sigma}\mathrm{d}^3\mathbf{u}\,\mathrm{d}^3\mathbf{v}\,\mathcal{G}(\mathbf{u},\mathbf{v})\mathbb{T}_{\mathbf{u}}\mathbb{T}_{\mathbf{v}} + \int_{\Sigma}\mathrm{d}^3\mathbf{u}\,\beta(\mathbf{u})\mathbb{T}_{\mathbf{u}}\right]O_{\mathrm{LO}}[\phi_0]$$

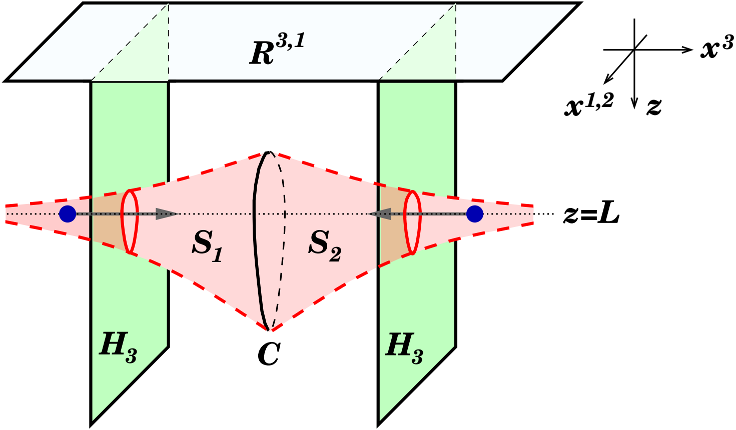
$$= \int\mathcal{D}\alpha\,\exp\left[-\frac{1}{2}\int_{\Sigma}\mathrm{d}^3\mathbf{u}\,\mathrm{d}^3\mathbf{v}\,\alpha(\mathbf{u})\mathcal{G}^{-1}(\mathbf{u},\mathbf{v})\alpha(\mathbf{v})\right]O[\phi_0+\alpha+\beta]$$

Particle production at strong coupling: nuclear collisions

- Particle production from thermally equilibrated fireball created in high energy nuclear collisions.
- In AdS language formation of this fireball (thermalization) is described by formation of AdS black hole in a collision of two high energy objects
- Multiplicity is proportional to the surface area of the black hole.
- In practice one calculates the area of the trapped surface (surface with null normals going inwards).

Particle production at strong coupling: nuclear collisions

S. Gubser, S. Pufu, A. Yarom (2009)



- Formation of a trapped surface in a high energy collision of two particles

Particle production at strong coupling: nuclear collisions

S. Gubser, S. Pufu, A. Yarom (2009)

- The shock wave is a solution of 5-d Einstein equations in AdS

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \frac{6}{L^2} g_{\mu\nu} = 8\pi G_5 J_{\mu\nu}$$

with the source

$$J_{--} = E \frac{z^3}{L^3} \delta(x^1) \delta(x^2) \delta(z - z_*) \delta(x^-)$$

- Solution:

$$ds^2 = ds^2_{AdS_5} + \frac{L}{z} \Phi(x^1, x^2, z) \delta(x^-) (dx^-)^2$$

Particle production at strong coupling: nuclear collisions

- Solution:

$$\Phi = \frac{G_5 E z_*}{8L^2} \frac{1}{q^3} \, {}_2F_1(3, 5/2; 5; -1/q)$$

where

$$q = \frac{(x^1)^2 + (x^2)^2 + (z - z_*)^2}{4 z z_*}$$

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$$\langle T_{--} \rangle = \frac{L^2}{4\pi G_5} \delta(x^-) \lim_{z \to 0} \frac{\Phi(x^1, x^2, z)}{z^3} = \frac{2 E z_*^4}{\pi (x_\perp^2 + z_*^2)^3} \delta(x^-)$$

-

$$\int d^3x \, \langle T_{--} \rangle = E \qquad \text{and} \qquad \frac{\int d^3x \, x_\perp^2 \langle T_{--} \rangle}{\int d^3x \, \langle T_{--} \rangle} = z_*^2$$

Particle production at strong coupling: nuclear collisions

- Peripheral collision of two shock waves:

$$ds^2_{AdS_5} + \frac{L}{z}\Phi_-(x^1,x^2,z)\delta(x^-)(dx^-)^2 + \frac{L}{z}\Phi_+(x^1,x^2,z)\delta(x^+)(dx^+)^2$$

where

$$\begin{aligned}
\Phi_\pm &= \frac{G_5 E_\pm z_\pm}{8L^2}\frac{1}{q_\pm^3}\,{}_2F_1(3,5/2;5;-1/q_\pm) \\
q_\pm &= \frac{(x^1-b_\pm)^2+(x^2)^2+(z-z_\pm)^2}{4zz_\pm}
\end{aligned}$$

- $$\langle T_{--}\rangle = \frac{2E_- z_-^4}{\pi\left[(x^1-b_-)^2+(x^2)^2+z_-^2\right]^3}\delta(x^-)$$

- $b_\pm$ mimick impact parameters

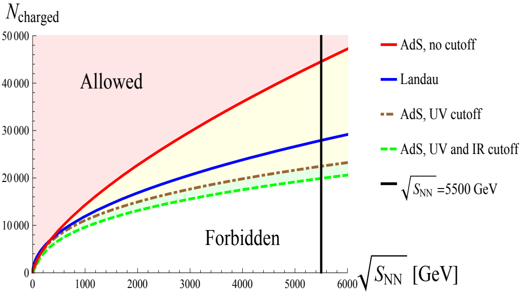
Particle production at strong coupling: nuclear collisions

- Solution: area of the trapped surface

$$A_{\rm trapped} = 4\pi G_5 \left(\frac{4E^2 z_*^2}{G_5/L^3}\right)^{1/3} \frac{\sinh^{-1}\beta}{\beta\sqrt{1+\beta^2}} \qquad \beta = b/2z_*$$

- Solution: entropy

$$S \geq S_{\rm trapped} \equiv \frac{A_{\rm trapped}}{4G_5} = \left(\frac{4E^2 z_*^2}{G_5/L^3}\right)^{1/3} \frac{\sinh^{-1}\beta}{\beta\sqrt{1+\beta^2}}$$



Particle production at strong coupling: nuclear collisions

Problems to solve

- Confinement scale should have an adequate representation
- Transverse scale (saturation momentum) should have an adequate representation
- Impact parameter profile controlling inelastic cross section and ensuring Froissart bound should have an adequate representation.

- There is clear;y a long way to realistic theory. Still the existing state of art shows many interesting leads to follow and looks promising!