Main properties of the velocity distribution of dark matter particles on the outskirts of the Solar System

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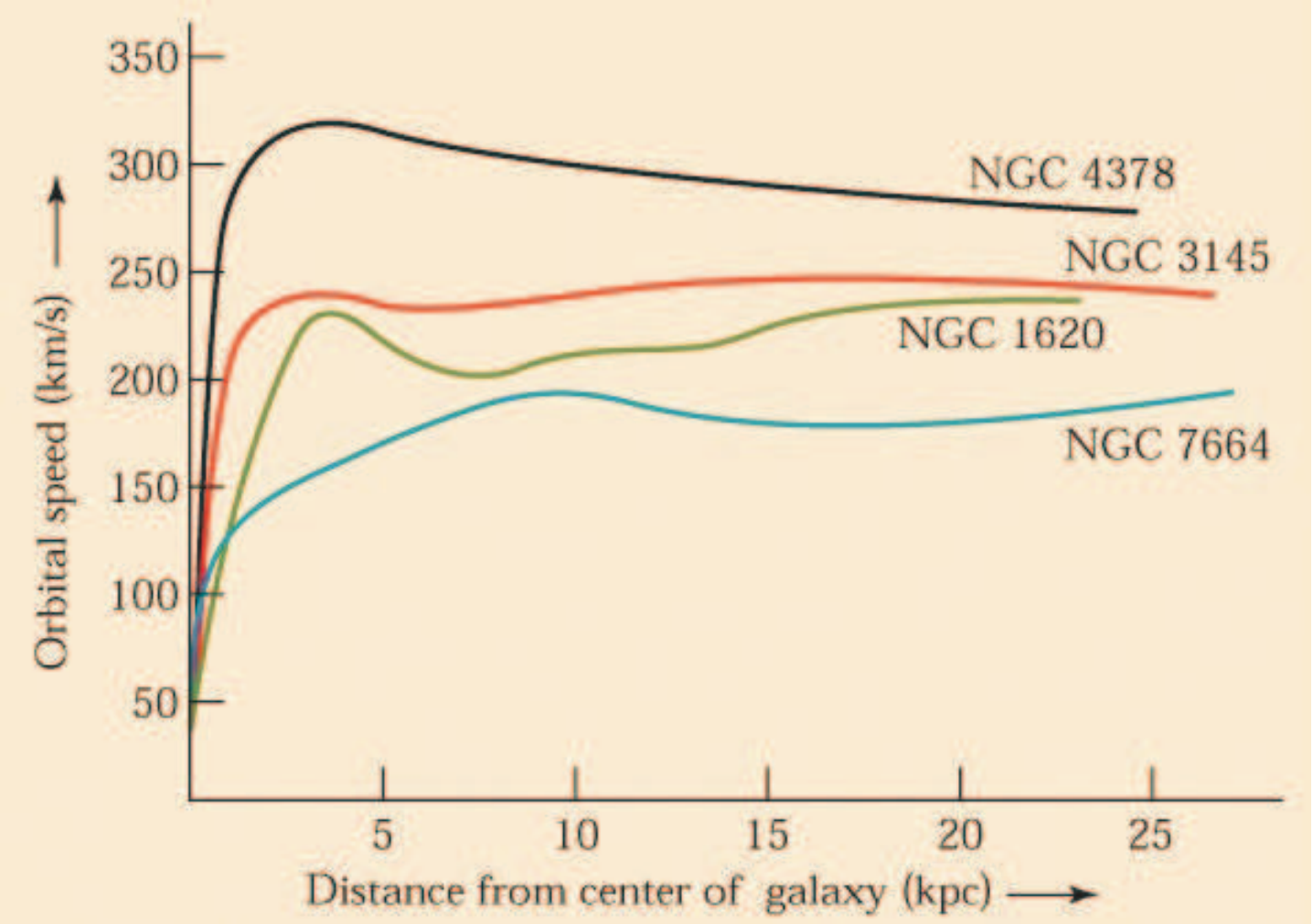
The halo

Universal halo density profile $\rho \sim r^{-2}$



Density profiles.

Isothermal profile

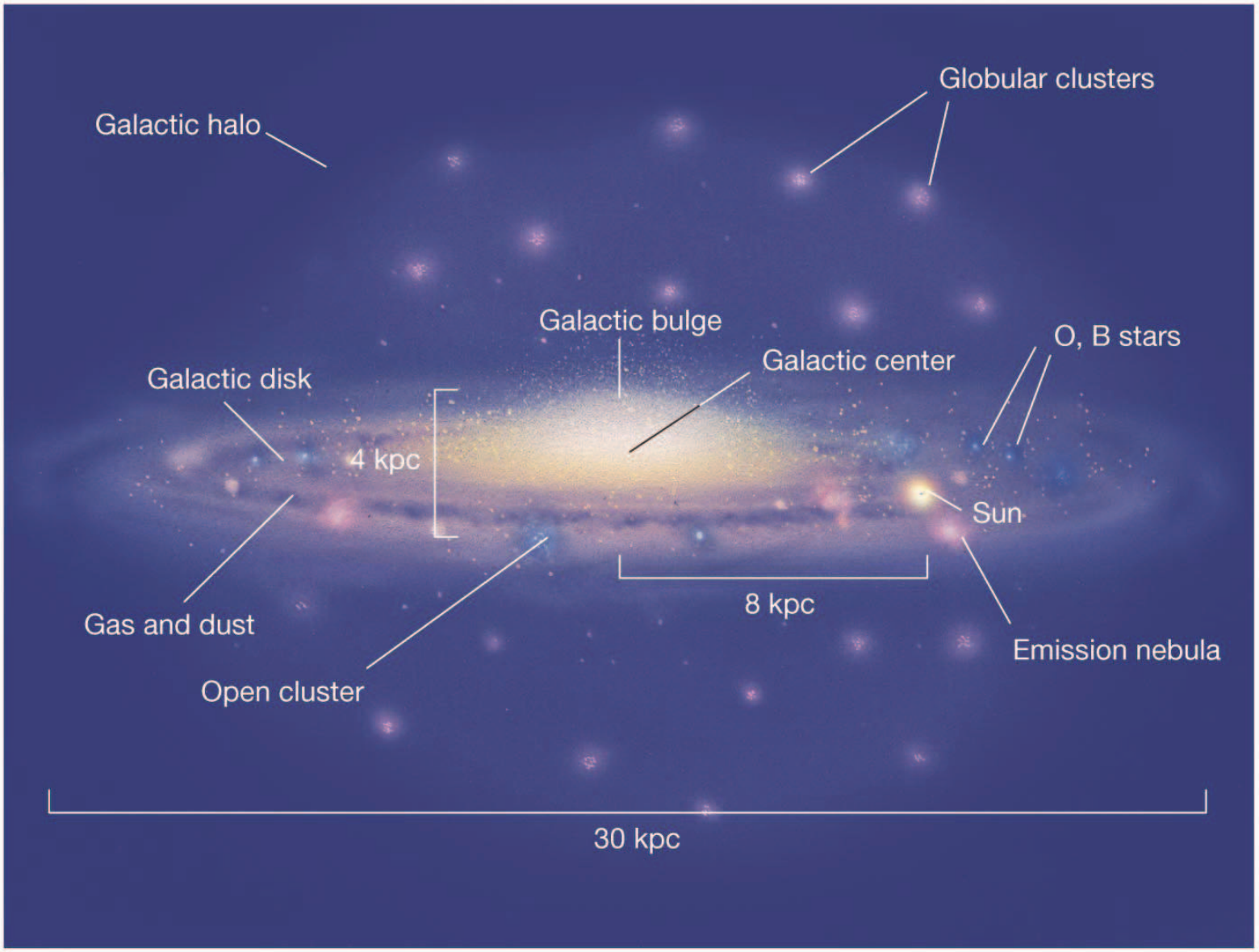
$$\rho \sim r^{-2}$$

$$\rho_{NFW} = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2} \qquad \rho_{Ei} = \rho_s \exp\left\{-2n\left[\left(\frac{r}{r_s}\right)^{\frac{1}{n}} - 1\right]\right\}$$

Maxwell velocity distribution, $\langle v \rangle \simeq 220$ km/s:

$$f(v) = \frac{4N}{\sqrt{\pi} v_\odot}\left(\frac{v}{v_\odot}\right)^2 \exp\left(-\frac{v^2}{v_\odot^2}\right), \quad v < v_{esc}$$

The Galaxy



Criticism

1) Why the dark matter particles have Maxwell distribution?

Relaxation mechanisms:

a) Pair collisions
b) Interaction with gas
c) Collective effects.

Only mechanism c) is effective for the dark matter particles, while all of them are effective for stars and star clusters.

Criticism

2) The majority of the dark matter particles has large specific root-mean-square angular momentum $\mu \equiv \mathfrak{M}/m_\chi = [\upsilon \times r]$

For the dark matter particles with Maxwell distribution
$\sqrt{\langle \mu^2 \rangle} \simeq 1800$ kpc $\cdot$ km/s at $r_\odot = 8$ kpc
$\sqrt{\langle \mu^2 \rangle} \sim 2 \cdot 10^4$ kpc $\cdot$ km/s at the edge of the halo ($r \sim 100$ kpc)

For the disc component $\sqrt{\langle \mu^2 \rangle} \sim 1800$ kpc $\cdot$ km/s

For subdwarf stars at $r_\odot$:
$\sigma_r \simeq 140$ km/s, $\sigma_\rho \simeq 80$ km/s
$\sqrt{\langle \mu^2 \rangle} \simeq 900$ kpc $\cdot$ km/s

All observable halo objects have very prolate orbits.

Assumptions

1) The Galactic halo is stationary and spherically symmetrical.

2) The density profile of the Galaxy is $\rho \propto r^{-2}$ up to some large enough radius R. Starting from some radius (we denote it by R), the halo density drops much faster. As we will see, this is the main parameter defining the velocity distribution of dark matter particles.

3) The dark matter mass outside the radius R is negligibly small as compared with the total halo mass.

4) Specific angular momentum $\mu \equiv \mathfrak{M}/m_\chi = [\upsilon \times r]$ of, at least, the main part of the particles is fairly small, and their orbits are rather prolate

Galaxy parameters

We symbolize the total mass of the Galaxy, the orbital radius of the Solar System and the average velocity of the Galaxy rotation at this radius by M, $r_\odot \simeq const$, $\upsilon_\odot$ respectively. The module of gravitational potential on the edge of the halo is equal $\Phi = GM/R$.

$$\Phi = \upsilon_\odot^2 \tag{1}$$

For our Galaxy we accept the following parameters: $R = 90$ kpc, $\upsilon_\odot = 220$ km/s, $r_\odot = 8$ kpc. These values correspond to the total mass of the Galaxy $M = 10^{12} M_\odot$.

$$\phi = -\Phi \left(1 + \ln \frac{R}{r}\right) \tag{2}$$

The zero-momentum case.

$\psi(r, \upsilon_r) dr d\upsilon_r$ gives the total mass of dark matter in the element of phase space $dr d\upsilon_r$.

$$4\pi r^2 \rho(r) = \int_{-\infty}^{\infty} \psi(r, \upsilon_r) d\upsilon_r = \frac{M}{R} \tag{3}$$

We denote by r_0 the maximum distance a dark matter moves off the centre.

$$|\upsilon_r| = \upsilon = \sqrt{2\Phi \ln \frac{r_0}{r}} \tag{4}$$

The zero-momentum case.

We introduce a distribution function ξ of the particles throughout parameter r_0, so that $\xi(r_0)dr_0$ is the total mass of DMPs which apoapsis lies in the interval $[r_0; r_0 + dr_0]$.

$$\psi(r, \upsilon) = \sqrt{\frac{2}{\pi\Phi}}\xi\left[r\exp\left(\frac{\upsilon^2}{2\Phi}\right)\right] \tag{5}$$

We can easily find function ξ if we suppose that the halo boundary is sharp.

$$\xi(r_0) = \frac{M}{\sqrt{\pi}R\sqrt{\ln\frac{R}{r_0}}} \tag{6}$$

This function has a peculiarity at $r_0 = R$. So the main part of DMPs comes to us from the very edge of the halo.

The zero-momentum case.

$$f(\upsilon) = \frac{2}{\pi\sqrt{\upsilon_{max}^2 - \upsilon^2}} \tag{7}$$

$$\upsilon_{max} = \sqrt{2\Phi \ln \frac{R}{r}} = \upsilon_{\odot}\sqrt{2 \ln \frac{R}{r}} \tag{8}$$

Near the Earth $\upsilon_{max} \simeq 2.2\upsilon_{\odot} \simeq 484$ km/s

The nonzero-momentum case.

The specific angular momentum $\mu \equiv \mathfrak{M}/m_\chi = [\upsilon \times r]$

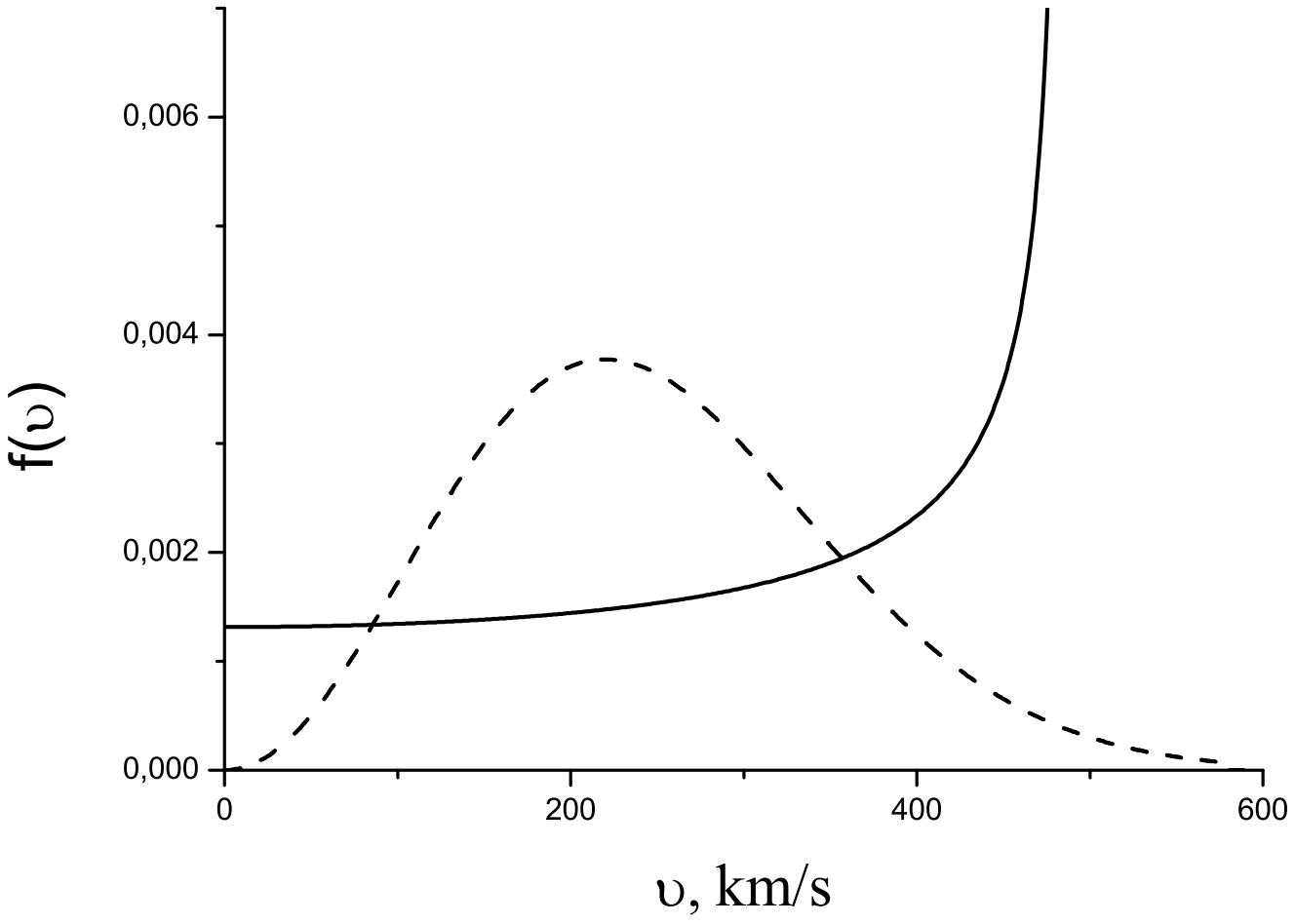
$$|\upsilon_r| = \sqrt{2\Phi \ln \frac{r_0}{r} - \mu^2 \left(\frac{1}{r^2} - \frac{1}{r_0^2}\right)} = \sqrt{2\upsilon_{orb}^2 \ln \frac{r_0}{r} - \mu^2 \left(\frac{1}{r^2} - \frac{1}{r_0^2}\right)}$$

$$\upsilon_\tau = \mu \frac{r_0}{r}$$

Near the Earth 500^2 km^2/s$^2 \gg 110^2$ km^2/s^2

$$f(\upsilon) = \frac{\exp\left(-\frac{\upsilon_\rho^2}{2\sigma_\rho^2}\right)}{2\pi^2\sigma_0^2\sqrt{\upsilon_{max}^2 - \upsilon_r^2}}, \qquad \sigma_\rho \simeq 80 \text{ km/s} \quad \upsilon_{max} \simeq 484 \text{ km/s}$$

Conventional velocity distribution (dash line) vs. the anisotropic one.



Direct and directional detection

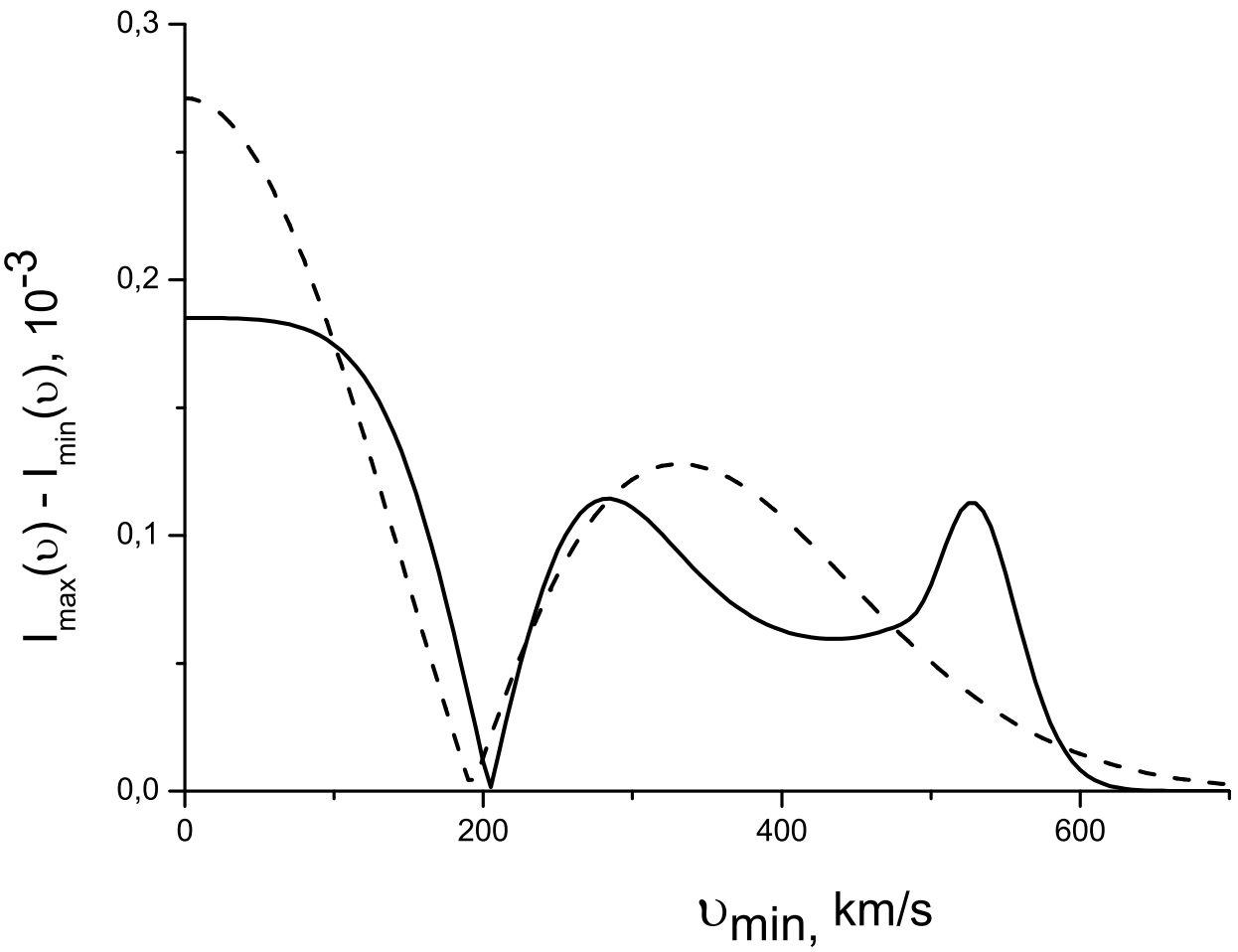
$$f(\upsilon) = \frac{\exp\left(-\frac{\upsilon_\rho^2}{2\sigma_\rho^2}\right)}{2\pi^2\sigma_0^2\sqrt{\upsilon_{max}^2 - \upsilon_r^2}}, \qquad \sigma_\rho \simeq 80 \text{ km/s} \quad \upsilon_{max} \simeq 484 \text{ km/s}$$

Direct detection:

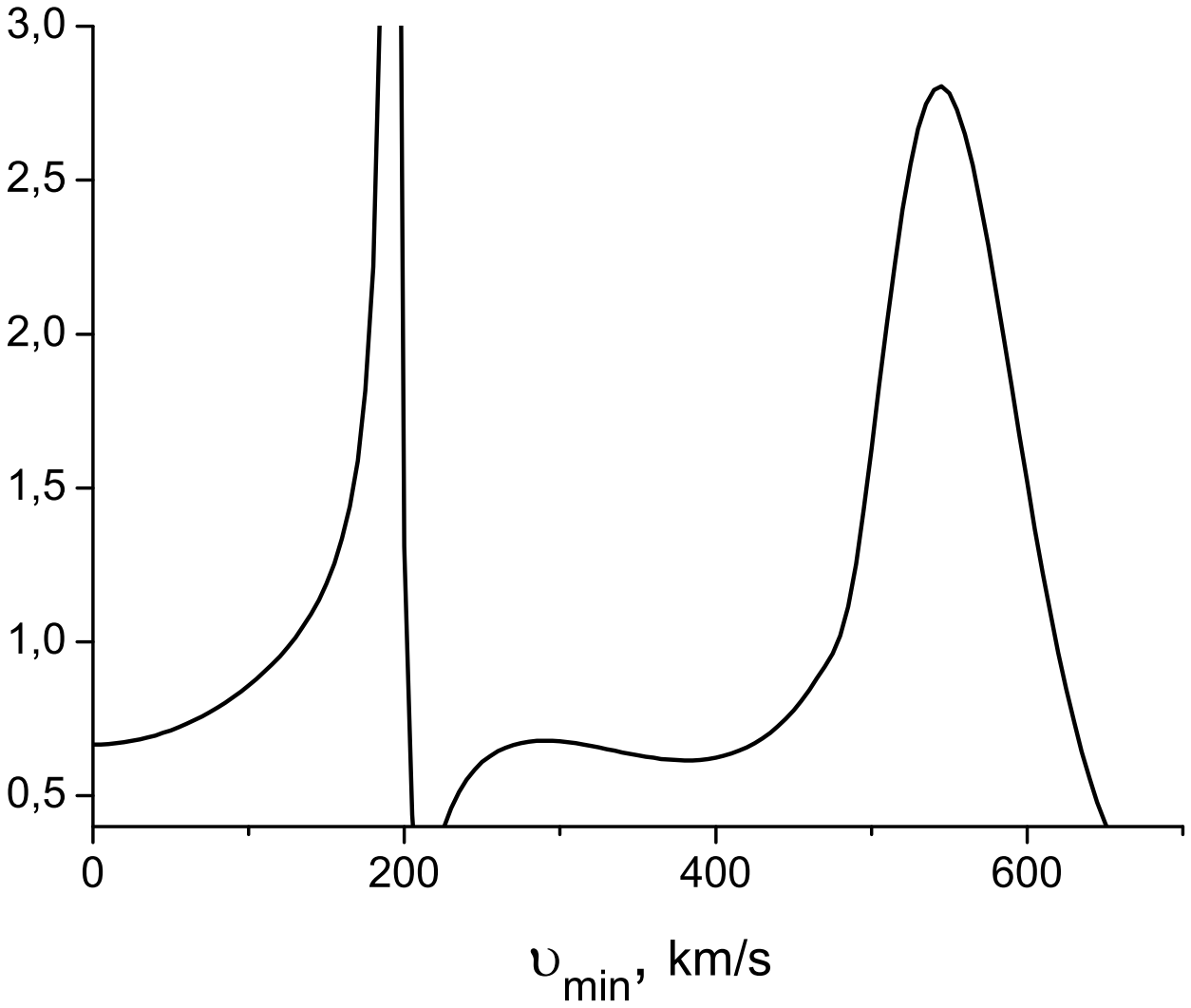
$$I = \int_{\upsilon_{min}}^{\infty} \frac{\tilde{f}(\upsilon)}{\upsilon} d\upsilon$$

Is of critical importance for the Directional detection! The Sky Map is completely different in this case: instead of one large hot spots in Cygnus (with the galactic coordinates of the centre (90°, 0°)) we have two small ($\sim 10°$) hot spot with coordinates $(90°, -65.5°)$ (Pisces) and $(90°, +65.5°)$ (Bootes).

The double amplitudes $2A = I_{max} - I_{min}$ of direct detection signals calculated for the anisotropic distribution (solid line) and the Maxwell distribution (dash line)



The ratio between the amplitudes of direct detection signals for the anisotropic and the Maxwell distributions



The overview of direct detection (Xenon collaboration)