Probing $\mathcal{N} = 4$ SYM rotating quark-gluon plasma using holography

Based on arXiv:2210:XXXXX

Tsegelnik Nikita, Golubtsova Anastasia

November 9, 2022

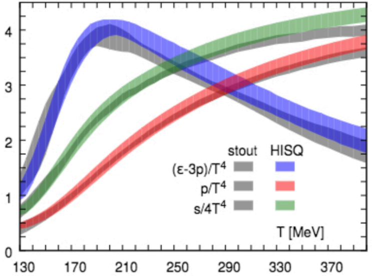
Joint Institute for Nuclear Research
tsegelnik@theor.jinr.ru

Outline

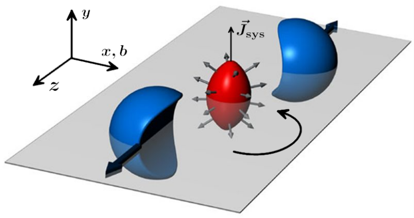
Introduction

Motivation

- QGP is the strongly interacting deconfined QCD matter
- The critical temperature in the non-rotating case at $\mu = 0$ is $T_c = 150 - 170$ MeV
- QCD at high T has a quasi-conformal behaviour $T^{\mu}_{\mu} \approx 0$ (Bazavov'14)



- The large initial orbital angular momentum of the ions is partially transferred to the medium, that leads to a non-vanishing averaged *vorticity* $\longrightarrow$ *rotating QGP*
- A probe of the medium rotation is the *polarization of Λ-hyperons* (Abelev'07; Adamczyk'17; Becattini'13)



The AdS/CFT conjecture

The conjecture

$4d$ $\mathcal{N}=4$ SYM with $SU(N)$ is dynamically equivalent to type IIB superstring theory(contains strings and D-branes) on $AdS_5 \times \mathbb{S}^5$ with a string length $\ell_s = \sqrt{\alpha'}$ and coupling constant g_s with the radius L and N units of $F_{(5)}$ flux on $\mathbb{S}^5$.

$$g_{YM}^2 = 2\pi g_s, \quad 2g_{YM}^2 N = \frac{L^4}{\alpha'^2}, \quad \lambda = g_{YM}^2 N$$

Forms of the AdS$_5$/CFT$_4$ correspondence

	$\mathcal{N}=4$ SYM	IIB theory on $AdS_5 \times \mathbb{S}^5$
Strongest form	any N and λ	Quantum string theory, $g_s \neq 0$, $\alpha'/L^2 \neq 0$
Strong form	$N \to \infty$, λ fixed but arbitrary	Classical string theory, $g_s \to 0$, $\alpha'/L^2 \neq 0$
Weak form	$N \to \infty$, λ large	Classical supergravity, $g_s \to 0$, $\alpha'/L^2 \to 0$

Classical (super)strings in asymptotically AdS_5 can predict results for strongly coupled 4d $\mathcal{N}=4$ SYM with $SU(N)$, $N \to \infty$

Holography at finite temperature

- Pure $AdS_5 \Leftrightarrow T = 0$ 4d $\mathcal{N} = 4$ SYM at strong coupling with $SU(N)$ (Maldacena'97)
  - the isometry group $SO(2,4)$ of AdS_5 is a symmetry group of the dual CFT
  - the field theory "lives" on the boundary of the gravity background
  - flat boundary $\Leftrightarrow$ CFT on $\mathbb{R}^4$; spherical boundary $\Leftrightarrow$ CFT on cylinder $\mathbb{R} \times \mathbb{S}^3$
- AdS_5 BH $\Leftrightarrow$ *thermal ensemble* of $\mathcal{N} = 4$ SYM $SU(N)$ at strong coupling (Witten'98)
  - T of the thermal ensemble of CFT is identified with the Hawking temperature T_{H} of BH
  - The Hawking-Page phase transition in the BH $=$ the first order phase transition in the dual theory
- $\mathcal{N} = 4$ SYM has 1 *vector field* + 4 *spinor fields* + 6 *scalar fields*

Sundborg'00: free $\mathcal{N} = 4$ SYM on $\mathbb{R} \times \mathbb{S}^3$ at $T \neq 0$ has a phase transition at the Hagedorn temperature (free energy on $\mathbb{S}^3$ at $N \to \infty, \lambda \to \infty$)

Harmark et al.'18'20: the Hagedorn temperature at any value of the 't Hooft coupling

Some results within AdS/CFT

- Shear viscosity bound for plasma $\frac{\eta}{s} = \frac{1}{4\pi}$ (Policastro/Kovtun, Son, Starinets'01/05)
- Second order transport coefficients (Son, Starinets'06; Herzog et.al.'07; Cherman'09; see review Aref'eva'14)
- **Quark-antiquark potential** (Maldacena'98; Sonnenschein et al.'98; Theisen'98; Kol, Sonnenschein'11; Chen, Hou'22)
- **Parton energy losses in plasma** (Sin, Zahed'05; Liu, Rajagopal, Wiedemann'06'07; Herzog'07; Ficnar, Gubser, Gyulassy'14; Rajagopal, Sadofyev'15; Golubtsova, Gourgoulhon, Usova'21)

- High-energy fixed-angle scattering of glueballs (Polchinski, Strassler'02)
- Chiral symmetry breaking and form factors (Gherghetta, Kapusta, Kelley'09)
- Meson mass spectra (Li, Huang, Yan'12)
- Heavy-ion collisions as shock waves (Grumiller, Romatschke'08)
- Initial conditions before hydro (Schee, Romatschke, Pratt'13)
- Holographic models of QCD (Erlich/Karch, Katz, Son, Stephanov'05/06; Aref'eva et al.'18'20)

The setup

$\mathbb{R}^1 \times \mathbb{S}^3$ black holes

The gravity action:

$$\mathcal{S} = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g}\, (R_5 - 2\Lambda)$$

Solutions to Einstein equations with $\mathbb{S}^3$-symmetry:

- Schwarzschild-Anti-de Sitter black hole (SAdS$_5$) with mass M $\Leftrightarrow$ non-rotating plasma
- Kerr-Anti-de Sitter black hole (Kerr-AdS$_5$) with mass M and angular momenta J_a, J_b $\Leftrightarrow$ rotating plasma

Non-rotating black hole

Schwarzschild-AdS$_5$ (SAdS$_5$)

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_3^2, \qquad d\Omega_3^2 = d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta d\psi^2,$$

$$0 \leq \theta \leq \pi/2, \qquad 0 \leq \phi, \psi \leq 2\pi,$$

$$f(r) = 1 + \frac{r^2}{\ell^2} - \frac{2M}{r^2}, \qquad \Lambda = -\frac{6}{\ell^2}$$

The horizon r_h and Hawking temperature $T_{\rm H}$ are defined by the following expressions:

$$T_{\rm H} = \frac{f'(r)}{4\pi}\Big|_{r=r_h} = \frac{2r_h^2 + \ell^2}{2\pi r_h \ell^2}, \qquad \Rightarrow \qquad \boxed{r_h = \frac{\ell^2}{2}\left(\pi T_{\rm H} \pm \sqrt{\pi^2 T_{\rm H}^2 - 2\ell^{-2}}\right)},$$

$$f(r_h) = 1 + \frac{r_h^2}{\ell^2} - \frac{2M}{r_h^2} = 0 \qquad \Rightarrow \qquad M = \frac{r_h^2}{2}\left(1 + \frac{r_h^2}{\ell^2}\right)$$

Rotating black hole

Asymptotic Kerr-AdS$_5$ metric in non-rotating at infinity frame[1]

$$ds^2 \simeq -\left(1+y^2\ell^{-2}\right)dT^2 + \frac{2M}{\Delta^3 y^2}\left(dT - a\sin^2\Theta d\Phi - b\cos^2\Theta d\Psi\right)^2 + \frac{dy^2}{1+y^2\ell^{-2}-\frac{2M}{\Delta^2 y^2}}$$
$$+ y^2\left(d\Theta^2 + \sin^2\Theta d\Phi^2 + \cos^2\Theta d\Psi^2\right),$$
$$\Delta = 1 - a^2\ell^{-2}\sin^2\Theta - b^2\ell^{-2}\cos^2\Theta$$

at a=b=0 the same as SAdS$_5$!

The horizon y_+ and Hawking temperature $T_{\rm H}$ of a stable *big BH*:

$$T_{\rm H} = \frac{1}{2\pi}\left(y_+(1+y_+^2\ell^{-2})\left(\frac{1}{y_+^2+a^2}+\frac{1}{y_+^2+b^2}\right)-\frac{1}{y_+}\right) \quad\Rightarrow\quad y_+ = y_+(T_{\rm H}),$$

$$1+\frac{y_+^2}{\ell^2}-\frac{2M}{\Delta^3 y_+^2} = 0 \quad\Rightarrow\quad M = \frac{\Delta^3 y_+^2}{2}\left(1+\frac{y_+^2}{\ell^2}\right)$$

[1] *G.W. Gibbons, M.J. Perry and C.N. Pope*, The First law of thermodynamics for Kerr-anti-de Sitter black holes, Class. Quant. Grav. **22**, 1503 (2005)

Conformal limit

- The physical temperature at the boundary is red-shifted via the Ehrenfest-Tolman effect:

$$T(y_0) = \frac{T_{\rm H}}{\sqrt{-G_{tt}(y_0)}} = \frac{T_{\rm H}}{\sqrt{1+\frac{y_0^2}{\ell^2}-\frac{2M}{\Delta^3 y_0^2}}} = \frac{T_{\rm H}\ell}{\sqrt{y_0^2-y_+^2}},$$

so rotation affect on the Hawking temperature $T_{\rm H}$ and horizon y_+, but there is no dependence on spatial coordinates, in opposite to the approach[2], where rotation warms up the periphery. For a rotating at infinity frame, this is also true, since $G_{tt}^{\rm BL} \approx G_{tt}$ at $y \gg y_+$.

- At $y_0 \to \infty$ the Kerr-AdS$_5$ approaches the same limit as SAdS$_5$:

$$ds^2 \to \frac{y_0^2}{\ell^2}(-dt^2 + \ell^2 d\Omega_3),$$

so $T \to 0$ at the conformal boundary $y \to \infty$ (in both rotating and non-rotating frames).

[2] *V.V. Braguta, A.Y. Kotov, D.D. Kuznedelev and A.A. Roenko*, Influence of relativistic rotation on the confinement-deconfinement transition in gluodynamics, Phys. Rev. D **103** (2021) 094515

Thermodynamics

- Angular momenta of a Kerr-AdS$_5$ BH in non-rotating frame[3]:

$$J_a = \frac{\pi M a}{2(1-a^2\ell^{-2})^2(1-b^2\ell^{-2})}, \qquad J_b = \frac{\pi M b}{2(1-b^2\ell^{-2})^2(1-a^2\ell^{-2})},$$

and corresponding angular velocities[4]:

$$\Omega_a = a\frac{1+\ell^{-2}y_+^2}{y_+^2+a^2}, \qquad \Omega_b = b\frac{1+\ell^{-2}y_+^2}{y_+^2+b^2}$$

- These definitions obey to the First law of thermodynamics:

$$dE = TdS + \Omega_a dJ_a + \Omega_b dJ_b$$

[3] *G.W. Gibbons, M.J. Perry and C.N. Pope*, The First law of thermodynamics for Kerr-anti-de Sitter black holes, Class. Quant. Grav. **22**, 1503 (2005)

[4] The rotation parameters are limited by the condition $a, b \leq \ell$ and the case of $a = \ell$ ($b = \ell$) corresponds to the critical rotation, when the entropy diverges

Phase transition without rotation ($SAdS_5$)

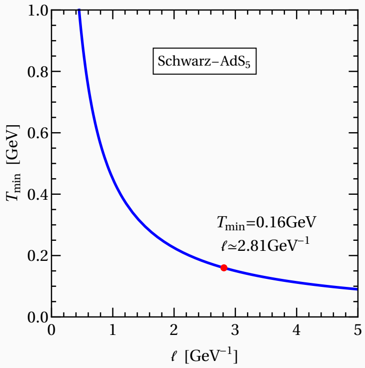


Figure 1: The minimum temperature $T_{\rm min}$ at which the Schwarzschild-AdS_5 black hole solution exists, depending on the AdS radius ℓ. We can *"tune" the temperature of the first-order phase transition* via selection of ℓ.

$$\Lambda = -6/\ell^2$$

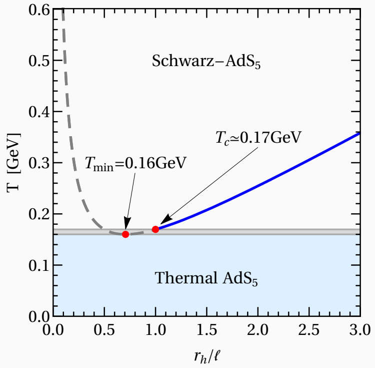


Figure 2: The Hawking temperature T dependence on the black hole horizon r_h. Below $T_{\rm min}$ a BH solution doesn't exist, and we need to consider the thermal AdS_5 spacetime. The first-order phase transition occurs at $r_h > \ell$ and $T > T_c = 3/(2\pi\ell)$, when $\Delta F < 0$. A small BH (left branch) is not allowed as a stable equilibrium, due to $\Delta F > 0$. A big BH (right branch) is not *globally* stable, so decays to the thermal AdS_5.

Phase transition in Kerr-AdS$_5$

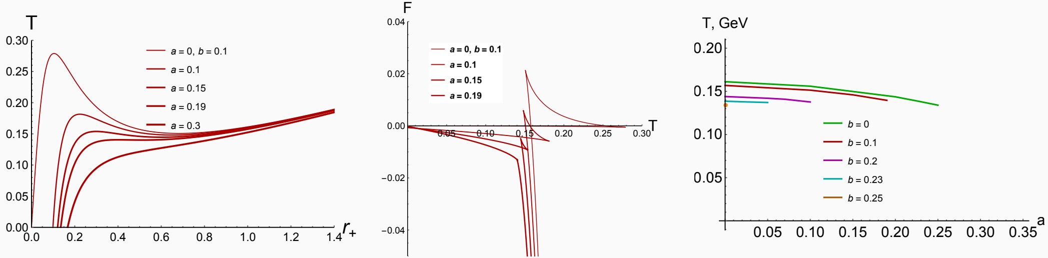


Figure 3: Temperature dependence on the horizon, free energy vs. temperature and critical temperature vs. a at different values of b. The parameter $\ell = 1$, while the temperature is scaled by the factor 2.9 to restore units. The minimal critical temperature $T_{\mathrm{CEP}} \approx 0.134\,$GeV[5].

[5] *I.Ya. Aref'eva, A.A. Golubtsova, E. Gourgoulhon*, Holographic drag force in 5d Kerr-AdS black hole, JHEP **04** (2021)

Heavy quark-qntiquark potential

Holographic Wilson loop

- $d=4$ $\mathcal{N}=4$ SYM with $SU(N)$:

$$W(\mathcal{C})=\frac{1}{N}\mathrm{Tr}\,\mathcal{P}\exp\left(\oint ds A_\mu \dot{x}^\mu+|\dot{x}^i|\Phi_i\theta^i\right)$$

- The AdS/CFT duality (*Maldacena*'98):

$$\langle W(\mathcal{C})\rangle = e^{-S_{\mathrm{NG,min}}-S_0},$$

where the Nambu-Goto action of an open string in asymptotically AdS_5 is

$$S_{\mathrm{NG}}=\frac{1}{2\pi\alpha'}\int \mathrm{d}\sigma\mathrm{d}\tau\sqrt{-\det(g_{\alpha\beta})},$$

with the induced metric on the string worldsheet

$$g_{\alpha\beta}=G_{MN}\partial_\alpha X^M\partial_\beta X^N,$$

G_{MN}– spacetime metric, X^M – embedding coordinates, α,β – indices on worldsheet
Zarembo et al.'98; *Gross et.al.'98*: $\langle W\rangle|_{\lambda\to\infty}\sim e^{\sqrt{\lambda}}$
Sonnenschein et al.'98; *Theisen'98*: finite T holographic WL for "planar" AdS BH

Heavy quark-antiquark potential

- The interquark potential is related to the expectation value of the temporal Wilson loop:

$$\langle W(\mathcal{C})\rangle \sim e^{-\mathcal{T}V(L)},$$

with the distance between quarks L and the temporal extent of the Wilson loop $\mathcal{T} \to \infty$.

- The quark-antiquark potential can be found in the following way:

$$V_{q\bar{q}} = \frac{S_{\rm NG}}{\mathcal{T}}\Big|_{\mathcal{T}\to\infty}.$$

- The *Cornell potential* is

$$V_{q\bar{q}} = \sigma L - \frac{\kappa}{L},$$

with σ and κ are the *string tension* and *Coulomb strength* parameters.

- In the *confined phase* the expectation value of the Wilson loop reproduces an *area law*

$$\langle W(\mathcal{C})\rangle \sim e^{-\sigma LT} = e^{-\sigma \mathrm{Area}(\mathcal{C})}.$$

Wilson loop configuration

Kerr-AdS$_5$ in non-rotating at infinity frame

$$ds^2 \simeq -\left(1+y^2\ell^{-2}\right)dT^2 + \frac{2M}{\Delta^3 y^2}\left(dT - a\sin^2\Theta d\Phi - b\cos^2\Theta d\Psi\right)^2 + \frac{dy^2}{1+y^2\ell^{-2}-\frac{2M}{\Delta^2 y^2}}$$

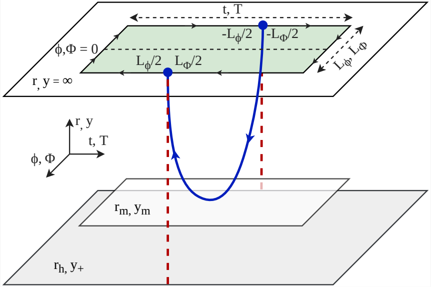
$$+ y^2\left(d\Theta^2 + \sin^2\Theta d\Phi^2 + \cos^2\Theta d\Psi^2\right), \qquad \Delta = 1 - a^2\ell^{-2}\sin^2\Theta - b^2\ell^{-2}\cos^2\Theta$$

The worldsheet parametrization:

$$\tau = T, \qquad \sigma = \Phi, \qquad y = y(\Phi), \qquad \Phi \in [0, 2\pi L_\Phi]$$

The boundary conditions:

$$y\left(-\frac{L_\Phi}{2}\right) = y\left(\frac{L_\Phi}{2}\right) = 0$$



At $a = b = 0$ the same as SAdS$_5$!

Wilson loop calculation

- The Nambu-Goto action is

$$S_{\rm NG} = \frac{T}{2\pi\alpha'}\int_{-\frac{L_\Phi}{2}}^{\frac{L_\Phi}{2}} d\Phi \sqrt{y'^2\frac{f_{\Delta^3}(y)}{f_{\Delta^2}(y)} + y^2 F_{\Delta^3}(y)\sin^2\Theta},$$

where we define

$$f_{\Delta^2}(y) \equiv 1 + y^2\ell^{-2} - \frac{2M}{\Delta^2 y^2}, \qquad f_{\Delta^3}(y) \equiv 1 + y^2\ell^{-2} - \frac{2M}{\Delta^3 y^2},$$

$$F_{\Delta^3}(y) = f_{\Delta^3}(y) + \frac{2Ma^2\sin^2\Theta}{y^4\Delta^3}\left(1 + y^2\ell^{-2}\right).$$

- The integral of motion:

$$\mathcal{H} = -\frac{y^2 F_{\Delta^3}(y)\sin^2\Theta}{\sqrt{y'^2\frac{f_{\Delta^3}(y)}{f_{\Delta^2}(y)} + y^2 F_{\Delta^3}(y)\sin^2\Theta}} = -\frac{\ell}{C}.$$

show induced metric

Wilson loop calculation

- The turning point is defined by $y'\big|_{\Phi=\Phi_m} = \frac{dy}{d\Phi}\big|_{\Phi=\Phi_m} = 0$, so we have

$$-y\sin\Theta\sqrt{F_{\Delta^3}(y)}\Big|_{y=y_m} = -\frac{\ell}{C}, \quad \text{with} \quad y_m = y(\Phi_m).$$

- The equation of motion is

$$y'^2 = y^2 F_{\Delta^3}(y)\frac{f_{\Delta^2}(y)}{f_{\Delta^3}(y)}\sin^2\Theta\left[\frac{C^2}{\ell^2}\sin^2\Theta y^2 F_{\Delta^3}(y) - 1\right].$$

- The distance between quarks L_Φ:

$$\frac{L_\Phi}{2} = \int_0^{\Phi(\frac{L_\Phi}{2})} d\Phi = \int_{y_m}^{\infty} dy\,\frac{\ell}{\sin\Theta y\sqrt{F_{\Delta^3}(y)}\sqrt{C^2\sin^2\Theta y^2 F_{\Delta^3}(y) - \ell^2}}\sqrt{\frac{f_{\Delta^3}(y)}{f_{\Delta^2}(y)}}.$$

The holographic renormalization

- Coming to the integration in terms of y we obtain

$$S_{\rm NG}=\frac{T}{\pi\alpha'}\int_{y_{\rm m}}^{\infty}dy\,\frac{C\sin\Theta y\sqrt{F_{\Delta^3}(y)}}{\sqrt{C^2\sin^2\Theta y^2F_{\Delta^3}(y)-\ell^2}}\sqrt{\frac{f_{\Delta^3}(y)}{f_{\Delta^2}(y)}}.$$

- The holographic renormalization

$$S_{\rm NG}^{\rm ren}=S_{\rm NG}-S_0$$

is a subtraction of the self-energy of two free quarks (straight strings from $y=\infty$ up to y_+)

$$S_0=\frac{T}{\pi\alpha'}\int_{y_+}^{\infty}dy\sqrt{-G_{TT}G_{yy}}=\frac{T}{\pi\alpha'}\left(\int_{y_{\rm m}}^{\infty}+\int_{y_+}^{y_{\rm m}}\right)\sqrt{\frac{f_{\Delta^3}(y)}{f_{\Delta^2}(y)}}dy.$$

- The quark-antiquark potential ($\lambda=\frac{\ell^4}{\alpha'^2}$):

$$\boxed{V_{q\bar q}=\frac{S_{\rm NG}^{\rm ren}}{T}=\frac{\sqrt{\lambda}}{\pi\ell^2}\left[\int_{y_{\rm m}}^{\infty}dy\sqrt{\frac{f_{\Delta^3}(y)}{f_{\Delta^2}(y)}}\left(\frac{C\sin\Theta y\sqrt{F_{\Delta^3}(y)}}{\sqrt{C^2\sin^2\Theta y^2F_{\Delta^3}(y)-\ell^2}}-1\right)-\int_{y_+}^{y_{\rm m}}dy\sqrt{\frac{f_{\Delta^3}(y)}{f_{\Delta^2}(y)}}\right]}.$$

$J = 0 \ (a = b = 0)$

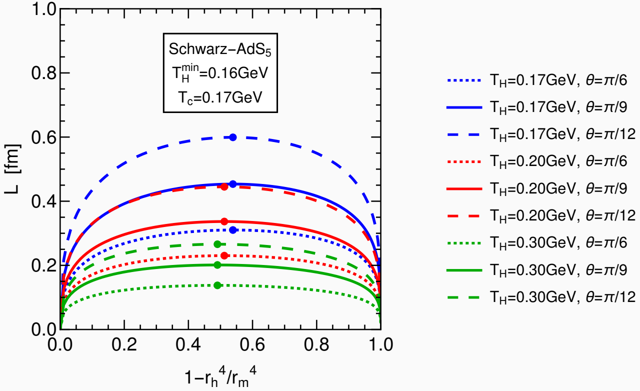


Figure 4: The distance L between quark and antiquark, depending on the string turning point r_{m}

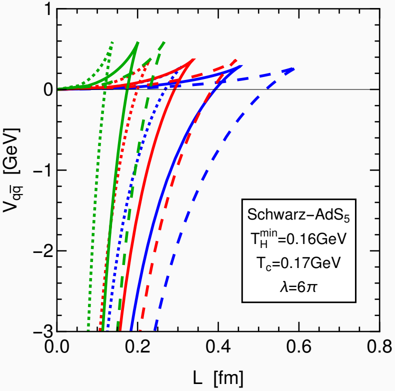


Figure 5: Numerical results for the dependence of $V_{q\bar{q}}$ on the distance between quarks L

- *With increasing of T or θ we receive smaller L and larger $V_{q\bar{q}}$!*
- Upper branch is unphysical (quarks become free)

$J \neq 0$

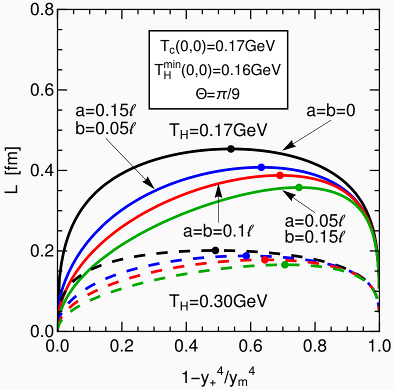


Figure 6: The distance L vs. turning point y_m

$$\Omega_a = a\frac{1+\ell^{-2}y_+^2}{y_+^2+a^2}$$

a/ℓ	Ω_a, MeV
0.05	35.107
0.10	68.811
0.15	100.175
0.20	128.823

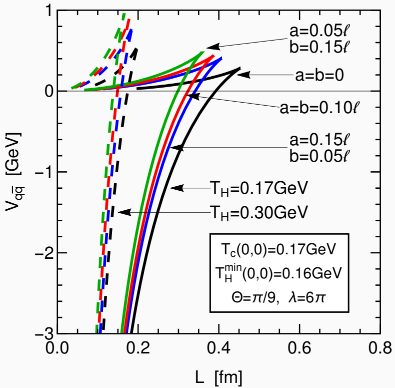


Figure 7: The potential $V_{q\bar{q}}$ vs. distance L

- *Rotation leads to decreasing of L and increasing of $V_{q\bar{q}}$!*
- *The influence of the parameter b is stronger!* due to string configuration on Φ

show angular velocity plot

The relation between $S_{\rm NG}^{\rm ren}$ and L_Φ

The relation between the string action and the quark-antiquark distance

$$S_{\rm NG}^{\rm ren} = \frac{T}{\pi\alpha'} I_1(y_{\rm m}, C), \qquad \frac{L_\Phi}{2} = I_2(y_{\rm m}, C).$$

We have the following relation:

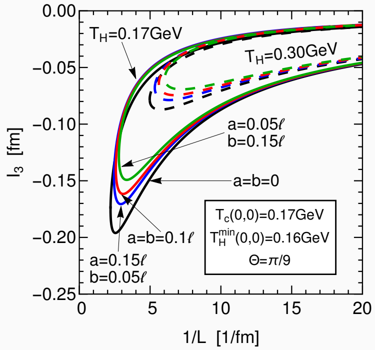
$$\frac{\partial I_2(y_{\rm m}, C)}{\partial C} = \frac{C}{\ell} \frac{\partial I_1(y_{\rm m}, C)}{\partial C}.$$

We find for the quark-antiquark potential

$$\boxed{V_{q\bar{q}} = \frac{\sqrt{\lambda}}{\pi \ell^2} y_{\rm m} \sin\Theta \sqrt{F_{\Delta^3}(y_{\rm m})} \left(\frac{L_\Phi}{2} + I_3 \right)},$$

where

$$I_3 = \int_{y_{\rm m}}^{\infty} dy \sqrt{\frac{f_{\Delta^3}(y)}{f_{\Delta^2}(y)}} \left(\frac{\sqrt{C^2 \sin^2\Theta y^2 F_{\Delta^3}(y) - \ell^2}}{y \sin\Theta \sqrt{F_{\Delta^3}(y)}} - C \right) - \frac{C}{\ell} \int_{y_+}^{y_{\rm m}} dy \sqrt{\frac{f_{\Delta^3}(y)}{f_{\Delta^2}(y)}}$$



$$V_{q\bar{q}}(L) = \sigma L - \frac{\kappa}{L} + \frac{\chi}{\sqrt{L}}$$

a/ℓ	b/ℓ	σ, GeV/fm	κ, GeV·fm	χ, GeV·fm$^{1/2}$
0	0	2.21704	1.28893	1.55392
0.15	0.05	2.76058	1.30234	1.71247
0.1	0.1	2.91683	1.31574	1.80647
0.05	0.15	3.13669	1.33658	1.95819

Table 1: Fitting coefficients at $T_{\rm H} = 0.17\,$GeV and $\theta = \pi/9$

- *We recognize linear and Coulomb terms*
- *With an increase in rotation, the string tension, the Coulomb strength, as well as the parameter χ increase*
- *The most simple additional term to fitting is $\frac{\chi}{\sqrt{L}}$. With $1/L^2$ gives large σ, small κ and worse precision. With $const$ gives negative σ*

Jet-quenching parameter

Jet-quenching parameter

- The jet-quenching parameter $\hat{q}$ gives the squared average transverse momentum exchange between the medium and the high-energy parton per unit path length:

$$\hat{q} = \frac{\langle p_\perp^2 \rangle}{L}.$$

Gyulassy, Plümer'90; Baier, Schiff, Zakharov'00: jet-quenching classical works

- The suppression of elliptic flows v_2 and hadrons yields with high transverse momentum p_{T}, as well as observed increase in the nuclear modification factor R_{AA} are related to the jet-quenching phenomenon (BRAHMS'05; PHENIX'05; STAR'05)

Light-like Wilson loop and jet-quenching parameter

• The expectation value of the the light-like WL on the contour $\mathcal{C}$ in the adjoint representation and the jet-quenching parameter $\hat{q}$ are related as follows (Rajagopal et. al.'06):

$$\langle W^A(\mathcal{C})\rangle \approx \exp\Big[-\frac{1}{4\sqrt{2}}\hat{q}L^-L^2\Big],$$

where L^- is a large side of the rectangular contour $\mathcal{C}$ and L is a short side.

• The WL operator in the adjoint representation is related to the the WL operator in the fundamental representation: $\langle W^A(\mathcal{C})\rangle \approx \langle W^F(\mathcal{C})\rangle^2.$

• Following the holographic dictionary, we have $\langle W^F(\mathcal{C})\rangle = e^{-S_{\rm NG}}.$

$$\Longrightarrow \quad \boxed{\langle W^A(\mathcal{C})\rangle \approx \exp\Big[-\frac{1}{4\sqrt{2}}\hat{q}L^-L^2\Big] \approx e^{-2S_{\rm NG}}}.$$

• Planar case (AdS on $\mathbb{R}^3$): $\boxed{\hat{q}_{\rm SYM} = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{\lambda}T^3}.$

Light-like Wilson loop in Schwarzschild-AdS$_5$

- "Light-cone" coordinates: $dx^+ = \ell^2(dt - \ell d\phi), \quad dx^- = \ell^2(dt + \ell d\phi).$
- The string parametrization

$$\tau = x^-, \quad \sigma = \psi, \quad x^\mu = x^\mu(\sigma), \quad \theta(\sigma) = const, \quad x^+(\sigma) = const.$$

- The Nambu-Goto action is

$$S = \frac{L^-}{2\pi\alpha'} \int_{-L/2}^{L/2} d\psi \, \frac{r}{2\ell^2} \sqrt{\left(\frac{f(r)}{r^2} - \ell^{-2} r^2 \sin^2\theta\right)\left(\cos^2\theta + \frac{r'^2}{f(r)}\right)}, \quad r' \equiv \partial r/\partial\psi$$

- The first integral is given by

$$\mathcal{H} = -\frac{\cos^2\theta\sqrt{f(r) - r^4\ell^{-2}\sin^2\theta}}{2\ell^2\sqrt{\cos^2\theta + \frac{r'^2}{f(r)}}} = -C.$$

- The equation of motion for $r(\sigma)$:

$$r'^2 = \frac{f(r)\cos^2\theta}{4C^2\ell^6}[\cos^2\theta(f(r)\ell^2 - r^4\sin^2\theta) - 4C^2\ell^6].$$

Jet-quenching parameter calculation

- Regularized action:

$$S_{\rm NG}^{\rm reg}=\frac{L^-}{\pi\alpha'}\int_{r_H+\epsilon}^{\infty}dr\,\frac{\sqrt{f(r)\ell^2-r^4\sin^2\theta}}{2\ell^3\sqrt{f(r)}}\left(\frac{\cos\theta\sqrt{f(r)\ell^2-r^4\sin^2\theta}}{\sqrt{\cos^2\theta(f(r)\ell^2-r^4\sin^2\theta)-4C^2\ell^6}}-1\right).$$

- Expanding for small C (low energy limit)

$$S_{\rm NG}^{\rm reg}=\frac{L^-}{\pi\alpha'}\frac{\ell^2C^2}{\cos^2\theta}\mathcal{I},\quad \mathcal{I}=\int_{r_m}^{\infty}\frac{dr}{\sqrt{f(r)}\sqrt{f(r)-r^4\ell^{-2}\sin^2\theta}}$$

and r_m is defined as a positive real solution to the equation

$$r^2+r^4\ell^{-2}\cos^2\theta-2M=0.$$

- To find the relation between L and C we remember that $r(\pm L/2)=\infty$:

$$\frac{L}{2}=\int_{r_H}^{\infty}\frac{dr}{r'}=\frac{2C\ell^3}{\cos\theta}\int_{r_H}^{\infty}\frac{dr}{\sqrt{f(r)}\sqrt{\cos^2\theta(f(r)\ell^2-r^4\sin^2\theta)-4C^2\ell^6}}.$$

Jet-quenching parameter calculation

- For small C we have

$$\frac{L}{2}=\frac{2\ell^2 C}{\cos^2\theta}\mathcal{I}$$

and we come to

$$S_{\rm NG}^{\rm reg}=\frac{L^-}{\pi\alpha'}\frac{L^2\cos^2\theta}{16\ell^2\int_{r_m}^{\infty}\frac{dr}{\sqrt{f(r)}\sqrt{f(r)-r^4\ell^{-2}\sin^2\theta}}}.$$

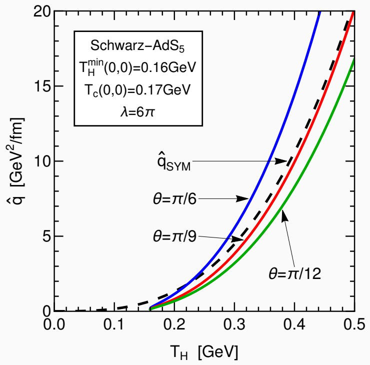
- Wilson loop and jet-quenching parameter relation (Rajagopal'06):

$$\langle W^A(\mathcal{C})\rangle\approx\exp\left[-\frac{1}{4\sqrt{2}}\hat{q}L^-L^2\right]\approx e^{-2S_{\rm NG}}.$$

Then the jet-quenching parameter is ($\lambda=\frac{\ell^4}{\alpha'^2}$)

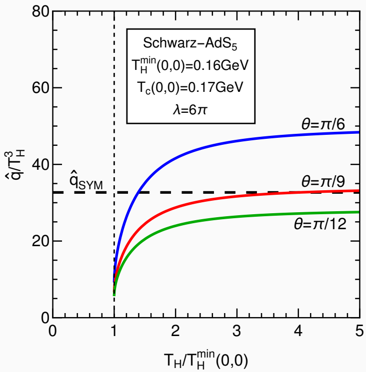
$$\boxed{\hat{q}=\frac{\sqrt{\lambda}}{\sqrt{2}\pi}\frac{\cos^2\theta}{\ell^4\int_{r_m}^{\infty}\frac{dr}{\sqrt{f(r)}\sqrt{f(r)-r^4\ell^{-2}\sin^2\theta}}}}.$$

The temperature dependence of JQ for the SAdS$_5$



Planar case (AdS on $\mathbb{R}^3$):

$$\hat{q}_{\mathrm{SYM}} = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{\lambda}T_{\mathrm{H}}^3$$



- *Near critical temperature T_{c} we have $\hat{q} < \hat{q}_{\mathrm{SYM}}$ (at some values of θ always $\hat{q} < \hat{q}_{\mathrm{SYM}}$!)*
- *We have $\hat{q} \sim T_{\mathrm{H}}^3$ at high temperatures T_{H} and $\hat{q} \approx \hat{q}_{\mathrm{SYM}}$ at $\theta \approx \pi/9$*

Light-like Wilson loop in Kerr-AdS$_5$

- "Light-cone" coordinates (Cvetic, Gao, Simon'05): *for simplicity* $\ell = 1$

$$dx^+ = dT - ad\Phi, \qquad dx^- = dT + ad\Phi.$$

- The string parametrization

$$\tau = x^-, \quad \sigma = \Psi, \quad x^\mu = x^\mu(\sigma), \quad \Theta(\sigma) = const, \quad x^+(\sigma) = const.$$

- We introduce the following notation:

$$\eta(y) = (1+y^2) - \frac{y^2}{a^2}\sin^2\Theta, \quad \zeta(y) = \eta(y) - \frac{2M}{\Delta^3 y^2}\cos^4\Theta,$$

$$\beta(y) = \cos^2\Theta\left(\eta(y)\frac{2M}{\Delta^3 y^2}b^2\cos^2\Theta + \zeta(y)y^2\right)$$

- The first integral and equation of motion ($y' = \frac{dy}{d\Psi}$) are given by

$$\mathcal{H} = \frac{\beta(y)}{2\sqrt{\beta(y) + \frac{y'^2\zeta(y)}{f_{\Delta^2}(y)}}} = -C, \qquad y'^2 = \frac{f_{\Delta^2}(y)\beta(y)}{\zeta(y)}\left(\frac{\beta(y)}{4C^2} - 1\right).$$

Light-like Wilson loop in Kerr-AdS$_5$

- The regularized action and distance between quarks:

$$S_{\rm NG}^{\rm reg} = \frac{L^-}{\pi\alpha'}\int_{y_+}^{\infty} dy\,\frac{\sqrt{\zeta(y)}}{2\sqrt{f_{\Delta^2}(y)}}\left(\frac{\sqrt{\beta(y)}}{\sqrt{\beta(y)-4C^2}}-1\right),$$

$$\frac{L}{2} = \int_{y_+}^{\infty}\frac{dy}{y'} = \int_{y_+}^{\infty} dy\,\frac{2C\sqrt{\zeta(y)}}{\sqrt{f_{\Delta^2}(y)\beta(y)}\sqrt{\beta(y)-4C^2}}.$$

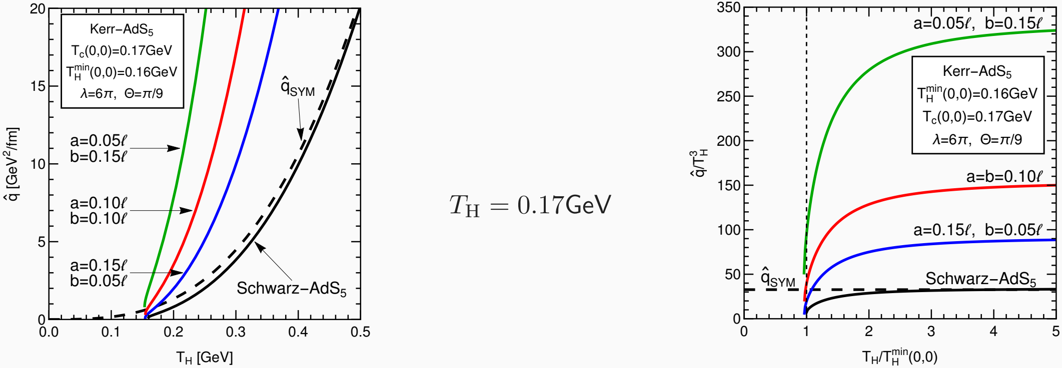
- In the low energy limit (small C):

$$S_{\rm NG}^{\rm reg} = \frac{L^-}{\pi\alpha'}C^2\mathcal{I}+\mathcal{O}(C^4), \qquad \frac{L}{2} = 2C\mathcal{I}+\mathcal{O}(C^3), \qquad \mathcal{I} = \int_{y_+}^{\infty} dy\,\frac{\sqrt{\zeta(y)}}{\beta(y)\sqrt{f_{\Delta^2}(y)}}.$$

- The final expression for the jet-quenching parameter (*with restored units*):

$$\boxed{\hat{q} = \frac{\sqrt{\lambda}}{\sqrt{2}\pi\ell^4\mathcal{I}}}$$

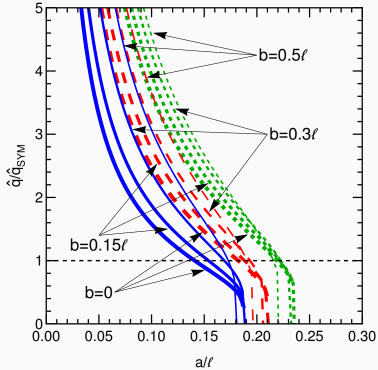
The temperature dependence of JQ for the Kerr-AdS$_5$



$$T_{\mathrm{H}} = 0.17\mathrm{GeV}$$

- *The rotating plasma is more strongly coupled!*
- *The phase transition temperature decreases with rotation!*
- *The behaviour* $\sim T_{\mathrm{H}}^3$ *at high* T_{H} *is saved in the rotating plasma.*
- *The singularity at* $a \to 0$ *is due to the metric transformation (string configuration).*

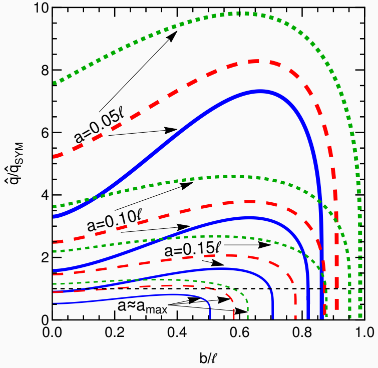
The rotational parameters dependence for the Kerr-AdS$_5$



$T_{\rm H} = 0.17$GeV (blue),
$a_{\rm max} = 0.18\ell$

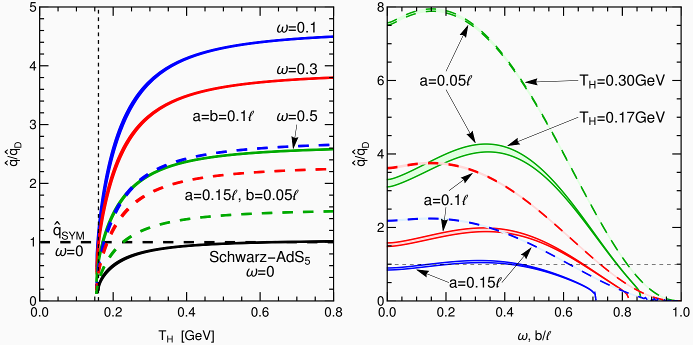
$T_{\rm H} = 0.20$GeV (red),
$a_{\rm max} = 0.19\ell$

$T_{\rm H} = 0.30$GeV (green),
$a_{\rm max} = 0.21\ell$



- *The singularity at $a \to 0$ is due to the metric transformation (string configuration).*
- *At some value of a near $a_{\rm max}$ we have $\hat{q} \approx \hat{q}_{\rm SYM}$ and weaker dependence on b*
- *The JQ parameter increases with b and sharply drops to zero at $b_{\rm max}$*
- *$b_{\rm max}$ decreases with increasing of a*

Comparison with D-instanton background



$$\hat{q}_{\rm D} = \frac{1}{\pi\alpha'}\frac{1}{\int_{r_t}^{\infty}\sqrt{\frac{1-\omega^2}{e^{\Phi}f(r)(1-f(r))}\frac{R^4}{r^4}}dr}$$

we set instanton density $q = [0,1]\ell^4$

The D-instanton background (Chen, Hou'22):

$$ds^2 = e^{\frac{\Phi}{2}}\frac{r^2}{R^2}\left[\frac{\omega^2 - f(r)}{1-\omega^2}dt^2 + \frac{1-\omega^2 f(r)}{1-\omega^2}d\varphi^2 + \frac{2\omega(1-f(r))}{1-\omega^2}dtd\varphi + \frac{1}{f(r)}\frac{R^4}{r^4}dr^2 + dx_1^2 + dx_2^2\right],$$

$$e^{\Phi} = 1 + \frac{q}{r_t^4}\log\frac{1}{f(r)}, \quad f(r) = 1 - \frac{r_t^4}{r^4},$$

radius to the rotating axis: $l = 1\text{GeV}^{-1}$; AdS radius $R = \ell$

In the dual picture the D-instanton density represents the vacuum expectation value of gluon condensation

Summary

Summary

We have computed the temporal (*heavy quark-antiquark potential*) and light-like (*jet-quenching parameter*) Wilson Loops in the $SAdS_5$ (*non-rotating plasma*) and Kerr-AdS_5 (*rotating plasma*) geometries:

- The analytical expression for the potential contains the linear and modified Coulomb parts.
- *With increasing of T or θ we receive smaller L and larger $V_{q\bar{q}}$!*
- *Rotation leads to decreasing of L and increasing of $V_{q\bar{q}}$!*
- With an increase in rotation, the string tension and Coulomb strength parameters increase

- Near critical temperature T_c we have $\hat{q} < \hat{q}_{\mathrm{SYM}}$ (at some values of θ always $\hat{q} < \hat{q}_{\mathrm{SYM}}$!)
- We have $\hat{q} \sim T_{\mathrm{H}}^3$ at high T_{H} and $\hat{q} \approx \hat{q}_{\mathrm{SYM}}$ at $\theta \approx \pi/9$
- The rotating plasma is more strongly coupled!
- The phase transition temperature T_c decreases with rotation!

Thank you for your attention!

Probing $\mathcal{N} = 4$ SYM rotating quark-gluon plasma using holography

Based on arXiv:2210:XXXXX

Tsegelnik Nikita, Golubtsova Anastasia

November 9, 2022

Joint Institute for Nuclear Research
tsegelnik@theor.jinr.ru

Induced metric

The Nambu-Goto action:

$$S_{\mathrm{NG}} = \frac{1}{2\pi\alpha'} \int \mathrm{d}\sigma \mathrm{d}\tau \sqrt{-\det(g_{\alpha\beta})}$$

The string configuration:

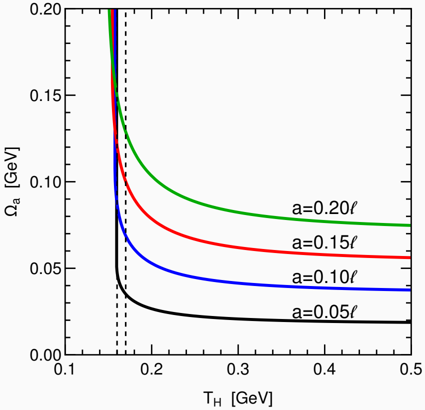
$$\tau = T, \qquad \sigma = \Phi, \qquad y = y(\Phi), \qquad \Phi \in [0, 2\pi L_{\Phi}]. \tag{1}$$

The induced metric:

$$g_{\tau\tau} = G_{TT} = -\left(1 + y^2 \ell^{-2} - \frac{2M}{\Delta^3 y^2}\right), \quad g_{\tau\sigma} = G_{T\Phi} = -\frac{2Ma\sin^2\Theta}{\Delta^3 y^2},$$
$$g_{\sigma\sigma} = G_{\Phi\Phi} + y'^2 G_{yy} = \sin^2\Theta\left(y^2 + \frac{2Ma^2\sin^2\Theta}{\Delta^3 y^2}\right) + \frac{y'^2}{1 + y^2\ell^{-2} - \frac{2M}{\Delta^2 y^2}}, \tag{2}$$

go back

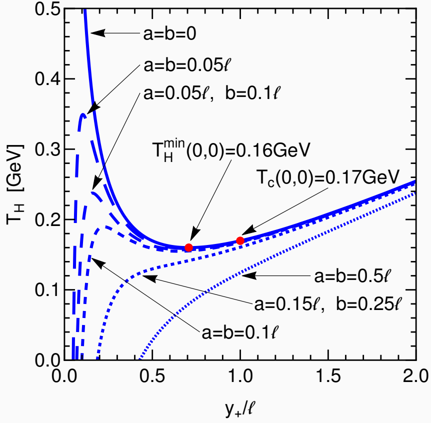
Angular velocity in Kerr-AdS_5

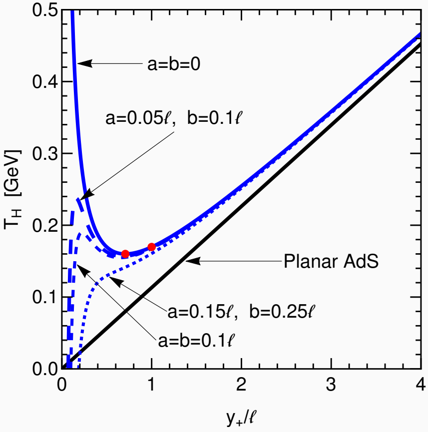


	$T_{\rm H} = 170\,$MeV	$T_{\rm H} = 300\,$MeV	$T_{\rm H} = 500\,$MeV
a/ℓ	Ω_a, MeV	Ω_a, MeV	Ω_a, MeV
0.05	35.107	20.7284	18.7286
0.10	68.811	41.3958	37.4411
0.15	100.175	61.9422	37.4411
0.20	128.823	82.3104	74.7549

go back

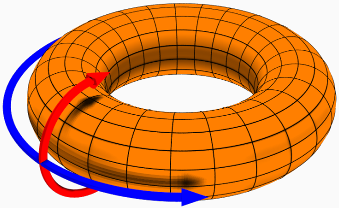
Temperature in Kerr-AdS_5





Hopf coordinates

$$d\Omega_3^2 = d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta d\psi^2,$$
$$0 \le \theta \le \pi/2, \qquad 0 \le \phi, \psi \le 2\pi$$



For any fixed value of θ the coordinates (ϕ, ψ) parametrize a 2-dimensional torus. Rings of constant ϕ and ψ form simple orthogonal grids on the torus. In the degenerate cases $\theta = 0$ and $\theta = \pi/2$ coordinates (ϕ, ψ) represent a circle

go back

Confinement in AdS/CFT

- Any CFT on $\mathbb{R}^n$ is scale invariant, so there is no confining phase
- If we deform the AdS spacetime, the gauge theory will also be deformed in some way
- *Some deformations or geometries of the $\mathcal{N} = 4$ SYM bring the theory "closer to QCD" and lead to the confinement*

Toy confinement model at $T = 0$

The holographic coordinate r refers to the gauge theory energy scale, so if we *cutoff*[6] AdS spacetime at $r = r_{\text{min}} \sim \Lambda \ell^2$ (Λ is the mass of the lightest glueball state and ℓ is the AdS radius), we receive the *confinement* behaviour near r_{min} and Coulomb-like potential at $r \gg r_{\text{min}}$

[6] *J. Polchinski, M.J. Strassler*, Hard scattering and gauge/string duality. Phys. Rev. Lett. **88**, 031601 (2002)

Confinement-deconfinement phase transition

Phase transitions[7] in AdS_5/CFT_4:

- D-branes (flat horizon) + background (dilaton, bulk scalar field): *hard-wall*[8] and *soft-wall*[9] AdS/QCD (or *KKSS model*)
- Black holes (compact spaces[10] and thermal ensembles): $\mathbb{R}^1 \times \mathbb{S}^3$ or $\mathbb{R}^3 \times \mathbb{S}^1$

$\mathbb{R}^3 \times \mathbb{S}^1$

- AdS soliton $\longleftrightarrow$ confining phase
- AdS $\mathbb{R}^3 \times \mathbb{S}^1$ black hole $\longleftrightarrow$ plasma

$\mathbb{R}^1 \times \mathbb{S}^3$ "precise"[11] correspondence

- thermal AdS[12] $\longleftrightarrow$ confining phase
- AdS $\mathbb{R}^1 \times \mathbb{S}^3$ black hole $\longleftrightarrow$ plasma

---

7 The phase transition in AdS black holes is called Hawking-Page phase transition. *S.W. Hawking, D.N. Page*, Commun. Math. Phys., **87**, 577 (1983)

8 *J. Erlich, E. Katz, D.T. Son and M.A. Stephanov*, Phys. Rev. Lett. **95**, 261602 (2005)

9 *A. Karch, E. Katz, D. T. Son and M. A. Stephanov*, Phys. Rev. D **74** 015005 (2006)

10 In gauge theory on the sphere we get "kinematic confinement", which comes from the Gauss law. Therefore, the physical states on a sphere cannot have any net color. So our "confinement" is not the same as QCD one, which comes from dynamics in a complicated way

6 *E. Witten*, series of works in 1998

12 The AdS spacetime with Euclidean time periodicity β

The Kerr-AdS$_5$ solution in the rotating at infinity frame

The Kerr-AdS$_5$ metric the in rotating at infinity frame[13]

$$
\begin{aligned}
ds^2 = & -\frac{\Delta_r}{\rho^2}\left(dt - \frac{a\sin^2\theta}{\Xi_a}d\phi - \frac{b\cos^2\theta}{\Xi_b}d\psi\right)^2 + \frac{\Delta_\theta \sin^2\theta}{\rho^2}\left(adt - \frac{\left(r^2+a^2\right)}{\Xi_a}d\phi\right)^2 \\
& + \frac{\Delta_\theta \cos^2\theta}{\rho^2}\left(bdt - \frac{\left(r^2+b^2\right)}{\Xi_b}d\psi\right)^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 \\
& + \frac{\left(1+r^2\ell^{-2}\right)}{r^2\rho^2}\left(abdt - \frac{b\left(r^2+a^2\right)\sin^2\theta}{\Xi_a}d\phi - \frac{a\left(r^2+b^2\right)\cos^2\theta}{\Xi_b}d\psi\right)^2 ,
\end{aligned}
$$

$$
\Delta_r = \frac{1}{r^2}\left(r^2+a^2\right)\left(r^2+b^2\right)\left(1+r^2\ell^{-2}\right) - 2M, \qquad \Xi_a = \left(1-a^2\ell^{-2}\right), \quad \Xi_b = \left(1-b^2\ell^{-2}\right)
$$

$$
\Delta_\theta = \left(1 - a^2\ell^{-2}\cos^2\theta - b^2\ell^{-2}\sin^2\theta\right), \qquad \rho^2 = \left(r^2 + a^2\cos^2\theta + b^2\sin^2\theta\right)
$$

---

[13] *S.W. Hawking, C.J. Hunter and M. Taylor-Robinson*, Rotation and the AdS/CFT correspondence, Phys.Rev. D **59** (1999) 064005;

Modified potential