Transport properties of matter created in heavy-ion collisions: A Kinetic theory approach

Jayanta Dey

BLTP JINR

PHYSICAL REVIEW D 110, 114051 (2024)
PHYSICAL REVIEW D 111, 014029 (2025)

28 May 2025

Overview of the talk

1. Electric field induction in QGP by thermoelectric effect
   - Motivation and Objective
   - Formalism
   - Results
   - Effect of external magnetic field
2. Diffusion of conserved charges
   - Introduction
   - Formalism
   - Results

Title: *Electric field induction in quark-gluon plasma due to thermoelectric effects*
Authors: Kamaljeet Singh, Jayanta Dey, and Raghunath Sahoo

PHYSICAL REVIEW D 110, 114051 (2024)

Motivation and Objective

- Seebeck effect: Induction of electric field due to thermal gradient
- Quarks are electrically charged
- Temperature gradients in QGP leads to thermoelectric phenomena
- Is the strength of the induced EM field comparable to Λ_{QCD}?
- Can it have significant impact in the final state observables?

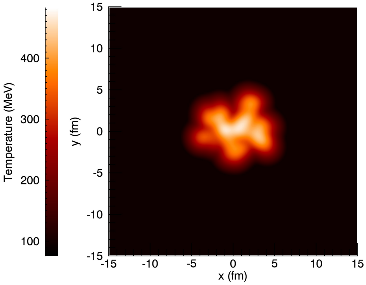


Figure: ECHO-QGP, Eur. Phys. J. C 73, 2524 (2013)

Formalism: Kinetic theory

$$\text{Current density} : J^\mu = N_s N_f N_c \; q \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{p^\mu}{E} f(x,p)$$

$p^\mu = (E, \vec{p})$; Chapman-Enskog expansion: $f = f_0 + f_1 + \mathcal{O}(Kn)$

$$\text{Boltzmann transport equation:} \quad p^\mu \partial_\mu f(x,p) + qF^{\mu\nu} p_\nu \frac{\partial f(x,p)}{\partial p^\mu} = \mathcal{C}(f)$$

Relaxation time approximation: $\mathcal{C}(f) = \frac{u^\mu p_\mu}{\tau_c}(f - f_0) = -\frac{u^\mu p_\mu}{\tau_c}\delta f$

$$\delta f = (\vec{p} \cdot \vec{\Omega}) \frac{\partial f^0}{\partial E}, \qquad \text{with } \vec{\Omega} = \alpha_1 \vec{E} + \alpha_2 \vec{\nabla} T + \alpha_3 \vec{\nabla} \dot{T}$$

Solving BTE we obtain expression of current density:

$$\vec{j} = \sum_i \frac{q_i g_i}{3} \int \frac{d^3\vec{p}}{(2\pi)^3} \left(\frac{p}{E_i}\right)^2 \tau_R^i \Big[- q_i \vec{E} + \frac{(E_i - \mathrm{b}_i h)}{T} \Big\{ \frac{\partial T}{\partial \vec{x}} - \tau_R^i \frac{\partial \dot{T}}{\partial \vec{x}} \Big\} \Big] \frac{\partial f^0}{\partial E_i}.$$

$$\begin{aligned} \text{For } \vec{J} = 0 : \sigma \ \mathbf{E} &= \kappa \ \mathbf{X} \\ \Rightarrow \mathbf{E} &= (\boldsymbol{\sigma}^{-1}\kappa)\mathbf{X} \end{aligned}$$

$$\Rightarrow \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \begin{pmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & S \end{pmatrix} \begin{pmatrix} \frac{dT}{dx} \\ \frac{dT}{dy} \\ \frac{dT}{dz} \end{pmatrix} . \tag{1}$$

Thermoelectric coefficient matrix.

Boltzmann equation and RTA

Seebeck coefficient: $S = \frac{\frac{\kappa}{T^2}}{\frac{\sigma_{el}}{T}}$

$$\kappa = \sum_i \frac{q_i g_i}{3T} \int \frac{d^3\vec{p}}{(2\pi)^3} \left(\frac{p}{E_i}\right)^2 (E_i - \mathrm{b}_i h)\tau_R^i f_i^0 (1 - f_i^0)\ (1 - \tau_R \frac{\partial}{\partial t})$$

$$\sigma = \sum_i \frac{q_i^2 g_i}{3T} \int \frac{d^3\vec{p}}{(2\pi)^3} \left(\frac{p}{E_i}\right)^2 \tau_R^i f_i^0 (1 - f_i^0)$$

Effective mass of partons

$$m_{u,d,s}(T,\mu_B) = m_{0\ u,d,s} + g(T)\sqrt{\frac{N_c^2-1}{8N_c}\left(T^2+\frac{\mu_B^2}{9\pi^2}\right)}.$$

$$m_g^2(T,\mu_B) = \frac{N_c}{6}g^2(T)T^2\left(1+\frac{N_f+\frac{\mu_B^2}{\pi^2T^2}}{6}\right).$$

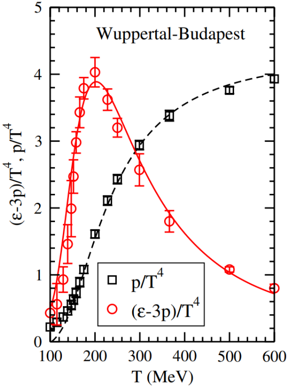


Figure: S. Plumari et al. Phys. Rev. D 84, 094004 (2011)

$$\text{Pressure}: P_{\mathrm{QGP}} = \sum_{i=u,d,s,g} P_{\mathrm{ideal}} - B(T)$$

$$\text{Thermodynamic consistency}: \left(\frac{\partial P}{\partial m_i}\right)_{T,\mu_B} = 0;\ \text{and},\ \epsilon + P = T\frac{dP}{dT}$$

Gorenstein and Yang, Phys. Rev. D 52, 5206 (1995); WB, JHEP 11 (2010) 077

Cooling rate: Gubser hydrodynamic flow

- Gubser generalizes the Bjorken flow
- Flow is constructed from symmetry consideration Boost, Rotation, and Reflection symmetry $(\eta = -\eta)$:

$$u^\tau = \cosh\kappa = \gamma_r, \quad \frac{u^r}{u^\tau} = \tanh\kappa = v_r, \quad u^\eta = u^\phi = 0.$$

$$T = \frac{1}{\tau f_*^{1/4}}\left[\frac{T_0}{(1+g^2)^{1/3}} + \frac{\mathrm{H}_0 g}{\sqrt{1+g^2}}\left\{1 - (1+g^2)^{1/6}\right.\right.$$
$$\left.\left. {}_2F_1\left(\frac{1}{2}, \frac{1}{6}; \frac{3}{2}; -g^2\right)\right\}\right].$$

Where, $g = \frac{1 - q^2\tau^2 + q^2 r^2}{2q\tau}$

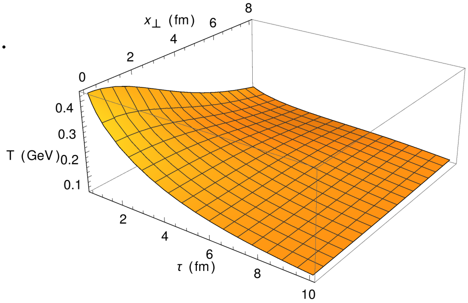
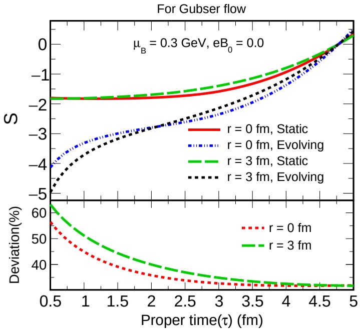
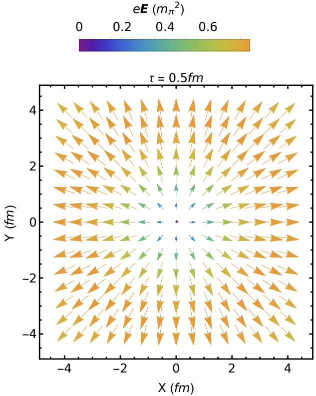


Figure: S. S. Gubser, Phys. Rev. D 82, 085027 (2010)

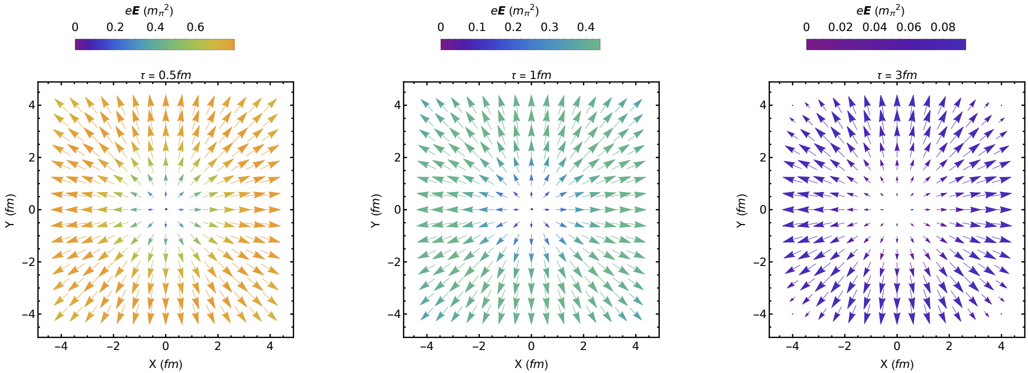
Understanding the Electric field





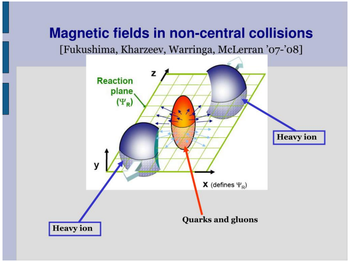
- Negative Seebeck: induced field is opposite to temperature gradient
- Enhancement of Seebeck in evolving case
- Electric field directed outward
- Field intensity increases from center to surface

Electric field: time evolution in isotropic QGP



- $E_z = 0$: because of boost invariant symmetry and isotropic transports
- Rapid decay of the induced field
- Maximum intensity $\approx 0.7 m_\pi^2$

Transient magnetic field in Peripheral HICs



Time-varying field: $\vec{B} = \vec{B}_0 \exp\left(-\frac{\tau}{\tau_B}\right)$,

Ansatz to solve BTE: $\delta f = (\vec{k}_i \cdot \vec{\Omega})\frac{\partial f_i^0}{\partial \omega}$, with

$$\vec{\Omega} = \alpha_1 \vec{E} + \alpha_2 \dot{\vec{E}} + \alpha_3 \vec{\nabla} T + \alpha_4 \vec{\nabla} \dot{T} + \alpha_5 (\vec{\nabla} T \times \vec{B}) + \alpha_6 (\vec{\nabla} T \times \dot{\vec{B}}) + \alpha_7 (\vec{\nabla} \dot{T} \times \vec{B}) + \alpha_8 (\vec{E} \times \vec{B}) + \alpha_9 (\vec{E} \times \dot{\vec{B}}) + \alpha_{10} (\dot{\vec{E}} \times \vec{B})$$

Thermoelectric effect in an external magnetic field

$$\mathbf{E} = (\boldsymbol{\sigma}^{-1}\kappa)\mathbf{X}$$

$$\Rightarrow \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \begin{pmatrix} S_B & \overline{NB} & NB \\ NB & S_B & \overline{NB} \\ \overline{NB} & NB & S_B \end{pmatrix} \begin{pmatrix} \frac{dT}{dx} \\ \frac{dT}{dy} \\ \frac{dT}{dz} \end{pmatrix}, \tag{2}$$

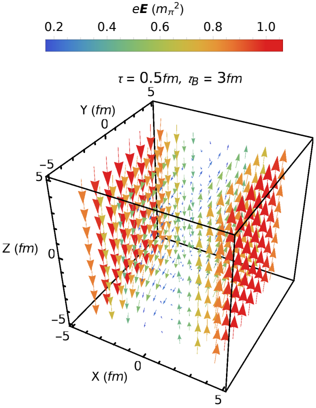
Dimensionless magneto-Seebeck and Nernst coefficients:

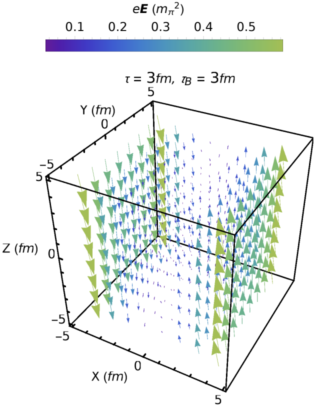
$$S_B = \frac{(\sigma_e^2 + \sigma_H^2)\kappa_0 + (2\sigma_e\sigma_H)\kappa_1}{T(\sigma_e^3 + 3\sigma_e\sigma_H^2)},$$

$$\overline{NB} = \frac{\sigma_H(\sigma_e + \sigma_H)\kappa_0 - \sigma_e(\sigma_e + \sigma_H)\kappa_1}{T(\sigma_e^3 + 3\sigma_e\sigma_H^2)},$$

$$NB = \frac{-\sigma_H(\sigma_e - \sigma_H)\kappa_0 + \sigma_e(\sigma_e - \sigma_H)\kappa_1}{T(\sigma_e^3 + 3\sigma_e\sigma_H^2)}. \tag{3}$$

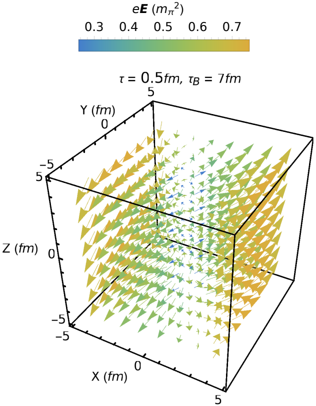
Induced Electric field in anisotropic QGP

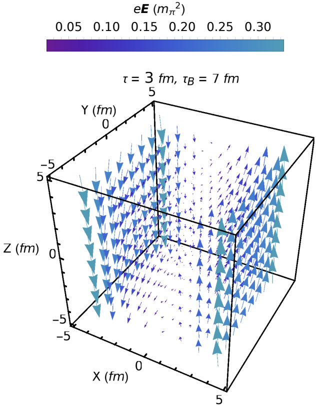




- $eB_0 = 5\mathrm{m}_\pi^2$ along Y
- $E_z \neq 0$. However, no Z dependency due to boost invariant T.
- Rotational symmetry is lost due to external magnetic field

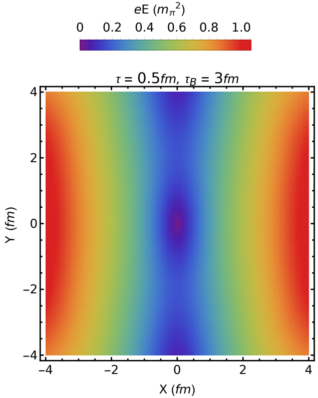
Induced Electric field in anisotropic QGP

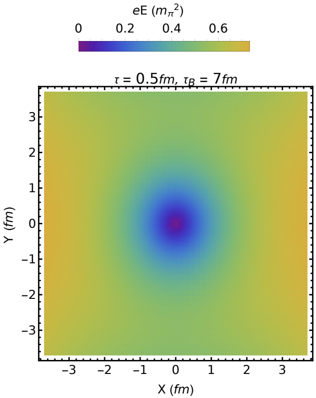




- $\tau_B = 7$ fm : Slow decay of the external magnetic field.
- Slower decay of eB imply higher σ. $S/S_B/N_B \sim \frac{1}{\sigma}$.
- Decrease in magnitude of induced field.

Magnitude Plot





- Initial strength $eE \approx 1\ \mathrm{m}_\pi^2$
- Rotational symmetry is broken in the induce field.
- Relatively isotropic field intensity for large τ_B.

Conclusion

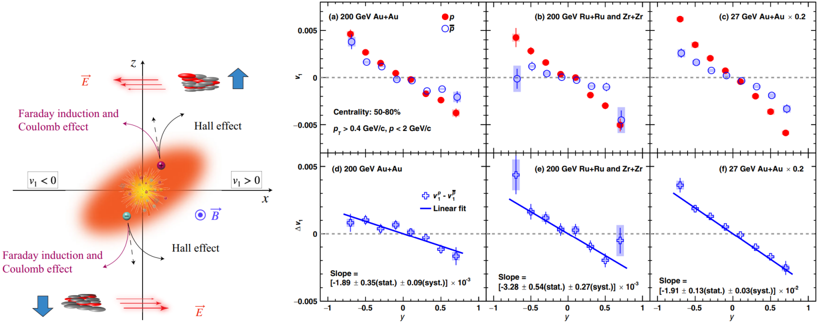


Figure: STAR: PHYSICAL REVIEW X 14, 011028 (2024)

- Induced field could enhance separation of v_1 for particle and anti-particle.

Part-II: Diffusion of conserved charges

Title: *Can charm fluctuation be a better probe to study the QCD critical point?*

Authors: Kangkan Goswami, Kshitish K. Pradhan, Dushmanta Sahu,
Jayanta Dey, and Raghunath Sahoo

PHYSICAL REVIEW D 111, 014029 (2025)

Motivation of diffusion in hadronic sector

- Fluctuations may survive till chemical freeze-out due to rapid expansion of fireball
- Fluctuations can be studied using higher order cumulants: baryon, strangeness etc.
- Non-monotonic behavior of cumulants: good probe for QCD critical point
- Diffusion affects the conserved charge fluctuations
- Conserved charge diffusion in hadronic phase can play major role in fluctuation studies
- Which conserved charge culumulant would be better proxy to trace critical point?

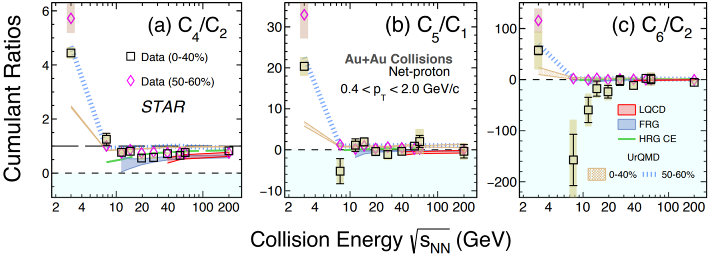


Figure: STAR: Phys. Rev. Lett. 130, 082301 (2023)

Diffusion: Fick's law

$$\text{Non}-\text{relativistic}: \;\; \Delta\vec{J}_q = -D_q \vec{\nabla} n_q(\alpha,\beta)$$

$$\text{Relativistic}: \Delta J_q^{\mu} = \kappa_q \nabla^{\mu}\alpha, \qquad \kappa_q = \frac{\partial n_q}{\partial \alpha_q} D_q$$

where, $\alpha = \frac{\mu_q}{T}$, $\beta = \frac{1}{T}$, $\nabla_\mu = \partial_\mu - u_\mu(u^\nu \partial_\nu)$

$$\text{Multicomponent system}: \begin{pmatrix} J_{\mathrm{B}}^{\nu} \\ J_{\mathrm{Q}}^{\nu} \\ J_{\mathrm{S}}^{\nu} \\ J_{\mathrm{C}}^{\nu} \end{pmatrix} = \begin{pmatrix} \kappa_{\mathrm{BB}} & \kappa_{\mathrm{BQ}} & \kappa_{\mathrm{BS}} & \kappa_{\mathrm{BC}} \\ \kappa_{\mathrm{QB}} & \kappa_{\mathrm{QQ}} & \kappa_{\mathrm{QS}} & \kappa_{\mathrm{QC}} \\ \kappa_{\mathrm{SB}} & \kappa_{\mathrm{SQ}} & \kappa_{\mathrm{SS}} & \kappa_{\mathrm{SC}} \\ \kappa_{\mathrm{CB}} & \kappa_{\mathrm{CQ}} & \kappa_{\mathrm{CS}} & \kappa_{\mathrm{CC}} \end{pmatrix} . \begin{pmatrix} \nabla^{\nu}\alpha_{\mathrm{B}} \\ \nabla^{\nu}\alpha_{\mathrm{Q}} \\ \nabla^{\nu}\alpha_{\mathrm{S}} \\ \nabla^{\nu}\alpha_{\mathrm{C}} \end{pmatrix}$$

Kinetic theory approach to transport coefficient

$$\text{Current}: J_q^\mu = n_q u^\mu + \Delta J_q^\mu .$$

$$J_q^\mu = \sum_{i=\text{hadrons}} q_i \int \frac{d^3 p_i}{(2\pi)^3} \frac{p_i^\mu}{E_i} f_i .$$

$$\text{Equilibrium distribution}: f_i^0 = \frac{1}{exp[\beta u \cdot p_i - \sum_q Q_{qi}\alpha_q] \pm 1}$$

$$\text{small deviation}: f_i = f_i^0 (1 + \Phi_i), \qquad \Phi_i = \sum_q B_i^q p_i^\mu \nabla_\mu \alpha_q .$$

Solving Boltzmann transport equation with RTA:

$$\kappa_{qq'} = \sum_i \int \frac{d^3 p_i}{(2\pi)^3} \frac{p_i^2}{3E_i} \left(q_i - \frac{n_q E_i}{\epsilon + P} \right) \frac{\tau_i}{E_i} \left(q_i' - \frac{n_{q'} E_i}{\epsilon + P} \right) f_i^0$$

HRG and VDWHRG

$$P_i^{id}(T,\mu) = \pm \frac{T g_i}{2\pi^2} \int_0^\infty p^2 dp \, \ln\{1 \pm \exp[-(E_i - \mu_i)/T]\},$$
$$\mu_i = B_i \mu_B + S_i \mu_S + Q_i \mu_Q + C_i \mu_C,$$

VDWHRG:

$$\mathrm{EoS}: P(T,n) = \frac{nT}{1-bn} - an^2.$$

$$For\ GCE: P_M(T,\mu) = \sum_{i \in M} P_i^{id}(T,\mu_M^*),$$

$$P_{B(\bar{B})}(T,\mu) = \sum_{i \in B(\bar{B})} P_i^{id}(T,\mu^*_{B(\bar{B})}) - a n^2_{B(\bar{B})}(T,\mu)$$

$$n(T,\mu) = \frac{\sum_i n_i^{id}(T,\mu^*)}{1 + b\sum_i n_i^{id}(T,\mu^*)}$$

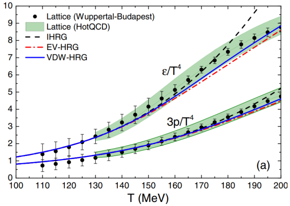


Figure: Vovchenko et al., PRL 118, 182301 (2017)

VDWHRD

$$\text{Effective chemical potential}: \mu^* = \mu - bP(T,\mu) - abn^2(T,\mu) + 2an(T,\mu)$$

$$\text{Total Pressure}: P(T,\mu) = P_M(T,\mu) + P_B(T,\mu) + P_{\bar{B}}(T,\mu).$$

Attractive parameter $a = 0.926$ GeV fm^3, Repulsive parameter $b = \frac{16}{3}\pi r^3$, $r_M = 0.2$ fm, $r_{B(\bar{B})} = 0.62$ fm **Phys. Rev. C 98, 014907 (2018)**.

- Isospin asymmetry of colliding nuclei $\frac{Z}{A} \approx 0.4$, maintained with $n_Q/n_B \sim 0.4$
- Strangeness neutrality condition: no net strange production.
- Charm-canonical ensemble for rare charm production.
- μ_Q, μ_S are related with μ_B by these conditions

Relaxation time

- Hard sphere scattering: $i(p_i) + j(p_j) \to i(p_k) + j(p_l)$

$$\text{In C.M. frame} : \tau_i^{-1}(p_i) = \sum_j \int \frac{d^3 p_j}{(2\pi)^3} \sigma_{ij} v_{ij} f_j^0$$

$$\sigma_{ij} = \pi(r_i + r_j)^2 \equiv \text{total scattering cross} - \text{section}, \qquad v_{ij} \equiv \textit{relative velocity}$$

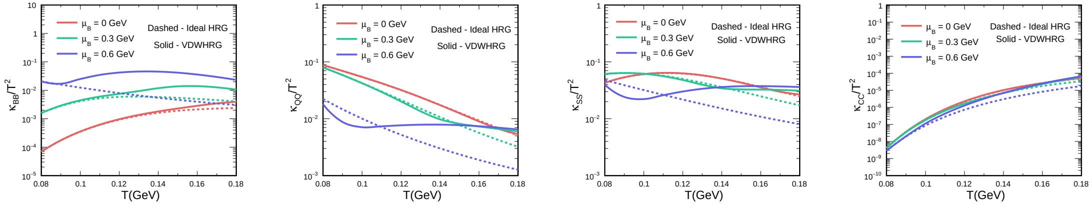
- Standard RTA do not satisfy energy-momentum conservation
- However, no difficulty with momentum-independent relaxation time

Thermal average average relaxation time:

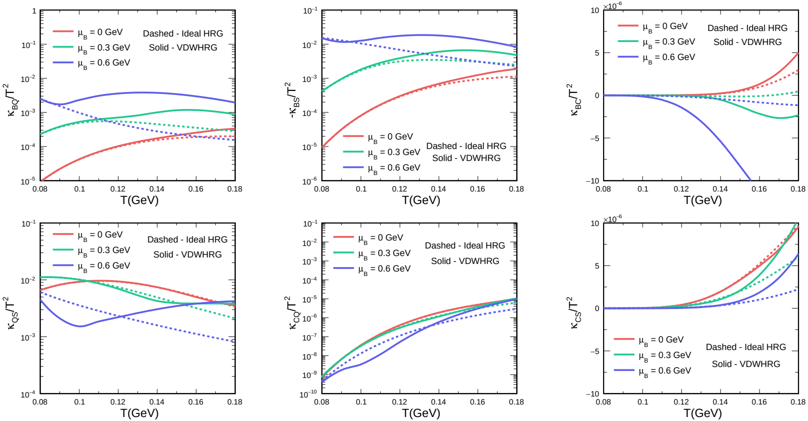
$$\tau_i^{-1} = \frac{\int \frac{d^3 p_i}{(2\pi)^3} f_i^0 \tau_i^{-1}(p_i)}{\frac{d^3 p_i}{(2\pi)^3} f_i^0} = \sum_j n_j \langle \sigma_{ij} v_{ij} \rangle.$$

$$\langle \sigma_{ij} v_{ij} \rangle = \frac{\sigma_{ij}}{8 T m_i^2 m_j^2 K_2(\frac{m_i}{T}) K_2(\frac{m_j}{T})} \int_{(m_i+m_j)^2}^{\infty} ds \times \frac{s - (m_i - mj)^2}{\sqrt{s}} (s - (m_i + m_j)^2) K_1\left(\frac{\sqrt{s}}{T}\right).$$

Diagonal components



- Interplay between relaxation time and thermodynamic phase space contribution
- τ_R decreases with T, while thermodynamics contribution increases
- At $\mu \to 0$, $\frac{\kappa_{qq}}{T^2} = \frac{\sigma_q}{T}$
- $n_{VDW} = \frac{n_{\rm id}}{1+bn_{\rm id}}$
- Small dip around $\mu_B = 0.6$ GeV, $T = 0.09$ GeV from liquid-gas phase transition.
- κ_{cc} small due to higher mass.

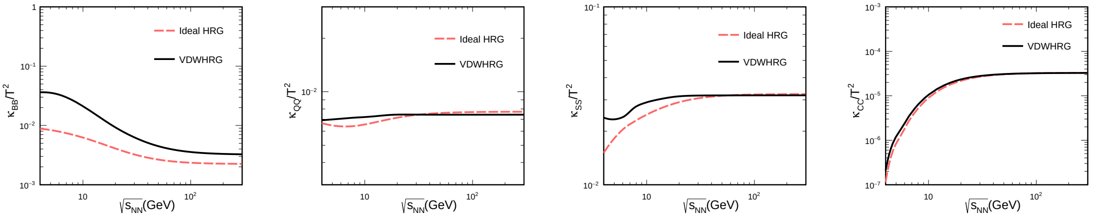


- Cross terms are as significant as diagonal terms
- By Onsager's symmetry principle $\kappa_{qq'} = \kappa_{q'q}$
- Negative for strange 'particle' contribution.

Collision energy dependence

$$\text{Freezeout parameters}: T(\mu_B) = q_1 - q_2\mu_B^2 - q_3\mu_B^4,$$
$$\mu_B(\sqrt{s_{NN}}) = \frac{q_4}{1 + q_5\sqrt{s_{NN}}}. \qquad \text{Phys. Rev. C } 73, 034905(2006).$$

$$q_1 = 0.166 \text{ GeV}, \; q_2 = 0.139 \text{ GeV}^{-1}, \; q_3 = 0.053 \text{ GeV}^{-3}, \; q_4 = 1.308 \text{ GeV}, \; q_5 = 0.273 \text{ GeV}^{-1}.$$



- κ_{BB} is high at low energy
- High value of κ_{QQ} and κ_{SS} can smeared the signal of fluctuations
- Small value of κ_{CC} could be better to find critical point.

Thanks for your attention!