Serendipitous discoveries in nonlocal gravity: DE and DM

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Advances in QFT, JINR, October 2011

Introduction

Cosmic acceleration and modifications of gravity theory:

explicit cosmological term
quintessence type models
f(R) models
higher-dimensional and braneworld models
massive gravity
nonlocal gravity
....

explicit DE scale encoded in the action

Cosmic coincidence aspect of the CC and DE problem

$$M_P^2\,\Lambda_{\rm eff} = \rho_{DE} \sim \rho_{\rm matter}$$

Fine tuning concrete value of effective Λ to matter density

Alternative idea – the model that has a stable dS or AdS background with an arbitrary value of Λ

Realization of an old idea of a scale-dependent gravitational coupling – nonlocal Newton constant $G_{Newton} \Rightarrow G_{\rm eff}(\Box)$

Model:

i) GR limit on flat space background
ii) Stable ghost-free (A)dS phase with arbitrary Λ
iii) Unexpected bonus – DM mechanism in this phase

Flat-space background setup

Idea of scale dependent gravitational coupling (noncovariant)

$$M_P^2 \;\Rightarrow\; M_{\rm eff}^2(\Box)$$
$$G_{Newton} \;\Rightarrow\; G_{\rm eff}(\Box)$$
$$G_{\mu\nu} = 8\pi G_{\rm eff}(\Box)\, T_{\mu\nu}$$

Arkani-Hamed, Dimopoulos, Dvali and Gabadadze (2002)

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

Einstein tensor

Einstein action on a flat-space background:

$$S_E = \frac{M_P^2}{2} \int dx\, g^{1/2} \left\{ -R^{\mu\nu} \frac{1}{\Box} G_{\mu\nu} + \mathrm{O}\,[R_{\mu\nu}^3] \right\},$$

A.Barvinsky (2003)

$$M_P^2\, R^{\mu\nu} \frac{1}{\Box} G_{\mu\nu} \;\Rightarrow\; R^{\mu\nu} \frac{M^2(\Box)}{\Box} G_{\mu\nu}$$

Correspondence principle with GR

$$R \;\Rightarrow\; R + R^{\mu\nu} F(\Box) G_{\mu\nu}$$

$$F(\Box) = \int d\mu^2\, \frac{\alpha(\mu^2)}{\mu^2 - \Box}$$

Trial choice

$$\alpha(\mu^2) \sim \delta(m^2 - \mu^2) \;\Rightarrow\; F(\Box) = \frac{\alpha}{m^2 - \Box}$$

Problem with $m^2 \neq 0$:

Structure of inverse propagator and characteristic equation for field modes

$$-\Box + \alpha \frac{\Box^2}{m^2 - \Box} = 0$$

$$\Downarrow$$

Massless and massive graviton

$$\Box = m_{\pm}^2,\; m_{-}^2 = 0,\; m_{+}^2 = O(m^2)$$

ghost

$$\Downarrow$$

$$m^2 = 0$$

First step towards nonlocal gravity:

$$\boxed{S=\frac{M^2}{2}\int dx\, g^{1/2}\left\{-R+\alpha\, R^{\mu\nu}\frac{1}{\Box}G_{\mu\nu}\right\}}$$

From post-Newtonian corrections $\quad |\alpha| \ll 1$

Linearized theory

$$S=-\frac{M^2(1-\alpha)}{2}\int dx\, g^{1/2}R+\alpha\, O[\, h^3_{\mu\nu}]$$

Small renormalization of the Planck mass

$$M^2=\frac{M_P^2}{1-\alpha}$$

Treatment of nonlocality

Schwinger-Keldysh technique and Euclidean QFT

In-in mean field

$$g_{\mu\nu} = \langle \mathrm{in} |\, \hat{g}_{\mu\nu} | \mathrm{in} \rangle$$

Quantum effective action of Euclidean QFT (nonlocal)

$$S = S_{\mathrm{Euclidean}}[\, g_{\mu\nu}\,]$$

Effective equations for *in-in* field

A.B. & G.A.Vilkovisky (1987)

$$\left.\frac{\delta S_{\mathrm{Euclidean}}}{\delta g_{\mu\nu}(x)}\right|^{\mathrm{retarded}}_{++++\,\Rightarrow\,-+++} = 0.$$

Causal, diffeomorphism and gauge invariant !

Tentative applications

$$S[g,\varphi] = \frac{M^2}{2}\int dx\, g^{1/2}\Big\{ -R - 2\alpha\,\varphi^{\mu\nu}R_{\mu\nu} \\ -\alpha\left(\varphi^{\mu\nu} - \frac{1}{2}g^{\mu\nu}\varphi\right)\Box\,\varphi_{\mu\nu}\Big\}$$

$$\Box\,\varphi^{\mu\nu} = -G^{\mu\nu}$$

$$\varphi^{\mu\nu} = -\left(\frac{1}{\Box}\right)_{\rm ret} G^{\mu\nu}$$

Example:

$$S[\varphi] \equiv -\int dx\,\varphi\Box\varphi \\ = -\int dx\,(\Box\varphi)\frac{1}{\Box}(\Box\varphi)$$

$$(\psi+\varphi)\Big|_{\infty} = 0$$

$$S[\varphi,\psi] = \int dx\,\left(2\psi\,\Box\varphi + \psi\,\Box\psi\right) \\ = \int dx\,\left(g\,\Box g - \varphi\,\Box\varphi\right), \quad g \equiv \psi + \varphi$$

fictitious ghost

Cosmological parameters $$H \equiv \frac{\dot{a}}{a}, \quad w \equiv -1 - \frac{2\dot{H}}{3H^2}$$

$$w(t_0) = -1$$
$$\dot{w}(t_0) = O(1) \times H(t_0) < 0$$

present moment

But

$$R_{\mu\nu}\Big|_{(A)dS} = \Lambda g_{\mu\nu}, \quad \Box g_{\mu\nu} = 0$$

$$R^{\mu\nu}\frac{1}{\Box}G_{\mu\nu}\Big|_{(A)dS} = \infty$$

No go!

Nonlocal gravity with a stable (A)dS background

$$S = \frac{M^2}{2}\int dx\, g^{1/2}\left\{-R + \alpha\, R^{\mu\nu}\frac{1}{\Box + \hat{P}}G_{\mu\nu}\right\},$$

$$\hat{P} \equiv P_{\alpha\beta}{}^{\mu\nu} = a R_{(\alpha\ \ \beta)}^{\ \ (\mu\ \ \nu)} + b\left(g_{\alpha\beta}R^{\mu\nu} + g^{\mu\nu}R_{\alpha\beta}\right)$$
$$+ c R_{(\alpha}^{(\mu}\delta_{\beta)}^{\nu)} + d R\, g_{\alpha\beta}g^{\mu\nu} + e R\delta_{\alpha\beta}^{\mu\nu}.$$

Operator $$\Box + \hat{P} \equiv \Box\, \delta_{\alpha\beta}{}^{\mu\nu} + P_{\alpha\beta}{}^{\mu\nu}$$

Green's function $$\frac{1}{\Box+\hat{P}}G_{\mu\nu} \equiv \left[\frac{1}{\Box+\hat{P}}\right]_{\mu\nu}^{\alpha\beta} G_{\alpha\beta}$$

a, b, c, d, e **-- parameters to be restricted by the requirement of a stable (A)dS solution**

Maximally symmetric background

$$R_{\alpha\mu\beta\nu} = \frac{\Lambda}{3}(g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\beta\mu})$$

$$\hat{P}\, g_{\mu\nu} \equiv P_{\alpha\beta}{}^{\mu\nu}\, g_{\mu\nu} = (A + 4B)\,\Lambda\, g_{\mu\nu},$$

$$\boxed{\begin{aligned} &A = a + 4\,b + c, \quad B = b + 4\,d + e, \\ &C = \frac{a}{3} - c - 4e \end{aligned}}$$

Maximally symmetric background

$$\left.\frac{\delta S}{\delta g_{\mu\nu}}\right|_{\text{(A)dS}} = \frac{1}{4}\, g^{\mu\nu} \left.\frac{\delta S}{\delta \sigma}\right|_{\text{(A)dS}} = -2M^2\Lambda\left(1 + \frac{\alpha}{A + 4B}\right) g^{1/2}$$

(A)dS solution with arbitrary Λ exists under the following restriction:

$$\boxed{\alpha = -A - 4B}$$

Another property: $\left. S \right|_{\text{(A)dS}} = 0$ **--- analogue of vacuum**

Stability of (A)dS background

$$g_{\mu\nu} \Rightarrow g_{\mu\nu}\Big|^{(A)dS} + h_{\mu\nu}, \quad S^{(2)} = ?$$

The hope for good $S^{(2)}$ – why ???

DeWitt gauge

$$\chi^\mu \equiv \nabla_\nu h^{\mu\nu} - \frac{1}{2}\nabla^\mu h = 0$$

Two nonlocal tensor structures

$$S^{(2)} \sim h^{\mu\nu} \times h_{\mu\nu} + h \times h$$

Nonlocal parts of these structures to be canceled by the parameter choice

$$h^{\mu\nu}\frac{1}{\Box + \hat{P}}h_{\mu\nu} \qquad h\,\frac{1}{\Box - \alpha\Lambda}\,h$$

Quadratic part of the action:

traceless part
$$\tilde{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{4} g_{\mu\nu} h$$

$$S^{(2)} = -\frac{1}{64\pi G_{\text{eff}}} \int d^4x g^{1/2} \left\{ \tilde{h}^{\mu\nu} \Box \tilde{h}_{\mu\nu} + \left(C - \frac{4}{3}\right) \Lambda \tilde{h}^2_{\mu\nu} + \Lambda^2 \left(C - \frac{2}{3}\right)^2 \tilde{h}^{\mu\nu} \frac{1}{\Box + \hat{P}} \tilde{h}_{\mu\nu} \right\},$$

Effective gravitational coupling constant vs Newton constant
$G_N = 1/8\pi M_P^2$

$$G_{\text{eff}} = \frac{\alpha(1-\alpha)}{8B(2B+\alpha)} G_N$$

Conditions of stability

$$C \equiv \frac{a}{3} - c - 4e = \frac{2}{3}, \quad G_{\text{eff}} > 0$$

$$\Downarrow$$

$$B < -\frac{\alpha}{2} \text{ and } B > 0, \quad \alpha > 0,$$
$$0 < B < -\frac{\alpha}{2}, \quad \alpha < 0$$

Gravitational potentials in the (A)dS phase and DM mechanism

$$S^{(2)}\Big|_{\rm non-gauged} = \frac{M_{\rm eff}^2}{2}\int d^4x\, g^{1/2}\left\{\frac{1}{4}h^{\mu\nu}\left(-\Box+\frac{2}{3}\Lambda\right)h_{\mu\nu}\right.$$
$$-\frac{1}{8}h\left(-\Box-\frac{2}{3}\Lambda\right)h-\frac{1}{2}\chi_\mu^2$$
$$\left.-\frac{1}{16}\Big[2\nabla_\mu\chi^\mu-(\Box+2\Lambda)\,h\Big]\frac{1}{\Box+2\Lambda}\Big[2\nabla_\nu\chi^\nu-(\Box+2\Lambda)h\Big]\right\}.$$

Free waves in the DeWitt gauge:

$$\left(-\Box+\frac{2}{3}\Lambda\right)h_{\mu\nu}+\frac{1}{2}\nabla_\mu\nabla_\nu h-\frac{\Lambda}{6}g_{\mu\nu}h=0\;\Rightarrow\;\Box h=0\;\Rightarrow$$

two physical transverse-traceless polarizations

Retarded gravitational potentials of matter sources:

$$h_{\mu\nu} = \frac{8\pi G_{\text{eff}}}{-\Box + \frac{2}{3}\Lambda} \left(T_{\mu\nu} + g_{\mu\nu} \frac{\Box - 2\Lambda}{\Box + 2\Lambda} \frac{\Lambda}{3\Box} T \right) + \text{gauge transform}$$

vs GR structure $\quad T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T$

DM mechanism: $\quad G_N \Rightarrow G_{\text{eff}} \sim \frac{G_N}{|\alpha|} \gg G_N$

$$G_{\text{eff}} \geq \frac{1-\alpha}{|\alpha|} G_N \gg G_N, \quad \alpha < 0$$

Attraction much stronger than in GR phase!

Range of validity of (A)dS phase:

$$|\delta R_{\mu\nu}| \sim |\nabla\nabla h_{\mu\nu}| \ll \Lambda |g_{\mu\nu}|,$$
$$|h_{\mu\nu}| \ll |g_{\mu\nu}| \;\; \Leftrightarrow \;\; |T_{\mu\nu}| \ll M_P^2 \Lambda$$

Local energy density $|T_{\mu\nu}| \gg M_P^2 \Lambda$ $\Rightarrow$ **GR regime**

two phases

Conclusions

i) GR limit on flat space background
ii) Stable ghost-free (A)dS phase with arbitrary Λ
iii) Unexpected bonus – DM mechanism in this phase

Problems for realistic cosmology and beyond:

i) PN corrections and effect of new type of nonlocality in the gravitational potentials;
ii) mechanism of crossover from GR to DE regime at a concrete scale;
iii) BH, AdS/CFT, etc. ramifications (zero entropy BH?)