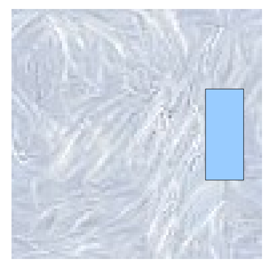
Supersymmetric QCD, Superconducting gaps and cyclic RG flows

A. Gorsky, to appear



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There is classical integrable system behind
the low energy sector of
N=2 supersymmetric gauge theories (GKMMM,95)

The number of degrees of freedom is fixed by the rank
of the gauge group SU(N)

What is the physics behind the integrable dynamics and
what is the meaning of its quantization?

We know that the color confinement implies the condensation of the magnetic degrees of freedom.

Which ones? Monopoles,strings or walls?

Propotype idea behind the integrable dynamics. Consider The propagation of the excitation at the top of the Inhomogenious condensate.

Integrability is a kind of consistency condition — the propagation of the particle excitation above the condensate does not destroy it. On the other hand the dispersion low for the excitation is stable under the integrable evolution of the condensate.

The simplified models of BCS-like superconductivity.
Finite number of degrees of freedom

Peierls model of 1D superconductivity — periodic Toda chain -
pure SUSY YM (A.G. 96)

Richardson model — generalized Gaudin model-
Supersymmetric QCD

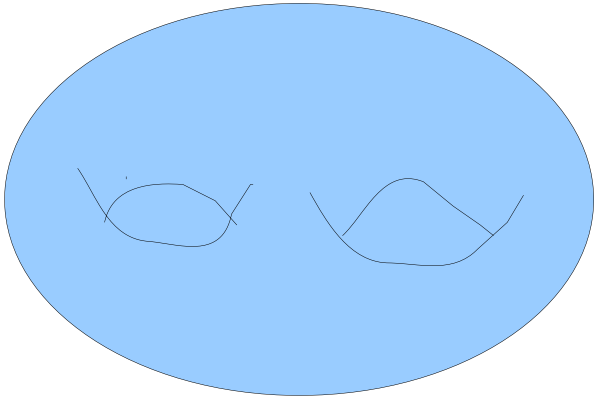
Russian Doll model — quantum twisted spin chain-
Supersymmetric QCD in Omega-background

Peierls model

Fermions at 1d lattice(=condensate) interacting
with phonons. Lax operator = Hamiltonian of the fermions.

$$H_{dis} = \sum_n \Psi_n^+ v_n \Psi_n + \Psi_n^+ c_n \Psi_{n+1} + \Psi_n^+ c_{n-1} \Psi_{n-1} + \sum_i \kappa_i I_i$$

$$c_n = exp(x_{n+1} - x_n) \qquad c_n \Psi_{n+i} + c_{n-1}\Psi_{n-1} + v_n \Psi_n = E\Psi_n$$



$$\Psi_{n+N}(E) = e^{iNp(E)}\Psi_n(E).$$

The dispersion law of the fermions F(p,E)=0 defines the Riemann surface. The gap equation follows from the minimization of the total Hamiltonian.

Richardson model- finite number N of electron levels and finite number of interacting Cooper pairs M

$$H_{BCS} = \sum_{j,\sigma=\pm} \epsilon_{j\sigma} c^{+}_{j\sigma} c_{j\sigma} - G \sum_{jj'} c^{+}_{j+} c^{+}_{j-} c_{j-} c_{j+}$$

The eigenfunctions of the Hamiltonian

$$|M >= \prod_{i}^{M} B_i |vac >, \qquad B_i = \sum_{j}^{N} \frac{1}{\epsilon_j - E_i} b^{+}_{j}$$

Equipped with the Bethe anzatz equations=BA for generalized Gaudin model

$$G^{-1} = \sum_{j}^{N} \frac{1}{\epsilon_j - E_i} - \sum_{j}^{M} \frac{1}{E_j - E_i}$$

(Witten,Gaiotto 11)

Russian Doll model of superconductivity (LeClair,Roman,Sierra,03-05)

The Hamiltonian of the model

$$H_{RD} = 2\sum^{N} \epsilon_i N_i - g \sum_{j<k} (e^{i\alpha} b_k^+ b_j + e^{-i\alpha} b_j^+ b_k)$$

$$[b_j^+ b_j'] = \delta_{ij}(2N_j - 1), \qquad b_j = c_{j-}c_{j+}, \qquad N_j = b_j^+ b_j$$

Bethe anzatz equations

$$\exp(-2i\alpha) \prod^{N} \frac{E_j - \epsilon_k - i\eta/2}{E_j - \epsilon_k + i\eta/2} = \prod^{M} \frac{\epsilon_j - \epsilon_k - i\eta}{\epsilon_j - \epsilon_k + i\eta}$$

$$\eta = g\sin(\alpha)$$

The key feature of the RD model is the infinite number of solutions to the gap equations.The infinite number of gaps are parameterized as follows

$$\Delta_n = \frac{\omega}{sinht_n}, \quad t_n = t_0 + \frac{\pi n}{\alpha} \tag{8.39}$$

where t_0 is solution to the following equation

$$tan(\theta t_0) = \frac{\theta}{g} \tag{8.40}$$

It has very unusual cyclic RG behavior! The examples of the cyclic RG behavior has been found in the system with finite number degrees of freedom(Glazek-Wilson 02) and In the field theory (LeClair, Sierra 03). In all cases there are additional resonance states in these models with the Regge-like spectrum.

The RG equation in the Russian Doll model corresponds to the decoupling of the single electronic level. The equation reads as.

$$g_{N-1} = g_N + \frac{1}{N}(g_N^2 + \theta^2), \theta_{N-1} = \theta_N$$

With the solution

$$g(s) = \theta tan(\theta s + tan^{-1}(\frac{g_0}{\theta}))$$

$$g(s+\lambda) = g(s), \qquad g(e^{-\lambda}N) = g(N)$$

With the period determind the «quantum parameter»

$$\lambda = \frac{\pi}{\theta}$$

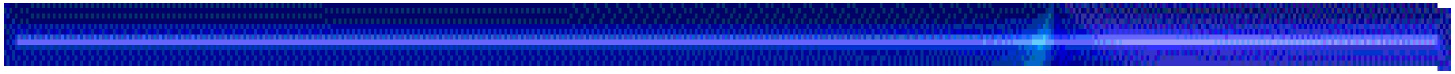
	Cyclic sine-Gordon	Russian doll BCS
RG-time	$L = e^{l} a$	$N(s) = e^{-s} N$
RG-period	$\lambda_{CSG} = \frac{2\pi}{h}$	$\lambda_{BCS} = \frac{\pi}{h}$
Energy scales	Resonances $m_n(L) \sim \frac{1}{L} e^{n\pi/h}$	Condensates $\Delta_n(N) \sim N A e^{-n\pi/h}$
Russian doll scaling	$m_n(e^{-\lambda_{CSG}} L) \approx m_{n+2}(L)$	$\Delta_n(e^{-\lambda_{BCS}} N) \approx \Delta_{n+1}(N)$
Finite systems	$n_{\rm res} \sim \frac{h}{\pi} \log(L/a)$	$n_c \sim \frac{h}{\pi} \log N$

What is going with the spectrum of the model during the RG cycle?

When solving the Bethe Anzatz equation there is the freedom corresponding to the selection of the branch of the logarithm. This freedom corresponds to the selection of the set of the numbers {N}

It turns out that these numbers get shifted during the cycle N -> N+1 corresponding to the change of the branch

Nonabelian strings and monopoles in Supersymmetric QCD



Hanany-Tong, Konishi-Auzzi-Evslin-Yung, Shifman-Yung 03-04

Solution to the classical equations of motion at the Higgs branch with very rich worldsheet theory. Contrary to the conventional ANO strings there are purely nonabeian degrees of freedom at the string worldsheet.

The nonabelian structure on the wourldsheet amounts to the kink-like excitations which are the 4d monopoles captured by the nonabelian string

Deformation of SUSY QCD by selfdual graviphoton background yields the quantization of the integrable system (Nekrasov,Shatashvili 09-10)

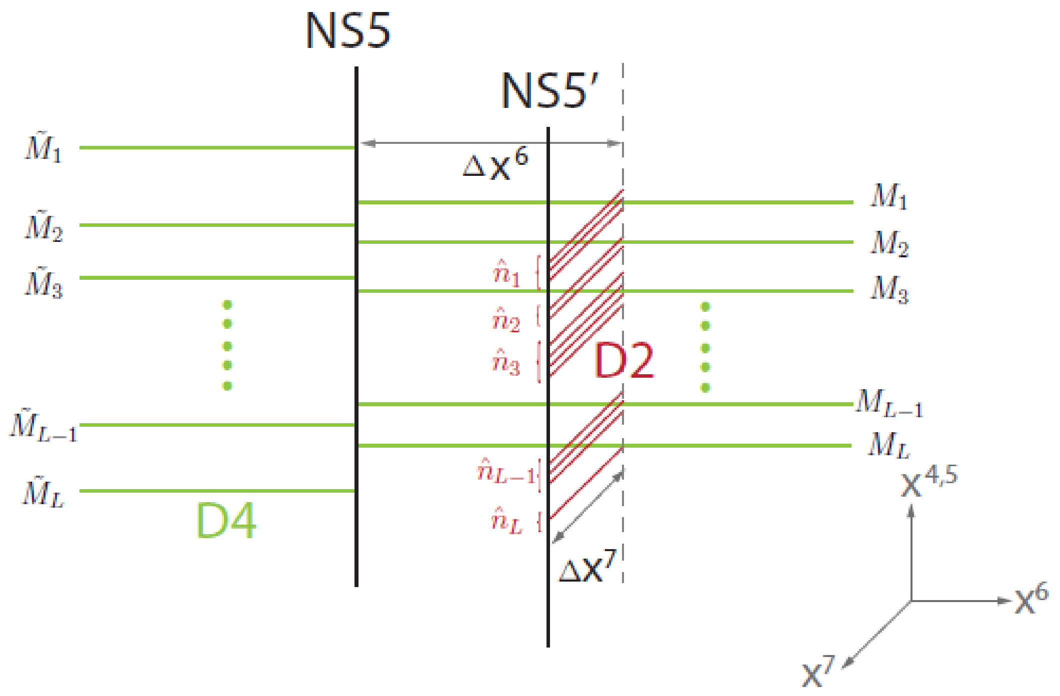
The deformed prepotential is derived via the following limit from Nekrasov partition function

$$\mathcal{F}(\vec{a},\epsilon) = \lim_{\epsilon_2\to 0}\left[\epsilon_1\epsilon_2 \log \mathcal{Z}(\vec{a},\epsilon_1,\epsilon_2)\big|_{\epsilon_1=\epsilon}\right] .$$

The theory has discrete set of vacua determined by minimization of the superpotential

$$\mathcal{W}^{(I)}(\vec{a},\epsilon) = \frac{1}{\epsilon}\mathcal{F}(\vec{a},\epsilon) - 2\pi i\,\vec{k}\cdot\vec{a} \qquad \frac{\partial \mathcal{W}^{(I)}}{\partial \vec{a}} = 0$$

There are nonabelian strings represented by D2 branes



The nontrivial matching conditions between 2d worldsheet Theory and 4d bulk theory

- beta function for the coupling constant
- BPS spectrum
- superpotential

$$\vec{M}_F = \vec{m}_F - \frac{3}{2}\vec{\epsilon}\,, \qquad \vec{M}_{AF} = \vec{m}_{AF} + \frac{1}{2}\vec{\epsilon}\,.$$

The extremization of the superpotential yields
the Bethe Anzatz equations for the positions
f the nonabelian strings

$$\prod_{l=1}^{L}\left(\frac{x_j-\theta_l-is_l\hbar}{x_j-\theta_l+is_l\hbar}\right) = q\prod_{k\neq j}^{N}\left(\frac{x_j-x_k+i\hbar}{x_j-x_k-i\hbar}\right)$$

The inhomogenities are identified with the
quark masses, the «Planck constant» with the
Graviphoton field and the twisting parameter
With the coupling constant . The number
of the Bethe root coincides with the number
of the nonabelian strings(Dorey,Lee,Hollowood 10)

The Bethe Anzatz equations for the Russian Doll Superconductivity Model and the deformed SUSY QCD coincide!

The identifications

- the one-particle energies in the RD model= quark masses

- the number of Cooper pairs= number of nonabelian strings

- the real interaction parameter= coupling constant in SQCD

- the imaginary parameter in RD model= graviphoton field

But we know the unusual cyclic RG behavior in RD model!

What is the RG step interpretation in SQCD?

Answer; The decoupling of the single flavour
Sending its mass M to infinity

There is periodicity in the number of flavors! Effectively the set of nonperturbative scales instead of the single one emerges!

$$\Delta_n(N) \sim NA\delta e^{-n\pi/h}$$

Such behaviour is due to two coupling constants in the model. The number of cycles is fixed by the value of the graviphoton field.

What happens with the spectrum of the model upon the single RG cycle?

The magnetic nonabelian string becomes dyonic!

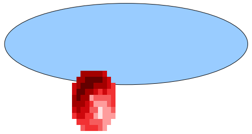
Hence we get upon the RG cycle the same spectrum of the BPS strings but with nontrivial identifications.

More phenomena are expected when the monopoles are included. Possible relation with the Seiberg duality in the worldsheet description (Shifman-Yung, 10)

Possible relation with the unexpected cyclic structure of the marginal stability curves in the worldsheet theory (Shifman-Yung 11)

Is the graviphoton deformation of the SQCD artificial?

Probably NO. The source of the graviphoton vector field is D0 brane — instanton. Hence the generation of the graviphoton field should be due to some collective instanton phenomena.



Good candidate- dyonic instanton. It generates the same effect as the instanton in the graviphoton field (A.G. 11)

However there are the troubles with the spontaneous violation of the Lorentz invariance. In the graviphoton background it is violated explicitly while the dionic instanton amounts to the spontaneous violation.

Conclusion

- The spectrum of the holomorphic sector of the deformed SQCD is equivalent to the spectrum of the Russian Doll model of superconductivity. Strings as Cooper pairs!

- Due to the cyclic RG behaviour in RD model the cyclic RG in the deformed SQCD. Interesting rearrangement of the spectrum at each step. Possible relation with Seiberg duality.

- Multiple scales. Possible role of the dyonic instantons instead of the external graviphoton field.