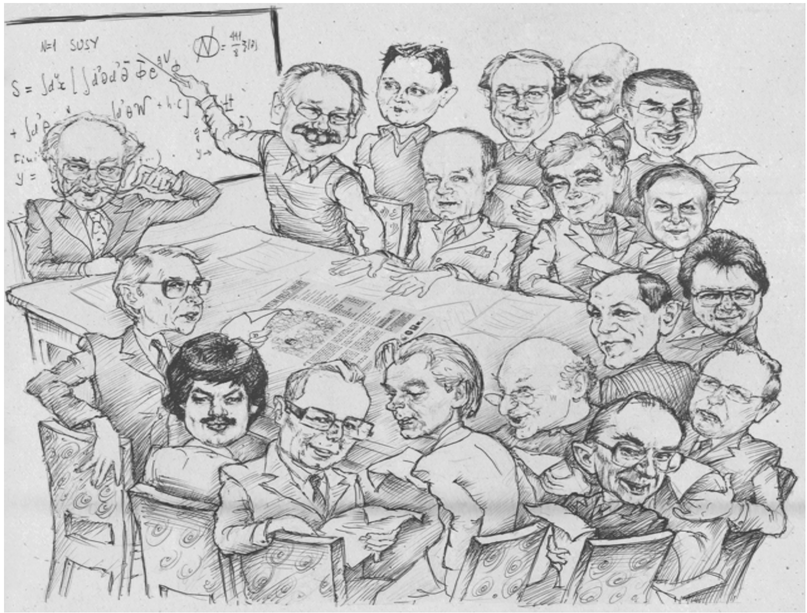
Cosmological Daemon

I. Aref'eva

Steklov Mathematical Institute, RAN, Moscow



Advances of Quantum Field Theory

October 4 — 7, 2011, Dubna

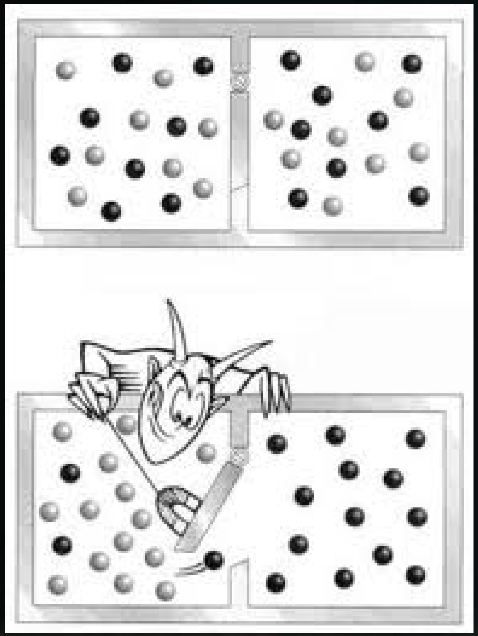
Cosmological Daemon

I. Aref'eva

Steklov Mathematical Institute, RAN, Moscow

Advances of Quantum Field Theory

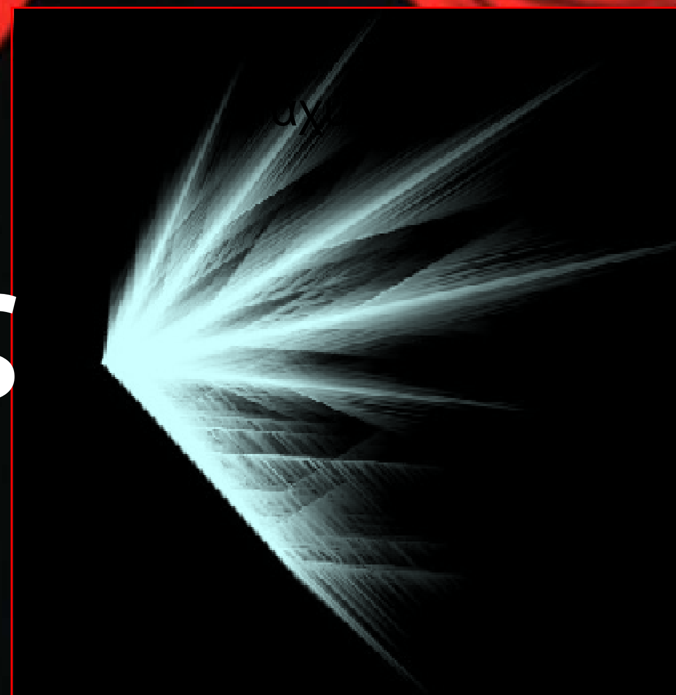
October 4 — 7, 2011, Dubna



Maxwell`s demon


ταχύς
Cosmo Daemon

ταχύς



Unstable

The OPERA collaboration has claimed the discovery of supeluminal neutrino propagation

However the superluminal interpretation of the OPERA result was refuted by Cohen and Glashow because superluminal neutrinos would lose energy rapidly via the bremsstrahlung of electron-positron pairs

$$\nu_{\mu} \rightarrow \nu_{\mu} + e^{-} + e^{+}$$

It is suggested that the superluminal interpretation is still possible if there exists a new dark neutrino which can propagate with a superluminal velocity and which couples with usual neutrinos only via the mass mixing leading to neutrino oscillations (B.Pontecorvo)

I.A., I.Volovich, arXiv:1110.0456

Superluminocity and Cosmology

Two possible pictures:

- **a ”conventional” tachyonic dark neutrino and**
- **the modification of the Lagrangian by adding perturbation with the maximum attainable speed of the dark neutrino which is larger than the speed of light in vacuum.**

“Physical laws are invariant under rotations and translations in a preferred reference frame, the rest frame of the cosmic background radiation” (Coleman, Glashow).

Cosmological Singularity

- Classical versions of the Friedmann Big Bang cosmological models of the universe

  contain a singularity at the start of time.

  Here we restrict ourself with a simple approach by considering the time variable t

  running over the half-line with regular boundary conditions at $t = 0$.

Model

$$S = \int d^4x\sqrt{-g}\left\{\frac{m_p^2}{2}R + \Phi F(\Box)\Phi - V(\Phi)\right\}$$

String tachyon

$$F(\Box) = e^{-\tau\Box}(\Box + \mu^2),$$

$$m_p^2 = \frac{1}{8\pi G}$$

$$V(\Phi) = \frac{\varepsilon}{4}\Phi^4$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu$$

Eq. with infinite number of derivatives

Main messages from SFT eqs with infinite number of derivatives

- On the half line in additional to the usual initial data a new arbitrary function (external source) occurs.

- Stretch of the kinetic energy

Nonlocal SFT Equations

$$F(\Box) = e^{-\tau\Box}(\Box + \mu^2), \quad \Box = -\partial_{tt}^2 + \partial_{x_i x_i}^2$$

$$\Psi(\tau, t) = e^{\tau \partial_t^2} \varphi(t)$$

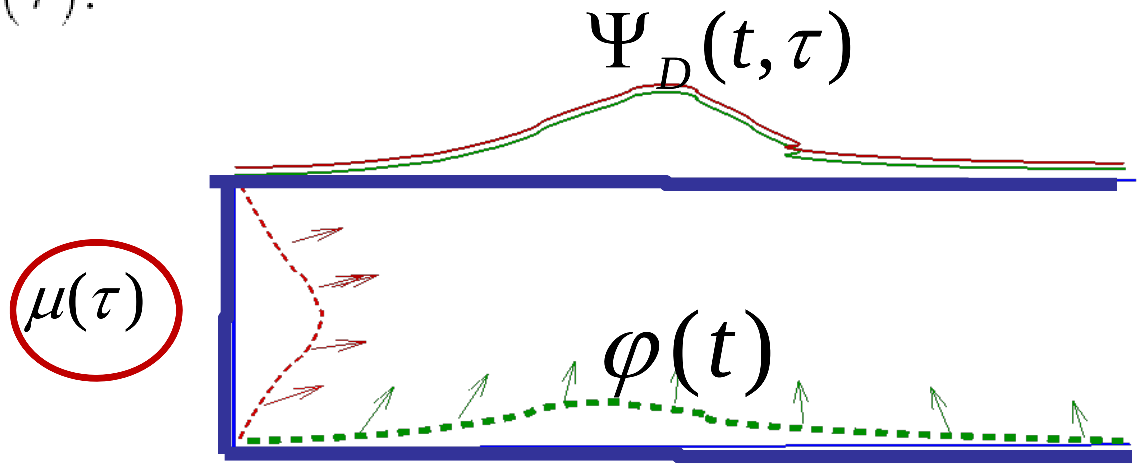
Fock (1937)
auxiliary parameter

$$\begin{cases} (\partial_\tau - \partial_t^2)\Psi(t,\tau) = 0, \\ \Psi(t,\tau)|_{\tau=0} = \varphi(t). \end{cases}$$

$$\boxed{e^{\partial_t^2}\varphi(t) = \Psi(t,\tau)|_{\tau=1}}$$

The heat equation of the half-line

$$
\begin{cases}
\frac{\partial}{\partial \tau}\Psi_D(t,\tau) = \frac{\partial^2}{\partial t^2}\Psi_D(t,\tau), \quad t>0, \quad \tau>0,\\
\Psi_D(t,0) = \varphi(t),\\
\Psi_D(0,\tau) = \mu(\tau).
\end{cases}
$$



$$
\Psi_D(t,\tau) = \frac{1}{\sqrt{4\pi\tau}} \int_0^\infty \varphi(t')\,[e^{-\frac{(t-t')^2}{4\tau}} - e^{-\frac{(t+t')^2}{4\tau}}]dt'
$$
$$
+ \frac{t}{\sqrt{4\pi}} \int_0^\tau \frac{\mu(\tau')}{(\tau-\tau')^{3/2}} e^{-\frac{t^2}{4(\tau-\tau')}}\, d\tau'
$$

Cosmological Daemon

I. A., I.Volovich, JHEP 08 (2011)102, arXiv:1103.0273

- The daemon function governs the evolution of the universe similar to Maxwell`s demon.
- In the simplest case the nonlocal scalar field reduces to the usual local scalar field coupled with an external source.

Semantic remarks

J(x) – Daemon external source.

- **According to Plato: daemons are good or benevolent "supernatural beings between mortals and gods"**
- **Judeo-Christian usage of demon: a malignant spirit.**
- **Socrates‘ daimon is analogous to the guardian angel.**

Daemon external source

$$S = \int d^4x\sqrt{-g}\left\{\frac{M_P^2}{2}R + \frac{1}{2}\phi\Box\phi - U(\phi) + J\phi\right\},$$

J(x) – **Daemon external source**

$$\epsilon = \frac{M_P^2}{2}\left(\frac{U'(\phi) - J}{U(\phi) - J\phi}\right)^2$$

$$U(\phi) = m^2\phi^2/2 \quad \epsilon = \frac{M_P^2}{2\phi^2}\left(\frac{m^2\phi - J}{\frac{m^2}{2}\phi - J}\right)^2 = \frac{M_P^2}{2\phi^2}\frac{\delta^2}{(\frac{m^2}{2}\phi - \delta)^2}$$

$$\delta = m^2\phi - J$$

Solutions to Nonlinear Nonlocal Equation on the Whole/Half-line

- Dirichlet's daemon **without** source

$$e^{\tau\partial^2}(\partial^2-\mu^2)\phi(t)=-\epsilon\phi^3(t)$$

$$\phi(t)=a\sinh(\Omega\,t)-\frac{\epsilon a^3}{32\mu^2}e^{-9\lambda\Omega^2}\,\sinh(3\Omega\,t)+\dots$$

$$\Omega^2-\mu^2-\frac{3}{4}\epsilon a^2\,e^{-\tau\Omega^2}=0$$

Generalization of the Bogolubov-Krylov

Solutions to Nonlinear Tachyon equation. Generalization of the Bogolubov-Krylov

$$
\begin{aligned}
\ddot{q} - \mu^2 q &= -\epsilon\, q^3, \quad \epsilon > 0, \\
I.D.: \quad q(0) &= q_0, \quad \dot{q}(0) = v_0
\end{aligned}
$$

$$E = \frac{1}{2} v_0^2 - \frac{1}{2} \mu^2 q_0^2 + \frac{1}{4} \epsilon q_0^4$$

$E > 0$ — motion in two holes

$$
\begin{aligned}
q(t) &= a\, \mathrm{cn}(\Omega t + b, k) \\
a^2 &= \frac{\mu^2}{\epsilon}\left(1 + \sqrt{1 + \frac{4\epsilon E}{\mu^4}}\right) \\
\Omega^2 &= \mu^2 \sqrt{1 + \frac{4\epsilon E}{\mu^4}} \\
k^2 &= \frac{1}{2} + \frac{1}{2} \frac{1}{\sqrt{1 + \frac{4\epsilon E}{\mu^4}}} \\
b &: \quad \text{from} \;\; q_0 = a\, \mathrm{cn}(b, k)
\end{aligned}
$$

Solutions to Nonlinear Tachyon equation. Generalization of the Bogolubov-Krylov

$$q(u) = \mathrm{cn}(u - \mathbf{K}, k) \tag{30}$$

Asymptotic expansion for $|u| < \mathbf{K}$ and small k'

$$\mathrm{cn}(u - \mathbf{K}, k) = \frac{\mathrm{sn}(u,k)}{\mathrm{dn}(u,k)} k' = \frac{\pi}{k\mathbf{K}'} \left\{ \frac{\sinh u'}{\cosh \rho'} - \frac{\sinh 3u'}{\cosh 3\rho'} + \frac{\sinh 5\rho'}{\cosh 5u'} + \ldots \right\} \tag{31}$$

where

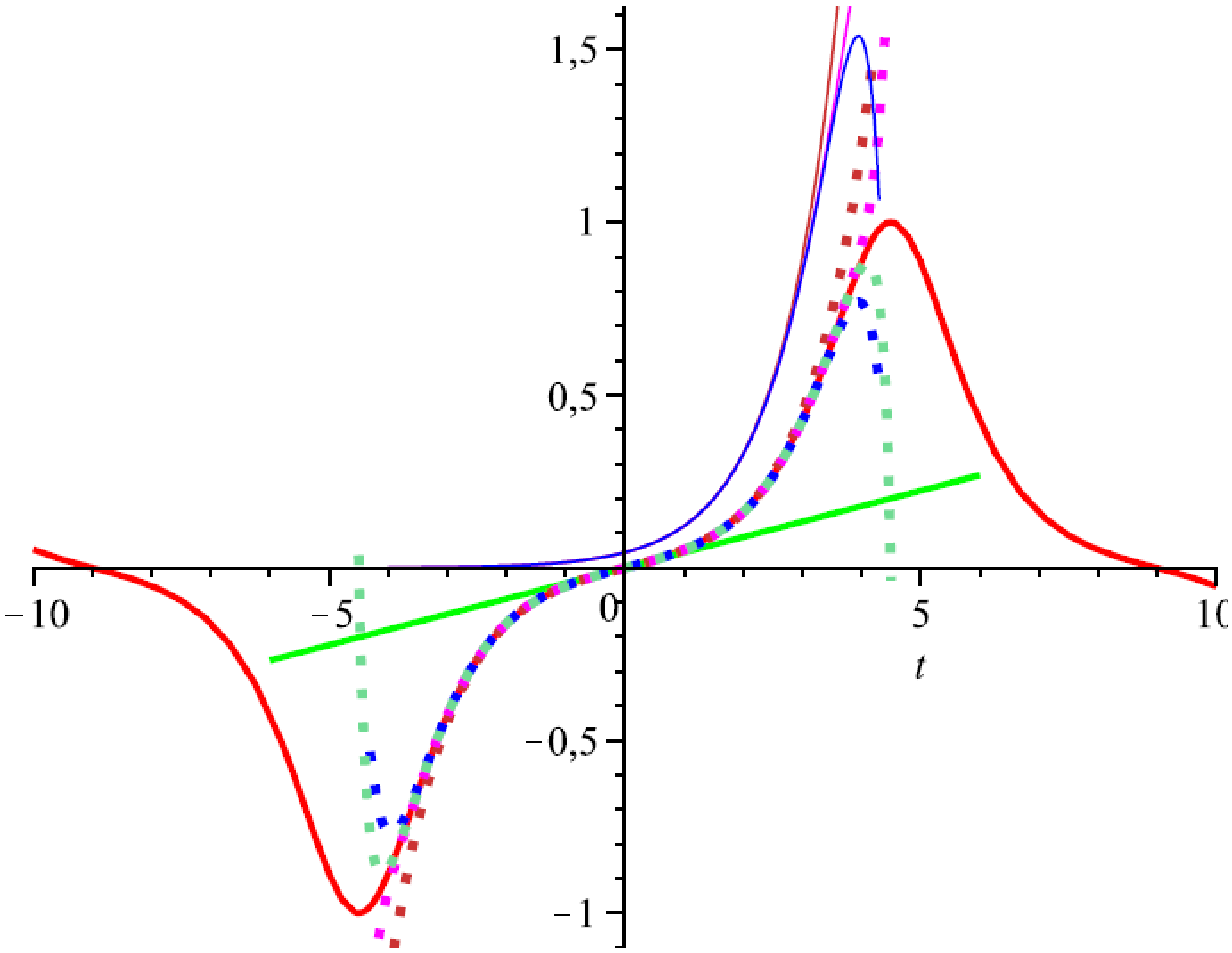
$$\rho' = \frac{\pi \mathbf{K}}{2\mathbf{K}'}, \quad u' = \frac{\pi u}{2\mathbf{K}'} \tag{32}$$

$$\frac{1}{\cosh \rho'} = \frac{2}{e^{\rho'} + e^{-\rho'}} = \frac{2}{\frac{1}{\mathrm{q}'^{1/2}} + \mathrm{q}'^{1/2}} = \frac{2\sqrt{\mathrm{q}'}}{1 + \mathrm{q}'}, \quad \mathrm{q}' = e^{-\frac{\pi \mathbf{K}}{\mathbf{K}'}}, \tag{33}$$

These series converge for $|\Re u| < \mathbf{K}$ and $-1 < k < 1$. Expansion of $1/\cosh \rho'$ for small k' is

$$\frac{1}{\cosh \rho'} = \frac{1}{2} k' + \frac{3}{32} k'^3 + \mathcal{O}(k'^5) \tag{34}$$

I.A, E.Piskowsky, I.Volovich



1,5
1
0,5
−0,5
−1
−10
−5
0
5
10
t

Stretch of the kinetic energy

$$e^{\tau(\partial^2+3H\partial)}(\partial^2+3H\partial-\mu^2)\phi(t)=-\epsilon\phi^3(t)$$

- **Approximation**

$$\ddot{\phi}+3\sqrt{\frac{8\pi G}{3}\left(e^{\tau\Omega^2}\left(\frac{1}{2}\dot{\phi}^2-\frac{\mu^2}{2}\phi^2\right)+\frac{\epsilon}{4}\phi^4+\Lambda\right)}\,\dot{\phi}=\mu^2\phi-\epsilon\phi^3$$

Consequences of the Stretch of the Kinetic Energy

- **Appearance of forbidden region**

$$\ddot{\phi} + 3\sqrt{\frac{8\pi G}{3}\left(e^{\tau\Omega^2}\left(\frac{1}{2}\dot{\phi}^2 - \frac{\mu^2}{2}\phi^2\right) + \frac{\epsilon}{4}\phi^4 + \Lambda\right)}\,\dot{\phi} = \mu^2\phi - \epsilon\phi^3$$

Assume Higgs potential

$$-\frac{\mu^2}{2}\phi^2 + \frac{\epsilon}{4}\phi^4 + \Lambda = \frac{\epsilon}{4}(\phi^2 - \phi_0^2)^2$$

$$\mu^2 = \epsilon\phi_0^2, \quad \Lambda = \frac{\epsilon\phi_0^4}{4}$$

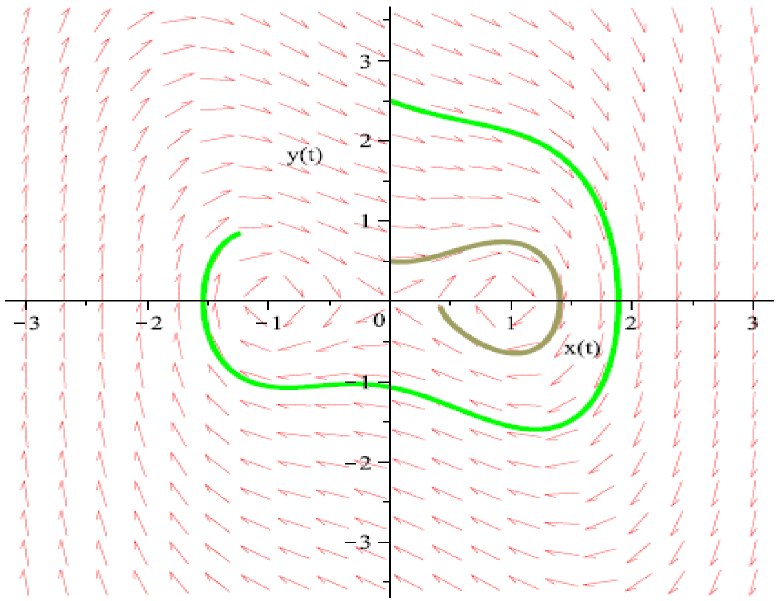
$$\left(-\frac{\mu^2}{2}e^{\tau\Omega^2}\phi^2 + \frac{\epsilon}{4}\phi^4 + \Lambda\right) = \frac{\epsilon}{4}(\phi^2 - \phi_0^2)^2 - \frac{\mu^2}{2}\left(e^{\tau\Omega^2} - 1\right)$$

Inflation with Higgs potential

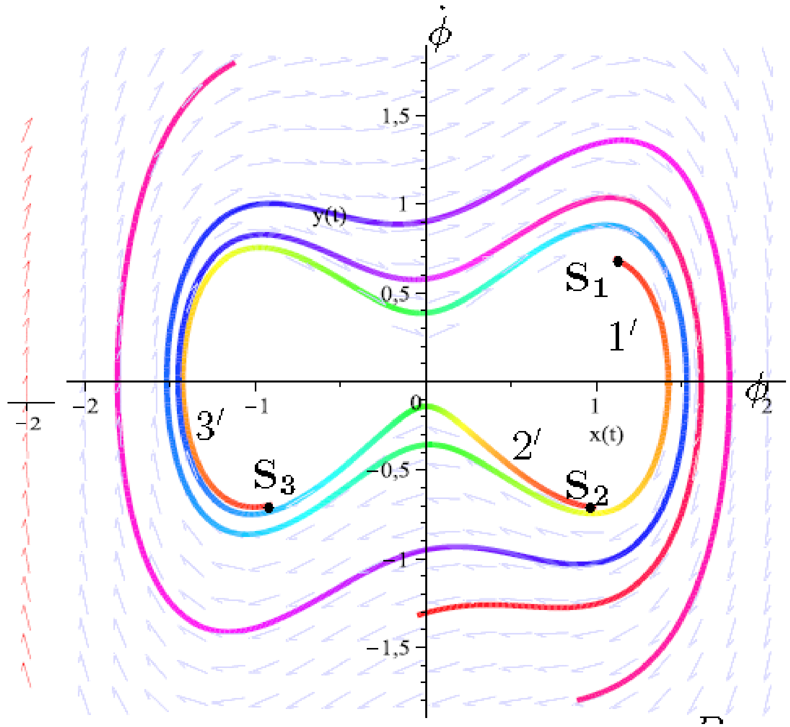
$$\ddot{\phi} + 3\sqrt{\frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + \frac{\epsilon}{4}\phi^4 - \frac{\mu^2}{2}\phi^2 + \Lambda\right)}\,\dot{\phi} = \mu^2\phi - \epsilon\phi^3$$

2 different situations. The potential energy is:

positive defined



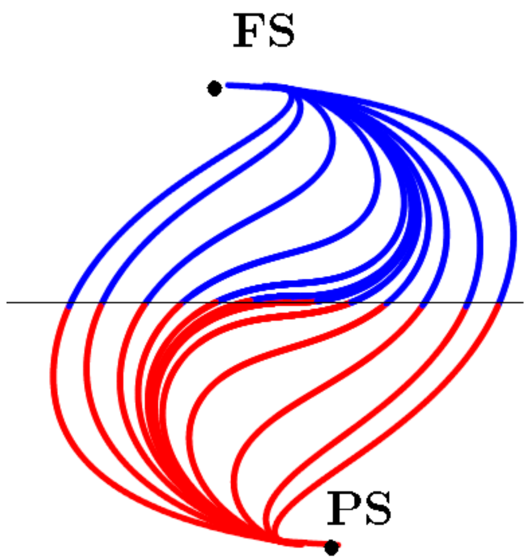
not positive defined



Higgs with a negative cosmological constant

$$\ddot{\phi} + 3\sqrt{\frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + \frac{\epsilon}{4}\phi^4 - \frac{\mu^2}{2}\phi^2 + \Lambda\right)}\,\dot{\phi} = \mu^2\phi - \epsilon\phi^3$$

In BGZK-variables



Nonlocal (SFT-inspired) inflation

- An extra current
- Effective negative cosmological constant


HAPPY
END
Врубель