Dualities in dense baryonic (quark) matter with chiral and isospin imbalance

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IHEP, MSU, IZMIRAN

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arXiv:1804.01014 Phys. Rev. D 98, 054030 (2018)
arXiv:1710.09706 Phys. Rev. D 97, 054036 (2018)
arXiv:1704.01477 Phys. Rev. D 95, 105010 (2017)

Seminar of sector of Hadron Matter Physics
BLTP JINR

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broad group

V. Ch. Zhukovsky, N. V. Gubina Moscow state University

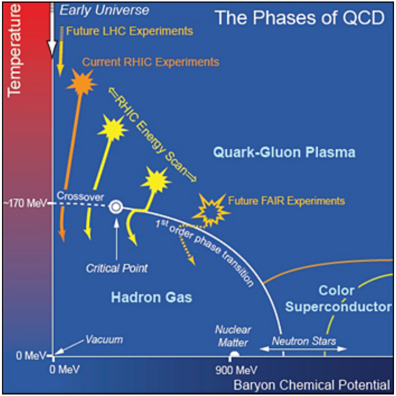
and

D. Ebert, Humboldt University of Berlin

QCD at finite temperature and nonzero chemical potential

QCD at nonzero temperature and baryon chemical potential plays a fundamental role in many different physical systems. (QCD at extreme conditions)

- neutron stars
- heavy ion collision experiments
- Early Universe



Methods of dealing with QCD

Methods of dealing with QCD

- First principle calculation – lattice Monte Carlo simulations, LQCD
- Effective models

  Nambu–Jona-Lasinio model NJL

Lattice QCD

non-zero baryon chemical potential μ_B

sign problem — complex determinant

$$(Det(D(\mu)))^\dagger = Det(D(-\mu^\dagger))$$

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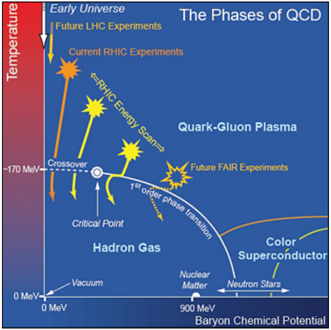
  Nambu–Jona-Lasinio model
  NJL



NJL model

NJL model can be considered as **effective field theory for QCD**.

the model is **nonrenormalizable**
Valid up to $E < \Lambda \approx 1$ GeV

Parameters G, Λ, m_0

NJL model

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Valid up to $E < \Lambda \approx 1$ GeV

Parameters G, Λ, m_0

dof– **quarks**
no gluons only **four-fermion interaction**
attractive feature — dynamical CSB
the main drawback – lack of confinement (PNJL)

Relative simplicity allow to consider hot and dense QCD in the framework of NJL model and explore the QCD phase structure (diagram).

chiral symmetry breaking

the QCD vacuum has non-trivial structure due to non-perturbative interactions among quarks and gluons

lattice simulations $\Rightarrow$ **condensation of quark and anti-quark pairs**

$$\langle \bar{q} q \rangle \neq 0, \quad \langle \bar{u} u \rangle = \langle \bar{d} d \rangle \approx (-250 MeV)^3$$

Nambu–Jona-Lasinio model

Nambu–Jona-Lasinio model

$$\mathcal{L} = \bar{q}\gamma^{\nu}\mathrm{i}\partial_{\nu}q + \frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5 q)^2\Big]$$

$$q \to e^{i\gamma_5\alpha}q$$

continuous symmetry

$$\widetilde{\mathcal{L}} = \bar{q}\Big[\gamma^{\rho}\mathrm{i}\partial_{\rho} - \sigma - \mathrm{i}\gamma^5\pi\Big]q - \frac{N_c}{4G}\Big[\sigma^2 + \pi^2\Big].$$

Chiral symmetry breaking

$1/N_c$ expansion, leading order

$$\langle\bar{q}q\rangle \neq 0$$

$$\langle\sigma\rangle \neq 0 \quad \longrightarrow \quad \widetilde{\mathcal{L}} = \bar{q}\Big[\gamma^{\rho}\mathrm{i}\partial_{\rho} - \langle\sigma\rangle\Big]q$$

Different types of chemical potentials: dense matter with isotopic imbalance

Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q,$$

Different types of chemical potentials: dense matter with isotopic imbalance

Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q,$$

Isotopic chemical potential μ_I

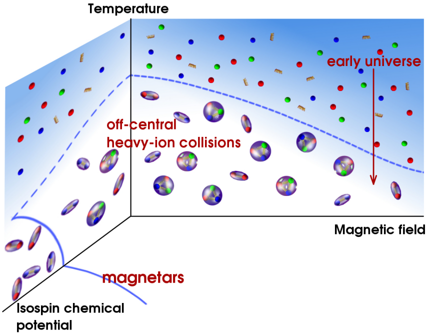
Allow to consider systems with isotopic imbalance.

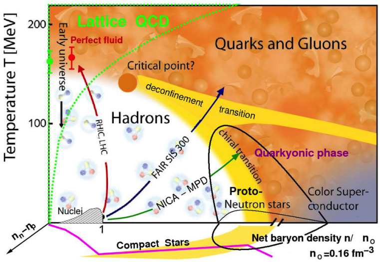
$$n_I = n_u - n_d \quad \longleftrightarrow \quad \mu_I = \mu_u - \mu_d$$

The corresponding term in the Lagrangian is $\frac{\mu_I}{2}\bar{q}\gamma^0\tau_3 q$

QCD phase diagram with isotopic imbalance

neutron stars, heavy ion collisions have isotopic imbalance





Different types of chemical potentials: chiral imbalance

chiral (axial) chemical potential

Allow to consider systems with chiral imbalance (difference between between densities of left-handed and right-handed quarks).

$$n_5 = n_R - n_L \quad \longleftrightarrow \quad \mu_5 = \mu_R - \mu_L$$

The corresponding term in the Lagrangian is

$$\mu_5 \bar{q} \gamma^0 \gamma^5 q$$

chiral (axial) isotopic chemical potential

Allow to consider systems with chiral isospin imbalance

$$\mu_{I5} = \mu_{u5} - \mu_{d5}$$

so the corresponding density is

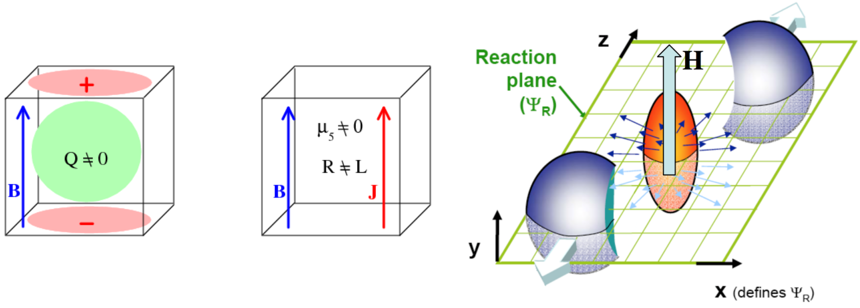
$$n_{I5} = n_{u5} - n_{d5}$$

$$n_{I5} \quad \longleftrightarrow \quad \mu_{I5}$$

Term in the Lagrangian — $\frac{\mu_{I5}}{2}\bar{q}\tau_3\gamma^0\gamma^5 q$

If one has all four chemical potential, one can consider different densities n_{uL}, n_{dL}, n_{uR} and n_{dR}

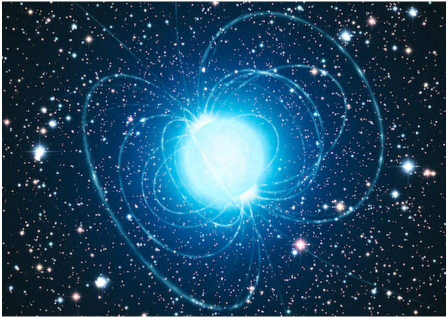
Chiral magnetic effect



$$\vec{J} = c\mu_5 \vec{B}, \qquad c = \frac{e^2}{2\pi^2}$$

A. Vilenkin, PhysRevD.22.3080,
K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D **78** (2008) 074033 [arXiv:0808.3382 [hep-ph]].

Generation of chiral imbalance in compact stars



Due to high baryon densities, magnetic fields and vorticity

- Chiral separation effect CSE
- Chiral Vortical effect CVE

Chiral separation effect

Chiral magnetic (CME) effect has the form

$$\vec{J} = c\mu_5\vec{H}$$

There is a dual effect so-called chiral sepration effect (CSE) (Son and Zhitnitsky 2004, Metlitski and Zhitnitsky 2005)

$$\vec{J_5} = c\mu\vec{H}, \quad J_5^\mu = \langle\bar{\psi}\gamma^\mu\gamma^5\psi\rangle$$

Then the phenomena looks very similar and dual.

$$\vec{J_V} = c\mu_A\vec{H}, \quad \vec{J_A} = c\mu_V\vec{H}$$

Chiral separation effect in a two flavoured system

Let us consider the system with u and d quark flavours

$$\vec{J}_{5u} = \frac{N_c q_u}{2\pi^2}\mu_u \vec{H}$$

and for d quark sector the axial current is

$$\vec{J}_{5d} = \frac{N_c q_d}{2\pi^2}\mu_d \vec{H}$$

Now let us calculate the chiral current

$$\vec{J}_5 = \vec{J}^5_u + \vec{J}^5_d = \frac{N_c}{2\pi^2}(q_u\mu_u + q_d\mu_d)\vec{H}$$

Now let us express it in terms of μ and ν, taking into account that $\mu_u = \mu + \nu$ and $\mu_d = \mu - \nu$ one has

$$\vec{J}_5 = \frac{N_c}{2\pi^2}[(q_u + q_d)\mu + (q_u - q_d)\nu)]\vec{H}$$

Chiral separation effect in a two flavoured system

Chiral isospin current and charge

$$\vec{J}_{I5} = \vec{J}_{5u} - \vec{J}_{5d} = \frac{N_c}{2\pi^2}(q_u\mu_u - q_d\mu_d)\vec{H}$$

Expressing it in terms of μ and ν

$$\vec{J}^{I5} = \frac{N_c}{2\pi^2}[(q_u - q_d)\mu + (q_u + q_d)\nu)]\vec{H}$$

Chiral separation effect in a two flavoured system

The chiral charge:

$$Q_5 = \int d^3x \langle \bar{\psi} \gamma^0 \gamma^5 \psi \rangle \Longleftrightarrow \mu_5$$

The chiral isospin charge

$$Q_{I5} = \int d^3x \langle \bar{\psi} \gamma^0 \gamma^5 \tau_3 \psi \rangle \Longleftrightarrow \mu_{I5}$$

Chiral separation effect in a two flavoured system

It is quite obvious that the ratio of charges is equal to the ratio of the currents

$$\frac{n^{I5}}{n^5} = \frac{Q^{I5}}{Q^5} = \frac{J_z^{I5}}{J_z^5}$$

$$\frac{Q^{I5}}{Q^5} = \frac{3+\delta}{1+3\delta}$$

where $\delta = \frac{\nu}{\mu}$

For example if $\nu = 0$ then

$$\frac{Q^{I5}}{Q^5} = 3$$

Chiral separation effect: real case

The full formula for CSE in the case of finite temperature and non-zero quark mass can be found by Zhitnitsky Metlitski

$$J_V^5 = \frac{e}{2\pi} n_m(T, \mu)\Phi$$

where $J_V^5 = \int d^2x J_3^5$ and $\Phi = \int d^2x B$
And the coefficient in front of the magnetic flux is

$$n_m(T, \mu) = \int \frac{dp_3}{2\pi} \left(\frac{1}{e^{\beta(\sqrt{p_3^2+m^2}-\mu)} + 1} - \frac{1}{e^{\beta(\sqrt{p_3^2+m^2}+\mu)} + 1} \right)$$

it is a baryon number density of one-dimensional fermions.

Chiral Vortical Effect (CVE)

Vorticity

$$\vec{\omega} = \frac{1}{2}\vec{\nabla} \times \vec{v}$$

Chiral Vortical Effect (CVE) quantifies the generation of a vector current J along the vorticity direction:

$$\vec{J} = \frac{1}{\pi^2}\mu\mu_5\vec{\omega}$$

Axial current can be generated by the rotation as well

$$\vec{J_5} = \left[\frac{1}{6}T^2 + \frac{1}{2\pi^2}(\mu^2 + \mu_5^2)\right]\vec{\omega}$$

Chiral imbalance generation due to CVE

$$\vec{J}_5 = \vec{J}_5^u + \vec{J}_5^d = \left[\frac{1}{3}T^2 + \frac{1}{2\pi^2}(\mu^2 + \nu^2)\right]\vec{\omega}$$

$$\vec{J}_{I5} = \vec{J}_5^u - \vec{J}_5^d = \left[\frac{2}{\pi^2}\mu\nu\right]\vec{\omega}$$

Model and its Lagrangian

We consider a NJL model, which describes dense quark matter with two massless quark flavors (u and d quarks).

$$\mathcal{L} = \bar{q}\Big[\gamma^\nu \mathrm{i}\partial_\nu + \frac{\mu_B}{3}\gamma^0 + \frac{\mu_I}{2}\tau_3\gamma^0 + \frac{\mu_{I5}}{2}\tau_3\gamma^0\gamma^5 + \mu_5\gamma^0\gamma^5\Big]q +$$

$$\frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5\vec{\tau}q)^2\Big]$$

q is the flavor doublet, $q = (q_u, q_d)^T$, where q_u and q_d are four-component Dirac spinors as well as color N_c-plets; τ_k $(k = 1, 2, 3)$ are Pauli matrices.

Equivalent Lagrangian

To find the thermodynamic potential we use a semi-bosonized version of the Lagrangian

$$\widetilde{L} = \bar{q}\Big[\gamma^\rho \mathrm{i}\partial_\rho + \mu\gamma^0 + \nu\tau_3\gamma^0 + \nu_5\tau_3\gamma^1 - \sigma - \mathrm{i}\gamma^5\pi_a\tau_a\Big]q - \frac{N_c}{4G}\Big[\sigma\sigma + \pi_a\pi_a\Big].$$

$$\sigma(x) = -2\frac{G}{N_c}(\bar{q}q); \quad \pi_a(x) = -2\frac{G}{N_c}(\bar{q}\mathrm{i}\gamma^5\tau_a q).$$

Condansates ansatz $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ do not depend on spacetime coordinates x,

$$\langle\sigma(x)\rangle = M, \quad \langle\pi_1(x)\rangle = \Delta, \quad \langle\pi_2(x)\rangle = 0, \quad \langle\pi_3(x)\rangle = 0. \quad (1)$$

where M and Δ are already constant quantities.

thermodynamic potential

the thermodynamic potential can be obtained in the large N_c limit

$$\Omega(M, \Delta)$$

Projections of the TDP on the M and Δ axes

No mixed phase $(M \neq 0, \Delta \neq 0)$

it is enough to study **the projections** of the TDP on the M and Δ

projection of the TDP on the M **axis** $F_1(M) \equiv \Omega(M, \Delta = 0)$

projection of the TDP on the Δ **axis** $F_2(\Delta) \equiv \Omega(M = 0, \Delta)$

Dualities

The TDP (phase daigram) is invariant

Interchange of condensates

matter content

$$\Omega(C_1, C_2, \mu_1, \mu_2)$$

$$\Omega(C_1, C_2, \mu_1, \mu_2) = \Omega(C_2, C_1, \mu_2, \mu_1)$$

Dualities

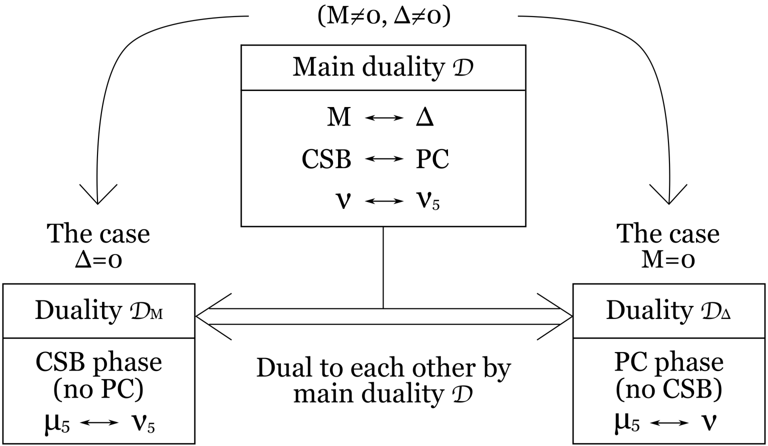


Figure: Dualities

Dualities in different approaches

- **Large N_c orbifold equivalences** connect gauge theories with different gauge groups and **matter content** in the large N_c limit.

  M. Hanada and N. Yamamoto,
  JHEP 1202 (2012) 138, arXiv:1103.5480 [hep-ph],
  PoS LATTICE **2011** (2011), arXiv:1111.3391 [hep-lat]

Dualities in large N_c orbifold equivalences

two gauge theories with gauge groups G_1 and G_2 with μ_1 and μ_2

Duality

$$G_1 \longleftrightarrow G_2,\ \mu_1 \longleftrightarrow \mu_2$$

G_2 is sign problem free

G_1 has sign problem, can not be studied on lattice

Dualities in large N_c limit of NJL model

$$\Omega(C_1, C_2, \mu_1, \mu_2)$$

Duality

$$C_1 \longleftrightarrow C_2,$$
$$\mu_1 \longleftrightarrow \mu_2$$

QCD with μ_1 —- sign problem free,
and with μ_2 has sign problem, can not be studied on lattice

Pion condensation history

In the early 1970s Migdal suggested the possibility of pion condensation in a nuclear medium
A.B. Migdal, Zh. Eksp. Teor. Fiz. 61, 2210 (1971) [Sov. Phys. JETP 36, 1052 (1973)]; A. B. Migdal, E. E. Saperstein, M. A. Troitsky and D. N. Voskresensky, Phys. Rept. 192, 179 (1990).
R.F. Sawyer, Phys. Rev. Lett. 29, 382 (1972);

From the results of the experiments concerning the repulsive πN interaction pion condensation is highly unlikely to be realized in nature A. Ohnishi D. Jido T. Sekihara, and K. Tsubakihara, Phys. Rev. C80, 038202 (2009), relativistic mean field (RMF) models. .

Pion condensation history

Pion condensation in NJL_4
K. G. Klimenko, D. Ebert J.Phys. G32 (2006) 599-608
arXiv:hep-ph/0507007
K. G. Klimenko, D. Ebert
Eur.Phys.J.C46:771-776,(2006) arXiv:hep-ph/0510222

also in **(1+1)- dimensional case**, NJL_2
K. G. Klimenko, D. Ebert, PhysRevD.80.125013 arXiv:0902.1861 [hep-ph]

$$\Downarrow$$

pion condensation in dense matter **predicted without certainty**

physical quark mass and neutral matter – no pion condensation in dense medium
H. Abuki, R. Anglani, R. Gatto, M. Pellicoro, M. Ruggieri
Phys.Rev.D79:034032,2009 arXiv:0809.2658 [hep-ph]

Phase structure of NJL model

Chiral isospin chemical potential μ_{I5} generates charged pion condensation in the dense quark matter.

$$\nu = \frac{\mu_I}{2}, \qquad \nu_5 = \frac{\mu_{I5}}{2}$$

(ν, ν_5) phase portrait of NJL_4

Duality between chiral symmetry breaking and pion condensation

$$\mathcal{D}: \; M \longleftrightarrow \Delta, \quad \nu \longleftrightarrow \nu_5$$

$$\text{PC} \longleftrightarrow \text{CSB} \quad \nu \longleftrightarrow \nu_5$$

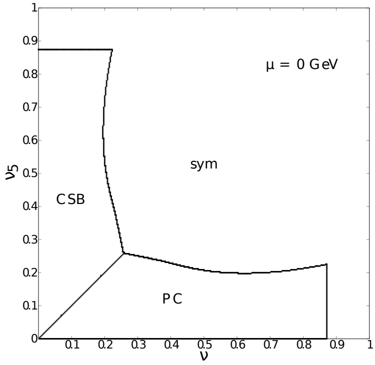


Figure: (ν, ν_5) at $\mu = 0$ GeV

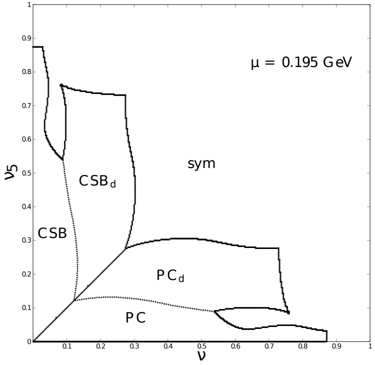


Figure: (ν, ν_5) at $\mu = 0.195$ GeV

Consideration of the general case μ, μ_I, μ_{I5} and μ_5

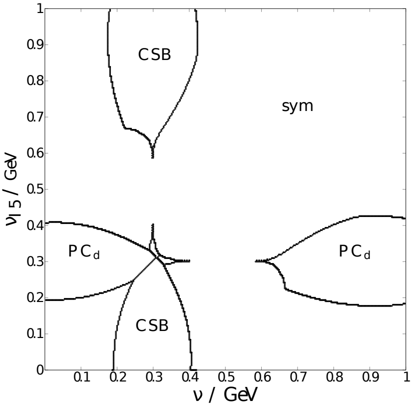


Figure: (ν, ν_5) phase diagram at $\mu_5 = 0.5$ GeV and $\mu = 0.3$ GeV.

generation of PC_d phase is even more widespread

possible even for zero isospin asymmetry

Comparison with lattice QCD

Comparison with lattice QCD, finite temperate and physical point

- Before that point we considered the **chiral limit**

$$m_0 = m_u = m_d = 0$$

$$m_0 \neq 0, \quad m_0 \approx 5 \text{ MeV}$$

- For that let us consider the finite temperature T

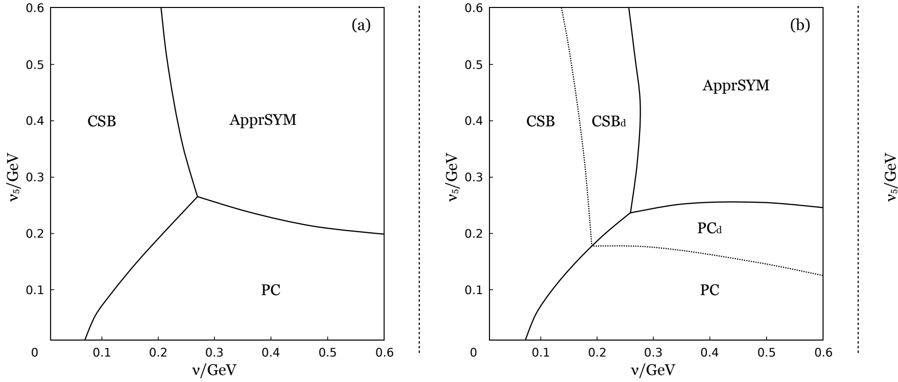
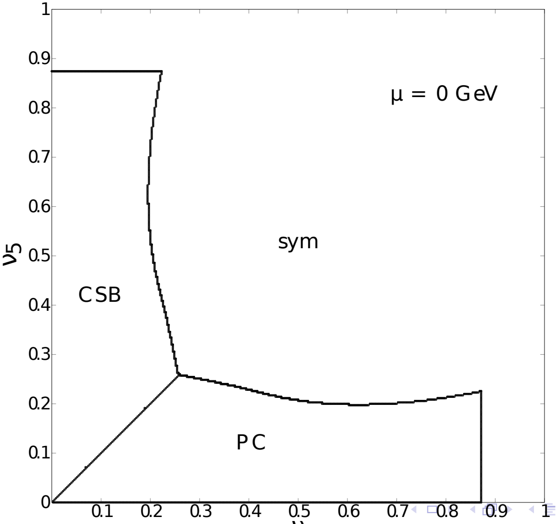


Figure: (ν, ν_5) phase diagram

(ν, ν_5) phase portrait of NJL_4 at $\mu = 0$

The case of $\mu = 0$ can be considered on lattice



Comparison with lattice QCD

Two cases have been considered in LQCD

– QCD at non-zero isospin chemical potential μ_I has been considered in arXiv:1611.06758 [hep-lat],
Phys. Rev. D 97, 054514 (2018) arXiv:1712.08190 [hep-lat]
Endrodi, Brandt et al

– QCD at non-zero chiral chemical potential μ_5 has been considered in Phys. Rev. D 93, 034509 (2016) arXiv:1512.05873 [hep-lat]
Braguta, ITEP lattice group

QCD at non-zero isospin chemical potential μ_I: (ν, T) phase portrait comparison between NJL model and lattice QCD

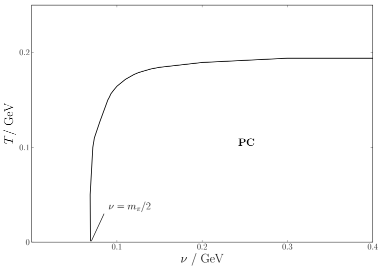


Figure: (ν, T) phase diagram at $\mu = 0$ and $\nu_5 = 0$ GeV

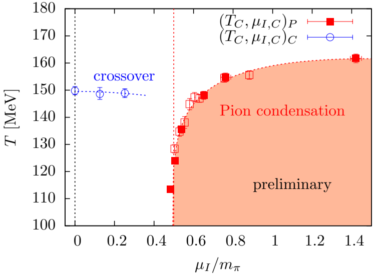


Figure: (ν, T) phase diagram arXiv:1611.06758 [hep-lat]

QCD at non-zero isospin chemical potential μ_I

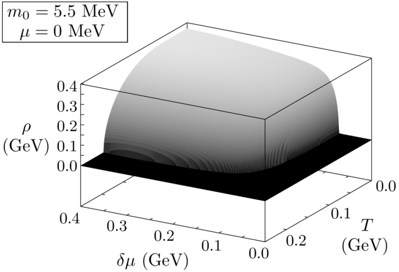


Figure: $\nu_5 = 0$ case from J. Phys. G: Nucl. Part. Phys. 37 015003 (2010), **Jens Andersen et al, Norwegian University of Science and Technology**

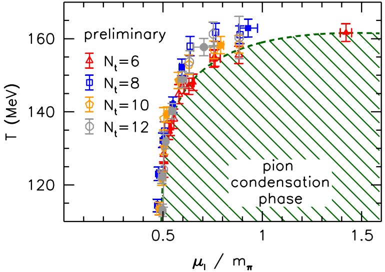


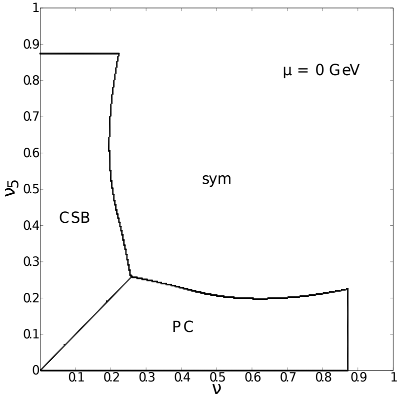
Figure: (ν, T) phase diagram at $\nu_5 = 0$ GeV arXiv:1611.06758 [hep-lat]

QCD with non-zero chiral chemical potential μ_5

QCD at zero baryon chemical potential $\mu = 0$
but with non-zero $\mu_5 \neq 0$ sign problem free

$\mu_5 \neq 0$ **no sign problem**

Braguta ITEP lattice, Ilgenfritz Dubna et al
– Catalysis of Dynamical Chiral Symmetry Breaking



μ_5 or ν_5 chemical potential, duality

CSB phenomenon is invariant under

$$\mathcal{D}_M \ : \ \nu_5 \leftrightarrow \mu_5$$

(μ_5, T) and (ν_5, T) are the same

QCD at non-zero chiral chemical potential μ_5, comparison between NJL model and lattice QCD

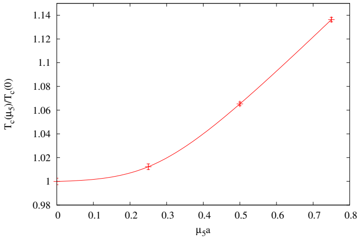


Figure: critical temperature T_c as a function of μ_5 in LQCD, from arXiv:1512.05873 [hep-lat

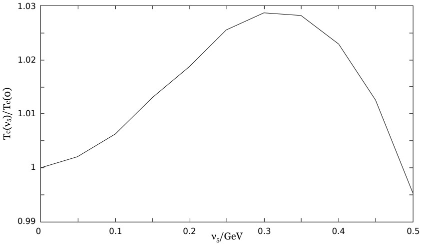


Figure: critical temperature T_c as a function of μ_5 in the framework of NJL model

(μ_I, T) is dual to (μ_{I5}, T)

duality is approximate in the physical point

two cases is in the agreement with LQCD

Lattice QCD supports duality

A number of papers predicted **anticatalysis** (T_c decrease with μ_5) of dynamical chiral symmetry breaking

A number of papers predicted **catalysis** (T_c increase with μ_5) of dynamical chiral symmetry breaking

(Could even depend on the scheme of regularization)

Braguta paper, lattice results show the **catalysis**

Phase diagram at μ_I is now well studied

simulations of Endrodi group, earlier lattice simulation,
ChPT has similar predictions
D.T. Son, M.A. Stephanov Phys.Rev.Lett. 86 (2001) 592-595
arXiv:hep-ph/0005225, Phys.Atom.Nucl.64:834-842,2001;
Yad.Fiz.64:899-907,2001 arXiv:hep-ph/0011365

Duality $\Rightarrow$ catalysis of chiral symmetry beaking

Charge neutrality condition

the general case (μ, μ_I, μ_{I5}, μ_5)

consider charge neutrality case $\quad\rightarrow\quad \nu = \mu_I/2 = \nu(\mu, \nu_5, \mu_5)$

Charge neutrality condition

-pion condensation in dense matter predicted without certainty, at ν there is a small region of PC_d phase
K. G. Klimenko, D. Ebert J.Phys. G32 (2006) 599-608
arXiv:hep-ph/0507007

-physical quark mass and electric neutrality - no pion condensation in dense medium
H. Abuki, R. Anglani, R. Gatto, M. Pellicoro, M. Ruggieri
Phys.Rev.D79:034032,2009 arXiv:0809.2658 [hep-ph]

-Chiral isospin chemical potential μ_{I5} generates PC_d

-can this generation happen in the case of neutrality condition

Charge neutrality condition

It can be shown that the PC_d phase can be generated by chiral imbalance in the case of charge neutrality condition

non-zero $\mu_5 \to \text{PC}_d$ phase in neutral quark matter

(1+1)-dimensional Gross-Neveu (GN) or NJL model consideration

(1+1)- dimensional GN, NJL model

(1+1)-dimensional Gross-Neveu (GN) or NJL model possesses a lot of common features with QCD

- renormalizability
- asymptotic freedom
- spontaneous chiral symmetry breaking in vacuum
- dimensional transmutation
- have the similar $\mu_B - T$ phase diagrams

NJL_2 model
laboratory for the qualitative simulation of specific properties of QCD **at arbitrary energies**

Phase structure of the (1+1) dim NJL model

Chiral isospin chemical potential μ_{I5} generates charged pion condensation in the dense quark matter.

Phys. Rev. D 95, 105010 (2017) arXiv:1704.01477 [hep-ph]
Phys. Rev. D 94, 116016 (2016) arXiv:1608.07688 [hep-ph]

Comparison of phase diagram of (3+1)-dim and (1+1)-dim NJL models

The phase diagrams obtained in two models that are assumed to describe QCD phase diagram are qualitatively the same

Conclusions

$\mu_B \neq 0$ - dense quark matter

$\mu_I \neq 0$ isotopically asymmetric

$\mu_5 \neq 0$ and $\mu_{I5} \neq 0$ chirally asymmetric

Phase diagram in NJL model

Dualities; duality between CSB and PC: $\nu_5 \leftrightarrow \nu$

CSB: $\nu_5 \leftrightarrow \mu_5$

PC: $\mu_5 \leftrightarrow \nu$

$\mu_{I5} \rightarrow \mathrm{PC_d}$

even with **neutrality condition**

Thanks for the attention

Comparison with lattice QCD will be discussed by Tamaz