NEW FORMULATION OF ELECTROWEAK MODELS

APPLICABLE BEYOND PERTURBATION THEORY.

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Equivalence theorems (A.A.S., 1991): Path integral representation for the scattering matrix

$$S = \int \exp\{i \int L(\varphi) dx\} d\mu(\varphi); \quad lim_{t \to \pm\infty} \varphi(x) = \varphi_{out,in}(x) \tag{1}$$

Let us make the change of variables depending on the time derivatives of the field:

$$\varphi \to \varphi'(\varphi, \dot{\varphi}) \tag{2}$$

If the change (2) does not change the asymptotic conditions, then the only effect of such transformation is the appearance of a nontrivial jacobian

$$L(\varphi) \to \tilde{L}(\varphi') = L[\varphi(\varphi')] + \bar{c}^a \frac{\delta \varphi^a}{\delta \varphi'^b} c^b \tag{3}$$

Unitarity?

The new Lagrangian is invariant with respect to the supertransformations

$$
\begin{aligned}
\delta\varphi'_a &= c_a\varepsilon \\
\delta c_a = 0; \quad \delta\bar{c}_a &= \frac{\delta L}{\delta\varphi_a}(\varphi')\varepsilon
\end{aligned}
\tag{4}
$$

On mass shell these transformations are nilpotent and generate a conserved charge Q. In this case there exists an invariant subspace of states annihilated by Q, which has a semidefinite norm. For asymptotic space this condition reduces to

$$
Q_0|\phi>_{as} = 0 \tag{5}
$$

The scattering matrix is unitary in the subspace which contains only excitations of the original theory. However the theories described by the L and the $\tilde{L}$ are different, and only expectation values of the gauge invariant operators coincide.

Using this method one can construct a renormalizable formulation of nonabelian gauge theories free of the Gribov ambiguity.

A.A.Slavnov,JHEP,0808(2008)047; Theor.Math.Phys,161(2009)204; A.Quadri, A.A.Slavnov, JHEP, 1007 (2010); A.Quadri, A.A.Slavnov, Theor.Math.Phys, 166(2011)201

To deal with gauge theories one should impose a gauge condition.

Differential gauge conditions: $L(A_\mu, \varphi) = 0 \rightarrow$ Gribov ambiguity.

Algebraic gauge conditions: $\tilde{L}(A_\mu, \varphi) = 0 \rightarrow$ absence of the manifest Lorentz invariance and other problems.

Coulomb gauge

$$
\begin{aligned}
\partial_i A_i &= 0 \\
A_i' &= (A^{\Omega})_i \\
\triangle \alpha^a + ig\varepsilon^{abc}\partial_i(A_i^b \alpha^c) &= 0
\end{aligned}
\tag{6}
$$

This equation has nontrivial solutions decreasing at spatial infinity→Gribov ambiguity. Any differential gauge does not choose a unique representative in the class of gauge equivalent fields!

In perturbation theory the only solution is $\alpha = 0$. **A problem of unambiguos quantization of nonabelian gauge theories beyond perturbation theory remains unsolved.**

Weinberg-Salam model

$$L = -1/4 F^a_{\mu\nu} F^a_{\mu\nu} - 1/4 G^a_{\mu\nu} G^a_{\mu\nu} + i\bar{L}\gamma^\mu(\partial_\mu + \frac{ig}{2}\tau^a A^a_\mu + \frac{ig_1}{2} B_\mu)L$$
$$+ i\bar{R}\gamma_\mu(\partial_\mu + ig_1 B_\mu)R + |\partial_\mu\varphi + \frac{ig}{2}\tau^a A^a_\mu\varphi + \frac{ig_1}{2} B_\mu\varphi|^2 -$$
$$-G\{(\bar{L}\varphi)R + \bar{R}(\varphi^* L)\} + \frac{m^2}{2}(\varphi^*\varphi) - \lambda^2(\varphi^*\varphi)^2 \quad (7)$$

where

$$\varphi(x) = (\varphi_1(x), \varphi_2(x)) = \sqrt{2}^{-1}(iB_1 + B_2, \sigma - iB_3 + \sqrt{2}\mu) \quad (8)$$

In perturbation theory all predictions fit the experiment very well.

However there are certain questions to be answered

1. Where is the Higgs meson? (LHC).

2. Is the model valid beyond perturbation theory?

3. Is it possible to derive the Weinberg-Salam model from some grand-unified model?

4. Quantization of the Weinberg-Salam model beyond the perturbation theory?

$SU(2)$ Higgs-Kibble model

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$$L = -1/4 F^a_{\mu\nu} F^a_{\mu\nu} + (D_\mu \varphi)^* (D_\mu \varphi) - \lambda^2 (\varphi^* \varphi - \mu^2)^2 \tag{9}$$

Gauge ransformations:

$$\delta A^a_\mu = \partial_\mu \eta^a + g \varepsilon^{abc} A^b_\mu \eta^c,$$
$$\delta B^a = \mu \sqrt{2} \eta^a + \frac{g}{2} \varepsilon^{abc} B^b \eta^c + \frac{g}{2} \sigma \eta^a . label4 \tag{10}$$

Unitary gauge $B^a = 0$ The spectrum: Three components of the massive vector field A^a_i. One scalar field (Higgs meson) σ.

Unitarity is obvious, but there is no renormalizability.

Renormalizable gauges: $\partial_\mu A^a_\mu = 0$.

The spectrum:

A^a_i, σ, unphysical components of A^a_μ, Faddeev-Popov ghosts $\bar{c}^a, c^a$, Goldstone bosons B^a.

The unitarity in the physical subspace should be proven!

Nonuniqueness of the gauge fixing does not allow to do that beyond perturbation theory.

An alternative formulation of the Higgs-Kibble model.

$$L = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + (D_\mu\varphi^+)^*(D_\mu\varphi^-) + (D_\mu\varphi^-)^*(D_\mu\varphi^+)$$
$$+(D_\mu\varphi)^*(D_\mu\varphi) - \lambda^2(\varphi^*\varphi - \mu^2)^2$$
$$-[(D_\mu b)^*(D_\mu e) + (D_\mu e)^*(D_\mu b)] \qquad (11)$$

Here the field φ is the complex doublet describing the Higgs meson, and the fields $\varphi^\pm$ are new auxiliary fields. The fields b, e have a similar structure, but correspond to the anticommuting fields. The shift

$$\varphi^-(x) \to \varphi^-(x) - \widehat{m}; \quad \varphi(x) \to \varphi(x) - \widehat{\mu} \qquad (12)$$

where $\widehat{m}$ and $\widehat{\mu}$ are the coordinate-independent condensates

$$\widehat{m} = (0, m/g); \quad \widehat{\mu} = (0, \mu/g) \qquad (13)$$

generates the mass term for the vector field.

The new Lagrangian describing the massive vector field is

$$\begin{aligned}L = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + (D_\mu\varphi^+)^*(D_\mu\varphi^-) + (D_\mu\varphi^-)^*(D_\mu\varphi^+) \\ -[(D_\mu\varphi^+)^*(D_\mu\widehat{m}) + (D_\mu\widehat{m})^* D_\mu\varphi^+] \\ -[(D_\mu b)^*(D_\mu e) + (D_\mu e)^*(D_\mu b)] + (D_\mu\varphi)^*(D_\mu\varphi) \\ -[(D_\mu\varphi)^*(D_\mu\widehat{\mu}) + (D_\mu\widehat{\mu})^*(D_\mu\varphi)] \\ +(D_\mu\widehat{\mu})^*(D_\mu\widehat{\mu}) - \lambda^2[(\varphi-\widehat{\mu})^*(\varphi-\widehat{\mu}) - \mu^2]^2 .\end{aligned} \tag{14}$$

After the shift both the fields φ and φ_- become the gauge fields:

$$\begin{aligned}\delta\varphi^a_- = m\eta^a + \frac{g}{2}\epsilon^{abc}\varphi^b_-\eta^c + \frac{g}{2}\varphi^0_-\eta^a \\ \delta\varphi^a = \mu\eta^a + \frac{g}{2}\epsilon^{abc}\varphi^b\eta^c + \frac{g}{2}\varphi^0\eta^a\end{aligned} \tag{15}$$

A gauge condition may be imposed on the fields $A^a_\mu, \varphi^a, \varphi^a_-$.

We choose the gauge $\varphi^a_- = 0$ This is an algebraic gauge, which is manifestly Lorentz invariant and, as we shall see, renormalizable.

To get rid off the ambiguity completely we introduce new variables

$$\begin{aligned} \varphi^0_- = \frac{2m}{g}(\exp\{\frac{gh}{2m}\} - 1); \quad \varphi^a_- = \tilde{M}\tilde{\varphi}^a_- \\ \varphi^a_+ = \tilde{M}^{-1}\tilde{\varphi}^a_+; \quad \varphi^0_+ = \tilde{M}^{-1}\tilde{\varphi}^0_+ \\ e = \tilde{M}^{-1}\tilde{e}; \quad b = \tilde{M}\tilde{b} \end{aligned} \tag{16}$$

$$\tilde{M} = 1 + \frac{g}{2m}\varphi^0_- = \exp\{\frac{gh}{2m}\} \tag{17}$$

At the surface $\varphi^a_- = 0$, the equation $(\tilde{\varphi}^\Omega_-)^a = 0$, implies $\eta^a = 0$.

No ambiguity!

The effective Lagrangian in the gauge $\varphi_-^a = 0$:

$$\tilde{L} = -\frac{1}{4}F_{\mu\nu}^a F_{\mu\nu}^a + \partial_\mu h \partial_\mu \tilde{\varphi}_+^0 - \frac{g}{2m}\partial_\mu h \partial_\mu h \tilde{\varphi}_+^0$$
$$+m\tilde{\varphi}_+^a \partial_\mu A_\mu^a - [((D_\mu \tilde{b})^* + \frac{g}{2m}\tilde{b}^* \partial_\mu h)(D_\mu \tilde{e} - \frac{g}{2m}\tilde{e}\partial_\mu h) + h.c.]$$
$$+\frac{mg}{2}A_\mu^2 \tilde{\varphi}_+^0 + g\partial_\mu h A_\mu^a \tilde{\varphi}_+^a + \frac{\mu^2}{2}A_\mu^a A_\mu^a + \mu\varphi^a \partial_\mu A_\mu^a + \dots \quad (18)$$

The mixed term may be eliminated by the change of variables

$$\tilde{\varphi}_+^a \to \tilde{\varphi}_+^a - \frac{\mu}{m}\varphi^a \quad (19)$$

The divergency index of a diagram with L_Φ external lines of the field Φ:

$$n = 4 - 2L_{\varphi_+^0} - 2L_{\varphi_+^a} - L_A - L_e - L_b - L_h - L_B - L_\sigma \quad (20)$$

The model is explicitly renormalizable!

Unitarity.

The model includes many unphysical (ghost) fields:$\varphi_+^\alpha, (\alpha = 0, 1, 2, 3,), h,$ $\varphi^a (B^a)(a = 1, 2, 3), e^\alpha, b^\alpha, A_0^a$. The unitarity in the physical subspace, including only A_i^a, σ should be proven.

The Lagrangian L was invariant with respect to the supersymmetry transformaions:

$$\begin{aligned} \delta\varphi_-^a &= -b^a \\ \delta\varphi_-^0 &= -b^0 \\ \delta e^a &= \varphi_+^a \\ \delta e^0 &= \varphi_+^0 \\ \delta b &= 0 \\ \delta\varphi_+^\alpha &= 0 \\ \alpha &= 0, 1, 2, 3. \end{aligned} \tag{21}$$

The nonrenormalized effective action in the gauge $\tilde{\varphi}_{-}^{a}=0$ is invariant with respect to simultaneous BRST -transformations and the transformations induced by the supersymmetry (eq.21). It may be written in the form:

$$A=\int\{\tilde{L}+\lambda^{a}\tilde{\varphi}_{-}^{a}+m\overline{c}^{a}c^{a}+\tilde{b}^{a}\overline{c}^{a}\}dx \tag{22}$$

Integrating over $\bar{c},c$ we get the effective action invariant with respect to the BRST transformation and supersymmetry transformation induced by (21)after the change $c^{a}=\tilde{b}^{a}/m$. This action is also invariant with respect to the global $SU(2)$ transformations of the variables $A_{\mu}^{a},\tilde{\varphi}_{+}^{a},\tilde{e},\tilde{b},\varphi$.

This symmetries are sufficient to guarantee the gauge invariance of the renormalized action and unitarity of the theory described by this action.

The "new BRST transformations"look as follows:

$$\delta A_\mu^a = \frac{1}{m}(D_\mu \tilde{b})^a; \quad \delta h = -\tilde{b}^0$$
$$\delta \tilde{\varphi}_+^a = \frac{g}{2m}\tilde{\varphi}_+^0 \tilde{b}^a + \frac{g}{2m}\varepsilon^{abc}\tilde{\varphi}_+^b \tilde{b}^c - \frac{g}{2m}\tilde{\varphi}_+^a \tilde{b}^0$$
$$\delta \tilde{\varphi}_+^0 = -\frac{g}{2m}(\tilde{\varphi}_+^a \tilde{b}^a + \tilde{\varphi}_+^0 \tilde{b}^0)$$
$$\delta \tilde{e}^a = \frac{g}{2m}(\tilde{e}^a \tilde{b}^0 - \tilde{e}^0 \tilde{b}^a - \varepsilon^{abc}\tilde{e}^b \tilde{b}^c) + \tilde{\varphi}_+^a$$
$$\delta \tilde{e}^0 = \frac{g}{2m}(\tilde{e}^a \tilde{b}^a + \tilde{e}^0 \tilde{b}^0) + \tilde{\varphi}_+^0$$
$$\delta \tilde{b}^a = -\frac{g}{2m}\varepsilon^{abc}\tilde{b}^b \tilde{b}^c$$
$$\delta \varphi^a = \mu \frac{\tilde{b}^a}{m} + \frac{g}{2m}\varepsilon^{abc}\varphi^b \tilde{b}^a + \frac{g}{2m}\tilde{b}^a \varphi^0$$
$$\delta \varphi^0 = -\frac{g}{2m}\tilde{b}^a \varphi^a \tag{23}$$

Two new counterterms compatible with the symmetry (23) and residual global $SU(2)$ invariance arise:

$$\mathcal{G} = \alpha \int d^4x \Big[\Big(\tilde{\varphi}^0_+ + \frac{g}{2m} \tilde{\varphi}^a_- \tilde{\varphi}^a_+ + \frac{g}{2m} (\tilde{e}^0 \tilde{b}^0 + \tilde{e}^a \tilde{b}^a) \Big)^2 + \Big(\tilde{\varphi}^a_+ - \frac{g}{2m} \tilde{\varphi}^0_+ \tilde{\varphi}^a_- - \frac{g}{2m} \varepsilon^{abc} \tilde{\varphi}^b_- \tilde{\varphi}^c_+ + \frac{g}{2m} (\tilde{e}^a \tilde{b}^0 - \tilde{e}^0 \tilde{b}^a - \varepsilon^{abc} \tilde{e}^b \tilde{b}^c \Big)^2 \Big] \quad (24)$$

$$\mathcal{G}_1 = \beta \int d^4x \Big(\Big(\tilde{\varphi}^0_+ + \frac{g}{2m} \tilde{\varphi}^a_- \tilde{\varphi}^a_+ \Big) [(\varphi - \hat{\mu})^* (\varphi - \hat{\mu}) - \mu^2] + \frac{g}{2m} (\tilde{e}^a \tilde{b}^a + \tilde{e}^0 \tilde{b}^0) [(\varphi - \hat{\mu})^* (\varphi - \hat{\mu}) - \mu^2] \Big) . \quad (25)$$

Here α and β are arbitrary constants. This new terms do not involve the derivatives and therefore do not change the structure of the nilpotent charge Q. This invariance provides the unitarity of the S-matrix in the space including only physical states A^a_i, σ

Now we show that the renormalized action possesses the same symmetry as the unrenormalized one up to renormalization of the parameters and a redefinition of the fields.

Let Γ is the generating functional for the vertex functions and Γ^0 the similar functional for the tree diagrams, including apart from the classical effective action also variation of the fields Φ connected with the antifields Φ^*.

The invariance with respect to the "new BRST transformations"may be expressed as the ST-identity for the Γ

$$\mathcal{S}(\Gamma) = \int d^4x \sum_{\Phi} \frac{\delta \Gamma}{\delta \Phi^*(x)} \frac{\delta \Gamma}{\delta \Phi(x)} = 0 \,. \tag{26}$$

If an invariant regularization of our theory exists, the effective action $\hat{\Gamma}$, including all the counterterms, satisfies the same equation

$$\mathcal{S}(\hat{\Gamma}) = 0 \tag{27}$$

The most general solution of this equation compatible with the degree of divergency and the residual $SU(2)$ invariance is obtained from Γ^0 by the following redefinition of the parameters:

$$\begin{aligned} g' &= Z_g g\,, \quad m' = Z_m m \, \alpha' = \frac{Z_\alpha}{Z_g^2}\alpha\,, \\ \beta' &= \frac{Z_\beta}{Z_g Z_m}\beta\,, \quad \lambda' = Z_\lambda \lambda \end{aligned} \tag{28}$$

The fields must be also redefined:

$$
\begin{aligned}
&\tilde{e}' = Z_1 \tilde{e}\,, \qquad \tilde{b}' = Z_m \tilde{b}\,, \qquad A_\mu^{a'} = Z_2 A_\mu^a\,, \qquad h' = Z_m Z_3 h\,, \\
&\varphi^{0'} = z_1 \varphi^0\,, \quad \varphi^{a'} = z_1 \varphi^a\,, \\
&\tilde{\varphi}_+^{a'} = Z_4 \tilde{\varphi}_+^a + Z_5 \partial A^a + Z_6 \frac{1}{m} \partial_\mu h A^{a\mu} + Z_7 (\tilde{e}^0 \tilde{b}^a - \tilde{e}^a \tilde{b}^0 - \varepsilon^{abc} \tilde{e}^b \tilde{b}^c)\,, \\
&\tilde{\varphi}_+^{0'} = Z_8 \tilde{\varphi}_+^0 + Z_9 \frac{1}{m} \Box h + Z_{10} \frac{1}{m^2} \partial_\mu h \partial^\mu h + Z_{11} A^2 + Z_{12} (\tilde{e}^0 \tilde{b}^0 + \tilde{e}^a \tilde{b}^a) \\
&\qquad\quad + z_2 \Big[(\varphi_0 + \mu)^2 + \varphi_a^2 - \mu^2 \Big]\,.
\end{aligned}
\tag{29}
$$

The functional $\hat{\Gamma}[g', m', \alpha', \beta', \lambda', \Phi'] =$
$\Gamma^0[Z_g g, Z_m m, Z_\alpha / Z_m^2 \alpha, Z_\beta / Z_g Z_m \beta, Z_\lambda \lambda, \Phi']$ is the most general solution of the equation (27)compatible with the power counting.

Conclusion

1.A unique covariant quantization of the Higgs-Kibble (Weinberg-Salam) model beyond the perturbation theory is possible.

The model is renormalizable in the ambiguity free Lorentz invariant gauge.

The necessary counterterms preserve the symmetries, which provide the unitarity of the renormalized theory and preserve the gauge invariance. However a redefinition of the parameters and the fields is needed.

The crucial role for all this consruction must be played by the nonperturbative calculations.