Non-Renormalization of Axial Anomaly in Perturbation Theory and Beyond

Advances of QFT (DIK-60)

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Outline

Recollection on 3 loop “renormalization” of Anomaly and its interpretation (80’s)

Anomaly sum rules (example – anomaly for virtual photons and transition formfactors)

in collaboration with

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Phys.Lett.B695:130-135,2011.
e-Print: **arXiv:1009.1120** [hep-ph]

Phys. Rev. D 84, 051901(R) (2011).
e-Print: **arXiv:1106.3855** [hep-ph]

Symmetries and conserved operators

- (Global) Symmetry -> conserved current ($\partial^{\mu} J_{\mu} = 0$)
- Exact:
- U(1) symmetry – charge conservation - electromagnetic (vector) current
- Translational symmetry – energy momentum tensor $\partial^{\mu} T_{\mu\nu} = 0$

Massless fermions (quarks) – approximate symmetries

- Chiral symmetry (mass flips the helicity)

$$\partial^{\mu} J^{5}{}_{\mu} = 0$$

- Dilatational invariance (mass introduce dimensional scale – c.f. energy-momentum tensor of electromagnetic radiation)

$$T_{\mu\mu} = 0$$

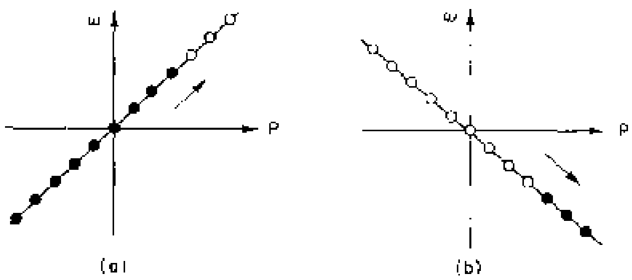
Quantum theory

- Currents -> operators
- Not all the classical symmetries can be preserved -> anomalies
- Enter in pairs (triples?...)
- Vector current conservation <-> chiral invariance
- Translational invariance <-> dilatational invariance

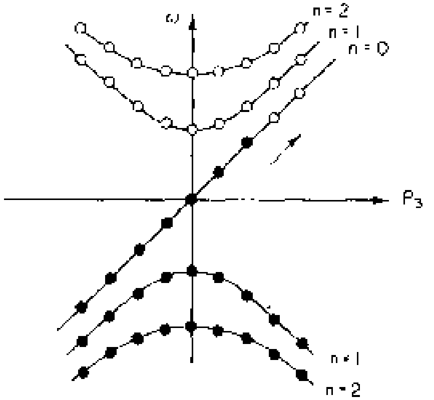
Calculation of anomalies

- Many various ways
- All lead to the same operator equation

$$\partial^\mu j^{(0)}_{5\mu} = 2i\sum_q m_q \bar{q}\gamma_5 q - \left(\frac{N_f\alpha_s}{4\pi}\right) G^a_{\mu\nu}\tilde{G}^{\mu\nu,a}$$



- UV vs IR languages- understood in physical picture (Gribov, Feynman, Nielsen and Ninomiya) of Landau levels flow (E||H)



Some history

- **Radiative Corrections To The Axial Anomaly.**
  A.A. Anselm, (Birkbeck Coll.) , A.A. Johansen, (St. Petersburg, INP) .
  Print-89-0622 (BIRKBECK), Jul 24, 1989. (Published Jul 1989). 16pp.
  Published in **JETP Lett.49:214-218,1989, Sov.Phys.JETP 69:670-682,1989, Pisma Zh.Eksp.Teor.Fiz.49:185-189,1989**.
- Cited 38 times

- **In the course of our work on anomaly for nucleon spin structure -**
- **On Renormalization Of Axial Anomaly.**
  A.V. Efremov, O.V. Teryaev, (Dubna, JINR) . JINR-E2-89-392, Jun 1989. 4pp.
  Published in **Sov.J.Nucl.Phys.51:943-944,1990, Yad.Fiz.51:1492-1494,1990**.
- Cited 6 times
- My seminar at BLTP (chaired by DIK) in the role of devil's attorney...
- Elucidating discussions with Dima

What (non-)renormalization means?

- Extra power of coupling in K
- Non-conservation ->MULTIPLICATIVE (K is gauge-dependent) renormalization of J
- Various ways to calculate the same anomalous dimension

$$\partial\mu(\mathrm{J}-K)\mu=0$$

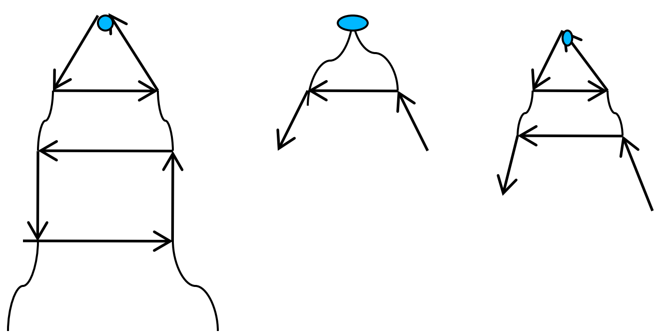
$$\dot{J}=\gamma J$$

$$\partial\dot{J}=\gamma\partial J-<e|....|e'>-2loops(\mathrm{Adler'69})$$

$$\partial\dot{J}=\gamma\partial K-<\gamma|....|\gamma'>-3loops(AJ\,'89)$$

$$\partial\dot{K}=\gamma\partial J-<e|....|e'>-1loop(DGLAP'70s)$$

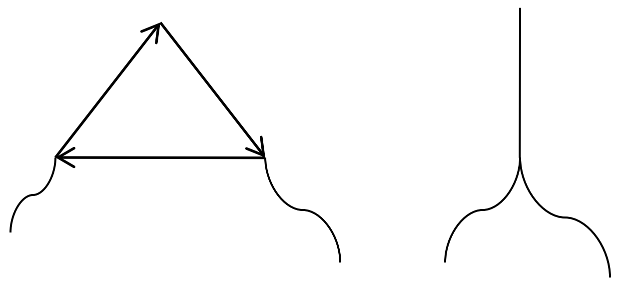
$$\partial\dot{K}=\gamma\partial K-<\gamma|....|\gamma'>-2loops(nobody\,?)$$



- ‘89 70’s ‘69

NP stability of anomaly – t'Hooft consistency principle

- Zero quark mass pole
- Baryons are massive ->
- Massless pion (complementary to NG – stable against small corrections
- Virtual photon – no pole in quark traingle, but puion pole persists – t'Hooft principle questioned (Achasov 90's)



- Principle is valid, but no pole (Horejsi, OT '95)!

Anomaly and virtual photons

- Often assumed that only manifested in real photon amplitudes
- Not true – appears at any Q^2
- Natural way – dispersive approach to anomaly (Dolgov, Zakharov'70) - anomaly sum rules
- One real and one virtual photon – Horejsi,OT'95

$$\int_{4m^2}^{\infty} A_3(t;q^2,m^2)dt = \frac{1}{2\pi}$$

- where

$$F_j(p^2) = \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{A_j(t)}{t-p^2}dt, \qquad j = 3,4$$

$$\begin{aligned} T_{\alpha\mu\nu}(k,q) \;&=\; F_1\,\varepsilon_{\alpha\mu\nu\rho}k^\rho + F_2\,\varepsilon_{\alpha\mu\nu\rho}q^\rho \\ &+\; F_3\,q_\nu\varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma + F_4\,q_\nu\varepsilon_{\alpha\mu\rho\sigma}k^\rho q^\sigma \\ &+\; F_5\,k_\mu\varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma + F_6\,q_\mu\varepsilon_{\alpha\nu\rho\sigma}k^\rho q^\sigma \end{aligned}$$

Dispersive derivation

- Axial WI $$F_2 - F_1 = 2mG + \frac{1}{2\pi^2}$$
- GI $$F_2 - F_1 = (q^2 - p^2)F_3 - q^2 F_4$$
- No anomaly for imaginary parts

$$(q^2 - t)A_3(t) - q^2 A_4(t) = 2mB(t)$$

$$F_j(p^2) = \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{A_j(t)}{t - p^2} dt, \qquad j = 3, 4$$

- Anomaly as a finite subtraction

$$F_2 - F_1 - 2mG = \frac{1}{\pi}\int_{4m^2}^{\infty} A_3(t) dt$$

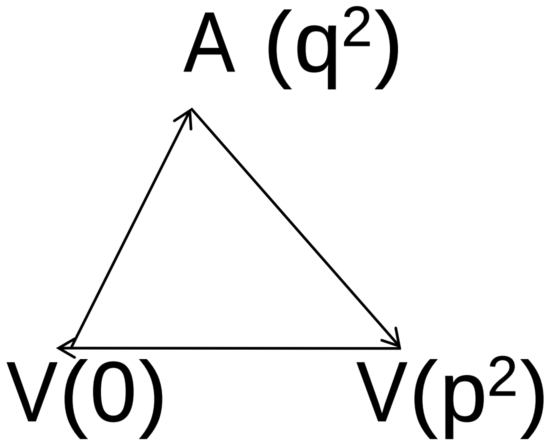
$$\int_{4m^2}^{\infty} A_3(t; q^2, m^2) dt = \frac{1}{2\pi}$$

Properties of anomaly sum rules

- Valid for any Q^2 (and quark mass)
- No perturbative QCD corrections (Adler-Bardeen theorem)
- No non-perturbative QCD correctioons (t'Hooft consistency principle)
- Exact – powerful tool
- For $Q^2=0$ – pole (no pole required for extra dimesnionful parameter – temperature, chemical potential?)

Anomaly in axial and vector channels

- Dispersion relations in q^2 or p^2



- Results coincide
- 2 ASRs – V-A global duality
- Local duality – Vainshtein relation ('02) between L and T (not related to anomaly!) V-A correlators in external field
- Corrections... AdS/QCD - Son, Yamamoto

Mesons contributions (Klopot, Oganesian, OT)

Phys.Lett.B695:130-135,2011 (1009.1120) and 1106.3855

- Pion – saturates sum rule for real photons $ImF_3 = \sqrt{2} f_\pi \pi F_{\pi\gamma\gamma*}(Q^2)\delta(s - m_\pi^2)$ $F_{\pi\gamma^*\gamma}(0) = \frac{1}{2\sqrt{2}\pi^2 f_\pi}$
- For virtual photons – pion contribution is rapidly decreasing $F_{\pi\gamma\gamma^*}^{\mathrm{asymp}}(Q^2) = \frac{\sqrt{2} f_\pi}{Q^2} + \mathcal{O}(1/Q^4)$
- This is also true also for axial and higher spin mesons (longitudianl components are dominant)
- Heavy PS decouple in a chiral limit

Anomaly as a collective effect

- One can never get constant summing finite number of decreasing function
- Anomaly at finite Q^2 is a collective effect of meson spectrum
- General situation –occurs for any scale parameter (playing the role of regulator for massless pole)
- For quantitative analysis – quark-hadron duality

Mesons contributions within quark hadron duality – transition FFs

- Pion:

$$F_{\pi\gamma\gamma*}(Q^2) = \frac{1}{2\sqrt{2}\pi^2 f_\pi}\frac{s_0}{s_0+Q^2}$$

- Cf Brodsky&Lepage, Radyushkin – comes now from anomaly!
- Axial mesons contribtion to ASR

$$\int_0^\infty A_3(s;Q^2)ds = \frac{1}{2\pi} = I_\pi + I_{a_1} + I_{cont}. \qquad I_{a_1} = \frac{1}{2\pi}Q^2\frac{s_1-s_0}{(s_1+Q^2)(s_0+Q^2)}$$

Content of Anomaly Sum Rule ("triple point")

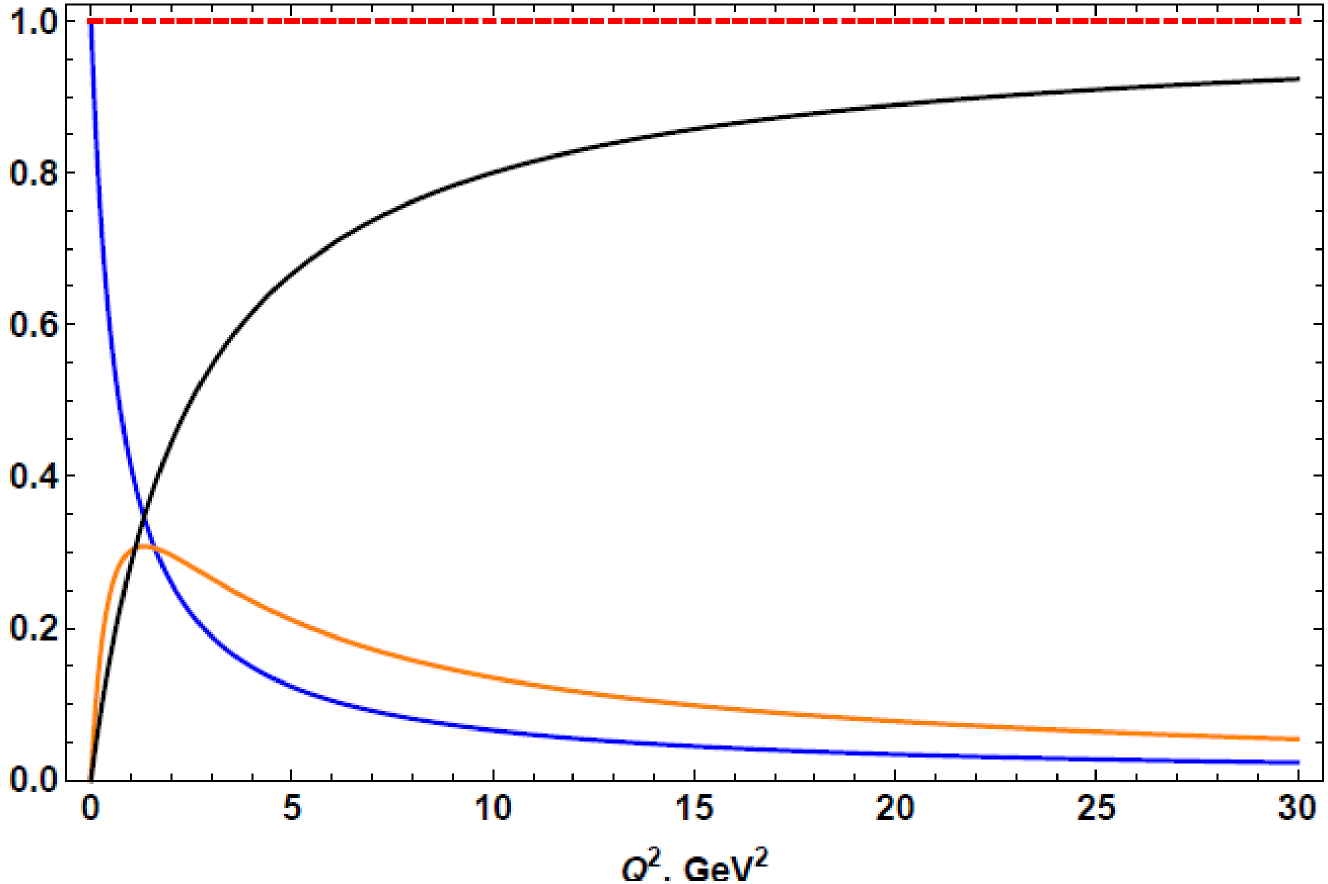


Figure 1: Relative contributions of π (blue line) and a_1 (orange line) mesons, intervals of duality are $s_0 = 0.7\ GeV^2$ and $s_1 - s_0 = 1.8\ GeV^2$ respectively, and continuum (black line), continuum threshold is $s_1 = 2.5\ GeV^2$

ASR and BaBar data

- In the BaBar(2009) region – main contribution comes from the continuum
- Small relative correction to continuum –due to exactness of ASR must be compensated by large relative contributions to lower states!
- Amplification of corrections

$$\frac{\delta I_{cont}/I^0_{cont}}{\delta I_\pi/I^0_\pi} = \frac{s_0}{Q^2} \simeq \frac{1}{30} \quad Q^2 = 20\ GeV^2,\ s_0 = 0.7\ GeV^2$$

- Smaller for eta because of larger duality interval (supported by BaBar)

Corrections to Continuum

- Perturbative – zero at 2 loops level (massive-Pasechnik&OT – however cf Melnikov; massless-Jegerlehner&Tarasov)
- Non-perturbative (e.g. instantons, short strings)
- The general properties of ASR require decrease at asymptotically large Q^2
  (and $Q^2=0$)

$$I_{cont} = \frac{1}{2\pi}\frac{Q^2}{s_0+Q^2} - c s_0 \frac{\ln(Q^2/s_0)+b}{Q^2},$$

$$I_{\pi} = \frac{1}{2\pi}\frac{s_0}{s_0+Q^2} + c s_0 \frac{\ln(Q^2/s_0)+b}{Q^2}.$$

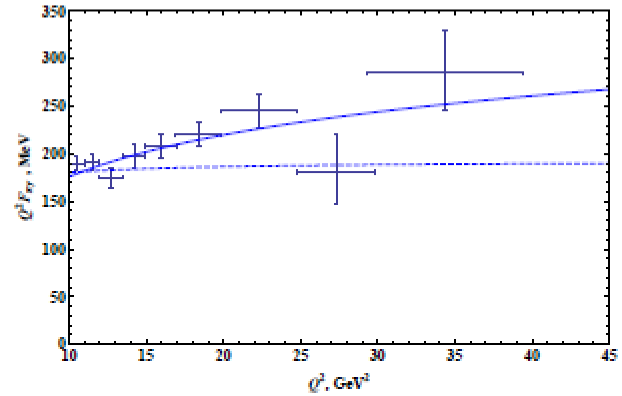
- Corresponds to logarithmic contribution
  (cf Radyushkin, Polyakov, Dorokhov).

Modelling of corrections

- Continuum vs pion

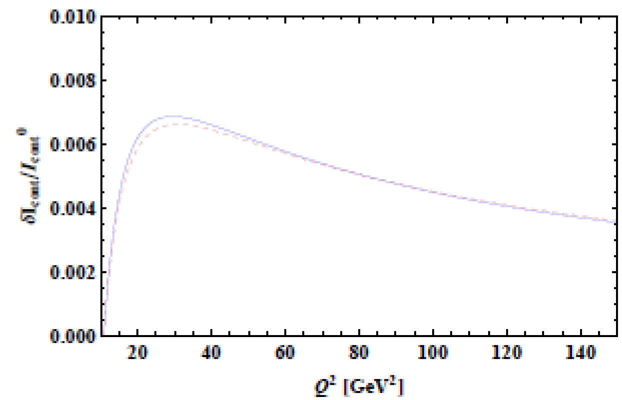
$$I_{cont} = \frac{1}{2\pi}\frac{Q^2}{s_0+Q^2} - cs_0\frac{\ln(Q^2/s_0)+b}{Q^2},$$

$$I_{\pi} = \frac{1}{2\pi}\frac{s_0}{s_0+Q^2} + cs_0\frac{\ln(Q^2/s_0)+b}{Q^2}.$$



- Fit $b = -2.74, \quad c = 0.045.$

- Continuum contribution similar for Radyushkin's approach

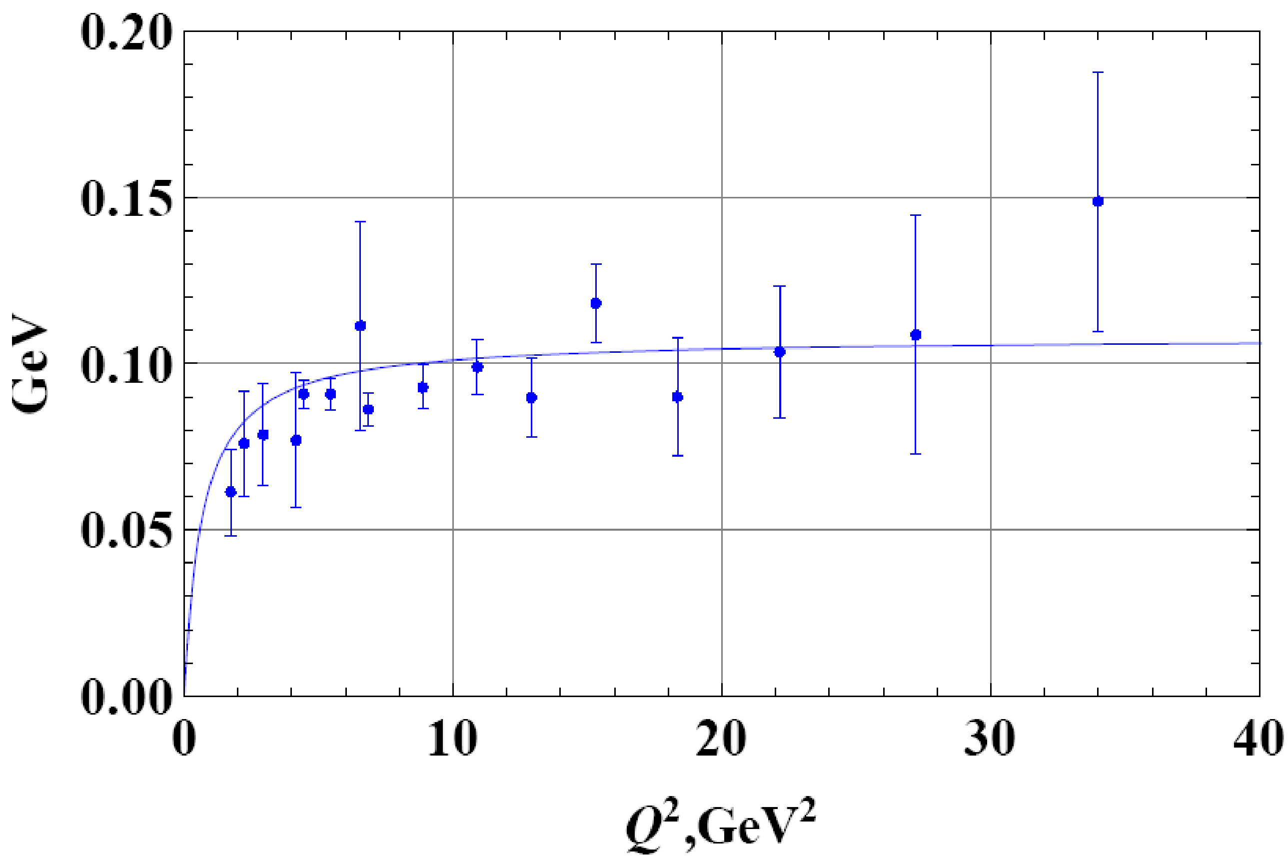


Interplay of pion with lower resonances

- Small (NP) corrections to continuum – interplay of pion with higher states
- A1 – decouples for real photons
- Relation between transition FF's of pion and A1 (testable!)

Generalization for eta(')

- Octet channel sum rule (gluon anomaly free)



Conclusions/Discussion-I

- New manifetsation of Axial Anomaly - Anomaly Sum Rule – exact NPQCD tool- do not require QCD factorization
- Anomaly for virtual photons – collective effect (with fast excitation of collective mode)
- Similar collective effect is expected for finite temperature and/or chemical potential
- Exactness of ASR – very unusual situation when small pion contribution can be studied on the top of large continuum – amplification of corrections to continuum
- BaBar data – small negative correction to continuum
- If continuum is precisely described by Born term– interplay with A1 (TO BE STUDIED THEORETICALLY AND EXPERIMENTALLY)

HAPPY

BIRTHDAY!