Hydrodynamics as an effective field theory

The Ideal hydrodynamics limit and its extensions

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UNICAMP

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Based on 1810.12468,1807.02796,1701.08263,1604.05291
with D.Montenegro,L.Tinti

What is this talk about

The necessity of a field theory perspective
Hydrodynamics is neither transport nor string theory!

Introduction to the field theory of hydrodynamics
Our knowledge of hydrodynamics rewritten as symmetries
Perhaps not ideal for solving problems, but worth thinking about!

Extending hydrodynamics I Polarization

Extending hydrodynamics II Gauge symmetries

Some experimental data warmup

(2004) Matter in heavy ion collisions seems to behave as a perfect fluid, characterized by a very rapid thermalization

RHIC Scientists Serve Up 'Perfect' Liquid

New state of matter more remarkable than predicted — raising many new questions

April 18, 2005

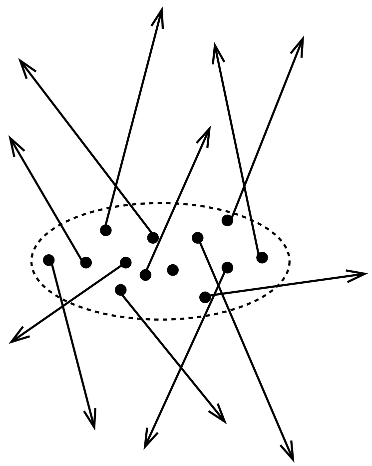
TAMPA, FL — The four detector groups conducting research at the Relativistic Heavy Ion Collider (RHIC) — a giant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory — say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In peer-reviewed papers summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a *liquid*.

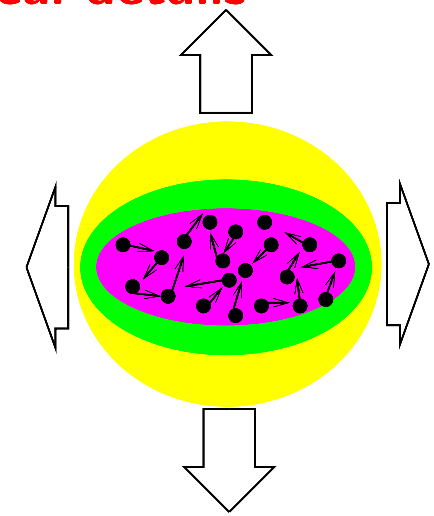
"Once again, the physics research sponsored by the Department of Energy is producing historic results," said Secretary of Energy Samuel Bodman, a trained chemical engineer. "The DOE is the principal federal funder of basic research in the physical sciences, including nuclear and high-energy physics. With today's announcement we see that investment paying off."

"The truly stunning finding at RHIC that the new state of matter created in the collisions of gold ions is more like a liquid than a gas gives us a profound insight into the earliest moments of the universe," said Dr. Raymond L. Orbach, Director of the DOE Office of Science.

The technical details

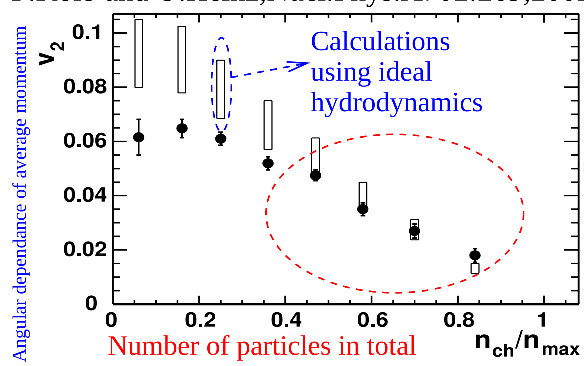
A "dust"
Particles ignore each other, their path is independent of initial shape



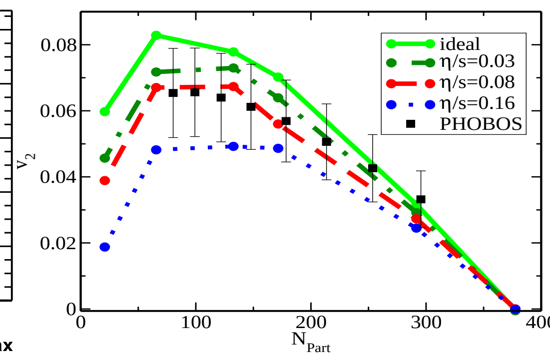


A "fluid"
Particles continuously interact. Expansion determined by density gradient (shape)

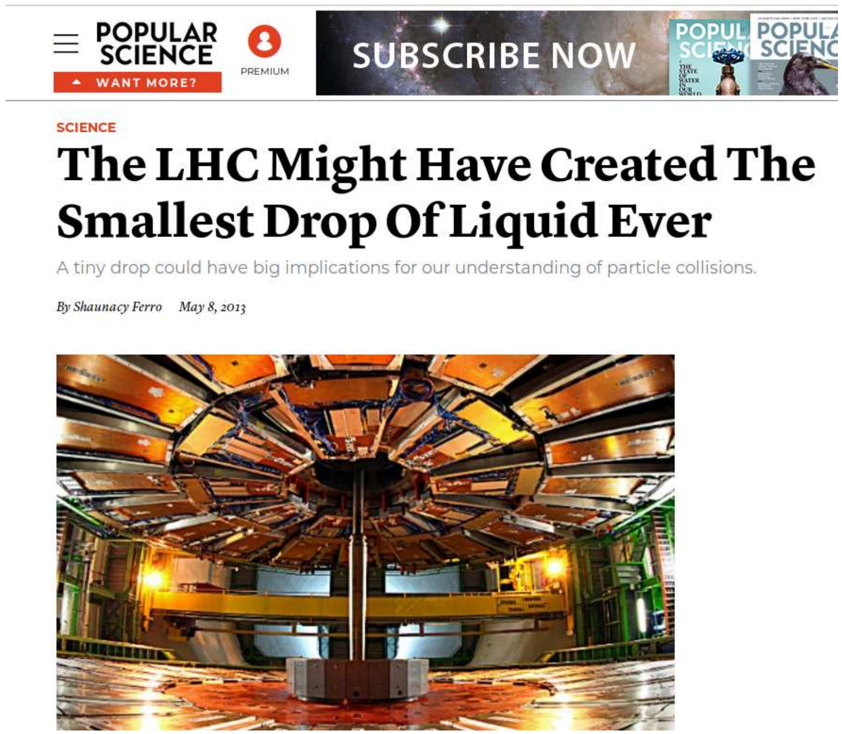
P.Kolb and U.Heinz,Nucl.Phys.A702:269,2002.



P.Romatschke,PRL99:172301,2007



2011-2013 the limit of this. FLuid-like behavior has been observed down to very low energies and small sizes.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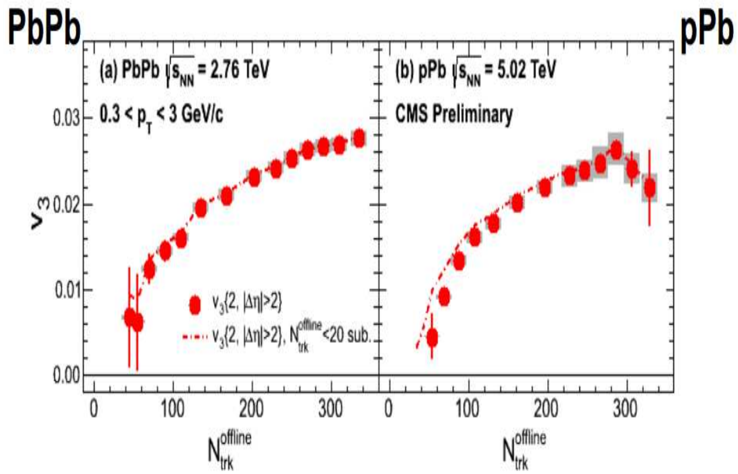
SCIENCE

The LHC Might Have Created The Smallest Drop Of Liquid Ever

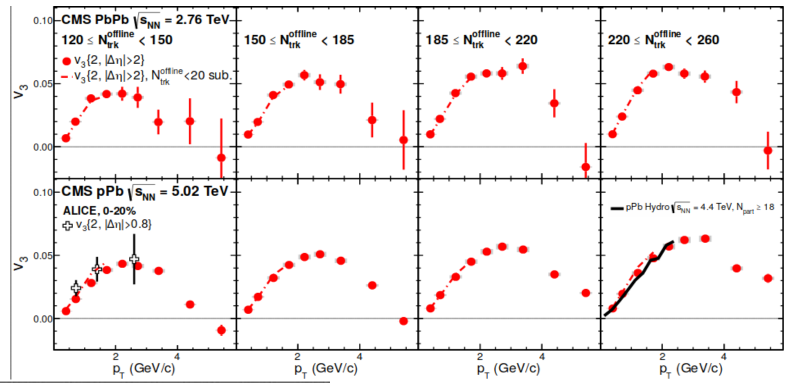
A tiny drop could have big implications for our understanding of particle collisions.

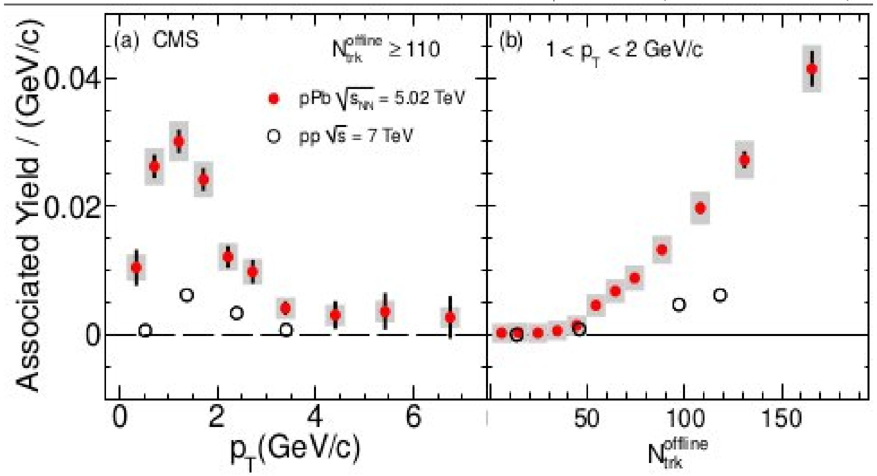
By Shaunacy Ferro *May 8, 2013*

The technical details

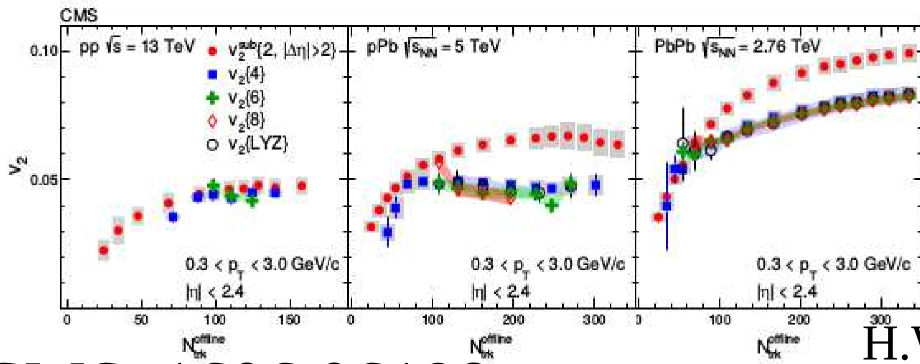


CMS collaboration



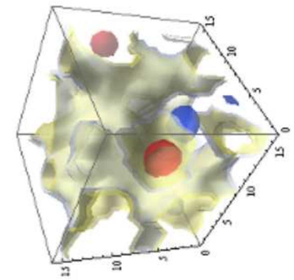


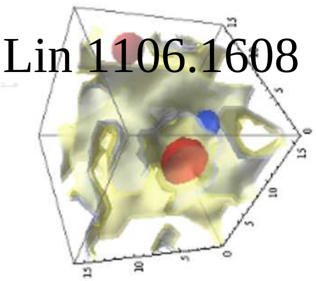
CMS 1305.0609

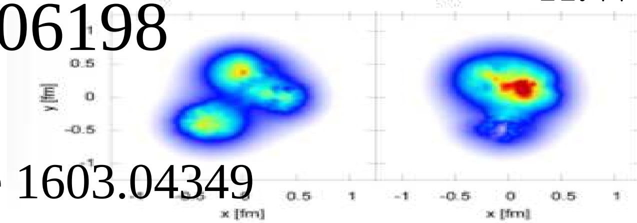


CMS 1606.06198

H.W.Lin 1106.1608







BSchenke 1603.04349

1606.06198 (CMS) : When you consider geometry differences, hydro with $\mathcal{O}(20)$ particles "just as collective" as for 1000. So mean free path is really small. What about thermal fluctuations? Nothing here is infinite, not even N_c Also hydro applicability scale below color domain scale. colored hydro?

2017 Macroscopic and microscopic degrees of freedom talk (polarization is transferred to spin DoFs).

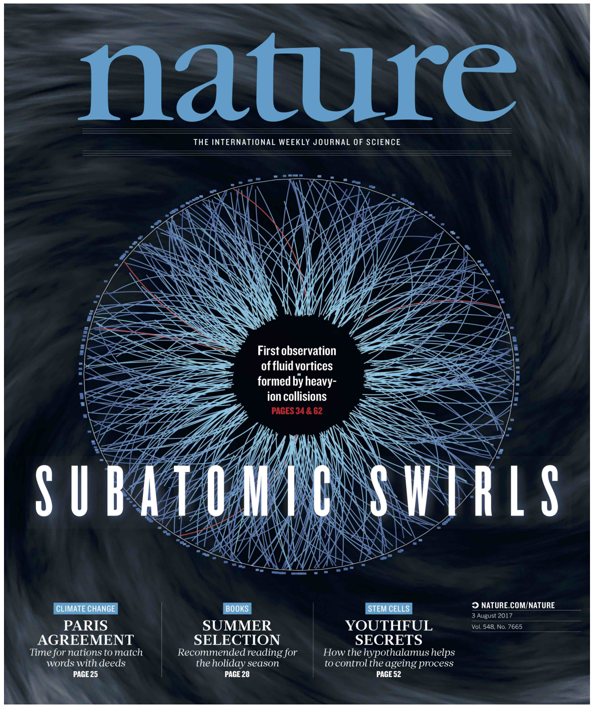
STAR collaboration

1701.06657

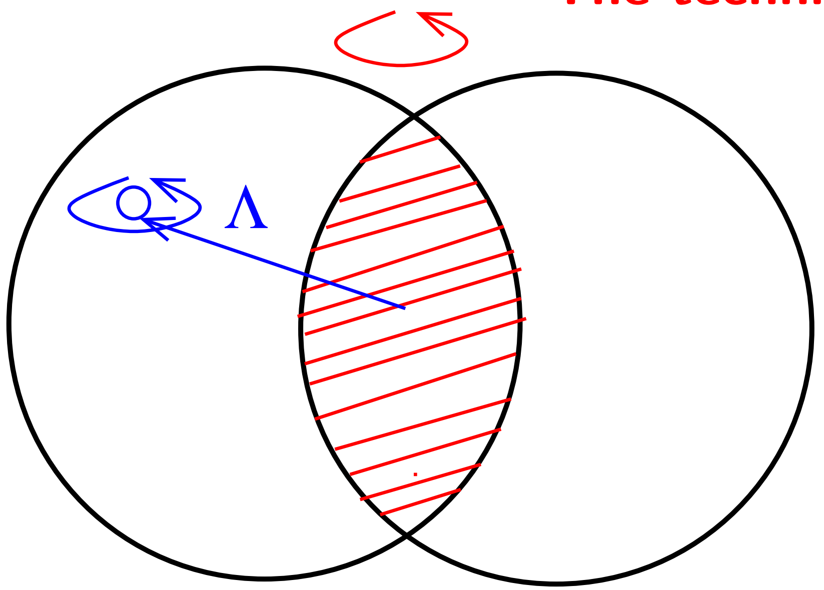
NATURE

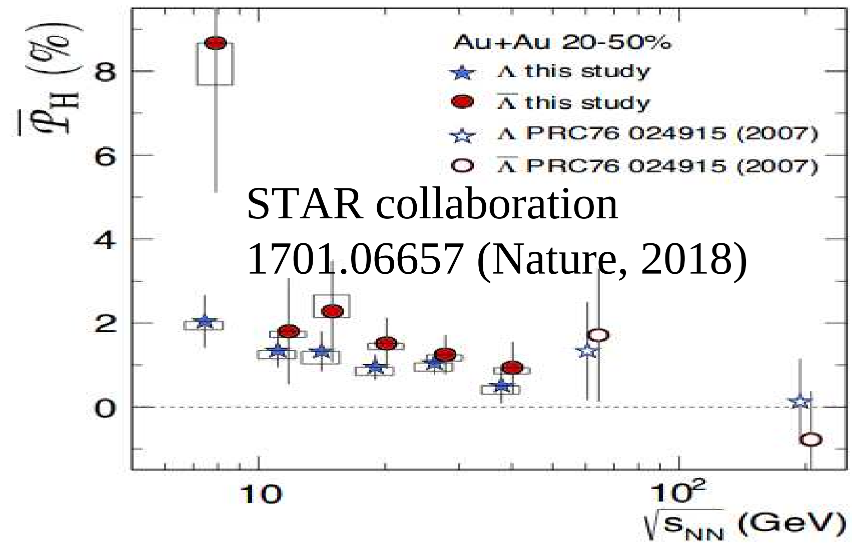
August 2017

Polarization by vorticity in heavy ion collisions



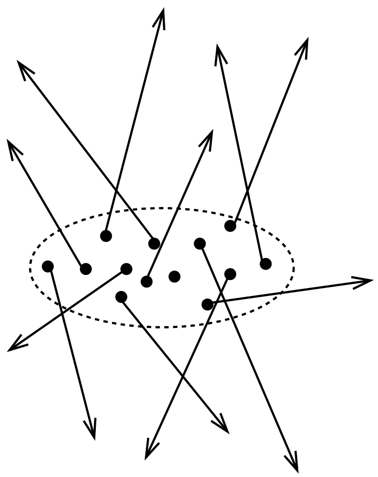
The technical details

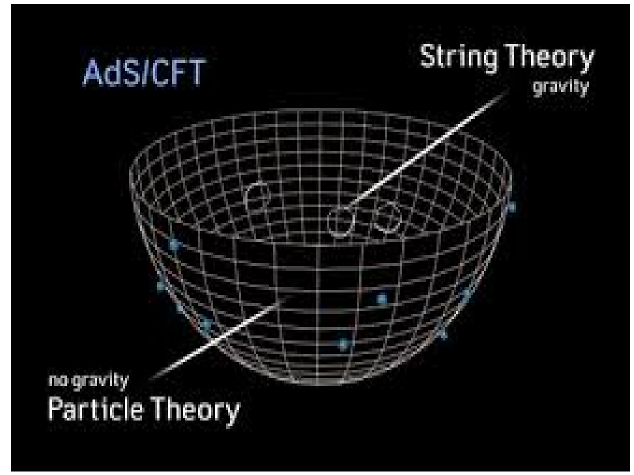




The conventional widsom

A "dust"
Particles ignore each other, their path is independent of initial shape





An interacting system is like a "gas of billiard balls" of microscopic particles, governed by the Boltzmann equation . When one has many particles, you get the perfect fluid limit. Microscopic correlations irrelevant. Or, if you are a string theorist , you use AdS/CFT and reduce things to a general relativity problem translated into large N_c conformal field theory . Not the same but similar in crucial respects.

What does all this mean?

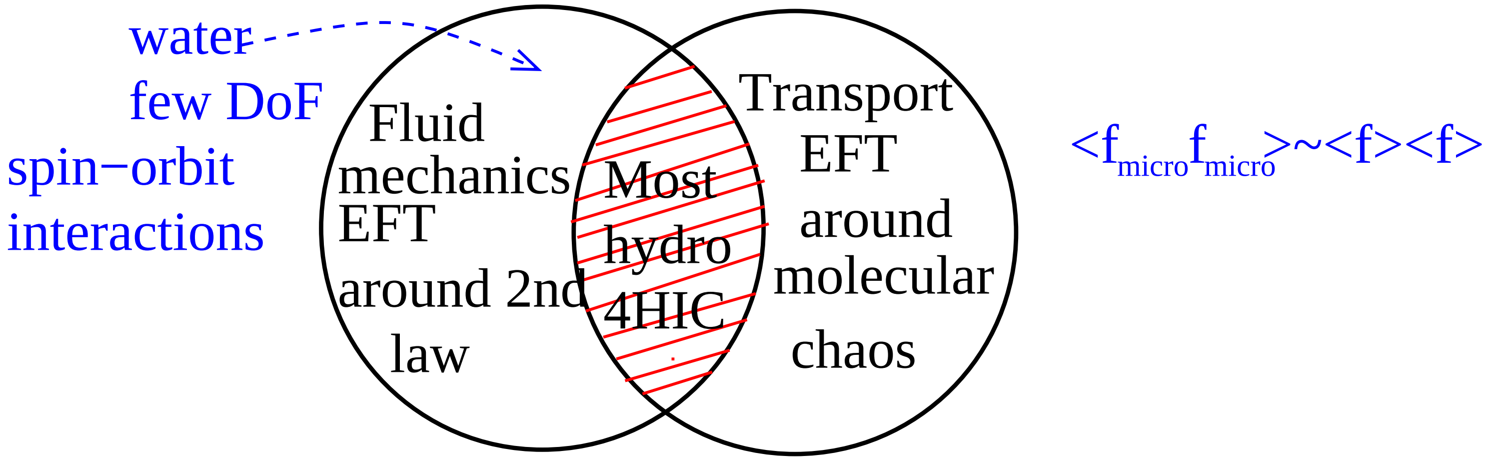
The last two points indicate that "hydrodynamic behavior" is intimately connected to the microscopic QCD effective theory. Separation between "microscopic" and "collective" scales problematic only 50 DoFs and they talk to each other!

In particular, molecular chaos (Boltzmann equation) and large N_c explicitly dubious as they define such scale separation!

Need a bridge between microscopic theory and thermal behavior, explain why "QCD looks thermal/hydro". no idea how to do this, so lets fake it!

More details: Hydro is not (just) transport!

Its constitutents are usually neither billiard balls not black holes!



Models based on microscopic distribution functions and 1-particle Wigner-functions most likely far away from ideal hydrodynamic limit because (unlike in non-polarized case) taking such a limit without spoiling convergence of the BBGKY hyerarchy impossible the fact that transport usually cannot handle stable circulation without a mean field should be a hint! Stochastic fluctuatons with finitely many particles destroy circulation

Hydro is not (just) transport: Hydrodynamics is based on three scales

$$\underbrace{l_{micro}}_{\sim s^{-1/3}, n^{-1/3}} \ll \underbrace{l_{mfp}}_{\sim \eta/(sT)} \ll L_{macro}$$

l_{micro} stochastic, l_{mfp} dissipative. If $l_{micro} \sim l_{mfp}$ soundwaves

Of amplitude so that momentum $P_{sound} \sim (area) \lambda \, (\delta \rho) \, c_s \gg T$

And wavenumber $k_{sound} \sim P_{sound}$

Survive (ie their amplitude does not decay to $E_{sound} \sim T$) $\tau_{sound} \gg 1/T$

fluctuating vortices, sound-waves, polarization mix. Creating an effective non-transport viscosity? (In abs/0708.0035 we confused l_{micro}, l_{mfp}).

$$\underbrace{l_{micro}}_{\sim s^{-1/3}, n^{-1/3}} \ll \underbrace{l_{mfp}}_{\sim \eta/(sT)} \ll L_{macro}$$

A lot of extensions of hydrodynamics should creep up at l_{micro}

FInite N_c/number of particles Thermal fluctuations and hydro perturbations mix

SPin/polarization Particles not billiard balls, have shape, absorb/emit angular momentum

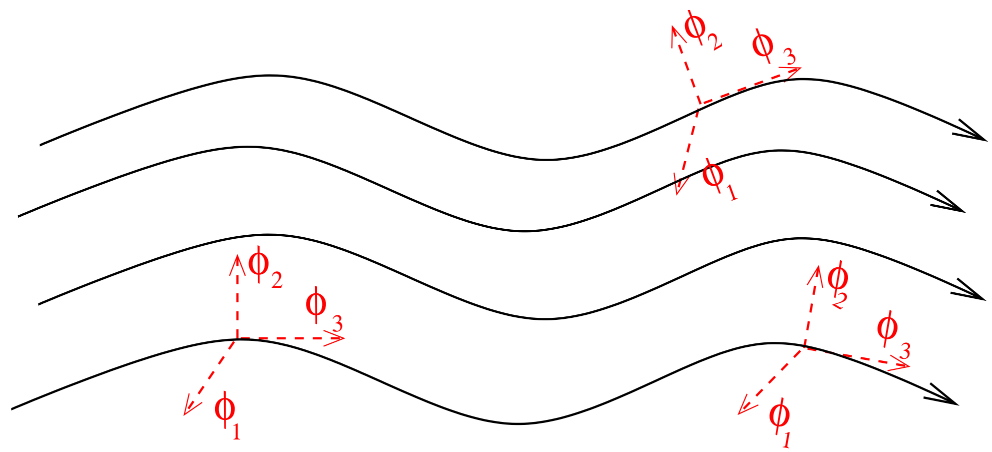
Gauge theories "spurious" correlations/redundancies across macroscopic scales

Impose local equilibrium at EFT level. Write the Lagrangian so its approximately the minimum of a free energy. "assume system thermalizes".

Havent we assumed the mystery we want to solve? yes but... there are microscopic structures which do not automatically combine with ideal hydrodynamics: fluctuations, spin and gauge theory . A good start is to see what an EFT combining these at the microscopic level and equilibrium looks like.

Hydro as an EFT (Nicolis et al,1011.6396 (JHEP))

Continuus mechanics (fluids, solids, jellies,...) is written in terms of 3-coordinates $\phi_I(x^\mu), I = 1...3$ of the position of a fluid cell originally at $\phi_I(t=0,x^i), I=1...3$. (Lagrangian hydro . NB: no conserved charges)



The system is a Fluid if it's Lagrangian obeys some symmetries (Ideal hydrodynamics $\leftrightarrow$ Isotropy in comoving frame) Solutions generally break these, Excitations (Sound waves, vortices etc) can be thought of as "Goldstone bosons".

Translation invariance at Lagrangian level $\leftrightarrow$ Lagrangian can only be a function of $B^{IJ} = \partial_\mu \phi^I \partial^\mu \phi^J$ Now we have a "continuus material"!

Homogeneity/Isotropy means the Lagrangian can only be a function of $B = \mathrm{det} B^{IJ}, \mathrm{diag} B^{IJ}$
The comoving fluid cell must not see a "preferred" direction $\Leftarrow SO(3)$ invariance

Invariance under Volume-preserving diffeomorphisms means the Lagrangian can only be a function of B (actually $b = \sqrt{B}$)
In all fluids a cell can be infinitesimally deformed
(with this, we have a fluid. If this last requirement is not met, Nicolis et all call this a "Jelly")

A few exercises for the bored public Check that L = -F(B) leads to

$$T_{\mu\nu} = (P+\rho)u_\mu u_\nu - P g_{\mu\nu}$$

provided that

$$\rho = F(B)\ , \qquad p = F(B) - 2F'(B)B\ , \qquad u^\mu = \frac{1}{6\sqrt{B}}\epsilon^{\mu\alpha\beta\gamma}\epsilon_{IJK}\,\partial_\alpha\phi^I\partial_\beta\phi^J\partial_\gamma\phi^K$$

(A useful formula is $\frac{db}{d\partial_\mu\phi_I}\partial_\nu\phi_I = u^\mu u^\nu - g^{\mu\nu}$)

Equation of state chosen by specifying $F(b)$. "Ideal": $\Leftrightarrow F(B) \propto b^{2/3}$

b is identified with the entropy and $b\frac{dF(B)}{dB}$ with the microscopic temperature.

u^μ fixed by $u^\mu\partial_\mu\phi^{\forall I} = 0$. Vortices become Noether currents of diffeomorphisms!

This is all really smart, but why?

Global conserved currents: chemical shift and u_μ

Within Lagrangian field theory a scalar chemical potential is added by adding a $U(1)$ symmetry to system.

$$\phi_I \to \phi_I e^{i\alpha} \quad , \quad L(\phi_I, \alpha) = L(\phi_I, \alpha + y) \quad , \quad J^\mu = \frac{dL}{d\partial_\mu \alpha}$$

generally flow of b and of J not in same direction. Can impose a well-defined u^μ by adding chemical shift symmetry

$$L(\phi_I, \alpha) = L(\phi_I, \alpha + y(\phi_I)) \to L = L\left(b, y = u_\mu \partial^\mu \alpha\right)$$

A comparison with the usual thermodynamics gives us

$$\mu = y \quad , \quad n = dF/dy$$

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Transport: Beyond Molecular chaos AdS/CFT: Beyond large N_c
It turns out Polarization, gauge symmetries mess this l_{micro} hyerarchy!

Ideal hydrodynamics and the microscopic scale

The most general Lagrangian is

$$L = T_0^4 F\left(\frac{B}{T_0^4}\right) \quad , \quad B = T_0^4 \det B^{IJ} \quad , \quad B^{IJ} = \left|\partial_\mu \phi^I \partial^\mu \phi^J\right|$$

Where $\phi^{I=1,2,3}$ is the comoving coordinate of a volume element of fluid.

NB: $T_0 \sim \Lambda g$ microscopic scale, includes thermal wavelength <u>and</u> $g \sim N_c^2$ (or μ/Λ for dense systems). $T_0 \to \infty \Rightarrow$ classical limit
It is therefore natural to identify T_0 with the microscopic scale!

Kn behaves as a gradient, T_0 as a Planck constant!!!

At $T_0 < \infty$ quantum and thermal fluctuations can produce sound waves and vortices, "weighted" by the usual path integral prescription!

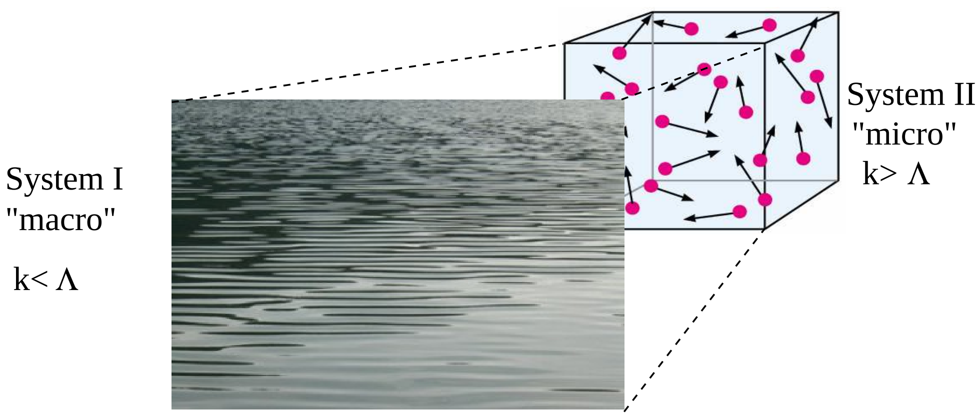
$$L \to \ln \mathcal{Z} \quad \mathcal{Z} = \int \mathcal{D}\phi_i \exp\left[-T_0^4 \int F(B) d^4x\right], \langle \mathcal{O} \rangle \sim \frac{\partial \ln \mathcal{Z}}{\partial \ldots}$$

$$\left(eg. \quad \left\langle T_{\mu\nu}^{x} T_{\mu\nu}^{x'} \right\rangle = \frac{\partial^2 \ln \mathcal{Z}}{\partial g_{\mu\nu}(x) \partial g_{\mu\nu}(x')} \right)$$

$T_0 \sim n^{-1/3}$, unlike Knudsen number, behaves as a "Planck constant".EFT expansion and lattice techniques should give all allowed terms and correlators. Coarse-graining will be handled here!

For analytical calculations fluid can be perturbed around a hydrostatic ($\phi_I = \vec{x}$) background

$$\phi_I = \vec{x} + \underbrace{(\vec{\pi}_L)}_{sound} + \underbrace{(\vec{\pi}_T)}_{vortex}$$



And we discover a fundamental problem: Vortices carry arbitray small energies but stay put! No S-matrix in hydrostatic solution!

$$L_{linear} = \underbrace{\dot{\vec{\pi}}_L^{\,2} - c_s^2(\nabla.\vec{\pi}_L)^2}_{sound\ \ wave} + \underbrace{\dot{\pi_T}^2}_{vortex} + Interactions(\mathcal{O}\left(\pi^3, \partial\pi^3, ...\right))$$

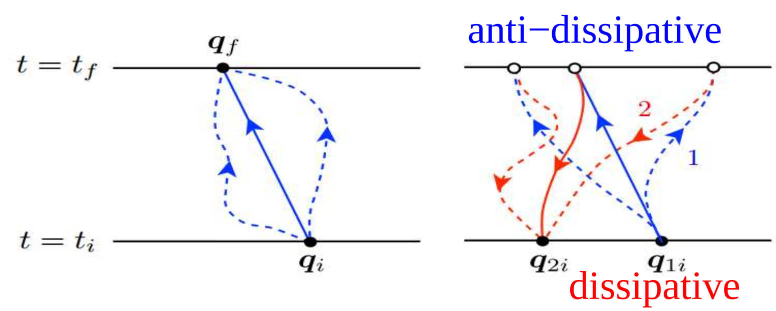
Unlike sound waves , Vortices can not give you a theory of free particles, since they do not propagate: They carry energy and momentum but stay in the same place! Can not expand such a quantum theory in terms of free particles.

Physically: "quantum vortices" can live for an arbitrary long time, and dominate any vacuum solution with their interactions. This does not mean the theory is ill-defined, just that its strongly non-perturbative!

Polarization might help here!

The big problem with Lagrangians... usually only non-dissipative terms
But there are a few ways to fix it. We focus on coordinate doubling (Galley,but before Morse+Feschbach)

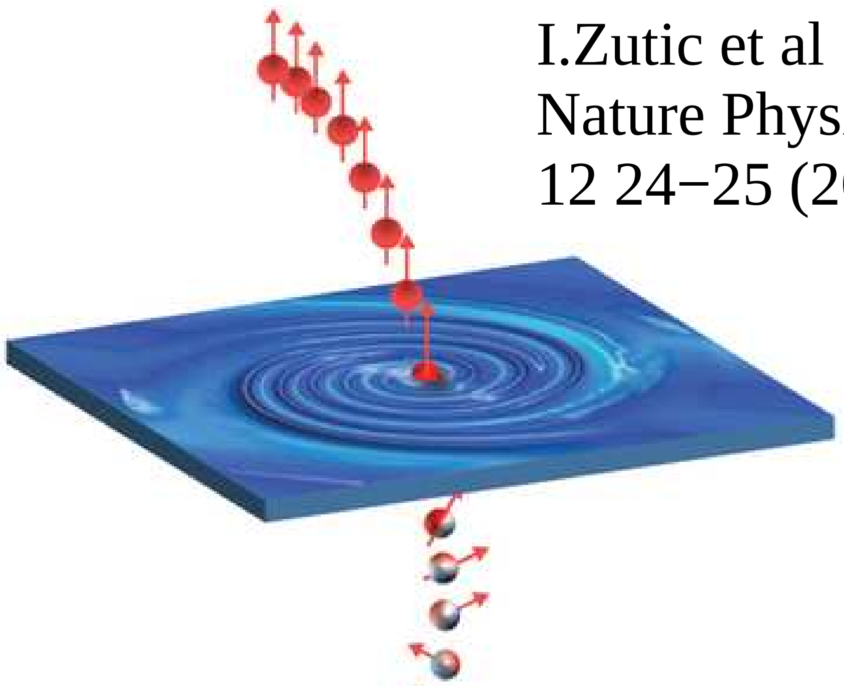
Dissipative
extension
of Hamiltons
principle



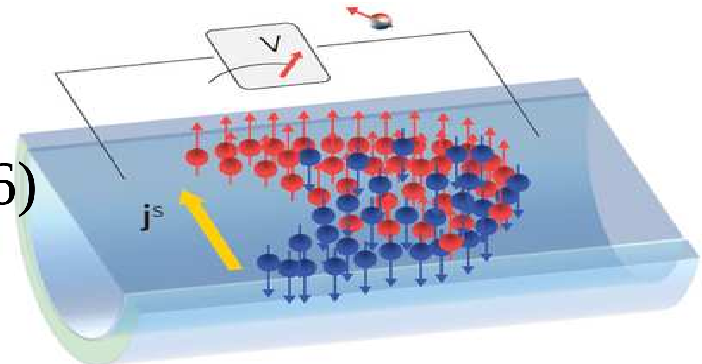
$$L = \frac{1}{2}\left(\underbrace{m\dot{x}^2 - wx^2}_{SHO}\right) \to \underbrace{\left(m\dot{x}_+^{\,2} - wx_+^2\right)}_{\mathcal{L}_1} - \underbrace{\left(m\dot{x}_-^{\,2} - wx_-^2\right)}_{\mathcal{L}_2} + \underbrace{\alpha\left(\dot{x}_+ x_- - \dot{x}_- x_+\right)}_{\mathcal{K}}$$

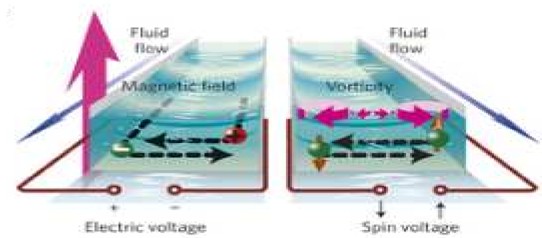
Standard techniques give you <u>two</u> sets of equations, one with a damped harmonic oscillator, the other “anti-damped”. Navier-Stokes and Israel-Steward (GT,D.Montenegro, PRD, in press)

Application 1: Hydro with polarization



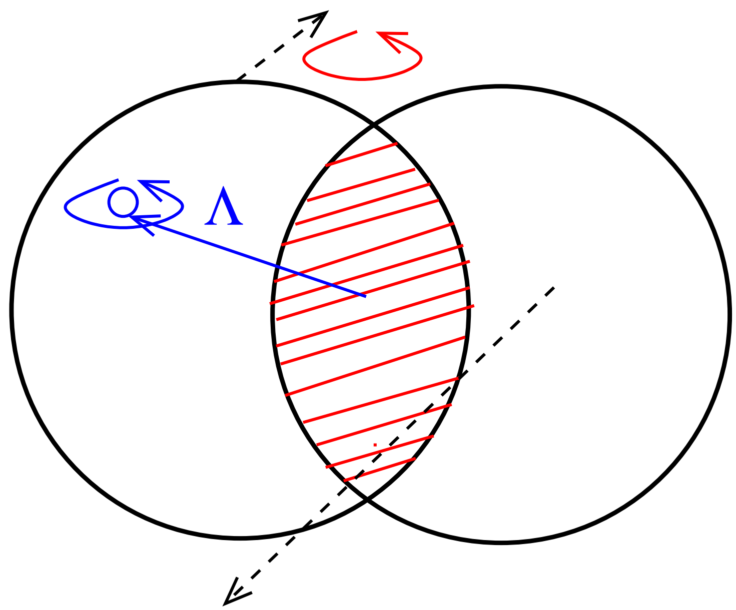
I.Zutic et al
Nature Physics
12 24−25 (2016)





Ultracold atoms: Zutic, Matos-Abiague, "Spin Hydrodynamics", Nature Physics **12** 24-25 Takahashi et al", Nature Physics **12** 52-56 (2016)

In the context of heavy ion physics...



Initial rapidity gradient and initial transparency could generate initial angular momentum. NB: Different from chiral magnetic/vortaic effects. Not anomalous and all DoFs in equilibrium (no B-field, "local" microscopic spin-orbit coupling, angular momentum follows local equilibrium)

What is ideal hydro? A conceptual difficulty!

Entropy conserved always at maximum at each point in spacetime

Local isotropy in the comoving frame

Vorticity is conserved (Kelvins theorem)

Continuum limit when you break up cells, intensive results stay the same

With polarization, only the first has a chance of being realized even in the ideal limit

Combining polarization with the ideal hydrodynamic limit, defined as

(i) The dynamics within each cell is faster than macroscopic dynamics, and it is expressible only in term of local variables and with no explicit reference to four-velocity u^μ (gradients of flow are however permissible, in fact required to describe local vorticity).

(ii) Dynamics is dictated by local entropy maximization, within each cell, subject to constraints of that cell alone. Macroscopic quantities are assumed to be in local equilibrium inside each macroscopic cell

(iii) Only excitations around a hydrostatic medium are sound waves,vortices

(i-iii) ,with symmetries and EFT define the theory

So how do we implement polarization?

In comoving frame, polarization described by a representation of a "little group" of the volume element.

Need local $\sim SO(3)$ charges and unambiguus definition of u^{μ} $(s^{\mu} \propto J^{\mu})$

$$\Psi_{\mu\nu}|_{comoving} = -\Psi_{\nu\mu}|_{comoving} = \exp\left[-\sum_{i=1,2,3} \alpha_i(\phi_I) \hat{T_i}^{\mu\nu} \right]$$

For particle spinor, vector, tensor... repreentations possible.

For "many incoherent particles" RPA means only vector representation remains

Chemical shift symmetry, $SO(3)_{\alpha_{1,2,3}} \rightarrow SO(3)_{\alpha_{1,2,3}(\phi^I)}$

$$\alpha_i \rightarrow \alpha_i + \Delta\alpha_i\left(\phi_I\right) \Rightarrow L(b, y_{\alpha\beta} = u_\mu \partial^\mu \Psi_{\alpha\beta})$$

$y_{\mu\nu} \equiv \mu_i$ for polarization vector components in comoving frame

This way we ensured spin current flows with u^μ.

Note that it is not a proper chemical potential (it it would be there would be 3 phases attached to each ϕ_I) as $y_{\mu\nu}$ not invariant under symmetries of ϕ_I. $y_{\mu\nu}$ "auxiliary" polarization field

How to combine polarization with local equilibrium?

Since polarization decreases the entropy by an amount proportinal to the DoFs and independent of polarization direction

$$b \to b\left(1 - c y_{\mu\nu} y^{\mu\nu} + \mathcal{O}\left(y^4\right)\right) \quad , \quad F(b) \to F(b,y) = F\left(b\left((1 - c y^2\right)\right)$$

.

Other terms break requirement (i)

First law of thermodynamics,

$$dE = TdS - pdV - Jd\Omega \to dF(b) = db\frac{dF}{db} + dy\frac{dF}{d(yb)}$$

Energy-momentum tensor

Not uniquely defined

Canonical defined as the Noether charge for translations, could be negative because of $\sim \frac{\partial L}{\partial(\partial\psi_i)}\partial\psi_j$

Belinfante-Rosenfeld $\sim \frac{\delta S}{\delta g_{\mu\nu}}$ symmetric independent of spin, no non-relativistic limit

Which is the source for $\partial_\mu T^{\mu\nu} = 0$? Not clear as...

The problem: Too many degrees of freedom

8 degrees of freedom,5 equations $(e, p, u_{x,y,z}, y^{\mu\nu})$. One can include the antisymmetric part of $T_{\mu\nu}$ and match equations but...

No entropy maximization If spin waves and sound waves separated, in comoving volume their ratio is arbitrary... but it should be decided by entropy maximization!

I suspect EFTs based on $T_{\mu\nu}$ (Hong Liu,Florkowski and collaborators) will have this problem

Solution clear: make polarization always proportional to vorticity,

$$y^{\mu\nu} \sim \chi(T)(e+p)\left(\partial^{\mu}u^{\nu} - \partial^{\nu}u^{\mu}\right)$$

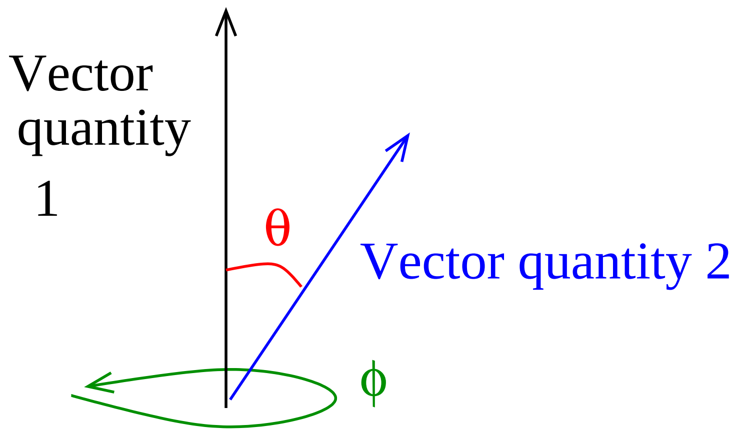
extension of Gibbs-Duhem to angular momentum uniquely fixes χ via entropy maximization. For a free energy $\mathcal{F}$ to be minimized

$$d\mathcal{F} = \frac{\partial \mathcal{F}}{\partial V} dV + \frac{\partial \mathcal{F}}{\partial e} de + \frac{\partial \mathcal{F}}{\partial \left[\Omega_{\mu\nu}\right]} d\left[\Omega_{\mu\nu}\right] = 0$$

where $[\Omega_{\mu\nu}]$ is the vorticity in the comoving frame.
THis fixes χ . It also constrains the Lagrangian to be a Legendre transform of the free energy just as in the chemical potential case, in a straightforward generalization of Nicolis,Dubovsky et al. Free energy always at (local) minimum! (requirement (ii))

A qualitative explanation

Instant thermalization means vorticity instantly adjusts to angular momentum, and is parallel to angular momentum. Corrections to this will be of the relaxation type a-la Israel-Stewart



Note that microscopic physics could allow an arbitrary angle between vorticity and polarization. but such systems would have no hydrodynamic limit due to requirement (iii) and the necessity for stability of relaxation dynamics

These techniques lead to a well-defined Euler-Lagrange equation of motion

$$\left\{ g_b(1-cy)\partial_\nu b + g_y 4y^\beta_\alpha \chi(T)\frac{\partial(\partial_\beta u^\alpha)}{\partial(\partial_\lambda \phi^I)}\partial^\nu(\partial_\lambda\phi^I)\right\}\times$$

$$\times[(1-cy)\frac{\partial b}{\partial(\partial_\nu\phi^I)} - (8cb)y^\beta_\alpha\chi(T)\frac{\partial(\partial_\beta u^\alpha)}{\partial(\partial_\nu\phi^I)}] + g(b,y)\times$$

$$\times(1-cy)\partial_\nu(\frac{\partial b}{\partial(\partial_\nu\phi)}) - 8c\chi(T)g(b,y)\frac{\partial(\partial_\beta u^\alpha)}{\partial(\partial_\lambda\phi^I)}\left[\frac{y^\beta_\alpha}{2}\partial_\nu\partial_\lambda\phi^I\times\right.$$

$$\times\frac{\partial b}{\partial(\partial_\nu\phi^I)} + (\partial^\nu b)4y^\beta_\alpha\delta^\lambda_\nu + b\chi(T)(\frac{\partial(\partial^\beta u_\alpha)}{\partial(\partial_\nu\phi^I)} + \frac{\partial(\partial_\alpha u^\beta)}{\partial(\partial_\nu\phi^I)})\times$$

$$\left.\times\partial_\nu(\partial_\lambda\phi^I) + by^\beta_\alpha\partial_\nu ln\frac{\partial(\partial_\beta u^\alpha)}{\partial(\partial_\nu\phi^I)}\right] = 0$$

NB depends on accelleration, so $\Delta S = 0 \Rightarrow \partial_\mu\partial_\nu\frac{\partial F}{\partial(\partial_\mu\partial_\nu\phi^I)} = \partial_\mu\frac{\partial F}{\partial(\partial_\mu\phi^I)}$

Which can be linearized, $\phi_I = X_I + \pi_I$

The "free" (sound wave and vortex kinetic terms) part of the equation will be

$$\mathcal{L} = (-F'(1))\left\{\frac{1}{2}(\dot{\pi})^2 - c_s^2[\partial\pi]^2\right\}+$$

$$+f\zeta\left\{\ddot{\pi}^i\partial_i\dot{\pi}_j + \ddot{\pi}_i\ddot{\pi}_j + \partial_j\dot{\pi}^i\partial_i\dot{\pi}_j + \partial_j\dot{\pi}_i\ddot{\pi}_j+\right.$$

$$\left.+(2\ddot{\pi}^i\partial_j\dot{\pi}_i - 2\ddot{\pi}_j\partial^i\dot{\pi}_j) + (\ddot{\pi}_i^2 - \ddot{\pi}_j^2) + (\partial_j\dot{\pi}_i^2 - \partial_i\dot{\pi}_j^2)\right\}$$

- Accelleration terms survive linearization
- Vortices and sound wave modes mix at "leading" order. Change in temperature due to sound wave changes polarizability, and that changes vorticity

We decompose perturbation into sound and vortex $\phi_I = \nabla\phi + \nabla \times \vec{\Omega}$

$$\begin{pmatrix} \varphi \\ \vec{\Omega} \end{pmatrix} = \int dw d^3k \begin{pmatrix} \varphi_0 \\ \vec{\Omega}_0 \end{pmatrix} \exp\left[i\left(\vec{k_{\phi,\Omega}}.\vec{x} - w_{\phi,\Omega}t\right)\right]$$

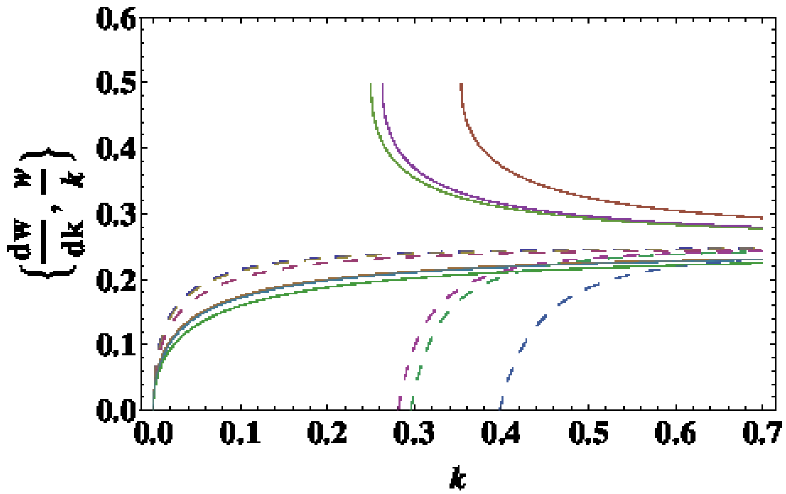
The part parallel to k ("sound-wave") will have a dispersion relation

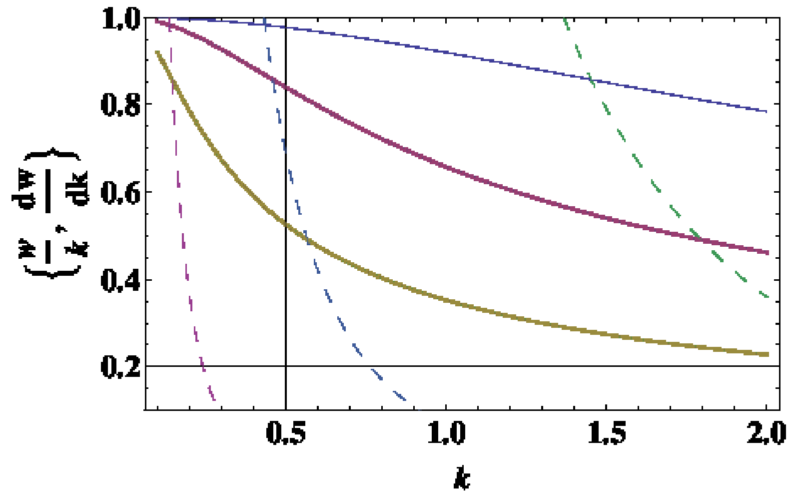
$$w_\phi^2 - c_s^2 k_\phi^2 + 2\beta k_\phi w_\phi^3 = 0$$

The vector part will be

$$(3k_\Omega^2 - 2k_\Omega w_\Omega)_j (\vec{k}_\Omega \times \vec{\Omega}_0)_i w_\Omega^2 + w^4\Omega = 0$$

Dispersion relations show violation of causality!





Both phase and group velocity will generally go above unity

What I think is going on I: A lower limit of viscosity for polarized hydro

the Free energy $\mathcal{F}$, and hence the local dynamics, is sensitive to an accelleration. As is well-known (Ostrogradski's theorem, Dirac runaway solutions) such Lagrangians are unstable and lead to causality violation. Note that one needs Lagrangians to see this!

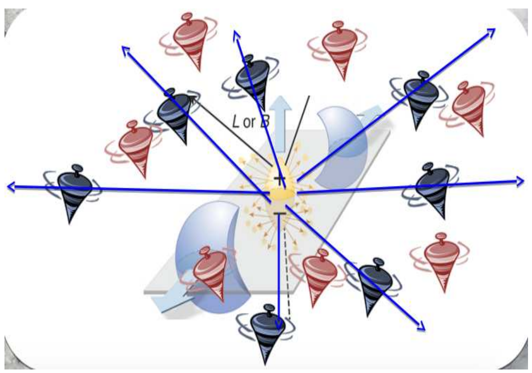
To fix this issue, one would need to update the proportionality of y on Ω to an Israel-Stewart type equation

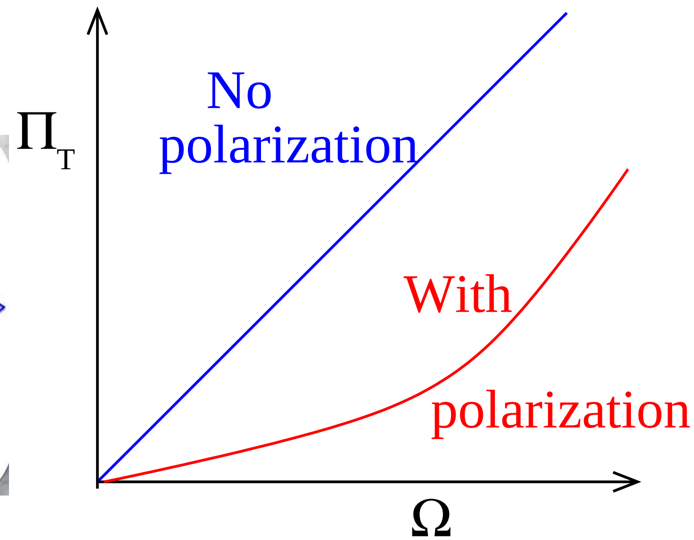
$$\tau_\Omega u_\alpha \partial^\alpha y_{\mu\nu} + y_{\mu\nu} = \chi(T, y)\Omega_{\mu\nu}$$

with an appropriate relaxation time τ_Ω would resolve this issue. Just like with Israel-Stewart, this requires the introduction of new DoFs with relaxation-type dynamics, but, unlike non-polarized hydro,such terms are required from the idea limit

What I <u>think</u> is going on II

Fluctuation-dissipation: $\tau_\Omega \sim \lim_{\omega\to 0} \omega^{-1} \int dt \, \langle y_{\mu\nu} \Omega_{\mu\nu} \rangle \exp(i\omega t)$





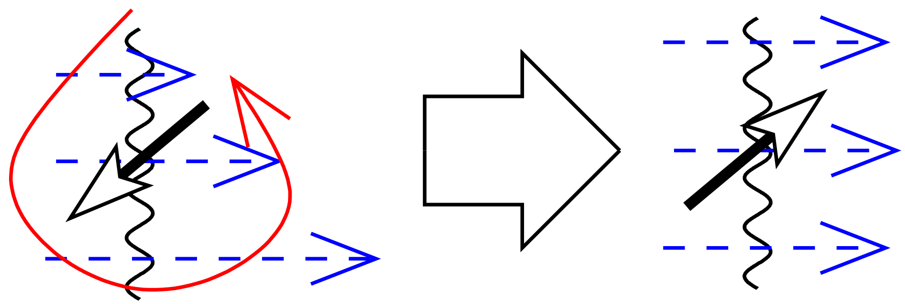
Polarization makes vorticity aquire a "soft gap" wrt angular momentum. At small amplitudes, creating polarization is more advantageus than creating vorticity. This means small amplitude vortices get quenched. Stabilizes theory against perturbations, might act as effective viscosity!

G Torrieri,D Montenegro, 1807.02796 : Polarization are independent DoFs which relax to vorticity

Anti-Ferromagnetic fluid non-causal mode ($|dw/dk| \geq 1$) in UV unless

$$\tau_Y^2 \geq \frac{8c\chi^2(b_o,0)}{(1-c_s^2)b_o F'(b_o)} \quad , \quad \frac{\eta}{s} \geq T\tau_Y$$

τ_Y regulates quenching of vortices into polarization!



A bottom-up limit on viscosity from polarization!

Ferromagnetic fluid Causal mode in IR remains
But remember that ferromagnetic vaccum is unstable and

$$\langle y_{\mu\nu} \rangle \equiv \lim_{k\to 0} \rho(\omega_{T,L}(k))$$

Banks-Casher mode for unstable vacua!

Thus this theory causal, incorporates electromagnetism and predicts bottom-up limit for viscosity with expected dependence (as $N_c \to \infty, T \neq 0$ limit goes to zero).

Are there any problems?

Gauge theory and local thermalization

The formalism we introduced earlier is ok for quark polarization but problematic for gluon polarization: Gauge symmetry means one can exchange locally angular momentum states for transversely polarized spin states. So vorticity vs polarization is ambiguus

Using the energy-momentum tensor for dynamics is even more problematic for spin $T_{\mu\nu}$ aquires a "pseudo-gauge" transformation

$$T_{\mu\nu} \to T_{\mu\nu} + \frac{1}{2}\partial_\lambda \left(\Phi^{\lambda,\mu\nu} + \Phi^{\mu,\nu\lambda} + \Phi^{\nu,\mu\lambda}\right)$$

where Φ is fully antisymmetric. $\delta S/\delta g_{\mu\nu}$ and canonical tensors are lmits of choice of Φ . But in a gauge theory, pseudo-Gauge transformations are gauge transformations! Affects EFTs based on $T_{\mu\nu}$ (Hong Liu,Florkowski and collaborators)

From global to gauge conserved currents

A reminder: Within Lagrangian field theory a scalar chemical potential is added by adding a $U(1)$ symmetry to system.

$$\phi_I \to \phi_I e^{i\alpha} \quad , \quad L(\phi_I, \alpha) = L(\phi_I, \alpha + y) \quad , \quad J^\mu = \frac{dL}{d\partial_\mu \alpha}$$

generally flow of b and of J not in same direction. Can impose a well-defined u^μ by adding chemical shift symmetry

$$L(\phi_I, \alpha) = L(\phi_I, \alpha + y(\phi_I)) \to L = L\left(b, y = u_\mu \partial^\mu \alpha\right)$$

A comparison with the usual thermodynamics gives us

$$\mu = y \quad , \quad n = dF/dy$$

Generalization from $U(1)$ to generic group easy

$$\alpha \to \{\alpha_i\} \quad , \quad \exp(i\alpha) \to \exp\left(i \sum_i \alpha_i \hat{T}_i \right)$$

One subtlety: Currents stay parallel to u_μ but chemical potentials become adjoint, since rotations in current space still conserved

$$y = J^\mu \partial_\mu \alpha_i \to y_{ab} = J_a^\mu \partial_\mu \alpha_b$$

Lagrangian still a function of $dF(b, \{\mu\})/dy_{ab}$, "flavor chemical potentials"

From global to gauge invariance! Lagrangian invariant under

$$\{y_{ab}\} \to y'_{ab} = U^{-1}_{ac}(x) y_{cd} U_{db}(x) \quad , \quad U_{ab}(x) = \exp\left(i \sum_i \alpha_i(x) \hat{T}_i\right)$$

However, gradients of x obviously change y .

$$y_{ab} \to U^{-1}_{ac}(x) y_{cd} U_{bd}(x) = U^{-1}(x)_{ac} J^\mu_f U_{cf} U^{-1}_{fg} \partial_\mu \alpha_g U_{bg} =$$

$$= U^{-1}(x)_{ac} J^\mu_f U_{cf} \partial_\mu \left(U^{-1}_{fg} \alpha_d U_{bd}(x)\right) - J^\mu_a \left(U \partial_\mu U\right)_{fb} \alpha_f$$

Only way to make lagrangian gauge invariant is

$$F\left(b, J^\mu_j \partial_\mu \alpha_i\right) \to F\left(b, J^\mu_j \left(\partial_\mu - U(x) \partial_\mu U(x)\right) \alpha_i\right)$$

Which is totally unexpected, profound and crazy

The swimming ghost!

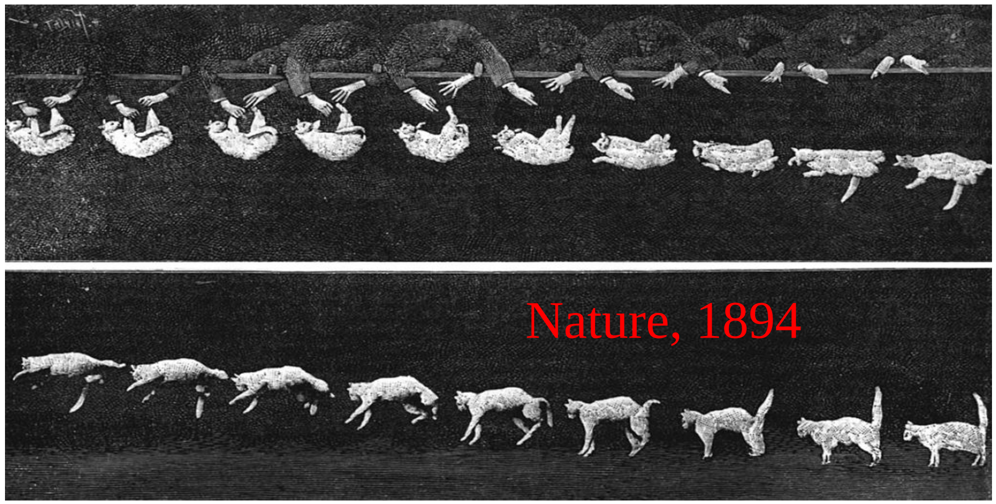
$$F\left(b, J_j^\mu \partial_\mu \alpha_i\right) \to F\left(b, J_j^\mu \left(\partial_\mu - U(x)\partial_\mu U(x)\right) \alpha_i\right)$$

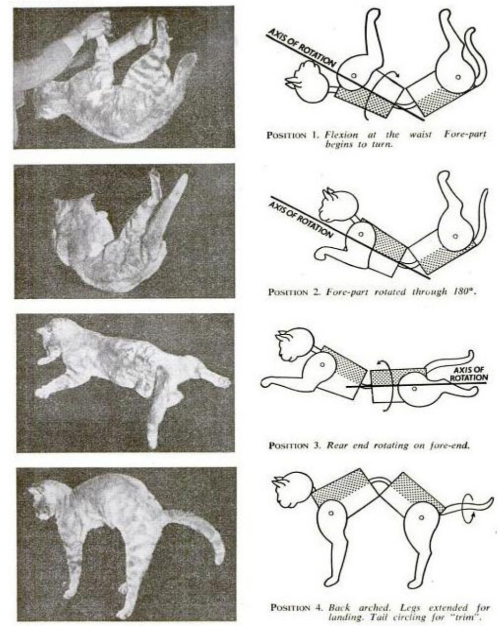
Means the ideal fluid lagrangian depends on velocity!. no real ideal fluid limit possibl the system "knows it is flowing" at local equilibrium! NB: For U(1)

$$\hat{T}_i \to 1 \quad , \quad y_{ab} \to \mu_Q \quad , \quad u_\mu \partial^\mu \alpha_i \to A_\tau$$

So second term can be gauged to a redefinition of the chemical potential (the electrodynamic potentials effect on the chemical potential).

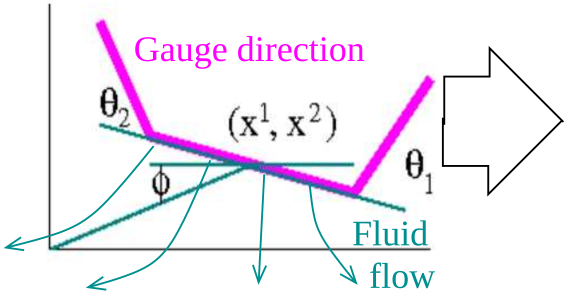
Cannot do it for Non-Abelian gauge theory, "twisting direction" in color space It turns out this has an old analogue...





S. Montgomery (2003): How does a cat always fall on its feet without anything to push themsevles against? The shape of spaces a cat can deform themselves into defines a “set of gauges” a cat can choose without change of angular momentum.

Purcell,Shapere+Wilczek,Avron+Raz : A similar process enables swimmers to move through viscous liquids with no applied force



Now imagine each fluid cell filled with a "swimmer", with arms and legs outstretched in "gauge" directions...

In ideal limit all currents proportional to u_μ . But gauge symmetry requires "ghost" excitations, proportional to gradients of currents, to not be physical. So free energy HAS to depend on flow.

Classic on this, B. Bistrovic, R. Jackiw, H. Li, V. P. Nair and S. Y. Pi, Phys. Rev. D **67**, 025013 (2003) , "NonAbelian fluid dynamics in Lagrangian formulation," missed this subtlety as no local equilibrium defined!

Whats going on? A more statistical mechanics perspective

We perturb the hydrostatic limit, where $\phi_I = X_I$, and isolate a transverse mode (vortex) and a longitudinal mode (sound wave)

$$\phi_I = X_I + \vec{\pi}_I^{sound} + \vec{\pi}_I^{vortex} \quad , \quad \nabla . \vec{\pi}_I^{vortex} = \nabla \times \vec{\pi}_I^{sound} = 0$$

Since the derivative of the free energy w.r.t. b is positive, sound waves and vortices do "work". Let us now assume the system has a "color chemical potential" in some direction Let us change the color chemical potential in space according to

$$\Delta\mu(x) = \sum_i \left(\mu_i(x)^{swim} + \mu_i(x)^{swirl} \right) \hat{T}_i \quad , \quad \nabla_i . \mu_i^{swim} = \nabla_i \times \mu_i^{swirl} = 0$$

Because of gauge redundancy, the derivatives of the free energy with respect to color ("color susceptibility") will typically be negative. So the two can balance!!!!

But this breaks the "hyerarchy" of statistical mechanics

It mixes micro and macro perturbations!

In statistical mechanics, what normally distinguishes "work" from "heat" is coarse-graining, the separation between micro and macro states. Quantitatively, probability of thermal fluctuations is normalized by $1/(c_V T)$ and microscopic correlations due to viscosity are $\sim \eta/(Ts)$. Since for a usual fluid, there is a hyerarchy between microscopic scale, Knudsen number and gradient

$$\frac{1}{c_V T} \ll \frac{\eta}{(Ts)} \ll \partial u_\mu$$

Gauge symmetry breaks it, since it equalizes perturbations at both ends of this!

Is there a Gauge-independent way of seeing this? Perhaps!

One can write the effective Lagrangian in a Gauge-invariant way using Wilson-Loops . But the effective Lagrangian written this way will have an infinite number of terms, in a series weighted by the characteristic Wilson loop size. For a locally equilibrated system, this series does not commute with the gradient. Just like with Polymers, the system should have multiple anisotropic non-local minima which mess up any Knuden number expansion. Some materials are inhomogeneus and anisotropic at equilibrium, YM could be like this!

Lattice would not see it , as there are no gradients there. There is an entropy maximum, and it is the one the lattice sees. The problems arise if you "coarse-grain" this maximum into each microscopic cell and try to do a gradient expansion around this equilibrium, unless you have color neutrality.

Gauge theory and polarization

Since $u^\mu \partial_\mu$ is in the Lagrangian,let us compare vorticity and Wilson loops!

$$\mathrm{Vorticity}: \oint J_\mu dx^\mu \neq 0 \quad , \quad \mathrm{Wilsonloop}: \oint dx_\mu \partial^\mu U_{ab} \equiv \int_\Sigma d\Sigma_{\mu\nu} F^{\mu\nu}_{ab}$$

Lagrangian will in general have gauge-invariant terms proportional to $Tr_a \omega_{\mu\nu a} F^{\mu\nu}_a$

$$F\left(b, J^\mu_j \left(\partial_\mu - U(x)\partial_\mu U(x)\right), Tr_a \omega_{\mu\nu a} F^{\mu\nu}_a\right) \quad , \quad \omega_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta}\partial^\alpha J^\beta_a$$

where as usual

$$F^a_{\mu\nu} = \partial_\mu A^a - \partial_\nu A^a + f_{abc} A^b_\mu A^c_\nu$$

$$F\left(b, J_j^{\mu}\left(\partial_\mu - U(x)\partial_\mu U(x)\right), Tr_a \omega_{\mu\nu a} F_a^{\mu\nu}\right) \quad , \quad \omega_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta}\partial^\alpha J_a^\beta$$

Unlike in Jackiw et al , $F_{\mu\nu}$ is not field strength but just a polarization tensor, whose value is set by entropy maximization.

$\chi(y,b)$ and τ_Y defineable as in hydrodynamics with polarization, but $\chi(y,b) = dF(...)/d\omega_{\mu\nu a}$ must be gauge invariant, and also depend on u_μ .

In analogy with hydrodynamics with polarization, most likely $F_i^{\mu\nu}$ relaxes to $\omega_i^{\mu\nu}$, with some time-scale τ from causality, with a "non-abelian" $f_{ijk}\epsilon\omega\omega$ correction.

But polarization and vorticity relaxation only up to gauge-dependence

$$\tau u^\mu \partial_\mu Tr_i \left[G^i_{\mu\nu}\right]^2 + Tr_i \left[G^i_{\mu\nu}\right]^2 = \chi Tr_i \left[\omega^i_{\mu\nu}\right]^2 + \mathcal{O}\left(f_{ijk}\omega_j\omega_k\right)$$

There should be “swirling modes” rotating in color space where vorticity and relaxation never relaxes.

Development of EoMs, linearization, etc. of this theory in progress!

A crazy guess, speculation Remember that all flow dependence through μ_{ab} color chemical potentials. What if local equilibrium happens when they go to zero, i.e. color density is neutral.

Could colored-swimming ghosts quickly be produced, and then locally thermalize and color-neutralize the QGP?

What about gauge-gravity duality?

Large N non-hydrodynamic modes go away in the planar limit
There are N ghost modes and N^2 degrees of freedom

Conformal fixed point most likely means ghosts non-dynamical
Not yet sure of this, but conformal invariance reduces pseudo-Gauge transformations to

$$\Phi_{\lambda,\mu\nu} \underbrace{\rightarrow}_{conformal} g_{\sigma\mu}\partial_\nu\phi - g_{\sigma\mu}\partial_\mu\phi$$

where ϕ is a scalar function. Irrelevant for dynamics.
As shown in Capri et al (1404.7163) Gribov copies for a Yang-Mills theory non-dynamical there. It would be a huge job to do this for hydrodynamics.

Conclusions

Hydrodynamics is not a limit of transport, AdS/CFT or any other microscopic theory

Hydrodynamics is an EFT built around symmetries and entropy maximization and should be treated as such

Once you realize this , generalizing it to theories with extra DoFs, symmetries etc. becomes straight-forward.

Lots of things to do Gauge symmetry looks particularly interesting!

Can we do better? Put theory on the lattice, work with Thiago Nunes