Vacuum viscosity and acceleration

speaker: **G. Yu. Prokhorov** [1,2]

D. D. Lapygin [3]

O. V. Teryaev [1,2]

V. I. Zakharov [2,1]

[1] Joint Institute for Nuclear Research (JINR), BLTP, Dubna

[2] NRC Kurchatov Institute, Moscow

[3] Southern Federal University, Rostov-on-Don

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Part 1

Result

Obtained result (spoiler)

- We consider quantum **fluid** living **above** a **membrane** describing the **Rindler** (stretched) horizon:

$$\text{Membrane}: \quad 0 \leqslant \rho \leqslant l_c$$

membrane **thickness** – fields live at $\rho > l_c$

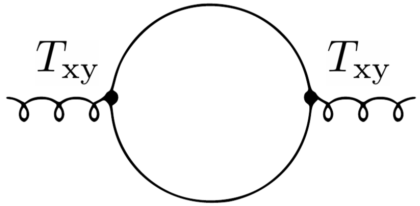
~ in fact, we are considering **Unruh radiation** with temperature:

$$T = a/2\pi$$

i.e. we are considering the *Minkowski vacuum*

- Use linear response theory - **Kubo formula** for **shear viscosity**:

$$\eta = \lim_{\omega \to 0} \int$$



Technical features:

- **Trivial** – use usual Minkowski massless propagators!
- **Non-trivial** - find the Fourier transform in the Rindler space.

Cases considered:

1) Free **scalar** fields (*discussed earlier*)

[Chirco, Eling, Liberati, PRD (2010), arXiv:1005.0475]

2) Free **Dirac** fields
3) Free **electromagnetic** fields (in covariant gauge, with ghosts...)

Naively expected:

Fields are **free** – **trivial** result?

Obtained:

Entanglement with states beyond horizon induces **viscosity** (as *believed*)

Obtained result (spoiler)

- We directly find shear viscosity - depends on the type of particles:

$$\eta^{\text{scalar}} = \frac{1}{6}\eta^{\text{Dirac}} = \frac{1}{12}\eta^{\text{photon}} = \frac{1}{1440\pi^2 l_c^2}$$

- Different approaches to find entropy: we use

[Becattini, Daher, Sheng, PLB (2024), arXiv:2309.05789]

$$s_{\text{loc}} = \frac{\partial p}{\partial T}\Big|_a$$

per unit area of horizon:

$$s = \int_{l_c}^{\infty} d\rho\, s_{\text{loc}}(\rho)$$

[Obukhov, Piskareva, Class. Quantum Grav.(1989)]

compare

- For example for spin 1 field:

$$p = \frac{1}{3}\left(\frac{\pi^2 T^4}{15} + \frac{T^2|a|^2}{6} - \frac{11|a|^4}{240\pi^2}\right)$$

Quantum corrections from acceleration

For Minkowski vacuum

$$T = T_U$$

$$s^{\text{scalar}} = \frac{1}{6}s^{\text{Dirac}} = \frac{1}{12}s^{\text{photon}} = \frac{1}{360\pi l_c^2}$$

In all considered cases:

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Locally – universal function for all spins:

$$\frac{\eta_{\text{loc}}}{s_{\text{loc}}}(\rho = l_c) = \frac{1}{8\pi}$$

compare

Part 1

Introduction and Motivation

Motivation: Unruh effect

[Blasone, (2018), e-Print: 1911.06002]

Formulation

The Minkowski **vacuum** is perceived by an **accelerated** observer as a medium with a finite (Unruh) **temperature**

$$T_U = \frac{a}{2\pi}$$

From the point of view of the quantum-statistical approach:

[Becattini, PRD (2018), arXiv:1712.08031]

Thus, the **mean values** of the thermodynamic quantities normalized to Minkowski vacuum should be **equal to zero** when the proper temperature, measured by comoving observer, equals to the **Unruh temperature**.

$$\langle \hat{T}^{\mu\nu} \rangle = 0 \qquad (T = T_U)$$

Example:

[Prokhorov, Teryaev, Zakharov, PRD (2019), arXiv:1903.09697]

$$\langle \hat{T}^{\mu\nu} \rangle^0_{\text{fermi}} = \Big(\frac{7\pi^2 T^4}{60} + \frac{T^2|a|^2}{24} - \frac{17|a|^4}{960\pi^2} \Big) u^\mu u^\nu - \Big(\frac{7\pi^2 T^4}{180} + \frac{T^2|a|^2}{72} - \frac{17|a|^4}{2880\pi^2} \Big) \Delta^{\mu\nu}$$

- Well-known in Rindler space. But can be obtained by a statistical method without switching to Rindler coordinates

- Supports the **"objective"** interpretation of the effect of the Unruh (in contrast to the fact that it is just the effect of the detector).

Minimal viscosity bound

Hydrodynamics in linear gradients - corrections to EMT with **dissipation**:

$$T_{\mu\nu} = T_{\mu\nu}^{\rm ideal} + T_{\mu\nu}^{\rm diss} \qquad T_{\mu\nu}^{\rm ideal} = (\varepsilon + p)u_\mu u_\nu - p g_{\mu\nu}$$

$$T_{\mu\nu}^{\rm diss} = -\eta(\nabla_\mu u_\nu + \nabla_\nu u_\mu - u_\mu u^\alpha \nabla_\alpha u_\nu - u_\nu u^\alpha \nabla_\alpha u_\mu) - \left(\zeta - \frac{2}{3}\eta\right) \nabla^\alpha u_\alpha (g_{\mu\nu} - u_\mu u_\nu) + \mathcal{O}(\nabla^2 u)$$

Bound inspired by string theory:

- **There are no completely ideal fluids!**
- It is believed that QGP near this limit
- **does not cover case of Rindler space!**

$$\frac{\eta}{s} \geqslant \frac{1}{4\pi}$$

KSS-bound

[Kovtun, Son, Starinets, PRL (2005), arXiv:hep-th/0405231]

- **Some "feeling"**: according to the holographic principle, the viscosity is associated with the scattering of gravitons on black brane, and entropy with the horizon area – their ratio will be finite.

- Plenty of work about KSS Bound
- The simplest illustration: the uncertainty principle for energy

$$\eta \sim \varepsilon \tau_{\rm free}$$
$$s \sim n$$

$$\frac{\eta}{s} \sim \frac{\varepsilon}{n}\tau_{\rm free} = E\,\tau_{\rm free} \gtrsim \hbar$$

A similar bound for bulk viscosity

[Buchel, PLB (2008), arXiv:0708.3459]

$$\frac{\zeta}{\eta} \geqslant 2\left(\frac{1}{p} - c_s^2\right)$$

[Dobado, Llanes-Estrada, Rincon, AIP Conf.Proc. (2008), e-Print: 0804.2601]

Emergent gravity and Membrane paradigm
(general idea and very superficial overview)

1 Scenario: Emergent gravity

[Jacobson, PRL (1995), e-Print: gr-qc/9504004]

[Eling, JHEP (2008), e-Print: 0806.3165]

The principle of equivalence: Ridler's local horizon at each point

+

Horizon area is related to entropy $S = \frac{k_B A}{4 l_p^2}$

equilibrium

$$\delta Q = T \delta S$$

→ Einstein equation

nonequilibrium

$$\delta Q = T \delta S + \delta W$$

→ Prediction for viscosity

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

2 Scenario: Membrane paradigm

Stretched horizon:

[Susskind, The Black Hole War, 2009]

- Due to the slowdown of the time near the horizon, the matter falling on it “stucks” at a certain distance from horizon
- “Spread” in the transverse direction.

$\rho = 0$ true horizon

$\rho = l_c$ stretched horizon

→ $\text{Membrane}: \quad 0 \leqslant \rho \leqslant l_c$

[Thorne, Price, Macdonald, Black holes: the membrane paradigm (1986)]

[Parikh, Wilczek, PRD (1998), arXiv:gr-qc/9712077]

- Membrane paradigm
- It has hydrodynamic properties
- It has viscosity $\frac{\eta}{s} = \frac{1}{4\pi}$

By integrating the action, we can obtain the Navier-Stokes equation

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} R + \frac{1}{8\pi} \int d^3x \sqrt{\pm h} K + S_{\text{matter}}$$

Motivation: statistical quantum mechanics

Zubarev density operator: statistical interaction with vorticity and acceleration

$$\hat{\rho} = \frac{1}{Z} \exp\Big\{ -\beta_\mu(x)\hat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}\hat{J}^{\mu\nu}_x + \xi\hat{Q} \Big\}$$

$$\varpi_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} w^\alpha u^\beta + \alpha_\mu u_\nu - \alpha_\nu u_\mu$$

$$\varpi_{\mu\nu}\hat{J}^{\mu\nu} \;=\; -2\alpha^\rho \hat{K}_\rho - 2w^\rho \hat{J}_\rho$$

- Plenty results on **vorticity** and **magnetic field** effects:

  -- quantum anomaly transport effects:

  chiral magnetic effect (**CME**), **[Fukushima, Kharzeev, Warringa, PRD (2008), e-Print: 0808.3382]**

  chiral vortical effect (**CVE**), **[Son, Surowka, PRL (2009), e-Print: 0906.5044]**

  kinematical vortical effect (**KVE**), **[Prokhorov, Teryaev, Zakharov, PRL (2022), e-Print: 2207.04449]**

  many other effects…

  -- vortical polarization **[STAR, Nature (2017), arXiv: 1701.06657]** **[Rogachevsky, Sorin, Teryaev, PRC (2010), e-Print: 1006.1331]** **[Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC (2017), e-Print: 1610.02506]**

  -- rotation on the lattice **[Braguta, Kotov, Kuznedelev, Roenko, PRC (2021), e-Print: 2102.05084]**

  …

- Modern development: **shear** effects **[Daher, Sheng, Wagner, Becattini, (2025), e-Print: 2503.03713]** **[Buzzegoli, (2025), e-Print: 2502.15520]**

Statement of the problem

- Does Unruh radiation have viscosity? How is it related to the KSS limit?

- Direct calculation for quantum fluid above the membrane (which considered mostly “classically”).

Part 2

Shear viscocity in Rindler space from Kubo formula

Method

Rindler coordinates and stretched horizon

Rindler's metric describes the accelerated reference system:

$$ds^2 = \rho^2 d\tau^2 - d\mathrm{x}^2 - d\mathrm{y}^2 - d\rho^2$$

- The relationship between Rindler coordinates and Minkowski coordinates:

$$t = \rho \sinh \tau$$
$$z = \rho \cosh \tau$$

$$\text{Horizon}: \quad g_{00}(\rho = 0) = 0$$

$$a = \frac{1}{\rho}$$

Acceleration - the inverse distance to the horizon.

$$a_\mu = u^\nu \nabla_\nu u_\mu$$

- As was said, the fields are stuck at a **certain distance** from the horizon:

$$\rho \in [l_c, \infty)$$

Kubo formula: Rindler space

Due to the fluctuation-dissipation theorem, dissipation coefficients can be found from fluctuations in equilibrium:

Kubo's formula for viscosity **[Zubarev, Nonequilibrium statistical thermodynamics, Studies in soviet science, 1974]**

$$\eta = \lim_{\omega \to 0} \frac{1}{\omega} \int d^4x e^{i\omega t} \theta(t) \langle [\hat{T}_{\rm xy}(x), \hat{T}_{\rm xy}(0)] \rangle$$

- Can be obtained from the interaction vertex with gravitons $\delta g_{\mu\nu} \hat{T}^{\mu\nu}$
- Contains a double limit $\omega, \vec{q} \to 0$

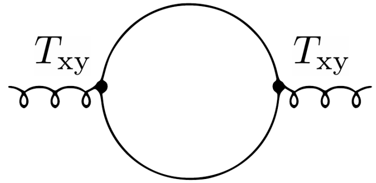
  First $\vec{q} \to 0$. Reflects the dissipative nature.

In the Rindler space: **[Chirco, Eling, Liberati, PRD (2010), arXiv:1005.0475]**

$$\eta = \pi \lim_{\omega \to 0} \int_{l_c}^{\infty} \rho' \, d\rho' \int_{l_c}^{\infty} \rho \, d\rho \int_{-\infty}^{\infty} d{\rm x} \, d{\rm y} \, d\tau e^{i\omega\tau} \langle 0 | \hat{T}_{\rm xy}(\tau, {\rm x}, {\rm y}, \rho) \hat{T}_{\rm xy}(0,0,0,\rho') | 0 \rangle_{\rm M}$$

- In the limit $\omega \to 0$, one can pass from the retarded Green's function to the Wightman function.
- We consider free fields:

$$\eta = \lim_{\omega \to 0} \int$$



per unit horizon area

$$\eta = \int_{l_c}^{\infty} d\rho' \, \eta_{\rm loc}(\rho')$$

Entropy derivation

Thermodynamic relations are modified in a medium with spin:

[Becattini, Daher, Sheng, PLB (2024), arXiv:2309.05789]

[Obukhov, Piskareva, Class. Quantum Grav.(1989)]

$$dp = s dT + n d\mu + \frac{1}{2} S^{\mu\nu} d\omega_{\mu\nu}$$

In a state of global equilibrium, it contains the vorticity tensor.
For an accelerated medium:

$$\omega_{\mu\nu} = a_\mu u_\nu - a_\nu u_\mu$$

$$\boxed{s_{\,\mathrm{loc}} = \left.\frac{\partial p}{\partial T}\right|_a}$$

$$p = -\frac{1}{3} \langle \hat{T}_{\mu\nu} \rangle \Delta^{\mu\nu}$$

- Unlike viscosity case, it is necessary to move away from the Minkowski vacuum

$$T = T_U + dT$$

$$\text{Minkowsky vacuum:} \quad s_{\,\mathrm{loc}}(T = T_U, |a|) \underset{|a| \to 1/\rho}{=} s_{\,\mathrm{loc}}(\rho)$$

Entropy per unit area of the horizon:

$$s = \int_{l_c}^{\infty} d\rho \, s_{\,\mathrm{loc}}(\rho)$$

Spin 0

[Chirco, Eling, Liberati, PRD (2010), arXiv:1005.0475]

Correlator with two EMTs

[Birrell, Davies, Quantum Fields in Curved Space, Cambridge University Press, 1982]

- Improved stress-energy tensor of a massless real scalar field:

$$T_{\mu\nu} = (1-2\xi)\partial_\mu\varphi\partial_\nu\varphi + (2\xi - \frac{1}{2})\eta_{\mu\nu}\partial_\alpha\varphi\partial^\alpha\varphi - 2\xi(\partial_\mu\partial_\nu\varphi)\varphi + \frac{\xi}{2}\eta_{\mu\nu}\varphi\partial^\alpha\partial_\alpha\varphi$$

- The correlator can be found in the Minkowski metric:

$$\langle 0|\hat{T}_{\mu\nu}(x)\hat{T}_{\alpha\beta}(y)|0\rangle_{\mathrm{M}} = \underbrace{\frac{4}{3\pi^4}\mathcal{I}_{\mu\nu\alpha\beta}(x-y)}_{} + \frac{240(\xi-1/6)^2}{\pi^4}\underbrace{\widetilde{\mathcal{I}}_{\mu\nu\alpha\beta}(x-y)}_{}$$

a piece universal for conformally symmetric theories

deviation from conformal symmetry

[Erdmenger, Osborn, Nucl.Phys.B (1997), arXiv:hep-th/9605009]

$$\mathcal{I}_{\mu\nu\alpha\beta}(b) = \frac{b_\mu b_\nu b_\alpha b_\beta}{\bar{b}^{12}} - \frac{\eta_{\mu\alpha}b_\nu b_\beta}{4\bar{b}^{10}} - \frac{\eta_{\nu\alpha}b_\mu b_\beta}{4\bar{b}^{10}} - \frac{\eta_{\mu\beta}b_\nu b_\alpha}{4\bar{b}^{10}} - \frac{\eta_{\nu\beta}b_\mu b_\alpha}{4\bar{b}^{10}} + \frac{\eta_{\mu\alpha}\eta_{\nu\beta}}{8\bar{b}^8} + \frac{\eta_{\mu\beta}\eta_{\nu\alpha}}{8\bar{b}^8} - \frac{\eta_{\mu\nu}\eta_{\alpha\beta}}{16\bar{b}^8}$$

- The general structure follows from symmetry and dimensional considerations

$$\widetilde{\mathcal{I}}_{\mu\nu\alpha\beta}(b) = \frac{b_\mu b_\nu b_\alpha b_\beta}{\bar{b}^{12}} - \frac{\eta_{\mu\alpha}b_\nu b_\beta}{10\bar{b}^{10}} - \frac{\eta_{\nu\alpha}b_\mu b_\beta}{10\bar{b}^{10}} - \frac{\eta_{\mu\beta}b_\nu b_\alpha}{10\bar{b}^{10}} - \frac{\eta_{\nu\beta}b_\mu b_\alpha}{10\bar{b}^{10}} + \frac{\eta_{\mu\alpha}\eta_{\nu\beta}}{80\bar{b}^8} + \frac{\eta_{\mu\beta}\eta_{\nu\alpha}}{80\bar{b}^8} + \frac{13\eta_{\mu\nu}\eta_{\alpha\beta}}{80\bar{b}^8} - \frac{3\eta_{\mu\nu}b_\alpha b_\beta}{10\bar{b}^{10}} - \frac{3\eta_{\alpha\beta}b_\mu b_\nu}{10\bar{b}^{10}}.$$

$\bar{b}^2 = b^2 - i\varepsilon b_0$ poles are shifted

Fourier transform in Rindler space

The dependence on ξ goes away after integration in the horizon plane:

$$\int dx\, dy\, \langle 0|\hat{T}_{\mu\nu}(t,\mathrm{x},\mathrm{y},\mathrm{z})\hat{T}_{\alpha\beta}(0,0,0,\mathrm{z}')|0\rangle_{\mathrm{M}} = -\frac{1}{30\pi^3(t^2-(\mathrm{z}-\mathrm{z}')^2-i\varepsilon t)^3}$$

Local viscosity – at a certain distance from the horizon

$$\eta^{\mathrm{scalar}}_{\mathrm{loc}}(\rho) = \frac{\rho\left[\rho^4+4\rho^2 l_c^2-5l_c^4-4l_c^2(2\rho^2+l_c^2)\ln\frac{\rho}{l_c}\right]}{240(\rho^2-l_c^2)^4\pi^2}$$

Viscosity per unit area of the horizon:

$$\eta = \int_{l_c}^{\infty} d\rho'\, \eta_{\mathrm{loc}}(\rho') \quad \Rightarrow \quad \boxed{\eta^{\mathrm{scalar}} = \frac{1}{1440\pi^2 l_c^2}}$$

- Diverges in the limit $l_c \to 0$

Typical for Rindler space

[Solodukhin, Living Rev. Rel. (2011), arXiv:1104.3712]

Entropy

[Page, PRD 25, 1499 (1982)]

[Dowker, Class. Quant. Grav. (1994), arXiv:hep-th/9401159]

The energy-momentum tensor of accelerated scalar fields is well known

$$\langle \hat{T}^{\text{scalar}}_{\mu\nu} \rangle = \left(\frac{\pi^2 T^4}{30} - \frac{|a|^4}{480\pi^2} \right) \left(u_\mu u_\nu - \frac{\Delta_{\mu\nu}}{3} \right)$$

For the case $\xi = 1/6$

$$\Delta_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$$

Corresponding pressure:

$$p^{\text{scalar}}(T, a) = \frac{1}{3} \left(\frac{\pi^2 T^4}{30} - \frac{|a|^4}{480\pi^2} \right)$$

Local entropy

$$s_{\text{loc}} = \left. \frac{\partial p}{\partial T} \right|_a \Rightarrow s^{\text{scalar}}_{\text{loc}}(T) = \frac{2\pi^2 T^3}{45} \xRightarrow[a = 1/\rho]{T = a/2\pi} s^{\text{scalar}}_{\text{loc}}(\rho) = \frac{1}{180\pi\rho^3}$$

Entropy per unit area of the horizon:

$$s^{\text{scalar}} = \frac{1}{360\pi l_c^2}$$

Shear viscosity/entropy ratio

- Viscosity and entropy diverge in the limit $l_c \to 0$

$$\eta^{\text{scalar}} = \frac{1}{1440\pi^2 l_c^2} \qquad s^{\text{scalar}} = \frac{1}{360\pi l_c^2}$$

- But their ratio is finite and does not depend on l_c

$$\left.\frac{\eta}{s}\right|_{\text{scalar}} = \frac{1}{4\pi}$$

Saturates KSS bound

- The ratio of local viscosity to local entropy is described by a function depending on l_c :

$$\frac{\eta_{\text{loc}}}{s_{\text{loc}}}(\rho) = f(\rho/l_c)$$

$$f(x) = \frac{x^4(x^4 + 4x^2 - 5 - 4(2x^2+1)\ln x)}{4\pi(x-1)^4}$$

Spin ½

Correlator with two EMTs

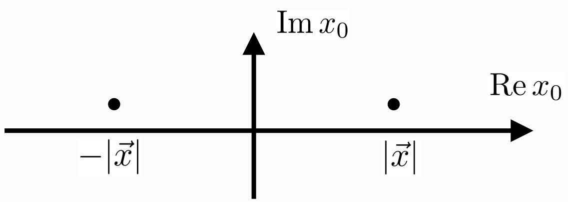
Belinfante energy-momentum tensor for free massless Dirac fields:

$$T_{\mu\nu} = \frac{i}{4}(\bar{\psi}\gamma_\mu\partial_\nu\psi - \partial_\nu\bar{\psi}\gamma_\mu\psi + \bar{\psi}\gamma_\nu\partial_\mu\psi - \partial_\mu\bar{\psi}\gamma_\nu\partial_\nu\psi)$$

Propagator (Wightman function)

$$S_{ab}(x) = \langle 0|\psi_a(x)\bar{\psi}_b(0)|0\rangle_M = \frac{i}{2\pi^2}\frac{(\gamma x)_{ab}}{(x^2 - i\varepsilon x_0)^2}$$

The poles are shifted upward relative to the real time axis:



For convenience, we split the point (not a regularization - no external fields):

$$\langle 0|\hat{T}_{\mu\nu}(x)\hat{T}_{\alpha\beta}(y)|0\rangle_{\mathrm{M}} = \lim_{\substack{x_1, x_2 \to x \\ y_1, y_2 \to y}} \mathcal{D}^{ab}_{\mu\nu}(\partial_{x_1}, \partial_{x_2})\mathcal{D}^{cd}_{\alpha\beta}(\partial_{y_1}, \partial_{y_2})\langle 0|\bar{\psi}_a(x_1)\psi_b(x_2)\bar{\psi}_c(y_1)\psi_d(y_2)|0\rangle_{\mathrm{M}}$$

Wick's theorem for Wightman functions **[Bogoliubov, Shirkov, Quantum Fields, 1983]**

$$\langle 0|\bar{\psi}_a(x_1)\psi_b(x_2)\bar{\psi}_c(y_1)\psi_d(y_2)|0\rangle_{M,\mathrm{connected}} = \langle 0|\bar{\psi}_a(x_1)\psi_d(y_2)|0\rangle_{\mathrm{M}}\langle 0|\psi_b(x_2)\bar{\psi}_c(y_1)|0\rangle_{\mathrm{M}}$$

We take into account only connected contributions

Correlator with two EMTs

Substitute Green's functions, take derivatives, and calculate traces with gamma matrices:

$$
\begin{aligned}
\langle 0|\hat{T}_{\mu\nu}(x)\hat{T}_{\alpha\beta}(y)|0\rangle_{\mathrm{M}} = \frac{1}{16}\mathrm{tr}\Big\{ & \gamma_\mu\partial_\nu S(b)\gamma_\alpha\partial_\beta S(b) - \gamma_\mu\partial_\beta\partial_\nu S(b)\gamma_\alpha S(b) + \gamma_\mu\partial_\nu S(b)\gamma_\beta\partial_\alpha S(b) \\
& -\gamma_\mu\partial_\alpha\partial_\nu S(b)\gamma_\beta S(b) - \gamma_\mu S(b)\gamma_\alpha\partial_\nu\partial_\beta S(b) + \gamma_\mu\partial_\beta S(b)\gamma_\alpha\partial_\nu S(b) \\
& -\gamma_\mu S(b)\gamma_\beta\partial_\alpha\partial_\nu S(b) + \gamma_\mu\partial_\alpha S(b)\gamma_\beta\partial_\nu S(b) + \gamma_\nu\partial_\mu S(b)\gamma_\alpha\partial_\beta S(b) \\
& -\gamma_\nu\partial_\beta\partial_\mu S(b)\gamma_\alpha S(b) + \gamma_\nu\partial_\mu S(b)\gamma_\beta\partial_\alpha S(b) - \gamma_\nu\partial_\alpha\partial_\mu S(b)\gamma_\beta S(b) \\
& -\gamma_\nu S(b)\gamma_\alpha\partial_\beta\partial_\mu S(b) + \gamma_\nu\partial_\beta S(b)\gamma_\alpha\partial_\mu S(b) - \gamma_\nu S(b)\gamma_\beta\partial_\alpha\partial_\mu S(b) \\
& +\gamma_\nu\partial_\alpha S(b)\gamma_\beta\partial_\mu S(b)\Big\},
\end{aligned}
$$

The result is:

$$
\langle 0|\hat{T}_{\mu\nu}(x)\hat{T}_{\alpha\beta}(y)|0\rangle_{\mathrm{M}} = \frac{8}{\pi^4}\mathcal{I}_{\mu\nu\alpha\beta}(x-y)
$$

Up to a common coefficient, the same as for conformally symmetric scalar field:

$$
\mathcal{I}_{\mu\nu\alpha\beta}(b) = \frac{b_\mu b_\nu b_\alpha b_\beta}{\bar{b}^{12}} - \frac{\eta_{\mu\alpha}b_\nu b_\beta}{4\bar{b}^{10}} - \frac{\eta_{\nu\alpha}b_\mu b_\beta}{4\bar{b}^{10}} - \frac{\eta_{\mu\beta}b_\nu b_\alpha}{4\bar{b}^{10}} - \frac{\eta_{\nu\beta}b_\mu b_\alpha}{4\bar{b}^{10}} + \frac{\eta_{\mu\alpha}\eta_{\nu\beta}}{8\bar{b}^8} + \frac{\eta_{\mu\beta}\eta_{\nu\alpha}}{8\bar{b}^8} - \frac{\eta_{\mu\nu}\eta_{\alpha\beta}}{16\bar{b}^8}
$$

Fourier transform in Rindler space

Let us perform **integration in the Rindler horizon plane**:

let's move on to polar coordinates

$$\mathrm{x} = r\cos\phi\,, \qquad \mathrm{y} = r\sin\phi$$

Integration can be done explicitly (poles are shifted from the real axis).

We obtain:

$$\int_0^\infty r dr \int_0^{2\pi} d\phi \, \langle 0|\hat{T}_{\mathrm{xy}}\hat{T}_{\mathrm{xy}}|0\rangle_{\mathrm{M}} = \frac{1}{5\pi^3\alpha^3}$$

where $\alpha = -t^2 + (z - z')^2 + i\varepsilon t$

Fourier transform in Rindler space

Let's move on to **integration by Rindler time** $d\tau$

We move on to the Rindler coordinates in the integrand

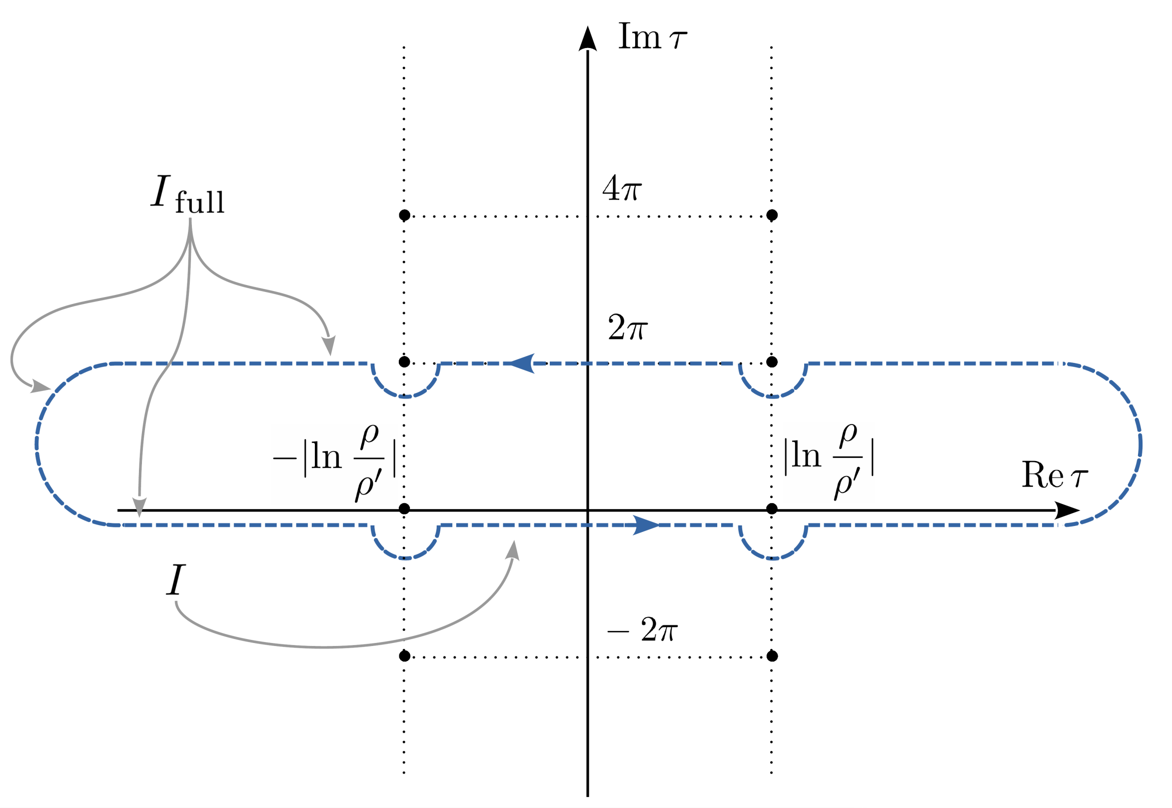
$$I = \pi \int_{-\infty}^{\infty} d\tau e^{i\tau\omega} \frac{1}{5\pi^3\alpha^3} = \int_{-\infty}^{\infty} \frac{e^{i\tau\omega}}{5\pi^2(\rho^2 + \rho'^2 - 2\rho\rho' \cosh(\tau) + i\varepsilon\tau)^3} d\tau$$

An infinite number of periodic poles located parallel to the imaginary axis:

$$\tau = \pm \ln \frac{\rho}{\rho'}(1 + i\varepsilon) + 2\pi i n \quad n = 0, \pm 1, \pm 2...$$

Fourier transform in Rindler space

Using the periodicity of the integrand with respect to the shift in the direction of the imaginary axis, we can close the integral:



- The relationship between the desired integral and the integral over a closed contour:

$$I = (1 - e^{-2\pi\omega})^{-1} I_{\text{full}}$$

- Only two poles fall inside the circuit.

$$\tau = \pm \ln \frac{\rho}{\rho'}$$

Let's use **Cauchy's theorem** and find the residues at the poles:

$$I_{\text{full}} = 2\pi i \sum_{\tau_0 = \pm \ln \frac{\rho}{\rho'}} \operatorname*{Res}_{\tau \to \tau_0} \frac{e^{i\tau\omega}}{5\pi^2[\rho^2 + \rho'^2 - 2\rho\rho' \cosh(\tau)]^3}$$

Fourier transform in Rindler space

Finding residues at the poles and passing to the limit of zero frequency, we obtain:

$$\lim_{\omega\to 0} I = \frac{3\rho'^4 - 3\rho^4 + 2[\rho^4 + 4\rho^2\rho'^2 + \rho'^4]\ln\frac{\rho}{\rho'}}{5\pi^2(\rho^2-\rho'^2)^5}$$

Taking the last integral over the distance to the horizon in the Fourier transform, we obtain the local viscosity:

$$\eta_{\text{loc}}^{\text{Dirac}}(\rho) = \frac{\rho\left[\rho^4 + 4\rho^2 l_c^2 - 5 l_c^4 - 4 l_c^2(2\rho^2 + l_c^2)\ln\frac{\rho}{l_c}\right]}{40(\rho^2 - l_c^2)^4\pi^2}$$

By directly integrating over the distance to the horizon, we obtain the viscosity per unit area of the horizon:

$$\eta^{\text{Dirac}} = \frac{1}{240\pi^2 l_c^2}$$

Entropy

The energy-momentum tensor is known:

[Page, PRD 25, 1499 (1982)]
[Prokhorov, Teryaev, Zakharov, PRD (2019), arXiv:1903.09697]
[Buzzegoli, Grossi, Becattini, JHEP (2017), arXiv:1704.02808]

$$\langle \hat{T}_{\mu\nu}^{\text{Dirac}} \rangle = \left(\frac{7\pi^2 T^4}{60} + \frac{T^2 |a|^2}{24} - \frac{17 |a|^4}{960\pi^2} \right) \left(u_\mu u_\nu - \frac{\Delta_{\mu\nu}}{3} \right)$$

Unlike a scalar field, the quadratic acceleration term contributes to the entropy

$$s_{\text{loc}} = \left. \frac{\partial p}{\partial T} \right|_a \quad \Rightarrow \quad s_{\text{loc}}^{\text{Dirac}}(T, a) = \frac{7\pi^2 T^3}{45} + \frac{T |a|^2}{36}$$

Local entropy (for Minkowski vacuum):

$$s_{\text{loc}}^{\text{Dirac}}(\rho) = \frac{1}{30\pi\rho^3}$$

Entropy per unit area of the horizon:

$$s^{\text{Dirac}} = \frac{1}{60\pi l_c^2}$$

Shear viscosity/entropy ratio

Viscosity and entropy differ from the case of the spin 0

$$\eta^{\text{Dirac}} = \frac{1}{240\pi^2 l_c^2} \qquad \eta^{\text{scalar}} = \frac{1}{1440\pi^2 l_c^2}$$

But the relation satisfies the KSS bound:

$$\left.\frac{\eta}{s}\right|_{\text{Dirac}} = \frac{1}{4\pi}$$

The ratio of local viscosity to local entropy is described by the same **universal function** as for spin 0:

$$\frac{\eta_{\text{loc}}}{s_{\text{loc}}}(\rho) = f(\rho/l_c)$$

$$f(x) = \frac{x^4(x^4 + 4x^2 - 5 - 4(2x^2+1)\ln x)}{4\pi(x-1)^4}$$

Spin 1

Correlator with two EMTs

[Birrell, Davies, Quantum Fields in Curved Space, Cambridge University Press, 1982]

Let's consider electromagnetic fields in R_ξ gauge:

$$T_{\mu\nu} = T_{\mu\nu}^{\rm M} + T_{\mu\nu}^{\rm G} + T_{\mu\nu}^{\rm ghost}$$ EMT contains three contributions

$$T_{\mu\nu}^{\rm M} = -F_{\mu\alpha}F_\nu{}^\alpha + \frac{1}{4}\eta_{\mu\nu}F^2$$ Maxwell's contribution

$$T_{\mu\nu}^{\rm G} = \frac{1}{\xi}\Big\{A_\mu\partial_\nu(\partial A) + A_\nu\partial_\mu(\partial A) - \eta_{\mu\nu}\big[A^\lambda\partial_\lambda(\partial A) + \frac{1}{2}(\partial A)^2\big]\Big\}$$ Contribution from the gauge-fixing term

$$T_{\mu\nu}^{\rm ghost} = \partial_\mu\bar{c}\partial_\nu c + \partial_\nu\bar{c}\partial_\mu c - \eta_{\mu\nu}\partial_\rho\bar{c}\partial^\rho c$$ Faddeev-Popov ghosts

Propagators (Wightman function) in coordinate representation:

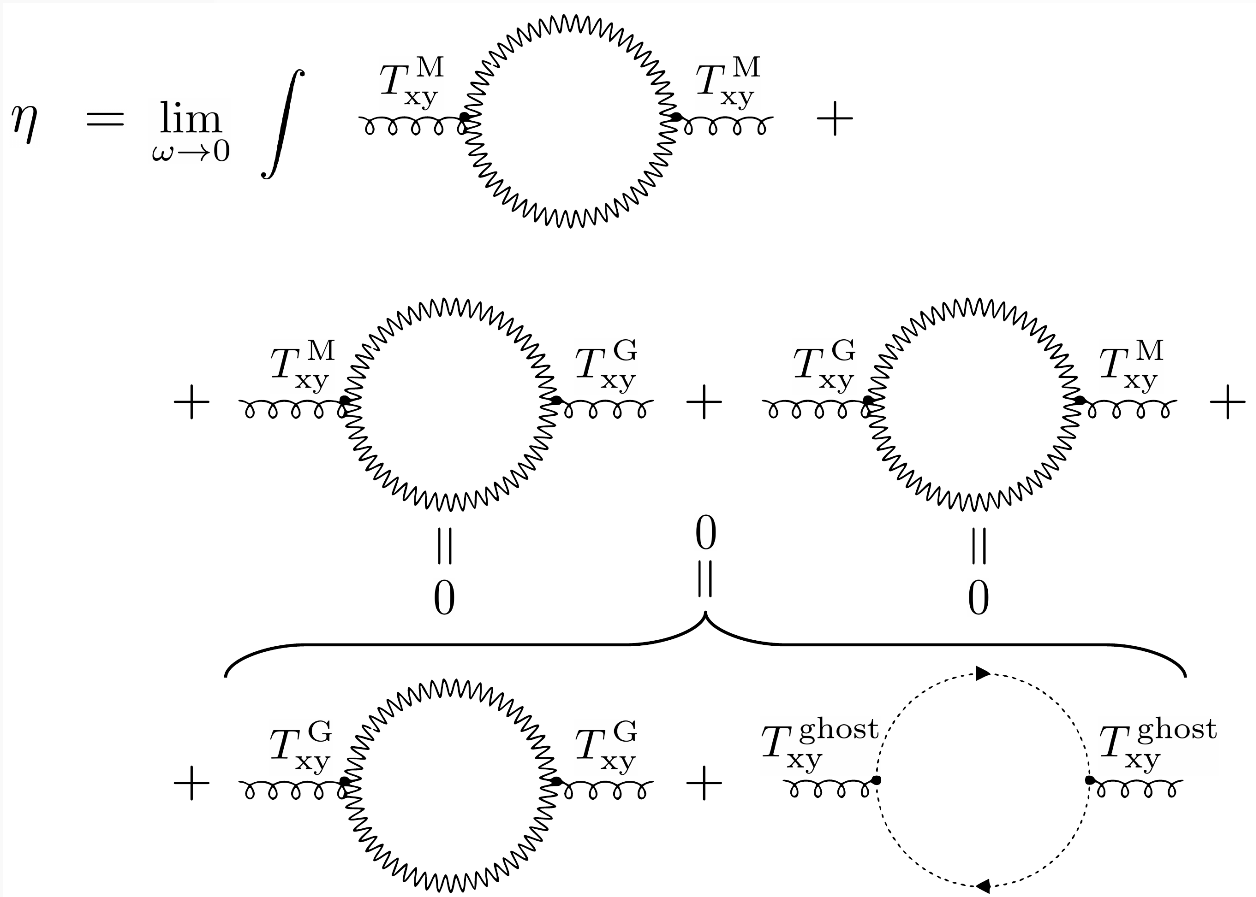
$$\langle 0|A_\mu(x)A_\nu(0)|0\rangle_M = \frac{1}{8\pi^2}\left(\frac{(1+\xi)\eta_{\mu\nu}}{x^2 - i\varepsilon x_0} + \frac{2(1-\xi)x_\mu x_\nu}{(x^2 - i\varepsilon x_0)^2}\right)$$

$$\langle 0|c(x)\bar{c}(0)|0\rangle_M = -\frac{1}{4\pi^2}\frac{1}{x^2 - i\varepsilon x_0}$$

Correlator with two EMTs

Expand the two-point correlator, selecting various contributions to the EMT operator

$$\langle 0|\hat{T}_{\mu\nu}(x)\hat{T}_{\alpha\beta}(y)|0\rangle_M = \langle 0|\hat{T}^{\rm M}_{\mu\nu}(x)\hat{T}^{\rm M}_{\alpha\beta}(y)|0\rangle_M + \langle 0|\hat{T}^{\rm M}_{\mu\nu}(x)\hat{T}^{\rm G}_{\alpha\beta}(y)|0\rangle_M + \langle 0|\hat{T}^{\rm G}_{\mu\nu}(x)\hat{T}^{\rm M}_{\alpha\beta}(y)|0\rangle_M$$
$$+\langle 0|\hat{T}^{\rm G}_{\mu\nu}(x)\hat{T}^{\rm G}_{\alpha\beta}(y)|0\rangle_M + \langle 0|\hat{T}^{\rm ghost}_{\mu\nu}(x)\hat{T}^{\rm ghost}_{\alpha\beta}(y)|0\rangle_M \,,$$



Correlator with two EMTs

The logic of calculations is similar to the case with the Dirac field.

The contributions of the ghosts and gauge-fixing terms cancel each other:

$$\langle 0|\hat{T}^{\,\mathrm{ghost}}_{\mu\nu}(x)\hat{T}^{\,\mathrm{ghost}}_{\alpha\beta}(y)|0\rangle_M = -\langle 0|\hat{T}^{\,\mathrm{G}}_{\mu\nu}(x)\hat{T}^{\,\mathrm{G}}_{\alpha\beta}(y)|0\rangle_M$$

The entire contribution is determined by the Maxwell term: the universal function

$$\langle 0|\hat{T}_{\mu\nu}(x)\hat{T}_{\alpha\beta}(y)|0\rangle_M = \langle 0|\hat{T}^{\,\mathrm{M}}_{\mu\nu}(x)\hat{T}^{\,\mathrm{M}}_{\alpha\beta}(y)|0\rangle_M = \frac{16}{\pi^4}\mathcal{I}_{\mu\nu\alpha\beta}(x-y)$$

Since the correlator differs only by the factor, the subsequent calculations are similar to the case of scalar and Dirac fields.

Since $\langle\hat{T}\hat{T}\rangle\Big|_{\mathrm{photon}} = \frac{1}{2}\,\langle\hat{T}\hat{T}\rangle\Big|_{\mathrm{Dirac}}$

then $\eta^{\,\mathrm{photon}} = \frac{1}{2}\,\eta^{\,\mathrm{Dirac}}$

We finally obtain:

$$\eta^{\,\mathrm{photon}} = \frac{1}{120\pi^2 l_c^2}$$

Does not depend on the gauge-parameter ξ

The result is gauge invariant.

Entropy

Entropy can be found similarly to the case of spins 0 and 1/2

The energy-momentum tensor is known: [Page, PRD 25, 1499 (1982)]

$$\langle \hat{T}_{\mu\nu}^{\text{photon}} \rangle = \left(\frac{\pi^2 T^4}{15} + \frac{T^2 |a|^2}{6} - \frac{11 |a|^4}{240\pi^2} \right) \left(u_\mu u_\nu - \frac{\Delta_{\mu\nu}}{3} \right)$$

$$s_{\text{loc}} = \frac{\partial p}{\partial T} \bigg|_a$$

Also, the quadratic acceleration term contributes to the entropy

Local entropy (for Minkowski vacuum):

$$s_{\text{loc}}^{\text{photon}}(T = T_U, |a| = 1/\rho) = \frac{1}{15\pi\rho^3}$$

Entropy per unit area of the horizon:

$$s^{\text{photon}} = \frac{1}{30\pi l_c^2}$$

Shear viscosity/entropy ratio

Viscosity and entropy differ from the case of spins 0 and ½

$$\eta^{\,\mathrm{photon}} = \frac{1}{120\pi^2 l_c^2} \qquad s^{\,\mathrm{photon}} = \frac{1}{30\pi l_c^2}$$

The ratio satisfies the KSS bound

$$\left.\frac{\eta}{s}\right|_{\mathrm{photon}} = \frac{1}{4\pi}$$

The ratio of local viscosity to local entropy is described by the same universal function as for spins 0 and ½:

$$\left.\frac{\eta_{\,\mathrm{loc}}}{s_{\,\mathrm{loc}}}(\rho)\right|_{\mathrm{photon}} = f(\rho/l_c)$$

$$f(x) = \frac{x^4(x^4 + 4x^2 - 5 - 4(2x^2 + 1)\ln x)}{4\pi(x-1)^4}$$

Part 3

Bulk viscosity

Bulk viscosity

Kubo formula for bulk viscosity: [Jeon, PRD (1995), arXiv:hep-ph/9409250]

$$\zeta = \pi \lim_{\omega\to 0} \int_{l_c}^{\infty} \rho\, d\rho \int_{l_c}^{\infty} \rho'\, d\rho' \int_{-\infty}^{\infty} dx\, dy\, d\tau e^{i\omega\tau} \langle 0|\hat{\mathcal{P}}(\tau,x,y,\rho)\hat{\mathcal{P}}(0,0,0,\rho')|0\rangle_{\mathrm{M}}$$

where $\hat{\mathcal{P}} = c_s^2 \hat{T}_0^0 + \frac{1}{3}\hat{T}_i^i$

In all cases considered:

Using equality

$$\mathcal{I}_\mu{}^\mu{}_{\alpha\beta} = \mathcal{I}_{\mu\nu\alpha}{}^\alpha = 0$$

$$\zeta = 0$$
$$\zeta_{\mathrm{loc}}(\rho) = 0$$

- The membrane paradigm problem – negative bulk viscosity of the black hole membrane
- Translation invariance of the Rindler horizon - it should be expected that it will not be negative

Part 4

Discussion

“Entanglement” viscosity?

- Thus, the view of the Unruh effect as an objective effect associated with the emergence of the media is strengthened:

  -- In an accelerated frame, the Minkowski vacuum behaves like a fluid

  Temperature of “vacuum fluid” $T = T_U$

  Viscosity of the “vacuum liquid” $\eta/s = 1/4\pi$

[Buchel, Liu and Starinets, Nucl.Phys.B (2005) arXiv:hep-th/0406264]

$$\frac{\eta}{s} = \frac{1}{4\pi}[1 + \frac{135\zeta(3)}{8(2g^2N_c)^{3/2}} + \ldots]$$

From string theory: KSS-bound is saturated for strong coupling (big ’t Hooft coupling)

- In our case, the **opposite situation** – KSS-bound is saturated for **free fields**.

  Free fields - what is the source of viscosity?

  Naively: $\eta \sim l_{\text{free}}$

  $l_{\text{free}} \to \infty$ ⇒ $\eta \to \infty$

“Entanglement” viscosity?

- Indirect indication of a connection with entanglement:

Entropy is in the denominator

$$s^{\,\mathrm{scalar}} = \frac{1}{6} s^{\,\mathrm{Dirac}} = \frac{1}{12} s^{\,\mathrm{photon}} = \frac{1}{360\pi l_c^2}$$

is related to entanglement

Rindler space has a horizon – an open system

+

Entanglement

⇩

Mixed states, entanglement entropy, decoherence - “**dissipation of information**”

Energy dissipation - viscosity?

Species problem

Bekenstein-Hawking: $$S_{\rm BH} = \frac{A}{4G\hbar}$$

Entanglement entropy: $$S_{entangl} \sim A$$

BUT depends on the number and type of fields

In particular, in accordance with that, we obtain:

$$s^{\rm scalar} = \frac{1}{6} s^{\rm Dirac} = \frac{1}{12} s^{\rm photon} = \frac{1}{360\pi l_c^2}$$

But the same for “entanglement” viscosity:

$$\eta^{\rm scalar} = \frac{1}{6}\eta^{\rm Dirac} = \frac{1}{12}\eta^{\rm photon} = \frac{1}{1440\pi^2 l_c^2}$$

Their relation will be universal:

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Let us consider viscosity in the membrane paradigm as an analogue of the Beknestein-Hawking entropy. Then:

$$\frac{\eta_{\rm membrane}}{S_{\rm BH}} = \frac{1}{4\pi}$$

[Parikh, Wilczek, An Action for black hole membranes, PRD (1998), arXiv:gr-qc/9712077]

The “species problem” exists at the level of entropy and viscosity separately, but disappears for their ratio.

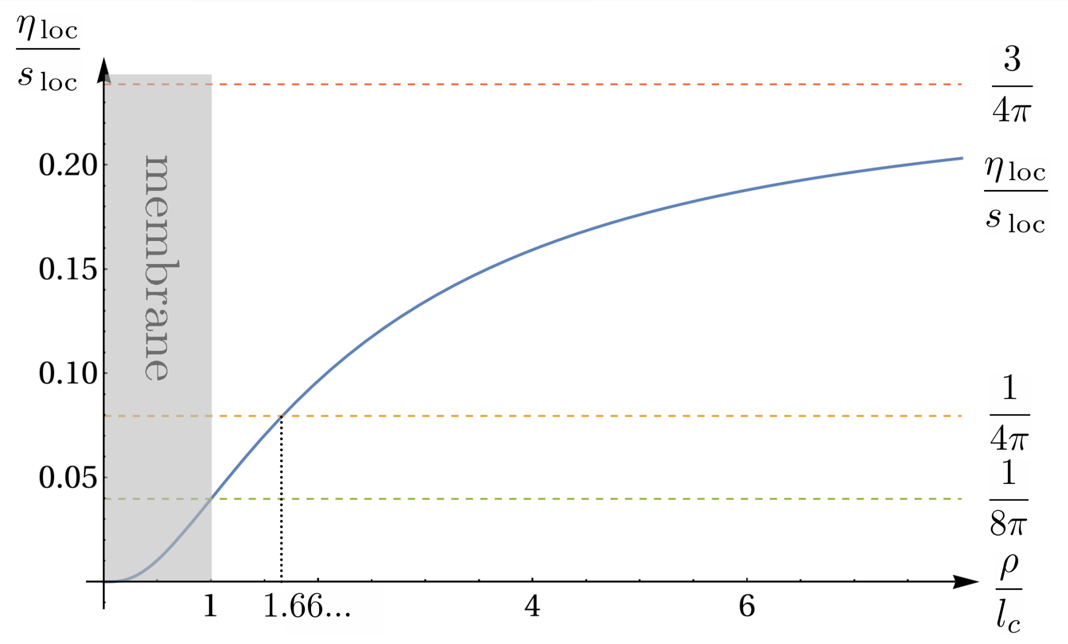
Local vs global

For all cases considered, the ratio of local shear viscosity and entropy is described by the universal function

$$\frac{\eta_{\rm loc}}{s_{\rm loc}}(\rho) = f(\rho/l_c)$$

where

$$f(x) = \frac{x^4(x^4 + 4x^2 - 5 - 4(2x^2+1)\ln x)}{4\pi(x-1)^4}$$



- Analytical continuation to the real horizon:

$$\frac{\eta_{\rm loc}}{s_{\rm loc}}(\rho \to 0) \to 0$$

- The viscosity to entropy ratio can be below the KSS bound:

$$\eta_{\rm loc}/s_{\rm loc}(\rho) < 1/4\pi$$

$$\rho < 1.66\ldots l_c$$

- On the surface of the membrane:

$$\frac{\eta_{\rm loc}}{s_{\rm loc}}(\rho = l_c) = \frac{1}{8\pi}$$

- On the contrary, far away from the membrane, the ratio is higher than the KSS bound:

$$\frac{\eta_{\rm loc}}{s_{\rm loc}}(\rho \to \infty) \to \frac{3}{4\pi}$$

Generalization

The two-point function has a universal form for conformal field theory:

[Erdmenger, Osborn, Nucl.Phys.B (1997), arXiv:hep-th/9605009]

$$\langle 0|\hat{T}_{\mu\nu}(x)\hat{T}_{\mu\nu}(0)|0\rangle_{\mathrm{M}} = c\,\mathcal{I}_{\mu\nu\alpha\beta}(x)$$

c is defined by conformal central charge

So, in general case: $\eta \sim c/l_c^2$

What can be said about entropy?

$s \sim c/l_c^2$ - for example, in theories with AdS/CFT duality

[Kovtun, Ritz, PRL (2008), arXiv:0801.2785]

If performed in our case, then for any conformal field theories:

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Conclusion and Outlook

Conclusion

- The obtained results support the “objective” interpretation of the Unruh effect – a medium arises that has finite **temperature** $T = T_U$ and **viscosity** $\eta/s = 1/4\pi$.

- The viscosity in the Rindler space for fields with spins ½ and 1 is calculated directly. This viscosity is not related to the interaction, and therefore, apparently, is a manifestation of **entanglement**.

- The average values of shear viscosity and entropy are different for different fields.

- However, their ratio satisfies the **KSS bound** for **all** considered **fields**: $\eta/s = 1/4\pi$. Could such universality be useful to understand the correspondence between Bekenstein-Hawking and entanglement entropies?

- **Locally**, the viscosity-to-entropy ratio may **violate KSS bound**. On the stretched horizon $\eta_{\rm loc}/s_{\rm loc} = 1/8\pi$. In general, the ratio is described by a **universal** function that is the same for different types of fields.

Outlook

- Beyond **Unruh temperature** $T \neq T_U$?
  - A more complicate analysis – conical space.
  - It would make it possible to demonstrate explicitly that $\frac{\eta}{s} = \frac{1}{4\pi}$ is a lower bound.
  - The role of **phase transition** at the Unruh temperature?

    [Prokhorov, Teryaev, Zakharov, Novel phase transition at the Unruh temperature, (2023), arXiv:2304.13151]

- **Higher spins** (work in progress)?
- Explicitly show the relationship with **entanglement** (by averaging over states inside the “black hole”)?

Emergent gravity and Membrane paradigm
(general idea and very superficial overview)

1 Scenario: Emergent gravity

[Jacobson, PRL (1995), e-Print: gr-qc/9504004]

[Eling, JHEP (2008), e-Print: 0806.3165]

Принцип эквивалентности: локальный горизонт Ридлера в каждой точке

+

Horizon area is related to entropy

$$S = \frac{k_B A}{4 l_p^2}$$

equilibrium

$$\delta Q = T\delta S$$

nonequilibrium

$$\delta Q = T\delta S + \delta W$$

EMT of matter contributes to the heat flux (and entropy increase) inside horizon

$$\delta Q = \int T_{\mu\nu}\xi^{\mu} d\Sigma^{\nu}$$

Raychaudhuri equation relates horizon area (and entropy) increase to Einstein tensor

Einstein equation

Raychaudhuri equation relates horizon area (and entropy) increase to shear (constructed from tangent vectors to geodesics)

Work of shear forses in hydrodynamics

$$\delta W = 2\eta \int \sigma_{\mu\nu}\sigma^{\mu\nu} d\Sigma$$

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Bulk viscosity

The membrane paradigm problem – negative bulk visco

Translation invariance of the Rindler horizon - it should

[Jeon, PRD (1995), arXiv:hep-ph/9409250]

Kubo formula for shear viscosity:

$$\zeta = \pi \lim_{\omega \to 0} \int_{l_c}^{\infty} \rho\, d\rho \int_{l_c}^{\infty} \rho'\, d\rho' \int_{-\infty}^{\infty} dx\, dy\, d\tau e^{i\omega\tau} \langle 0|\hat{\mathcal{P}}(\tau, x, y, \rho)\hat{\mathcal{P}}(0,0,0,\rho')|0\rangle_{\mathrm{M}}$$

where $\hat{\mathcal{P}} = c_s^2 \hat{T}_0^0 + \frac{1}{3}\hat{T}_i^i$

In all cases considered:

For example, for photons:

$$\langle \hat{T}_{\mu\nu}^{\mathrm{photon}} \rangle = \left(\frac{\pi^2 T^4}{15} + \frac{T^2 |a|^2}{6} - \frac{11|a|^4}{240\pi^2} \right) \left(u_\mu u_\nu - \frac{\Delta_{\mu\nu}}{3} \right)$$

$$\varepsilon = 3p$$

$$c_s^2 = \frac{\partial p}{\partial \varepsilon} = \frac{1}{3}$$

Then the correlator contains the trace of EMT

$$c_s^2 = 1/3 \quad \Rightarrow \quad \hat{\mathcal{P}} = \hat{T}_\mu^\mu$$

Using equality

$$\mathcal{I}_\mu{}^\mu{}_{\alpha\beta} = \mathcal{I}_{\mu\nu\alpha}{}^\alpha = 0$$

$$\zeta = 0$$

$$\zeta_{\mathrm{loc}}(\rho) = 0$$

We obtain the vanishing of the bulk viscosity for spins 0, ½ and 1

Satisfies the bound

$$\frac{\zeta}{\eta} \geqslant 2\left(\frac{1}{p} - c_s^2\right)$$

$$c_s^2 = 1/3$$

$$\zeta \geqslant 0$$