Transport Coefficients from PLσM

BLTP Seminar

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Outline

- Sigma model and symmetries
- SU(3) LσM with Polyakov-Loop Potential
- Electrical and Heat Conductivity
- Bulk and Shear Viscosity

Sigma Models

Sigma-Model is a Physical system with the Lagrangian

$$\mathcal{L}(\phi_1, \phi_2, \ldots, \phi_n) = \sum_{i=1}^{n} \sum_{j=1}^{n} g_{ij}\, \mathrm{d}\phi_i \wedge * \mathrm{d}\phi_j$$

The fields φ_i represent **map** from a **base manifold** spacetime (worldsheet) to a **target** (Riemannian) **manifold** of the scalars linked together by internal symmetries.

The scalars **g**ij determines linear and non-linear properties.

It was introduced by **Gell-Mann** and **Levy** in **1960**. The name **σ-model** comes from a field corresponding to the spinless meson ***σ***, scalar introduced earlier by **Schwinger**.

Symmetries

- **LσM** is an effective theory for QCD dof at low-energy and incorporates global $SU(Nf)_r \times SU(Nf)_\ell \times$ **$U(1)_A$** symmetry (not local $SU(3)_c$)
- For Nf=2 massless quarks, the phase transition can be of
    - 2nd-order, if **$U(1)_A$** symmetry is explicitly broken by instantons
    - 1st-order (fluctuations), if **$U(1)_A$** symmetry is restored at Tc
- For Nf = 3 massless quarks, the transition is always of 1st-order
- In last case, the term which breaks **$U(1)_A$** symmetry explicitly drives 1st-order phase-transition
- In absence of explicit **$U(1)_A$** symmetry breaking, the transition is fluctuation-induced of 1st-order

Pisarski and Wilczek, PRD29, 338 (1984).

Importance of LσM

- LσM is one of lattice QCD alternatives
- Various symmetry-breaking scenarios can be investigated in a more easy way
- Various properties of strongly interacting matter can be studied
- But, finite-T LσM requires many-body resummation schemes, because the IR divergences cause perturbation theory to break down

LSM Symmetries

- Again, for Nf massless quarks, QCD Lagrangian has $SU(Nf)_r \times SU(Nf)_\ell \times \mathbf{U(1)}_A$ symmetry
- In vacuum, a non-vanishing expectation value of the quark-antiquark condensate, spontaneously breaks this symmetry to diagonal $SU(Nf)_V$ group of vector transformations, $V = r + \ell$

- **For Nf=3, effective low-energy dof of QCD are scalar and pseudoscalar mesons. Since mesons are quark-antiquark states, they fall in singlet and octet representations of $SU(3)_V$.**

- The $SU(Nf)_r \times SU(Nf)_\ell \times \mathbf{U(1)}_A$ symmetry of QCD Lagrangian is explicitly broken by nonvanishing quark masses
- For M≤Nf degenerate quarks, $SU(M)_V$ symmetry is preserved
- If M>Nf , mass eigenstates are mixtures of singlet and octet states

Jonathan T. Lenaghan,, Dirk H. Rischke, Jurgen Schaffner-Bielich, **Phys.Rev. D62 (2000) 085008**

Symmetry and Conservations

Symmetries imply conservation laws:

invariance of Lagrangian under translations in space and time → momentum and energy conservation

QCD Lagrangian for massless quarks shows symmetry under vector and axial transformation.

equally (vector)
left- and right-handed parts treated
differently (Axial)

For example: symmetry of vector transformations leads to Isospin conservation

Transformation

Chiral symmetry of vector field under unitary transformation $\vec{\Phi} \Longrightarrow e^{-i\ \theta^a\ T^a_{ij}}\ \vec{\Phi}$

θ^a corresponding the rotational angle, T^a_{ij} matrix generates the transformation and a index indicating several generators associated with the symmetry transformation.

Vector transformation Λ_V

Axil transformation Λ_A

$$\Psi \Longrightarrow e^{-i\ \frac{\vec{\tau}}{2}\vec{\theta}}\ \Psi \approx (1 - i\frac{\vec{\tau}}{2}\vec{\theta})\ \Psi$$

$$\bar{\Psi} \Longrightarrow e^{+i\ \frac{\vec{\tau}}{2}\vec{\theta}}\ \bar{\Psi} \approx (1 + i\frac{\vec{\tau}}{2}\vec{\theta})\ \bar{\Psi}$$

conjugate

$$\Psi \Longrightarrow e^{-i\gamma_5\ \frac{\vec{\tau}}{2}\vec{\theta}}\ \Psi \approx (1 - i\gamma_5\frac{\vec{\tau}}{2}\vec{\theta})\ \Psi$$

$$\bar{\Psi} \Longrightarrow e^{-i\ \gamma_5\frac{\vec{\tau}}{2}\vec{\theta}}\ \bar{\Psi} \approx (1 - i\gamma_5\frac{\vec{\tau}}{2}\vec{\theta})\ \bar{\Psi}$$

Fermions **Dirac Lagrangian which describes free Fermion particle of mass m**

$$\mathcal{L}_D = \bar{\psi}(i\gamma_\mu\partial^\mu - m^2)\psi$$

Under vector transformation Λ_V L_D is invariant.
BUT axial-voctor transformation Λ_A reads

$$\Lambda_A : \quad m\ \bar{\psi}\ \psi \Longrightarrow e^{-i\ \gamma^5\frac{\vec{\tau}}{2}\vec{\theta}}\ m\ \bar{\psi}\ \psi \approx (1 - i\gamma^5\frac{\vec{\tau}}{2}\vec{\theta})\ m\ \bar{\psi}\ \psi,$$
$$= m\ \bar{\psi}\ \psi - 2im\bar{\theta}(\bar{\psi}\gamma_5\frac{\vec{\tau}}{2}\psi)$$

ϕ are component fields such as π's

Sigma fields

Combination of quarks (q# of mesons), a meson-like state

(scalar Meson)
Sigma like state $J^p = 0^+$

(pseudoscalar Meson)
Pion like state $J^p = 0^-$

$$\sigma = \bar{\psi}\psi$$

$$\pi = i\bar{\psi}\bar{\tau}\gamma_5\psi$$

Gell-Mann & Levy **obtained an invariant form if squares of the two states are summed**

(Vector Meson)
Sigma like state $J^p = 0^+$

(Axial-Vector Meson)
Pion like state $J^p = 0^-$

$$\Lambda_V: \quad \pi^2 \rightarrow \pi^2$$
$$\sigma^2 \rightarrow \sigma^2$$

$$\Lambda_A: \quad \pi^2 \rightarrow \pi^2 + 2\sigma\theta\pi$$
$$\sigma^2 \rightarrow \sigma^2 - 2\sigma\theta\pi$$

$$(\pi^2 + \sigma^2) \xleftrightarrow{\Lambda_V, \Lambda_A} (\pi^2 + \sigma^2)$$

Vector transformation

$$\pi_i: \qquad i\bar{\psi}\bar{\tau}\gamma_5\psi \longrightarrow i\bar{\psi}\bar{\tau}\gamma_5\psi + \theta_j\left[\bar{\psi}\tau_i\gamma_5\frac{\tau_j}{2}\psi - \bar{\psi}\frac{\tau_j}{2}\tau_i\gamma_5\psi\right]$$

$$= \quad i\bar{\psi}\bar{\tau}\gamma_5\psi + i\theta\epsilon_{ijk}\bar{\psi}\gamma_5\tau_k\psi,$$

Vector transformation

$$\left[\tau_i, \tau_j\right] = 2i\epsilon_{ijk}\tau_k$$

Levi-Civita Symbols

$$\epsilon_{ijk} = \begin{cases} +1 & for\ even\ permutation\ 1\,2\,3\,, \\ -1 & for\ odd\ permutation\ 1\,2\,3\,, \\ 0 & Otherwise \end{cases}$$

$$\boxed{\bar{\pi} \longrightarrow \bar{\pi} + \epsilon_{ijk}\bar{\theta}\bar{\pi}_k}$$

Axial-Vector transformation

$$\pi_i : \qquad i\bar{\psi}\bar{\tau}\gamma_5\psi \longrightarrow i\bar{\psi}\bar{\tau}\gamma_5\psi + \theta_j \left[\bar{\psi}\tau_i\gamma_5\gamma_5\frac{\tau_j}{2}\psi - \bar{\psi}\gamma_5\frac{\tau_j}{2}\gamma_5\tau_i\psi\right]$$

$$= \quad i\bar{\psi}\bar{\tau}\gamma_5\psi + \theta_j\bar{\psi}\psi\delta_{ij},$$

$\gamma_5\gamma_5 = 1$ and the commutation relation between matrices

$$\left[\tau_i, \tau_j\right] = 2\delta_{ij}$$

$$\delta_{ij} = \begin{cases} +1 & for\, i = j, \\ 0 & for\, i \neq j \end{cases}$$

$$\boxed{\bar{\pi} \longrightarrow \bar{\pi} + \theta\bar{\pi}}$$

LSM Lagrangian

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial_\mu\psi + \frac{1}{2}(\partial_\mu\pi\partial^\mu\pi + \partial_\mu\sigma\partial^\mu\sigma) - g_\pi\left[(i\bar{\psi}\gamma_5\bar{\tau}\psi)\bar{\pi} + (i\bar{\psi}\psi)\sigma\right] - \frac{\lambda}{4}\left((\bar{\pi}^2 + \sigma^2) - f_\pi^2\right)$$

K. E Of nucleons

K. E Of Mesons

interaction term between nucleons and the mesons

Pion nucleon Potential

Nucleon mass term

The chiral part of LσM-Lagrangian has $SU(3)_R \times SU(3)_L$ symmetry

where fermionic part $$\mathcal{L}_q = \sum_f \overline{\psi}_f(i\gamma^\mu D_\mu - gT_a(\sigma_a + i\gamma_5\pi_a))\psi_f$$

and mesonic part $$\begin{aligned}\mathcal{L}_m = {} & \mathrm{Tr}(\partial_\mu\Phi^\dagger\partial^\mu\Phi - m^2\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 \\ & -\lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2 + c[\mathrm{Det}(\Phi) + \mathrm{Det}(\Phi^\dagger)] \\ & +\mathrm{Tr}[H(\Phi + \Phi^\dagger)],\end{aligned}$$

- m^2 is tree-level mass of the fields in the absence of symmetry breaking
- λ_1 and $\lambda_2$are the two possible quartic coupling constants,
- c is the cubic coupling constant,
- g flavor-blind Yukawa coupling of quarks to mesons and of quarks to background gauge field $A_\mu = \delta_{\mu 0}A_0$

$c = 4.80; g = 6.5; \lambda_1 = 5.90; \lambda_2 = 46.48; m^2 = (0.495)$^2;

SU(3) LSM

ϕ is a complex 3×3 matrix and parameterizing scalar σ_a and pseudoscalar π_a (nonets) mesons

$$\Phi = T_a \, \phi_a = T_a \, (\sigma_a + i\pi_a)$$

where σ_a are the scalar fields and π_a are the pseudoscalar fields. The 3×3 matrix H breaks the symmetry explicitly and is chosen as

$$H = T_a \, h_a$$

where h_a are nine external fields and T_a=$\hat{\lambda}_a$/2 are generators of U(3) with $\hat{\lambda}_a$ are Gell-Mann matrices $\hat{\lambda}_0 = \sqrt{\frac{2}{3}}\,1$

The T_a are normalized such that $\mathrm{Tr}(T_a T_b) = \delta_{ab}/2$ and obey the $U(3)$

$$[T_a, T_b] = i \, f_{abc} \, T_c \; ,$$
$$\{T_a, T_b\} = d_{abc} \, T_c \; ,$$

where f_{abc} and d_{abc} for $a, b, c = 1, \ldots, 8$ are the standard antisymmetric and symmetric structure constants of $SU(3)$ and

$$f_{ab0} \equiv 0 \quad , \quad d_{ab0} \equiv \sqrt{\frac{2}{3}} \, \delta_{ab}$$

SU(3) LSM

Gell-Mann matrices with $\lambda_0 = \sqrt{\frac{2}{3}}\mathbf{I}$

$$\hat{\lambda}_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \hat{\lambda}_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \hat{\lambda}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \hat{\lambda}_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\hat{\lambda}_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \hat{\lambda}_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \hat{\lambda}_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \hat{\lambda}_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

as required λ_a span all traceless Hermitian matrices, then the generators follow1

$$[T_a, T_b] = i \sum_{c=1}^{8} f_{abc} T_c \{T_a, T_b\} = \frac{1}{3}\delta_{ab} + \sum_{c=1}^{8} d_{abc} T_c$$

where f are structure constant given by

$$f_{123} = 1 f_{147} = -f_{156} = f_{246} = f_{257} = f_{345} = -f_{367} = \frac{1}{2} f_{458} = f_{678} = \frac{\sqrt{3}}{2},$$

$$d_{118} = d_{228} = d_{338} = -d_{888} = \frac{1}{\sqrt{3}} d_{448} = d_{558} = d_{668} = d_{778} = -\frac{1}{2\sqrt{3}}$$

SU(3) LSM

$$H = \begin{pmatrix} \sqrt{\frac{2}{3}}h0 + h3 + \frac{h8}{\sqrt{3}} & h1 - ih2 & h4 - ih5 \\ h1 + ih2 & \sqrt{\frac{2}{3}}h0 - h3 + \frac{h8}{\sqrt{3}} & h6 - ih7 \\ h4 + ih5 & h6 + ih7 & \sqrt{\frac{2}{3}}h0 - 2\frac{h8}{\sqrt{3}} \end{pmatrix};$$

$$T_a\sigma_a = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}a_0^0 + \frac{1}{\sqrt{6}}\sigma_8 + \frac{1}{\sqrt{3}}\sigma_0 & a_0^- & \kappa^- \\ a_0^+ & -\frac{1}{\sqrt{2}}a_0^0 + \frac{1}{\sqrt{6}}\sigma_8 + \frac{1}{\sqrt{3}}\sigma_0 & \bar{\kappa}^0 \\ \kappa^+ & \kappa^0 & -\frac{2}{\sqrt{3}}\sigma_8 + \frac{1}{\sqrt{3}}\sigma_0 \end{pmatrix},$$

$$T_a\pi_a = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\pi_8 + \frac{1}{\sqrt{3}}\pi_0 & \pi^- & K^- \\ \pi^+ & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\pi_8 + \frac{1}{\sqrt{3}}\pi_0 & \bar{K}^0 \\ K^+ & K^0 & -\frac{2}{\sqrt{3}}\pi_8 + \frac{1}{\sqrt{3}}\pi_0 \end{pmatrix}.$$

SU(3) LSM

When shifting Φ field by vacuum expectation value,

$$\begin{aligned}\mathcal{L} = &\frac{1}{2}\left[\partial_\mu \sigma_a \partial^\mu \sigma_a + \partial_\mu \pi_a \partial^\mu \pi_a - \sigma_a (m_S^2)_{ab} \sigma_b - \pi_a (m_P^2)_{ab} \pi_b\right] \\ &+ \left(\mathcal{G}_{abc} - \frac{4}{3}\mathcal{F}_{abcd}\,\bar{\sigma}_d\right)\sigma_a \sigma_b \sigma_c - 3\left(\mathcal{G}_{abc} + \frac{4}{3}\mathcal{H}_{abcd}\,\bar{\sigma}_d\right)\pi_a \pi_b \sigma_c \\ &- 2\,\mathcal{H}_{abcd}\,\sigma_a \sigma_b \pi_c \pi_d - \frac{1}{3}\mathcal{F}_{abcd}\left(\sigma_a \sigma_b \sigma_c \sigma_d + \pi_a \pi_b \pi_c \pi_d\right) - U(\bar{\sigma})\ ,\end{aligned}$$

where the tree-level potential is

$$U(\bar{\sigma}) = \frac{m^2}{2}\,\bar{\sigma}_a^2 - \mathcal{G}_{abc}\bar{\sigma}_a\bar{\sigma}_b\bar{\sigma}_c + \frac{1}{3}\mathcal{F}_{abcd}\bar{\sigma}_a\bar{\sigma}_b\bar{\sigma}_c\bar{\sigma}_d - h_a\bar{\sigma}_a$$

$\bar{\sigma}_a$ is determined from

$$\frac{\partial U(\bar{\sigma})}{\partial \bar{\sigma}_a} = m^2\,\bar{\sigma}_a - 3\,\mathcal{G}_{abc}\bar{\sigma}_b\bar{\sigma}_c + \frac{4}{3}\mathcal{F}_{abcd}\bar{\sigma}_b\bar{\sigma}_c\bar{\sigma}_d - h_a = 0$$

SU(3) LSM

coefficients $\mathcal{G}_{abc}$, $\mathcal{F}_{abcd}$, and $\mathcal{H}_{abcd}$ are given by

$$\mathcal{G}_{abc} = \frac{c}{6}\left[d_{abc} - \frac{3}{2}\left(\delta_{a0}d_{0bc} + \delta_{b0}d_{a0c} + \delta_{c0}d_{ab0}\right) + \frac{9}{2}d_{000}\delta_{a0}\delta_{b0}\delta_{c0}\right] ,$$

$$\mathcal{F}_{abcd} = \frac{\lambda_1}{4}\left(\delta_{ab}\delta_{cd} + \delta_{ad}\delta_{bc} + \delta_{ac}\delta_{bd}\right) + \frac{\lambda_2}{8}\left(d_{abn}d_{ncd} + d_{adn}d_{nbc} + d_{acn}d_{nbd}\right)$$

$$\mathcal{H}_{abcd} = \frac{\lambda_1}{4}\delta_{ab}\delta_{cd} + \frac{\lambda_2}{8}\left(d_{abn}d_{ncd} + f_{acn}f_{nbd} + f_{bcn}f_{nad}\right) .$$

where tree-level masses, $(m_S^2)_{ab}$ and $(m_P^2)_{ab}$ are given by

$$(m_S^2)_{ab} = m^2\,\delta_{ab} - 6\,\mathcal{G}_{abc}\,\bar{\sigma}_c + 4\,\mathcal{F}_{abcd}\,\bar{\sigma}_c\bar{\sigma}_d$$

$$(m_P^2)_{ab} = m^2\,\delta_{ab} + 6\,\mathcal{G}_{abc}\,\bar{\sigma}_c + 4\,\mathcal{H}_{abcd}\,\bar{\sigma}_c\bar{\sigma}_d$$

The masses are not diagonal, thus σ_a and π_a fields are not mass generators in standard basis of SU(3). As, the mass matrices are symmetric and real, diagonalization is achieved by an orthogonal transformation

$$\tilde{\sigma}_i = O_{ia}^{(S)}\,\sigma_a ,$$

$$\tilde{\pi}_i = O_{ia}^{(P)}\,\pi_a ,$$

$$\left(\tilde{m}_{S,P}^2\right)_i = O_{ai}^{(S,P)}\,\left(m_{S,P}^2\right)_{ab}\,O_{bi}^{(S,P)}$$

SU(3) LSM

The expectation values $\langle\Phi\rangle = T_0\,\bar{\sigma}_0 + T_8\,\bar{\sigma}_8$

where

$$h_0 = \left[m^2 - \frac{c}{\sqrt{6}}\,\bar{\sigma}_0 + \left(\lambda_1 + \frac{\lambda_2}{3}\right)\bar{\sigma}_0^2\right]\bar{\sigma}_0 + \left[\frac{c}{2\sqrt{6}} + (\lambda_1+\lambda_2)\,\bar{\sigma}_0 - \frac{\lambda_2}{3\sqrt{2}}\,\bar{\sigma}_8\right]\bar{\sigma}_8^2$$

$$h_8 = \left[m^2 + \frac{c}{\sqrt{6}}\,\bar{\sigma}_0 + \frac{c}{2\sqrt{3}}\,\bar{\sigma}_8 + (\lambda_1+\lambda_2)\,\bar{\sigma}_0^2 - \frac{\lambda_2}{\sqrt{2}}\,\bar{\sigma}_0\,\bar{\sigma}_8 + \left(\lambda_1 + \frac{\lambda_2}{2}\right)\bar{\sigma}_8^2\right]\bar{\sigma}_8$$

From PCAC relations

$$\bar{\sigma}_0 = \frac{f_\pi + 2\,f_K}{\sqrt{6}}\ ,$$

$$\bar{\sigma}_8 = \frac{2}{\sqrt{3}}\,(f_\pi - f_K)$$

$$f_\pi = 92.4\ \mathrm{MeV},\ f_K = 113\ \mathrm{MeV}$$

LSM involving Polyakov-Loop Potential

Why Polyakov loop?

- the chiral model does NOT describe effects of QCD gluonic dof
- absence of confinement results in a non-zero quark number density even in confined phase

- The functional form of the potential is motivated by the QCD symmetries of in the pure gauge limit

$$\frac{\mathcal{U}(\phi,\phi^*,T)}{T^4} = -\frac{b_2(T)}{2}|\phi|^2 - \frac{b_3}{6}(\phi^3+\phi^{*3}) + \frac{b_4}{4}(|\phi|^2)^2,$$

$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3.$$

$$a_0 = 6.75, \qquad a_1 = -1.95, \qquad a_2 = 2.625, \qquad a_3 = -7.44$$

$$b_3 = 0.75 \qquad b_4 = 7.5$$

LSM involving Polyakov-Loop Potential

The thermal expectation value of color traced Wilson loop in the temporal direction determines Polyakov-loop potential

$$\Phi(\vec{x}) = \frac{1}{N_c}\langle \mathcal{P}(\vec{x})\rangle,$$

Polyakov-loop potential and its conjugate

$$\begin{aligned} \phi &= (\mathrm{Tr}_c\,\mathcal{P})/N_c, \\ \phi^* &= (\mathrm{Tr}_c\,\mathcal{P}^\dagger)/N_c, \end{aligned}$$

This can be represented by a matrix in the color space

$$\mathcal{P}(\vec{x}) = \mathcal{P}\mathrm{exp}\left[i\int_0^\beta d\tau A_4(\vec{x},\tau)\right],$$

$\beta = 1/T$ Temperature

$A_4 = iA^0$ Polyakov gauge

LSM involving Polyakov-Loop Potential

The coupling between Polyakov loop and quarks is given by the covariant derivative

$$D_\mu = \partial_\mu - iA_\mu \text{ in PLSM Lagrangian}$$

$$A_\mu = \delta_{\mu 0} A_0$$ **in the chiral limit**

$$\mathcal{L}_{PLSM} = \mathcal{L}_{meson} + \mathcal{L}_{quark} + \bar{q}\gamma_0 \mathcal{A}_0 - \mathcal{U}(\phi, \phi^*, T),$$

invariant under chiral flavor group (like QCD Lagrangian)

$$\boxed{\mathcal{L}_{chiral} = \mathcal{L}_{meson} + \mathcal{L}_{quark} + \bar{q}\gamma_0 \mathcal{A}_0}$$

$U(\phi, \phi^*, T)$ is T-dependent Polyakov Potential

In case of no quarks, then $\phi = \phi^*$ and the Polyakov loop is considered as an order parameter for the deconfinement phase-transition

Partition Function

In thermal equilibrium, the grand partition function can be defined by using a path integral over quark, antiquark and meson fields

$$
\begin{aligned}
\mathcal{Z} &= \mathrm{Tr}\, \exp[-(\hat{\mathcal{H}} - \sum_{f=u,d,s} \mu_f \hat{\mathcal{N}}_f)/T] \\
&= \int \prod_a \mathcal{D}\sigma_a \mathcal{D}\pi_a \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left[\int_x (\mathcal{L} + \sum_{f=u,d,s} \mu_f \bar{\psi}_f \gamma^0 \psi_f) \right],
\end{aligned}
$$

where $\int_x \equiv i \int_0^{1/T} dt \int_V d^3x$ **and** μ_f **chemical potential**

Thermodynamic potential density

$$
\boxed{\Omega(T,\mu) = \frac{-T \ln \mathcal{Z}}{V} = U(\sigma_x, \sigma_y) + \mathcal{U}(\phi, \phi^*, T) + \Omega_{\bar{\psi}\psi}.}
$$

LSM involving Polyakov-Loop Potential

The quarks and antiquarks Potential contribution

$$\Omega_{\bar{\psi}\psi} = -2TN\int_0^\infty \frac{d^3\vec{p}}{(2\pi)^3}\left\{\ln\left[1+3(\phi+\phi^* e^{-(E-\mu)/T})\times e^{-(E-\mu)/T}+e^{-3(E-\mu)/T}\right]\right.$$
$$\left. +\ln\left[1+3(\phi^*+\phi e^{-(E+\mu)/T})\times e^{-(E+\mu)/T}+e^{-3(E+\mu)/T}\right]\right\},$$

where **N** gives the number of quark flavors, $E = \sqrt{\vec{p}^2+m^2}$

$$m_q = g\frac{\sigma_x}{2},$$
$$m_s = g\frac{\sigma_y}{\sqrt{2}}.$$

Mesonic potential $U(\sigma_x,\sigma_y) = \frac{m^2}{2}(\sigma_x^2+\sigma_y^2) - h_x\sigma_x - h_y\sigma_y - \frac{c}{2\sqrt{2}}\sigma_x^2\sigma_y$

$$+\frac{\lambda_1}{2}\sigma_x^2\sigma_y^2 + \frac{1}{8}(2\lambda_1+\lambda_2)\sigma_x^4 + \frac{1}{4}(\lambda_1+\lambda_2)\sigma_y^4.$$

Vandermonde determinant is found negligibly small

Thermodynamical Potential

The thermodynamic potential

$$\Omega(T,\mu)=\frac{-T\ln\mathcal{Z}}{V}=U(\sigma_x,\sigma_y)+\mathcal{U}(\phi,\phi^*,T)+\Omega_{\bar{\psi}\psi}.$$

has the parameters

$m^2, h_x, h_y, \lambda_1, \lambda_2, c$ and g

σ_x and σ_y Condensates (chiral order parameters)

ϕ and ϕ^* (deconfinement order parameters

$m^2, h_x, h_y, \lambda_1, \lambda_2$ and c can be fixed, experimentally

σ_x, σ_y, ϕ and ϕ^* minimizing the potential

$$\frac{\partial\Omega}{\partial\sigma_x}=\frac{\partial\Omega}{\partial\sigma_y}=\frac{\partial\Omega}{\partial\phi}=\left.\frac{\partial\Omega}{\partial\phi^*}\right|_{min}=0,$$

refined by lattice QCD,

$\sigma_x=\bar{\sigma_x}, \sigma_y=\bar{\sigma_y}, \phi=\bar{\phi}$ and $\phi^*=\bar{\phi}^*$ are the global minimum

Transport Coefficients from PLSM

Electrical and Heat Conductivity

Fine structure etc.

Number density

$$\sigma_e(T,\mu) = \sum_k \frac{4\pi}{137} q_k^2 \frac{n_k(T,\mu)\,\tau_k(T,\mu)}{m_k(T,\mu)}$$

Quark mass

Quarks flavors

Decay time

$$\kappa(T,\mu) = \frac{1}{3} v_{rel}\, c_V(T,\mu) \sum_k \tau_k(T,\mu)$$

Relative velocity

Specific heat

Bulk and Shear Viscosity

$$\xi = \frac{1}{9T}\left[T^5 \frac{\partial}{\partial T}\left(\frac{\epsilon - 3p}{T^4}\right) + 16\,|\epsilon_v|\right] = \frac{1}{9T}\left[-16\,\epsilon + 9\,T\,S + T\,c_V + 16\,|\epsilon_v|\right]$$

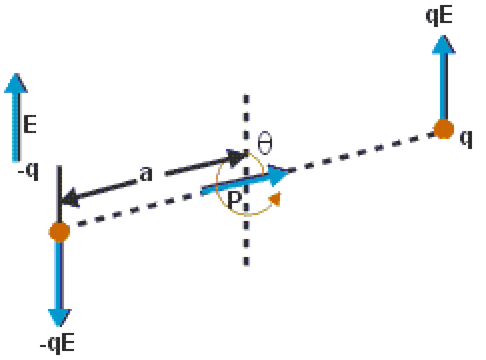
Vacuum energy density

$$\eta \sim \frac{\xi}{-0.45(c_s^2 - \frac{1}{3})}$$

Pressure, energy density, entropy, speed of sound

Electrical Conductivity

Based on parton-hadron-string dynamics transport approach



$$\frac{d}{dt}p_z^j = q_j e E_z$$

an additional force causes the propagation of charge.

The electrical current density

$$j_z(t) = \frac{1}{V}\sum_j q_j e \frac{\boxed{p_z^j(t)}}{\boxed{M_j(t)}}$$

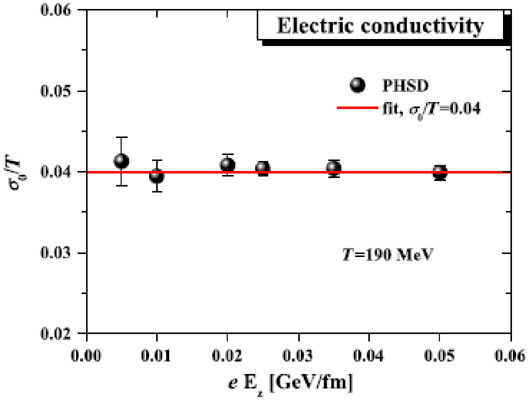
z-momentum of j-th particle at time t

Mass of j-th particle at time t

In natural units, the ratio of current density and electric field strength ➔ electric conductivity

$$\frac{\sigma_0}{T} = \frac{j_{\mathrm{eq}}}{E_z T}$$

proportionality between e-current and e-field



F. Reif, Fundamentals of Statistical and Thermal Physics, (McGraw-Hill, New York, 1965).
W. Cassing, O. Linnyk, T. Steinert, and V. Ozvenchuk, Phys. Rev. Lett. 110, 182301 (2013).

Electrical Conductivity

in relaxation time approximation, σ is described in Gases, Liquids and Solid State,

$$\sigma_0 = \frac{e^2 n_e \boxed{\tau}}{\boxed{m_e^*}}$$

n density of nonlocalized charges

τ relaxation time of charge carriers

m_e^* effective masses

for partonic degrees of freedom within the dynamical quasiparticle model (DQPM), the thermal dependence reads

$$\frac{\sigma_0(T)}{T} \approx \boxed{\frac{2}{9}} \frac{e^2 n_q(T)}{\boxed{M_q(T)}\boxed{\Gamma_q(T)}T}$$

Γ_q width of quasiparticle spectral function

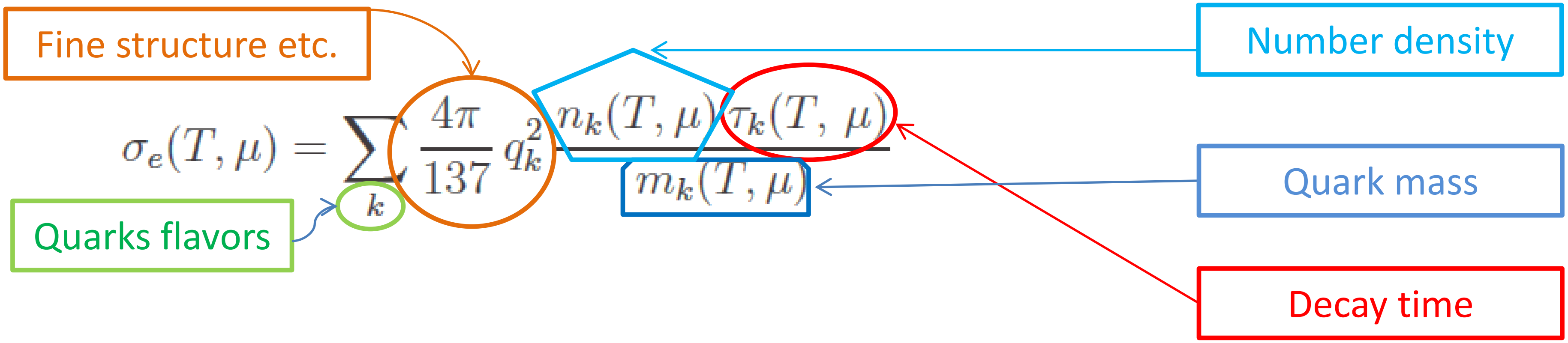
M_q pole mass=spectral dist. of quark-mass

flavor averaged fractional quark charge squared

In PHSD: DQPM matches quasiparticles properties to lattice QCD results in equilibrium for EOS, electromagnetic correlator, among others.

Electrical Conductivity

Durde-Lorentz conductivity



Fine structure etc.

Number density

$$\sigma_e(T,\mu) = \sum_k \frac{4\pi}{137} q_k^2 \frac{n_k(T,\mu)\tau_k(T,\mu)}{m_k(T,\mu)}$$

Quark mass

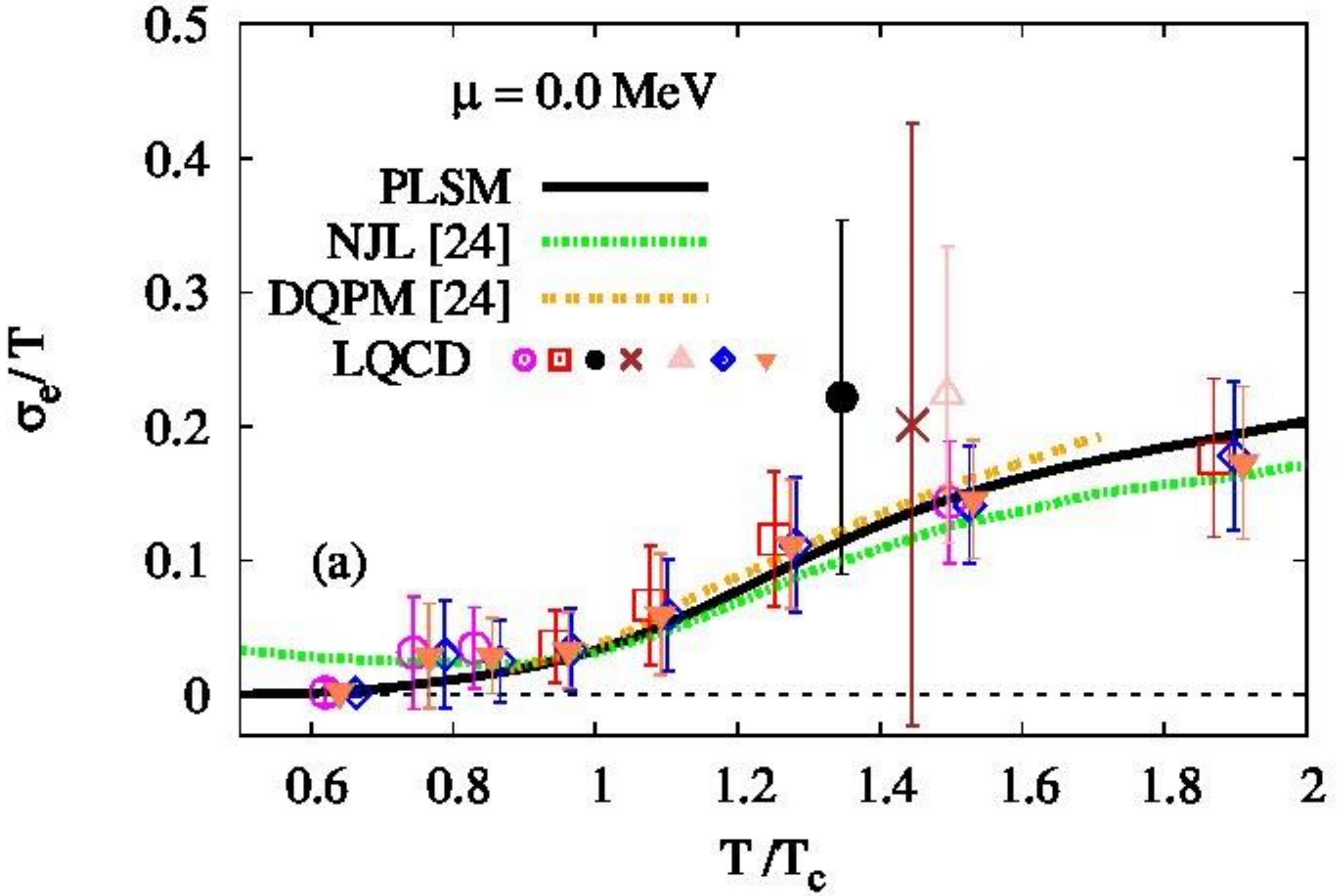
Quarks flavors

Decay time

σ is related to flow of charges in presence of an electric field (decay constant & relaxation time) **response of the strongly interacting system in equilibrium to an external e-field**

- external e-field is applied on flowing charges, the induced electric current J is related to the e-field. σ is the proportionally constant.
- self-interaction between quarks and gluons, Green-Kubo corrector

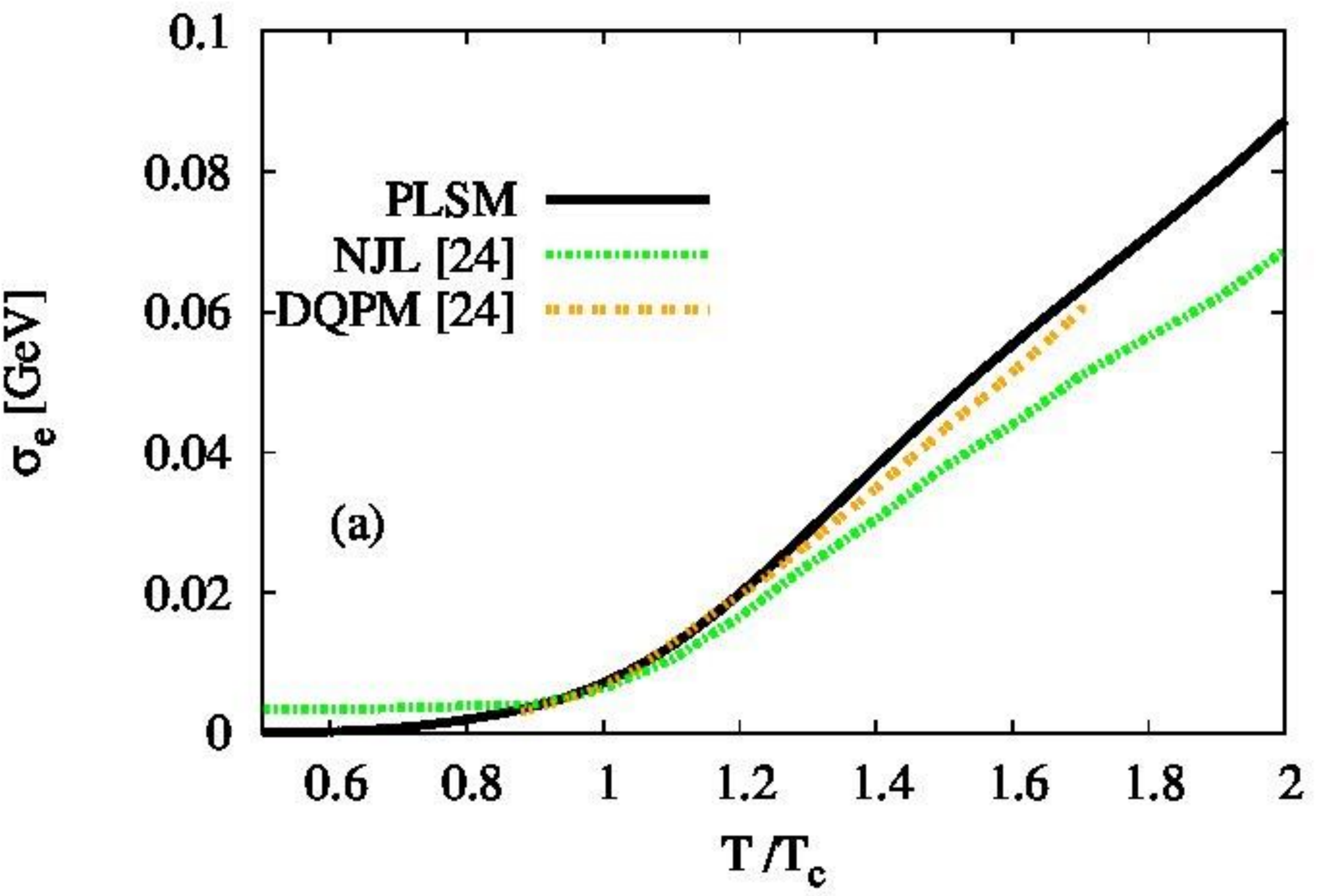
Normalized Electrical Conductivity



NJL/DQPM: PRC88, 045204 (2013)

LQCD: PRL111,172001 (2013), PRD83,034504 (2011), JHEP1303,100 (2013), 1412.6411, 1501.0018

Non-Normalized Electrical Conductivity



NJL/DQPM: PRC88, 045204 (2013)

Heat Conductivity

From relativistic Navier-Stokes ansatz, heat flow is proportional to the gradient of thermal potential

$$q^\mu = -\kappa \frac{nT^2}{\epsilon + p} \nabla^\mu \alpha = \kappa \left(\nabla^\mu T - \frac{T}{\epsilon + p} \nabla^\mu p \right)$$

PRE87, 033019 (2013)

$$q^x = \kappa(\nabla^x T) = -\kappa \partial_x \boxed{T(x)} \xleftarrow{\quad} \boxed{\text{Temperature profile}}$$

Modeling

$$\kappa = q^x \frac{(ax+b)^2}{ap}$$

Alternatively, linearizing Boltzmann Eq. ➔

PRD48, 2916 (1993)

$$f_i = f_i^{le} + \frac{\partial f_i^0}{\partial \varepsilon_i} \Phi_i \frac{\nabla T}{T}$$

Non-Equilibrium distribution function

$$f_i^{le} = \{\exp[(\varepsilon_i - \mu)/T(z)] + 1\}^{-1}$$

Equilibrium distribution function

Then, the thermal current reads

$$J_T = \nu_q \sum_{\mathbf{p}} (\varepsilon_p - \mu_q) v_z \frac{\partial f^0}{\partial \varepsilon_p} \Psi_{\mathbf{p}} = \frac{1}{3} \mu_q^2 T^2$$

$$\frac{1}{\kappa} = \frac{24}{\pi^3} \alpha_s^2 T^{-2} I_\kappa(T/q_D)$$

$$I_\kappa(T/q_D) = \begin{cases} \frac{1}{3} \ln(T/q_D) + 0.30\,, & T \gg q_D\,, \\ 2\zeta(3) \left(\frac{T}{q_D}\right)^2\,, & T \ll q_D\,. \end{cases}$$

α_s running strong coupling

q_D Debye wave number $g^2 N_q \mu^2 / (2\pi^2)$

Heat Conductivity

$$\kappa = \frac{1}{3} v_F^2 \, c_v \, \tau_\kappa$$

Fermi velocity

Specific heat

Relaxation time

Relaxation time, specific heat are T- and mu-dependent

Relative velocity $\nu_{rel} = \sqrt{(p_1 p_2)^2 - (m_1 m_2)^2} / E_1 E_2$

$$\kappa(T,\mu) = \frac{1}{3} \nu_{rel} \, c_V(T,\mu) \sum_k \tau_k(T,\mu)$$

Relative velocity

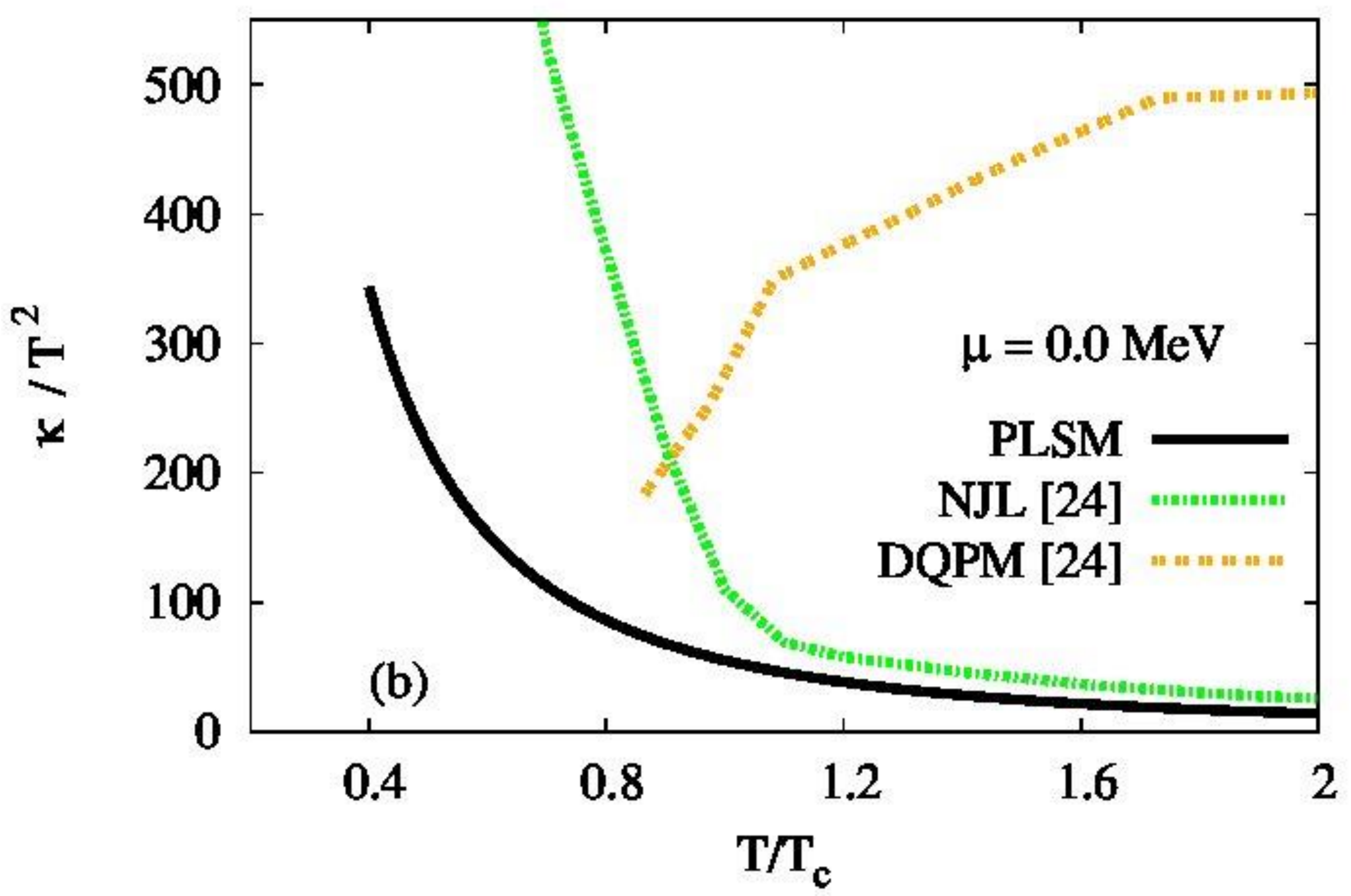
Specific heat

Decay time

κ is related to heat flow of relativistic fluid (rate of energy change)

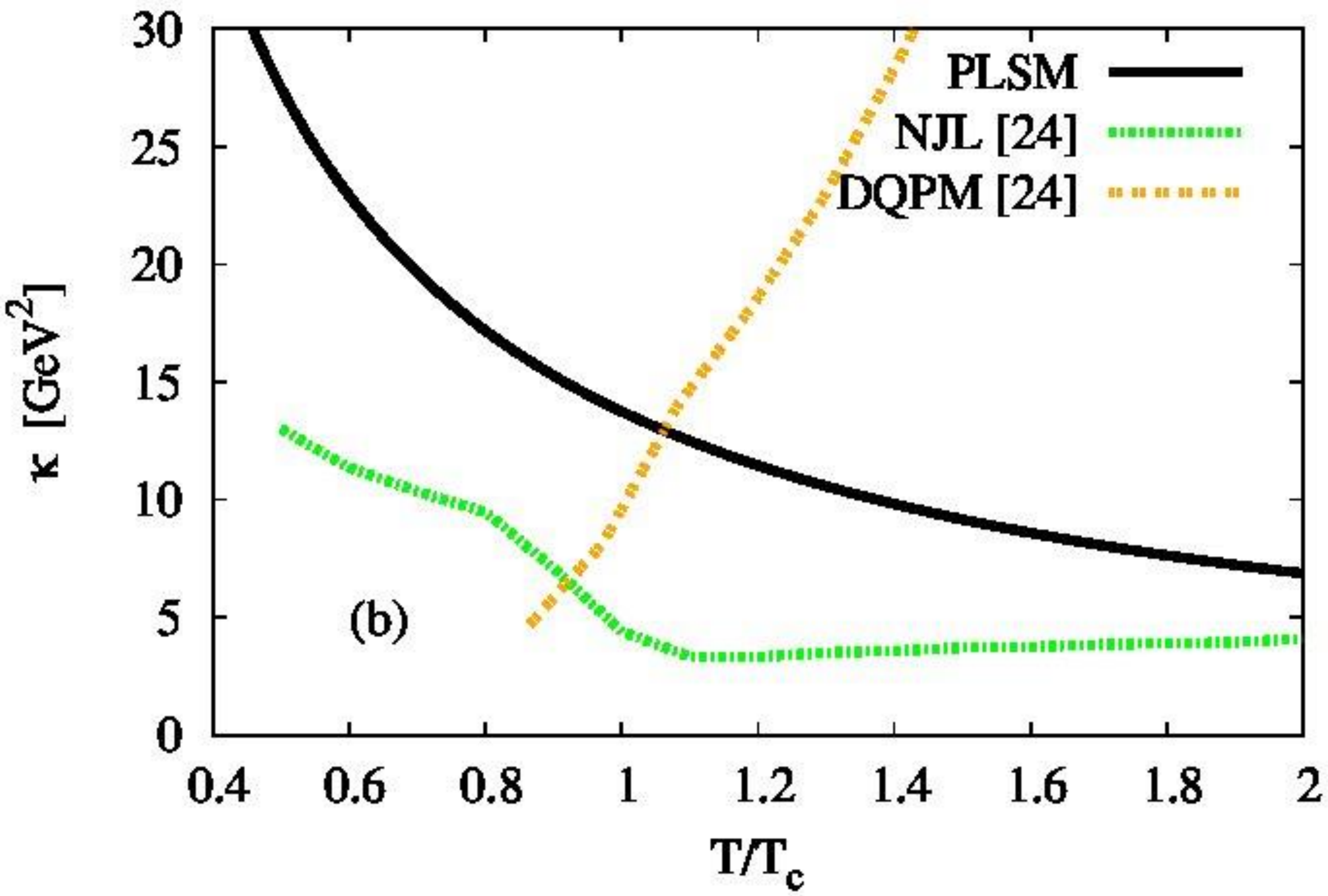
κ can be estimated through irradiation caused by energetic ions

Normalized Heat Conductivity



NJL/DQPM: PRC88, 045204 (2013)

Non-Normalized Heat Conductivity



NJL/DQPM: PRC88, 045204 (2013)

Bulk Viscosity

Kubo’s formula: shear η and bulk ζ viscosities are related to the **correlation function of stress tensor**

$$\zeta = \frac{1}{9} \lim_{\omega \to 0} \frac{1}{\omega} \int_0^\infty dt \int d^3r \, e^{i\omega t} \boxed{\langle [\theta_{ii}(x), \theta_{kk}(0)] \rangle}$$

PLB663, 217 (2008)

$$\zeta = \frac{1}{9} \lim_{\omega \to 0} \frac{1}{\omega} \int_0^\infty dt \int d^3r \, e^{i\omega t} \, \langle [\theta^\mu_\mu(x), \theta^\mu_\mu(0)] \rangle$$

LI- operators

In low energy theorems: bulk viscosity is a measure for violation of conformal invariance

$$(\mathcal{E} - 3P)^* = \langle m\bar{q}q \rangle^* + \langle m\bar{q}q \rangle_0 - 4|\epsilon_v|$$

$$\langle m\bar{q}q \rangle_0 = -M_\pi^2 \, f_\pi^2 - M_K^2 \, f_K^2$$

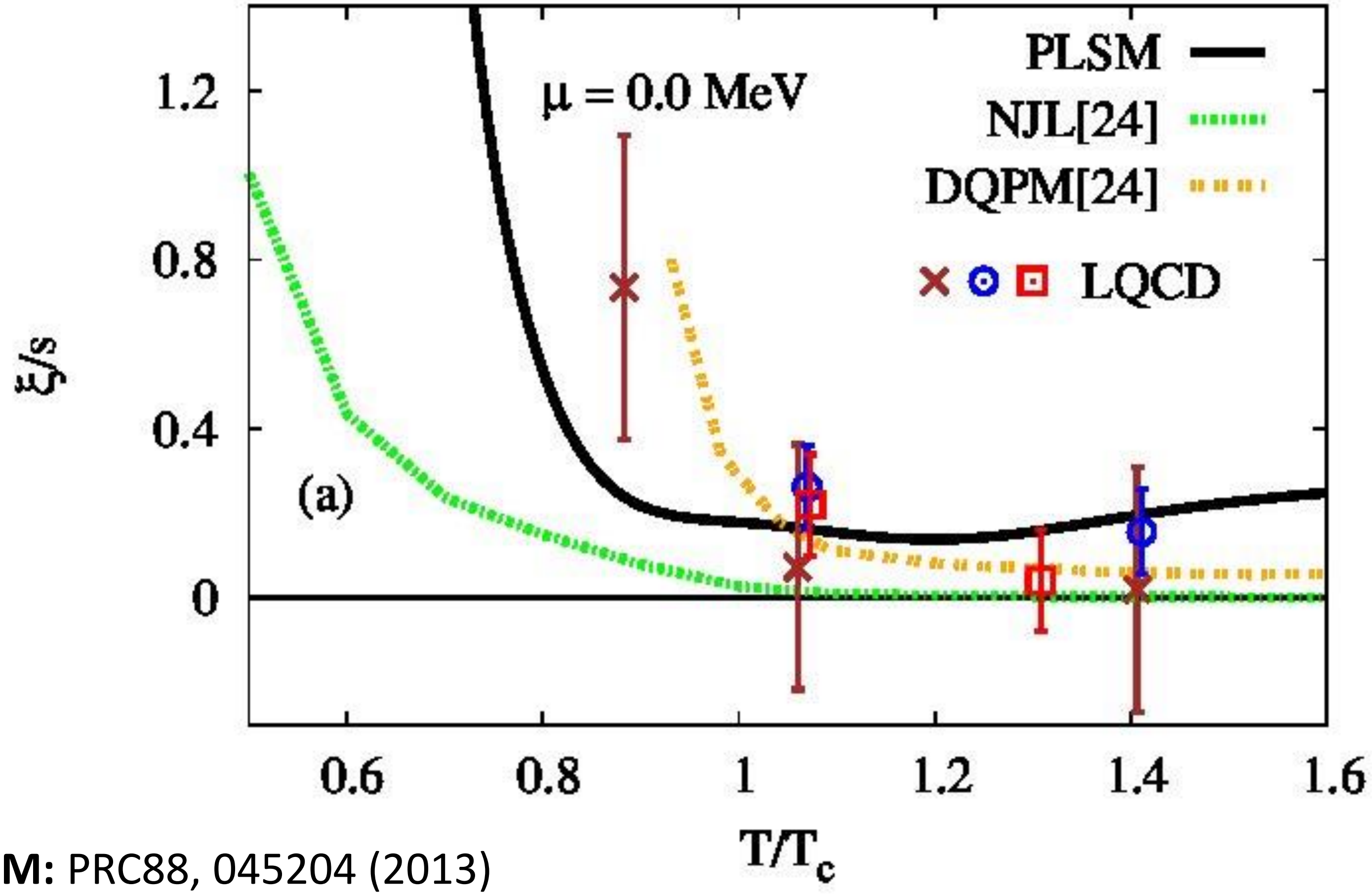
PCAC relations

$$9\,\omega_0\,\zeta = T\,s \left(\frac{1}{c_s^2} - 3\right) - 4(\mathcal{E} - 3P) + \left(T \frac{\partial}{\partial T} - 2\right) \langle m\bar{q}q \rangle^* + 16|\epsilon_v| + 6(M_\pi^2 \, f_\pi^2 + M_K^2 \, f_K^2)$$

Bulk Viscosity

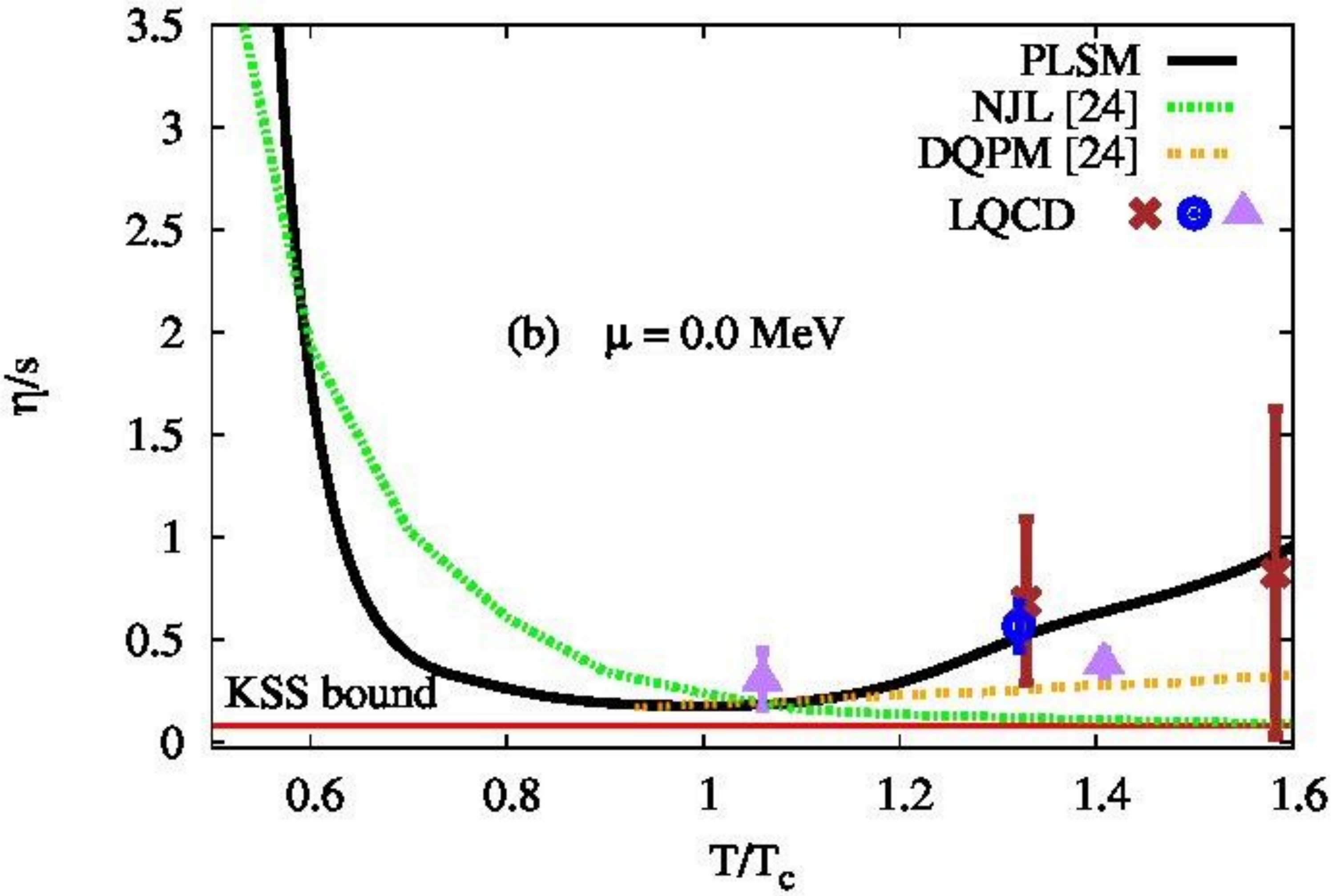
$$\xi = \frac{1}{9T}\left[T^5 \frac{\partial}{\partial T}\left(\frac{\epsilon - 3p}{T^4}\right) + 16\boxed{|\epsilon_v|}\right] = \frac{1}{9T}\left[-16\,\epsilon + 9\,T\,S + T\,c_V + 16\boxed{|\epsilon_v|}\right]$$

Vacuum energy density



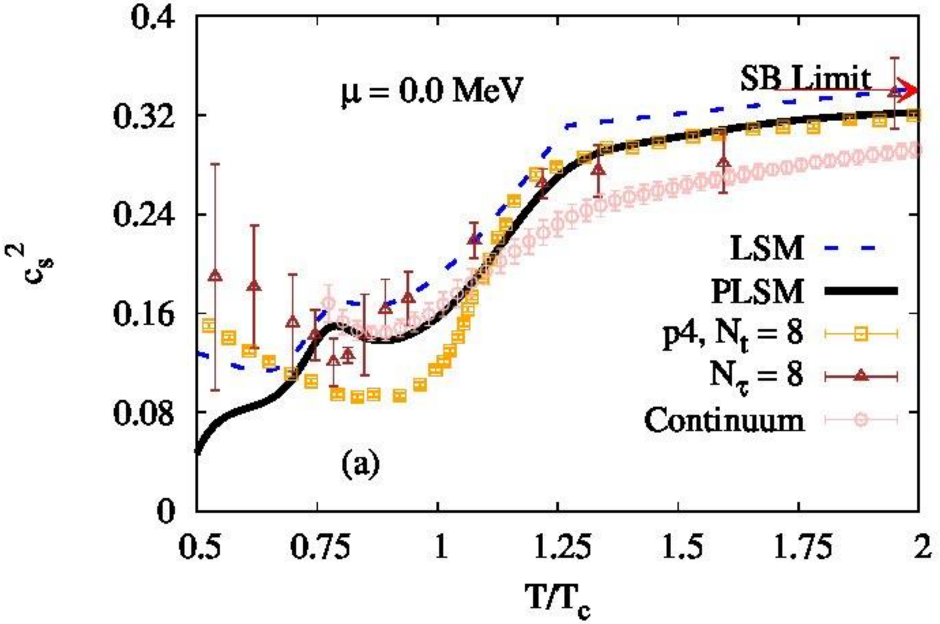
NJL/DQPM: PRC88, 045204 (2013)
PRD76, 101701(2007); PRL100, 162001(2008); PoS LAT2007, 221(2007); PRL94, 072305(2005).

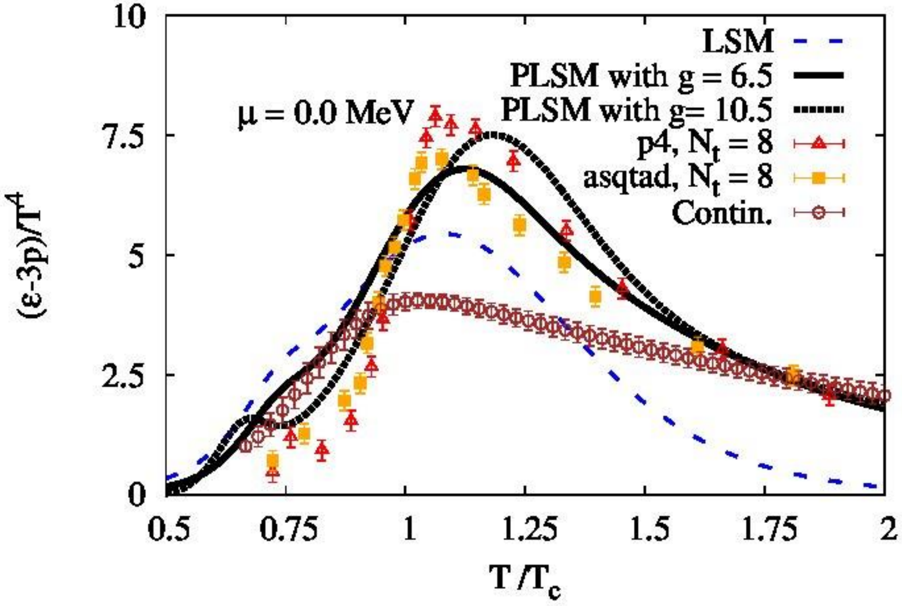
Shear Viscosity

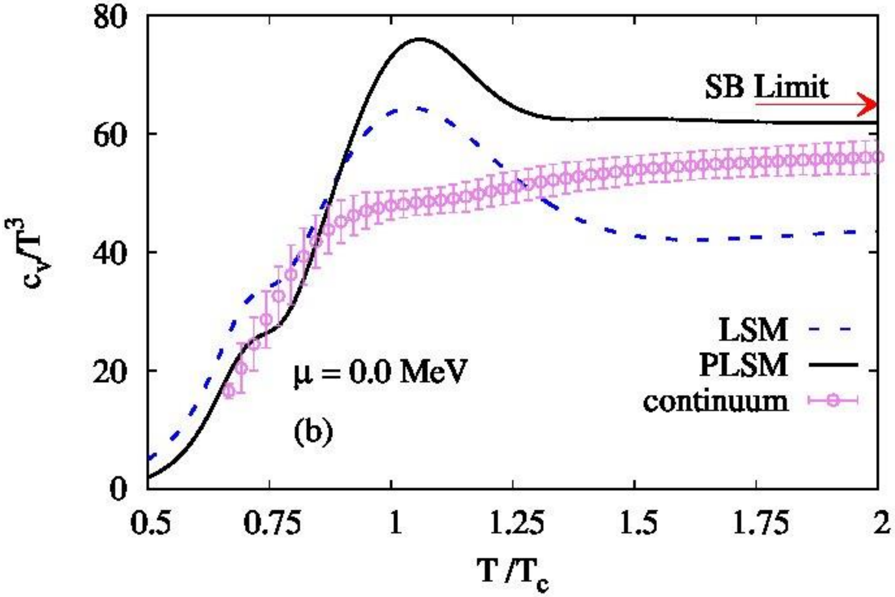


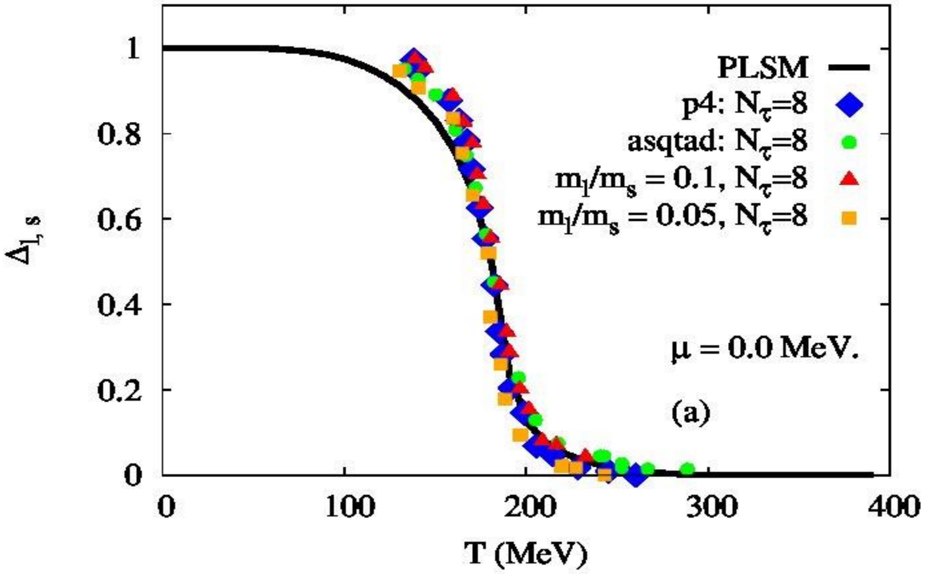
NJL/DQPM: PRC88, 045204 (2013)
KSS: Kovtun, Son, Starinets, PRL94, 111601 (2005).

PLSM and LQCD

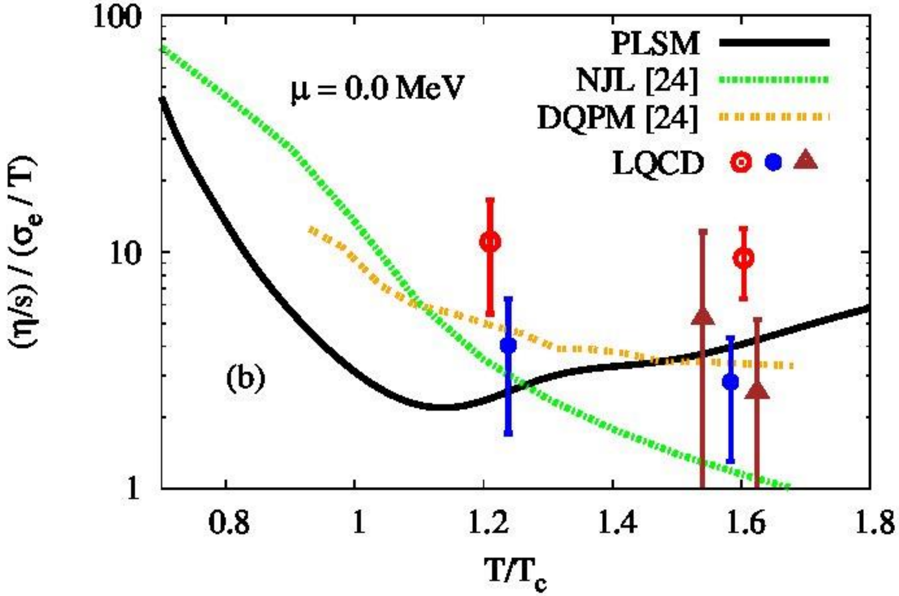


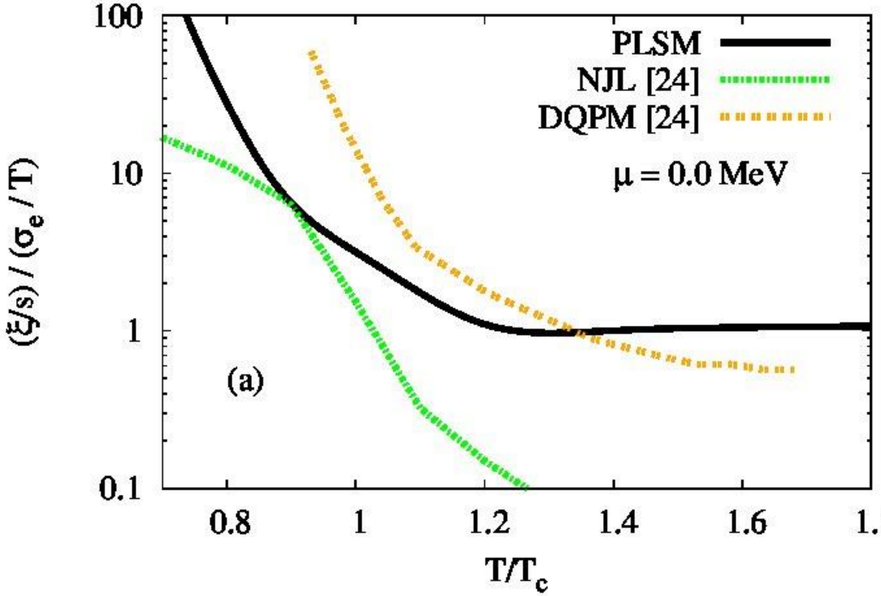


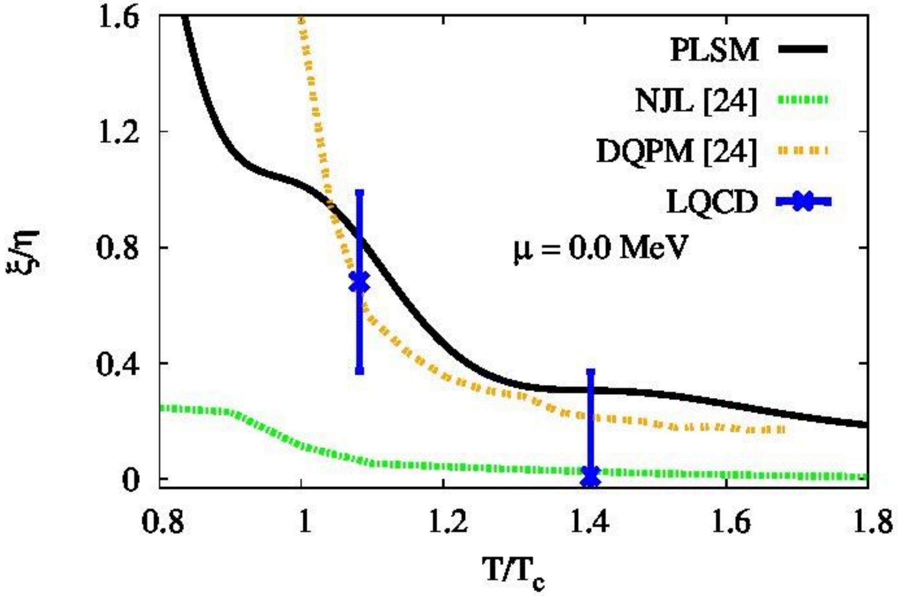




Transport Confidents







Summary

- PLSM seems to be able to generate lattice QCD transport confidents
- Approaches a wide horizon in understanding QGP properties at finite T and mu

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Спасибо!

Thanks!

Danke!

http://atawfik.net/