Imaginary Acceleration

V.I. Zakharov

Talk 26.12.18,
Seminar at JINR (Dubna)

26 декабря 2018 г.

Explanation of the title of the talk

Based on:

"On axial current in rotating and accelerating medium"

G. Prokhorov, O. Teryaev, V. Zakharov,
arXiv:1805.12029 [hep-th] published in
"Rapid Communications" PRD

one evaluates matrix element $< \vec{J}_5 >_{statistical}$

Result can be reproduced through the substitution:

$$\mu \;\to\; \mu + \frac{\Omega}{2} - i\frac{a}{2}$$

where μ is the chemical potential, Ω - angular velocity, a-acceleration (Signs are rather $\pm$)

Alexander Vilenkin, PRD 21 (1980) 2260

The field is started by A. Vilenkin, in case of rotation:

$$< J^{\mu}(\vec{x}) > = \; Tr\Big(\rho J^{\mu}(\vec{x}, t)\Big)$$

where $J^{\mu} \;=\; \frac{1}{2}[\bar{\Psi}, \gamma^{\mu}\Psi]$ is the current density operator and

$$\rho \;=\; C \exp\Big(-\beta\big(H - \vec{M}\cdot\vec{\Omega} - \Sigma_i \mu_i N_i\big)\Big)$$

$\beta = T^{-1}$, ρ is the statistical operator (see Landau&Lifshitz)
$\vec{M}$ is the angular momentum, $\vec{\Omega}$ is the angular velocity
μ_i is chemical potential, N_i is number of charged particles
ρ is built on conserved operators

Vilenkin, cnt'd

One-loop effect in finite-temperature QFT

$$< J^\mu(\vec{x}) > = \ -Tr\gamma^\mu S(\vec{x},\tau;\vec{x},\tau+\epsilon)_{\epsilon\to 0})$$

reduces to one of the so called Sommerfeld integrals (M.Stone (2018))

$$< J_\Omega > = \ \frac{1}{4\pi^2}\int_{-\infty}^{+\infty}\epsilon^2 d\epsilon\cdot\Big(\frac{1}{1+e^{\beta(\epsilon-(\mu+\Omega/2))}}-\frac{1}{1+e^{\beta(\epsilon-(\mu-\Omega/2)}}\Big)$$

Finally, (for a single Weyl fermion of unit charge):

$$< \vec{J}(0) > = \ -\vec{\Omega}\big(\mu^2/4\pi^2 + T^2/12 + \Omega^2/(\pi^2 48)\big)$$

so called Chiral Vortical Effect

More recent generalizations

Basic points on ρ from Landau&Lifshitz:

$$\ln\rho_{12} \;=\ln\rho_1+\ln\rho_2 \qquad (\textit{additivity})$$

$$\partial\rho/\partial t \;=\; i[H,\rho],\;\; \textit{in equlibrium}\; [H,\rho]=0...$$

Recently:

- Relativistic formulation, $\beta \;\rightarrow\; \beta_\nu, \;\; \beta_\nu \;\equiv\; u_\nu/T_0$
- Possible "conflict" between massless and massive cases (generally speaking, non-trivial in view of the anomaly)
- Phenomenology (of heavy-ion collisions) suggests that the medium is accelerated (STAR Collaboration)

Francesco Becattini + coauthors, $\sim$ 2017

As a response to the challenge, these authors suggested

$$\rho = \frac{1}{Z}\exp[-\hat{H}/T_0 + a_z\hat{K}_z/T_0]$$

where a_z is acceleration and the operator $\hat{K}_z$ is the boost (both in z-direction). A basic generalization of L&L.

There exists prehistory, mostly for scalar particles

Not without problems, e.g.:

- explicit dependence on time is introduced
- should we factorize motion of the center of mass?

A strong case in favor of the extension of $\hat{\rho}$ has been made

Prokhorov et al. (2018)

It is in this framework (and Prokhorov+Teryaev (2017)) that imaginary acceleration has been introduced, see above A more precise formulation of Prokhorov et al. result:

$$2 < J^5_\Omega > =$$
$$\int \frac{d^3p}{(2\pi)^3}\Big(n_F(E_p - \mu - \Omega/2 + ia/2) - n_F(E_p - \mu + \Omega/2 - ia/2) +$$
$$+ n_F(E_p + \mu - \Omega/2 + ia/2) - n_F(E_p + \mu + \Omega/2 - ia/2) + c.c. \Big)$$

where n_F is the corresponding Fermi distribution (mass of the fermion is not necessarily vanishing)

Imaginary acceleration as a sign of instability

Because of the factor i the second order in ia is negative

As a result, there emerges instability at temperature $T < T_{Unruh}$:

- The axial current oscillates like mad at $T < a/(2\pi)$
- If the average energy-momentum is evaluated along similar lines, density of energy is negative at temperatures below the Unruh temperature (Becattini)
- The axial current is somewhat stabilized with increasing angular velocity (Prokhorov et al.(2018))

At least qualitatively, the picture is similar to the Hwking/Unruh effect

Conclusions to Part I ("Phenomenology")

- Following the phenomenology path we apparently arrive at another incarnation of the Hawking radiation, or Unruh effect
- It is better to become "more theoretical" at this point. Let's go back to FT

Part II: Is there place for ia in Field Theory?

The answer is defnite although sounds unexpected: "Yes, one could have readily dug out the imaginary acceleration long time ago"

The most straightforward way is to start with the chiral limit (L&L, Becattini... are rather non-relativistic)

Namely, it is a textbook statement that generators of the Lorentz transformation are realized on the fundamental (massless) fermion representation as:

$$\hat{J}_z = \frac{\hat{\sigma}_z}{2}, \hat{K}_z = \frac{i\hat{\sigma}_z}{2}$$

and we immediately come to $\mu \rightarrow \mu + \Omega/2 + ia/2$

Imaginary acceleration as signal of instability

In other words, we do not notice that $\hat{K}_z$ is anti-hermitian as far as work with Lagrangians and use $\psi^\dagger \gamma_0$ instead of $\psi^\dagger$

Statistics (or systems with finite densities) make $< \hat{K}_z >$ observable and reveal that boost is realized as

$$(\hat{K}_z)^\dagger \;=\; - \;\hat{K}_z$$

Imaginary acceleration ia, entering along with real μ looks as imaginary energy and, apparently, signals instability

What kind of instability?–Field-theoretic instability of finite-density, accelerated matter with $T = 0$.

Lorenzian signature vs Unruh effect

Back to text-books: one chooses $K_z = i\sigma_z/2$ to imitate commutators of the Hermitian boost operators.

Thus, in the second order we observe the minus-sign inherent to the commutators rather than $\sqrt{-1}\ a$.

As a result, the Unruh-instability gets related to the Lorenzian signature
(amazing!)

From FT to QFT : from Facts to Speculations

A fact: Quantizing allows to introduce $\psi(\vec{p})$, (and the corresponding $a^{\dagger}(\vec{p}), a(\vec{p})$) a kind of "infinite-component representation".

Boosts can transform density of fermions with momentum $\vec{p}$ into density with boosted value of the momentum. Restoring hermitian $\hat{K}_z$.

Apparently, this trick should work for integration over virtual momenta as well.

Except for the anomalous cases when shifts of momenta are not allowed. Hence, speculation:
in QFT the instability can come back with anomaly

Support for re-discovering the Unruh effect

- Proceeding to evaluate energy-density of the equilibrium, accelerated state one finds out (Becattini (2018)) that for temperatures below T_{Unruh} (better to say, below or order of) energy-density is negative
- Similarly (Prokhorov et al. (2018)), the axial current at temperatures below T_{Unruh} oscillates like mad
- The axial current is somewhat stabilized with increasing angular velocity (Prokhorov et al.(2018))

All these observations (qualitatively) agree with the Unruh effect/instability.

Quantum Field Theory: role of anomalies

Independently, the role of anomalies in generating thermal chiral effects was clarified by M Stone (2018), who generalized approach due to Wilzcek (2005)

$$\nabla_\mu T^{\mu\nu} \;=\; F^{\nu A} J_{N,A} - \frac{1}{384\pi^2} \frac{\epsilon^{\rho\sigma\alpha\beta}}{\sqrt{-g}} \nabla_\mu [F_{\rho\sigma} R^{\nu\mu}_{\alpha\beta}]$$

$$\nabla_\mu J^\mu_N = -\frac{1}{32\pi^2} \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} F_{\rho\nu} F_{\mu\sigma} - \frac{1}{768\pi^2} \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}} R^\alpha_{\beta\mu\nu} R^\beta_{\alpha\rho\sigma}$$

where J_N is the number-current, corresponds to a single right-handed Weyl spinor, and other notations are standard

Anomaiies, cnt'd

In all the cases the metric is factorized into a 2d black hole,

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 \,, \quad f(r = r_H) = 0$$

and transverse coordinates which are occupied by a uniform magnetic ($\vec{B} \neq 0$) or rotational ($\vec{\Omega} \neq 0$) fields

The $2d$ gravitational anomaly is quite readily integrated out in time-independent (equilibrium) case

$$\sqrt{|g|}\nabla_\mu(T^{\mu\nu}\eta_\nu) = \frac{c}{96\pi}\epsilon^{\nu\sigma}\eta_\nu\partial_\sigma R$$

to produce a flux of particles far off from the horizon (path to the Hawking radiation found by Wilzcek)

Anomalies, cnt'd

Reproduced thermal effects, with $T = T_{Black\ \ hole}$

Energy flux:

$$\vec{J}_{energy} \;=\; \vec{B}\Big(\frac{\mu^2}{8pi^2} + \frac{1}{24}T^2\Big)$$

Number-of-particles flux:

$$\vec{J}_N \;=\; \vec{\Omega}\Big(\frac{\mu^2}{4\pi^2} + \frac{|\vec{\Omega}|^2}{48\pi^2} + \frac{1}{12}T^2\Big)$$

The whole of thermal CVE given exactly by the anomalies.
No apriori mention of the temperature. Pure field theory.
Temperature is nothing else but acceleration (on the horizon)

Conclusions to Part II

QFT, through anomalies, clearly signals that matter occupying part of space– because of the horizon –is unstable against particle production. Quantitatively, Hawking radiation from a black hole is reproduced (F. Wilzcek, M. Stone....)

Hint on the instability, contained in classical (in Grassmann numbers) theory of fermions is promoted to a fully quantitative framework by QFT

Questions not answered

Many more questions to answer. E.g.:

- In a way, we have not started yet. Would like to consider imaginary part So far, quadratic in acceleration terms.
- What is analogy to

$$eA_\alpha \;\rightarrow\; eA_\alpha + \mu \cdot u_\alpha$$

- Possible dependence on IR, and so on
- Critique (?)

Part III: $T \leftrightarrow a$ Duality as Guiding Principle

Absolutely unfinished part.

To reiterate, duality between what and what:

- On one hand, one can apply thermal field theory in flat space and evaluate the thermal part of the chiral vortical effect. $\vec{j} = (T^2/12) \cdot \vec{\Omega}$
- On the other hand, one can evaluate flow of charge from the horizon in terms of the gravitational anomaly (no explicit temperature). The input is metric near the horizon, $g_{00} \to 0$
- Results coincide for (massless) spin-1/2 constituents, provided $T = a/(2\pi)$

Further tests of the duality

Chiral vortical effect for photons (in grav. field). The basic new ingredient is the bosonic chiral anomaly:

$$\nabla^\alpha K_\alpha \ = \ (const) R\tilde{R}$$

where $K_\alpha = \epsilon_{\alpha\beta\gamma\delta} A^\beta \partial^\gamma A^\delta$, $R\tilde{R}$- product of Riemann tensors

The two ways of evaluating the CVE for photons seem to disagree with each other by a factor of two.
Should we modify the field-theoretic calculation and impose the duality?

Also, composite particles are treated differently in the two approaches

Overall conclusion

Equations with imaginary acceleration seem to be true and shed new light on the Hawking radiation in FT

The crucial question is whether there are noval applications