Dyons and KvBLL Instantons in QCD

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December 12, 2012

Outline

KvBLL instantons

- Classical self-dual solutions of Yang-Mills eq. at finite temperature
- Generalizations of instantons with an additional parameter – holonomy $\exp(i \int A_0 d\tau)$
- When holonomy is "non-trivial", they disassociate into static objects called dyons

KvBLL instantons

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$A_0(r \to \infty) = v\frac{\tau \cdot \hat{\omega}}{2}$ – Acts like a an adjoint higgs field
$F_{\mu\nu}^2 = (D_i A_0)^2 + F_{ij}^2$ (Assuming time independent fields)

The dyon

In the usual radial ansatz it looks as

$$A_0^a = \mathcal{H}(r)\hat{r}^a \tag{1}$$

$$A_i^a = \mathcal{A}(r)\epsilon_{aij}\hat{r}^j \tag{2}$$

Imposing selfduality

$$F_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F_{\rho\sigma}$$

with boundary condition

$$\mathcal{A}(r \to \infty) = 0 \ ,$$
$$\mathcal{H}(r \to \infty) = v$$

One can see that functions

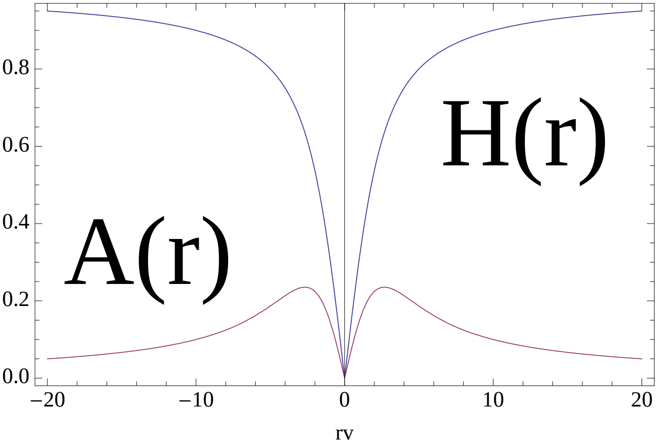
$$\mathcal{H} = -\frac{1 - vr\coth(vr)}{r} \tag{3}$$

$$\mathcal{A} = \frac{1}{r} - \frac{v}{\sinh(vr)} \tag{4}$$

satisfy the self duality equations, such that

$$\int d^3x\ F^2 = 4\pi v$$

which gives a fractional topological charge $Q = \frac{v\beta}{2\pi}$.



$$A_0^a = \mathcal{H}(r)\hat{r}^a \qquad A_i^a = \mathcal{A}(r)\epsilon_{aij}\hat{r}^j$$

The dyon

The fields look like

$$E_i = \frac{\hat{r}^i}{r^2}\frac{\hat{r}\cdot\boldsymbol{\tau}}{2} + o(e^{-rv}) \,, \qquad (5)$$

$$B_i = E_i \qquad (6)$$

Changing the gauge so that $\boldsymbol{\tau}\cdot\hat{r} \to \tau^3$ (which is not unique) reveals the abelian nature of the solutions, for example

$$A_r = 0 \,, \qquad A_\theta = o(e^{-vr})$$

$$A_0 = (v - \frac{1}{r})\frac{\tau^3}{2} + o(e^{-vr})$$

$$A_\varphi = \frac{\tan\frac{\theta}{2}}{r}\frac{\tau^3}{2} + o(e^{-vr})$$

In this form the Dirac monopole field is evident, with the Dirac string along $\theta = \pi$ direction.

The second dyon

Take $v \to 2\pi T - v$ and using the time dependent gauge transform

$$U(t) = e^{-it\pi\tau^3/\beta}$$

we get a new solution with same asymptotics

$$A_0 \sim (v + 1/r)\frac{\tau_3}{2}$$

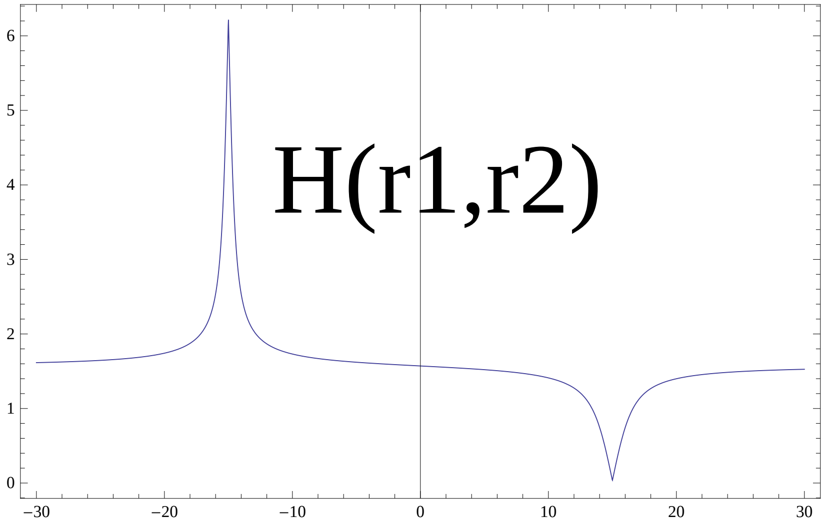
but opposite charge of magnetic and electric fields, i.e.

$$E_i = -\frac{\hat{r}^i}{r^2}\frac{\tau_3}{2} \,, \qquad\qquad B_i = E_i \,.$$

This configuration, however, has and action $\int F^2 = 4\pi(2\pi - v\beta)$ topological charge $\bar{Q} = 1 - v\beta/(2\pi)$, so that

$$Q + \bar{Q} = 1$$

The approximate higgs potential



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- Instanton has constituents! And their "masses" (action) depend on the holonomy parameter v.
- Indeed Kraan and van Baal found this solution!

The exact KvBLL Caloron

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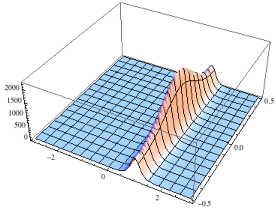

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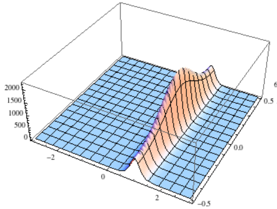

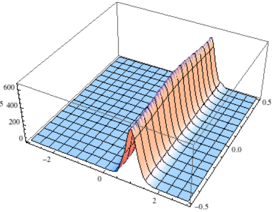

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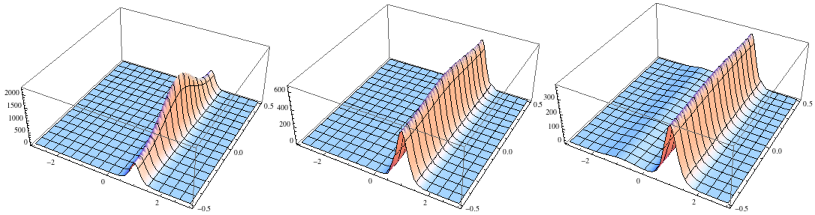

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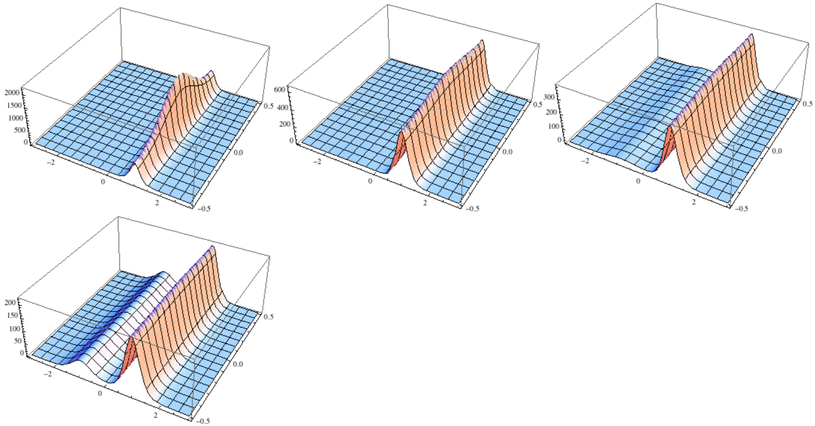

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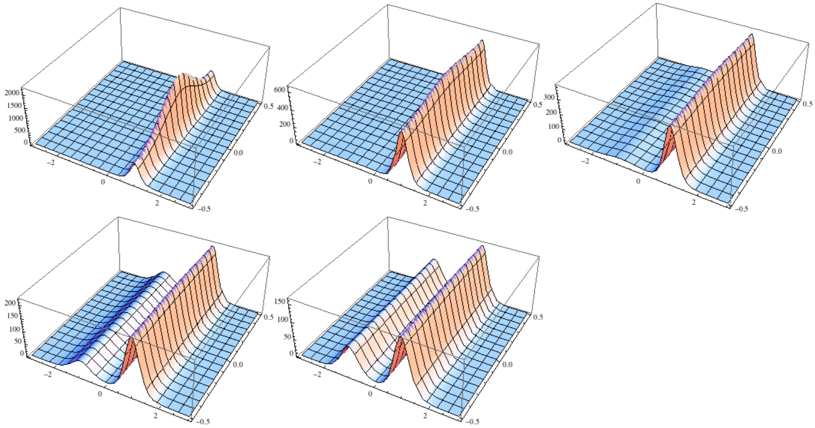

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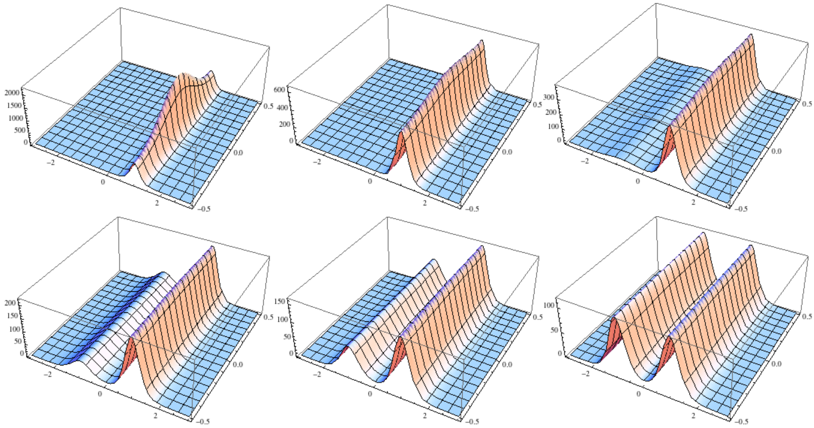


The fermionic zero modes

Fermionic zero modes

Now we look for the solution of Dirac equation in the background of a single dyon. The result is well known from the 70s for 3D theory. Writing the equation for a chiral fermion

$$\psi^{\alpha}_{A}(r) = \left[\left(\alpha_1(r)\mathbf{1} + \alpha_2(r)\hat{r}\cdot\boldsymbol{\tau}\right)\epsilon\right]_{A\alpha} e^{-i\varphi t/\beta} ,$$

we obtain for the Dirac equation

$$\frac{d\alpha_1(r)}{dr} + \frac{\mathcal{H}+2\mathcal{A}}{2}\alpha_1 + \varphi T\alpha_2 = 0 , \tag{7}$$

$$\frac{d\alpha_2(r)}{dr} + \left(\frac{\mathcal{H}-2\mathcal{A}}{2} + \frac{2}{r}\right)\alpha_2 + \varphi T\alpha_1 = 0 . \tag{8}$$

with $\phi = 0$ (periodic fermions for now) we can get that $\not{D}\psi = 0$ results in

$$\alpha_1(r) = \mathrm{const}\frac{\tanh(vr/2)}{\sqrt{vr\sinh(vr)}} \sim e^{-vr/2} , \qquad \alpha_2(r) = 0 .$$

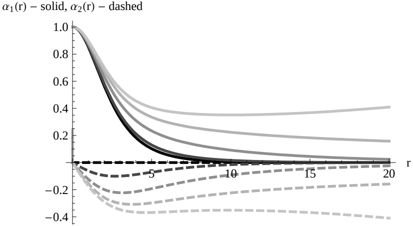
But we can do better! In fact one can solve the Dirac equation with general ϕ.

The solution turns out to be (Shuryak, Sulejmanpasic - Phys. Rev. D86 036001)

$$\alpha_{1,2}(r,\varphi) = c\,\frac{\chi_{1,2}(vr)}{\sqrt{vr\sinh(vr)}} \tag{9a}$$

$$\chi_1(rv) = \left[2\,\frac{\varphi}{v}\,\sinh(r\varphi/\beta) - \tanh(vr/2)\cosh(r\varphi/\beta)\right], \tag{9b}$$

$$\chi_2(rv) = \left[-\,2\,\frac{\varphi}{v}\,\cosh(r\varphi/\beta) + \coth(vr/2)\sinh(r\varphi/\beta)\right], \tag{9c}$$



Functions α_1 (solid) and α_2 (dashed) for $\varphi = 0$ (black) to $\varphi = 0.55v/\beta$

When $\varphi \geq v\beta/2$ the solution diverges. But since $v \in [0, \pi T]$, what happens to antieriodic fermions?

The answer lies in the second dyon! Recall that the second dyon differs from the first by a time dependent gauge transform (in stringy gauge)

$$\exp(i\pi t T \tau^3)$$

Therefore the solution has to be multiplied by an anti periodic gauge transformation! This means that the solution for anti periodic fermions has the same profiles, but with $v \to \bar{v}$

The adjoint zero modes

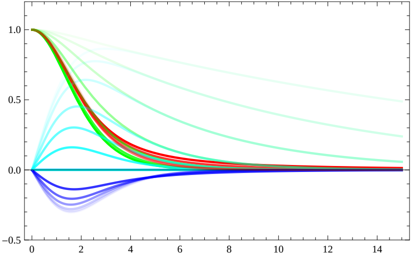
The Dirac equation:

$$\alpha_1' + \frac{2}{r}\alpha_1 - 2\frac{v}{\sinh(vr)}\beta_1 = \frac{\varphi}{\beta}\alpha_2 ,$$

$$\alpha_2' + 2\frac{v}{\sinh(vr)}\beta_2 = \frac{\varphi}{\beta}\alpha_1 ,$$

$$\beta_1' - \frac{v}{\sinh(vr)}\alpha_1 + v\coth(vr)\beta_1 = \frac{\varphi}{\beta}\beta_2 ,$$

$$\beta_2 + \frac{v}{\sinh(vr)}\alpha_2 + v\coth(vr)\beta_2 = \frac{\varphi}{\beta}\beta_1 .$$



$\varphi = (0, 0.2, 0.4, 0.6, 0.8, 0.9, 0.95)v\beta$

$$\psi_m = \hat{r}_m\alpha_2(r) - i(\hat{r}\times\vec{\sigma})_m\beta_2(r) + \alpha_1(r)\hat{r}_m(\vec{\sigma}\cdot\hat{r}) + ((\hat{r}\times\vec{\sigma})\times\hat{r})\beta_1(r)$$

and

$$\Psi_a = \psi_a\varepsilon e^{i\varphi t/\beta} \qquad \varepsilon \text{ - arbitrary 2-spinor}$$

The adjoint zero modes

The leading order

$$\Psi_M \sim e^{-(v-\varphi/\beta)r}$$

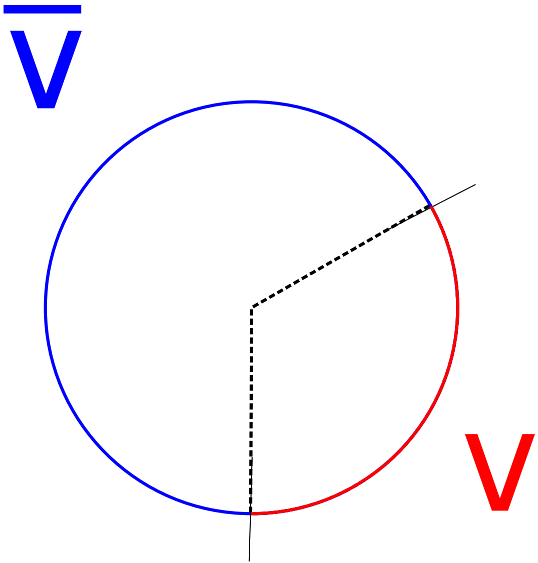
Similarly we can construct a zero mode on top of the "heavy" dyon

$$\Psi_L \sim e^{-(\bar{v}-\varphi/\beta)r} \ . \qquad \bar{v} = 2\pi - v$$

However there are also solutions with $\varphi \to \varphi - 2\pi$

$$\tilde{\Psi}_M \sim e^{-(\varphi/\beta-\bar{v})r} \qquad \tilde{\Psi}_L \sim e^{-(\varphi/\beta-v)r} \ .$$

The adjoint hopping



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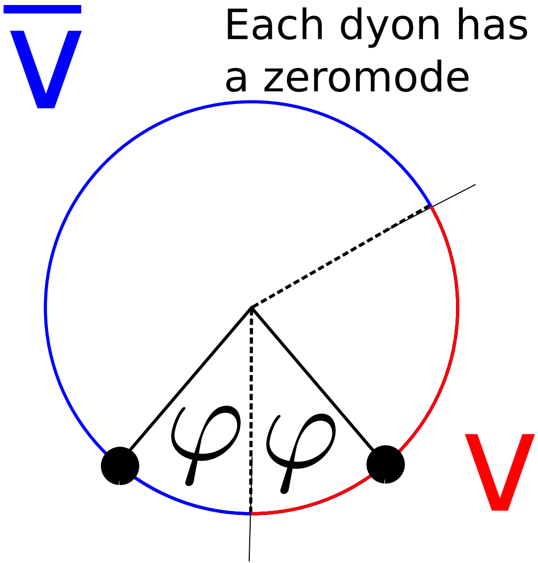

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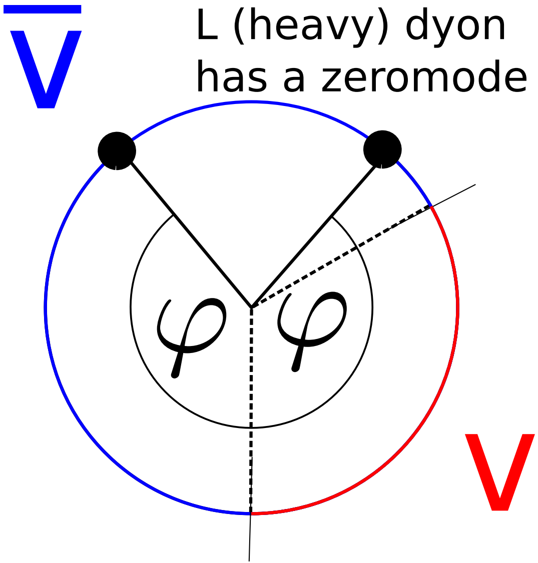

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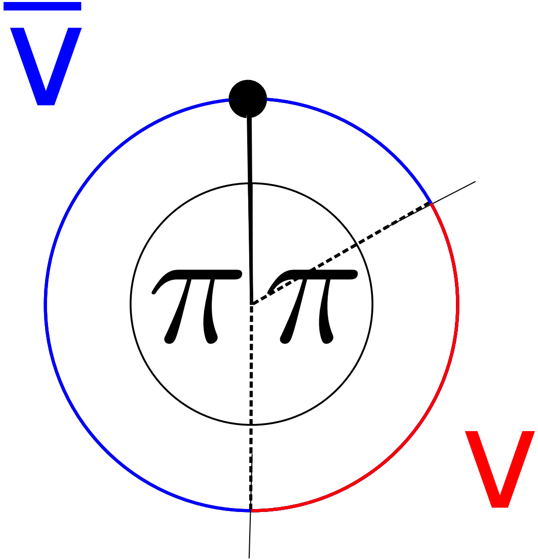
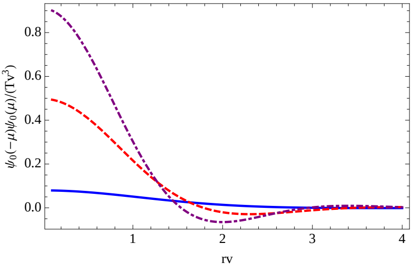


Zeromodes at finite μ (with Bruckmann and Rödl)

One needs to solve

$$(\not{D} + \mu\gamma_0)\psi = 0$$

This is the same as solving the Dirac equation with $\Psi = \psi e^{\mu t}$, so $\mu = i\varphi/\beta$. However, since the operator does not have any Hermiticity property, it turns out that one needs to redefine the bra vector and use $\psi^\dagger(-\mu)$ instead of $\psi^\dagger(\mu)$.



The full solution for the caloron is complicated, but will be published soon!

Interactions of dyons

Classical interactions

Partition function is schematically

$$Z \sim e^{-\sum S_i - \sum_{i \neq j} S_{ij}}$$

where

- S_i is the action of individual dyons or antidyons
- S_{ij} is the interaction between pairs of dyons

The self dual dyons are in fact BPS states, i.e. they do not interact classically. However dyons interact with antidyons (clearly they will annihilate to zero action if they come together)

Classical interactions

Recall that self dual dyons in SU(2) have electric and magnetic charge $(++)$ or $(--)$, and that their fields are abelian outside their cores $r > 1/v$ and $r > 1/\bar{v}$. It is then strange how these objects do not have any classical interaction, as naively one would think that they attract Coulombicaly.

Consider an action of two constituent dyons with actions $S_M = 4\pi v$ and $S_L = 4\pi \bar{v}$ where $v + v \bar{=} 2\pi$. Naively we would say that

$$S_{tot} = S_M + S_L + S_{int}(r_M, r_S) \ ,$$

where $S_{int} = \frac{1}{2} \int d^4x \left(E^M_{abel} \cdot E^L_{abel} + B^M_{abel} \cdot B^L_{abel} \right) = -\frac{8\pi\beta}{|\vec{r}_M - \vec{r}_S|}$

Classical interactions

Remember that the asymptotic "higgs" of L dyon

$$A_0 \sim v + 1/r_L$$

so close to M dyon we must replace $v \to v + 1/d$ where d is the distance between the two dyons, which replaces the action

$$S_M(v) \to S_M(v + 1/d)$$

Going to the stationary gauge of the L, we get that close to L dyon $A_0 \sim v + 1/d$, so the total action is

$$S_{tot} = 4\pi(v + 1/d)\beta + 4\pi(\bar{v} + 1/d)\beta - \frac{8\pi\beta}{d} = 8\pi^2$$

exactly the instanton action! (up to $1/g^2$ which is not written here)

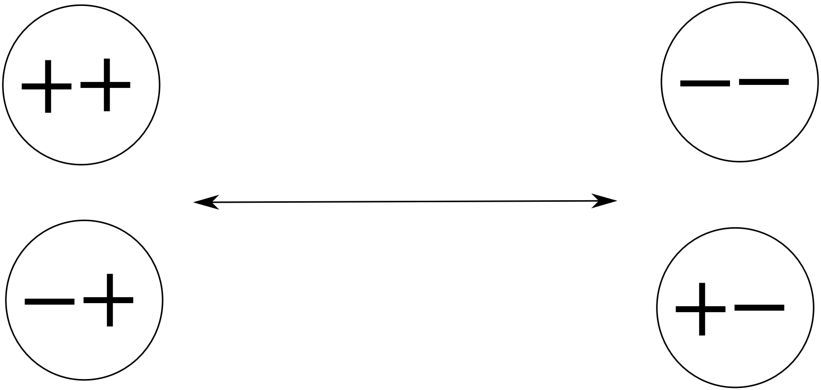
Classical interactions

The same reasoning can be applied to any combination of dyons. The self dual dyon are BPS states and they don't interact classically. For example two $(++)$ don't interact because their "higgs" fields are attractive, but their abelian fields are repulsive.

However for dyon-antidyon pair the situation is a bit different. In fact dyons of opposite magnetic charge and same electric attract coulombically with twice the strength, but of same magnetic charge repel with twice the strength due to the repulsive "higgs" field.

Classical interactions

So naively



Perturbative quantum effects

Perurbatively one has that (D.J. Gross, R.D. Pisarski and L.G. Yaffe, Rev. Mod. Phys. 53, 43 (1981).)

$$F(v) = -\ln Z(v) \sim V_3 \frac{v^2 \bar{v}^2}{3T(2\pi)^2}$$

This free energy has a minimum at $v = 0$ or at $\bar{v} = 0$. If one (perturbativellt) calculates the free energy in the presence of a KvBLL instanton, one gets (Diakonov, Petrov Phys.Rev. D70 (2004) 036003)

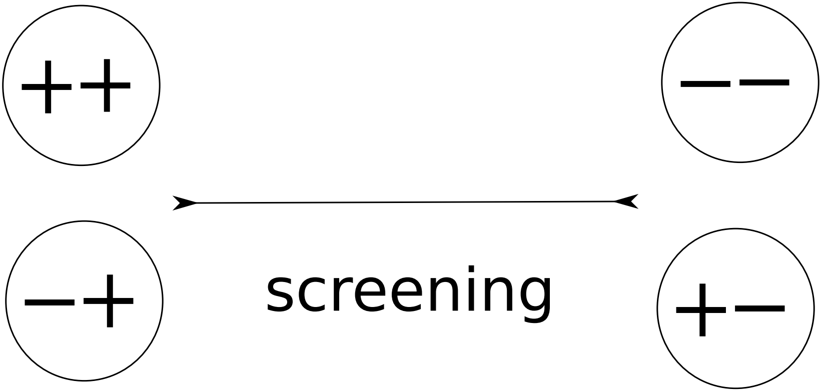
$$F_{KvBLL}(v) = VF(v) + 2\pi d F''(v) + \dots$$

where d is the distance between constituents. At trivial holonomy this is just the Pisarski-Yaffe term

$$\delta S_{PY} = \frac{4}{3}\pi^2 \rho^2 T^2 \ , \qquad d = \pi \rho^2 T$$

which, in turn, is related to Debye screening of the electric charge at finite temperature.

Screening



Zeromode facilitated interactions

The fermionic determinant for a dyon-antidyon pair

$$\det \not{D} = |T_{D\bar{D}}|^{2N_f}$$

We take it in zero mode basis

$$T_{D\bar{D}} = \int d^4x \psi^\dagger_{\bar{D}} \not{D} \psi_D \sim e^{-\bar{v}d/2}$$

because $\psi_D \sim e^{-\bar{v}r/2}$. So

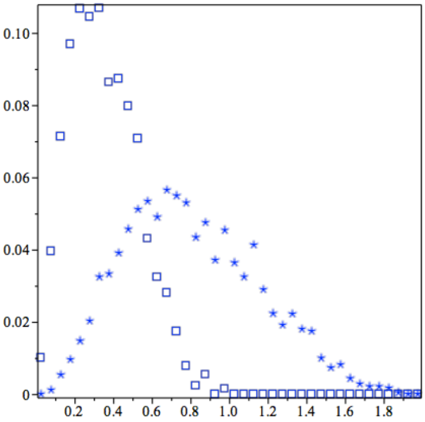
$$V_{eff}(d) = -d\frac{N_f \bar{v}}{2}$$

The dyonic molecule

We assemble all the pieces which we know for a partition function

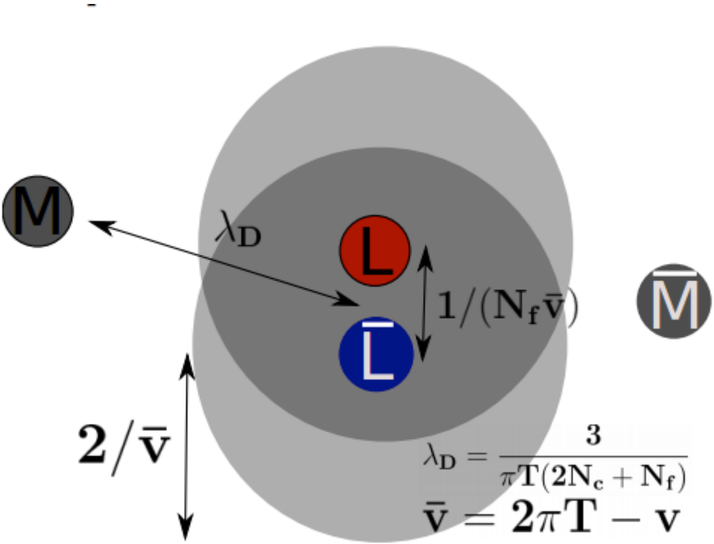
$$dZ_{mol} = dZ_{LM} dZ_{\bar{L}\bar{M}} \left[\frac{m^2 + |T_{D\bar{D}}|^2}{\Lambda^2} \right]^{N_f} C(N_f) \left(\frac{\pi^2 r_{LM} r_{\bar{L}\bar{M}} \Lambda^4}{T^2} \right)^{N_f/6} e^{-V_{scr} - V_{L\bar{L}}} \quad (10)$$

The dyonic molecule



stars – r_{LM} dyon distances
boxes – $r_{L\bar{L}}$ dyon distances

The dyonic molecule

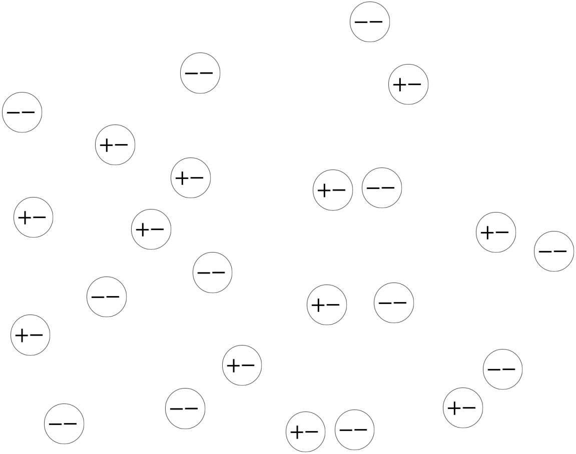


Chiral symmetry breaking via dyons

Some lattice observations

- Chiral symmetry differs for fundamental and adjoint fermions
  - For fundamental the transition is a crossover, and the chiral symmetry seems to go hand in hand with confinement
  - For adjoint fermions the confinement phase transition is at much lower temperatures then chiral symmetry breaking $T_\chi \approx 8T_c$.
- The dependence of the critical coupling $\beta_c = 6/g_c^2$ as a function of flavours N_f is inversely varying (larger N_f needs larger β_c)
- Varying periodicity conditions of fermions reveals interesting properties in the chiral condensate (Ilgenfritz, Bruckmann, Gattringer, etc.)

The ensemble of (heavy) dyons



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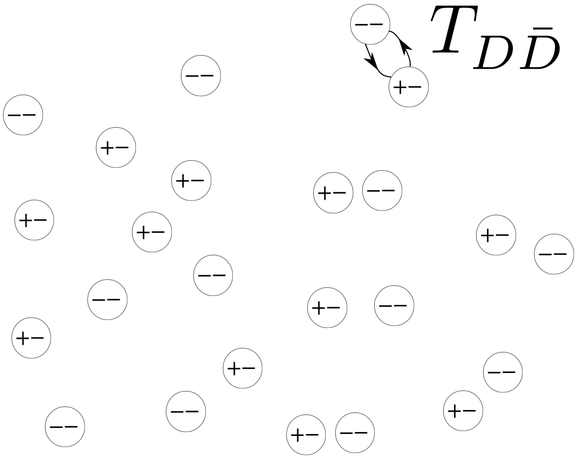


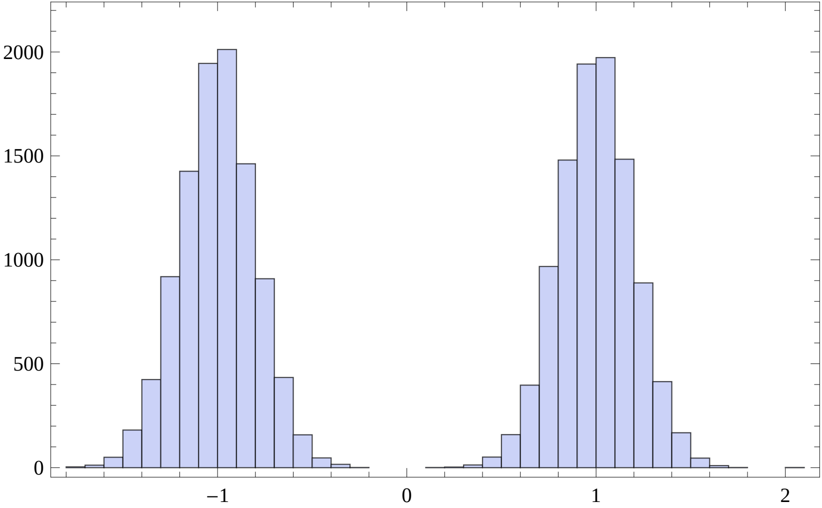
Dirac operator in zero mode basis

$$
[\not{D}] = \left(\begin{array}{cccc|cccc}
0 & \cdots & \cdots & 0 & T_{1\bar{1}} & T_{1\bar{2}} & \cdots & T_{1\bar{N}} \\
\vdots & \ddots & & \vdots & T_{2\bar{1}} & T_{2\bar{2}} & \cdots & T_{2\bar{N}} \\
\vdots & & & \vdots & \vdots & & \ddots & \vdots \\
0 & \cdots & \cdots & 0 & T_{N\bar{1}} & T_{N\bar{2}} & \cdots & T_{N\bar{N}} \\
\hline
T_{\bar{1}1} & T_{\bar{1}2} & \cdots & T_{\bar{1}N} & 0 & \cdots & \cdots & 0 \\
T_{\bar{2}1} & T_{\bar{2}2} & \cdots & T_{\bar{2}N} & \vdots & \ddots & & \vdots \\
\vdots & & \ddots & \vdots & \vdots & & & \vdots \\
T_{\bar{N}1} & T_{\bar{N}2} & \cdots & T_{\bar{N}N} & 0 & \cdots & \cdots & 0
\end{array}\right)
$$

Dirac operator with the pair ensemble

$$[\not{D}] = \left(\begin{array}{cccc|cccc}
0 & \cdots & \cdots & 0 & T_{1\bar{1}} & 0 & \cdots & 0 \\
\vdots & \ddots & & \vdots & 0 & T_{2\bar{2}} & \cdots & 0 \\
\vdots & & & \vdots & \vdots & & \ddots & \vdots \\
0 & \cdots & \cdots & 0 & 0 & 0 & \cdots & T_{N\bar{N}} \\
\hline
T_{\bar{1}1} & 0 & \cdots & 0 & 0 & \cdots & \cdots & 0 \\
0 & T_{\bar{2}2} & \cdots & 0 & \vdots & \ddots & & \vdots \\
\vdots & & \ddots & \vdots & \vdots & & & \vdots \\
0 & 0 & \cdots & T_{\bar{N}N} & 0 & \cdots & \cdots & 0
\end{array}\right)$$

Simple example: gaussian distributed pairs



The dyon ensambles

We need is to generate an ensemble. We study 3 models

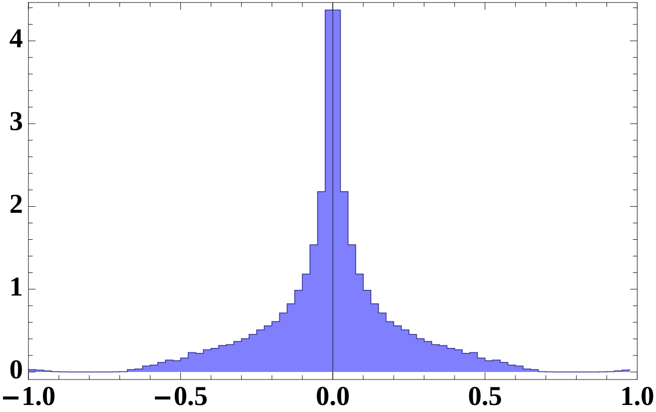
- The random dyon model RDM
- The random molecule model RMM
- The reweighed molecule model RWMM

We take that $T_{D\bar{D}} = c\frac{e^{-Mr}}{\sqrt{1+Mr}}$, and the molecules for the RMM and RWMM are generated with the distribution

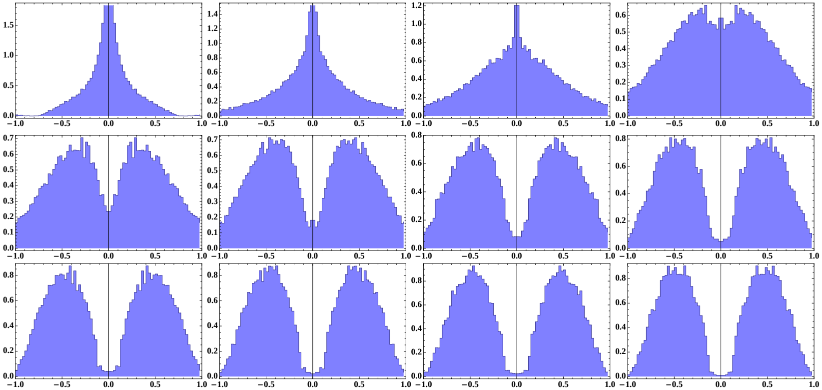
$$dist(r) = \text{norm} \times r^2 \left(\frac{e^{-Mr}}{\sqrt{1+Mr}}\right)^{2N_f}$$

where M is a parameter of our model, as well as N_f (although they are quite similar, so it is really one an the same parameter, so we set $N_f = 2$). All dimensional quantities are expressed in terms of dyon density $n = N/V$.

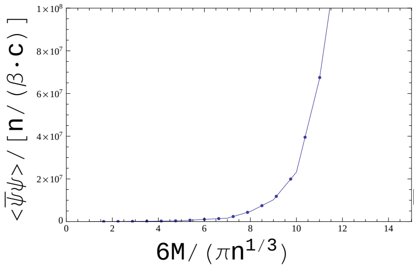
Dirac spectrum of the random dyon model

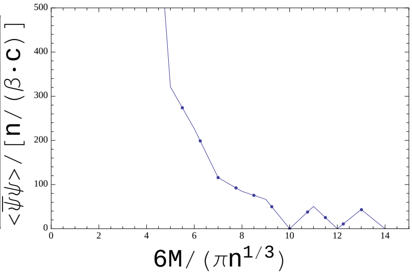


Dirac spectrum of the random molecule model



Above plots are for $\frac{M}{n^{1/3}} = (1 \ldots 12)\frac{\pi}{6}$

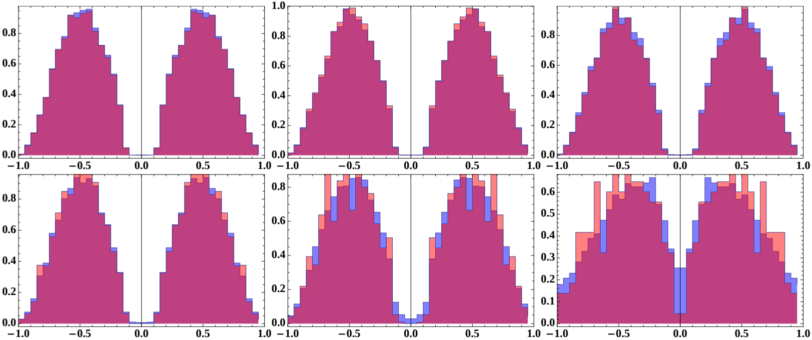




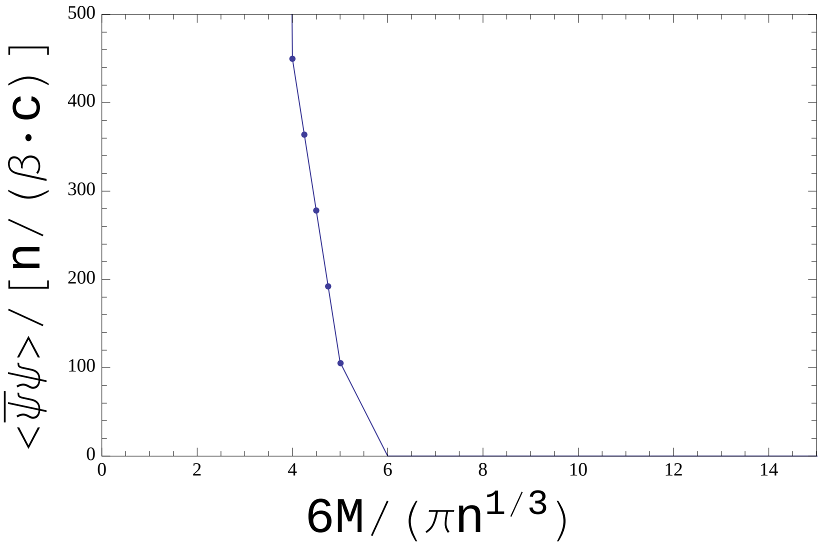
Random dyon model chiral condensate as a function of M

Random molecular model chiral condensate as a function of M

Dirac spectrum of the reweighed random molecule model



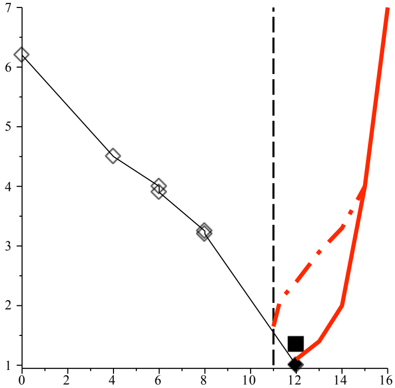
Above plots are for $\frac{M}{n^{1/3}} = (30 \ldots 5)\frac{\pi}{6}$


500
400
300
200
100
0
0
2
4
6
8
10
12
14
$<\overline{\psi}\psi> / [n/(\beta \cdot c)]$
$6M/(\pi n^{1/3})$

What is M in the dyon picture?

- For physical, anti periodic fermions $M = \bar{v}/2 = \frac{2\pi T - v}{2}$, so increasing the holonomy parameter v helps break chiral symmetry (possible connection between confinement and chiral symmetry breaking, more later)!
- For (anti periodic) adjoint fermions it turns out that $M = \pi - v$ (but there are more of them), so they have long tails close to $v = \pi$ (confining holonomy). This explains why it is more difficult to restore chiral symmetry for adjoint fermions then for fundamental.

Critical coupling as a function of N_f



K. Miura, M. P. Lombardo and E. Pallante, arXiv:1110.3152 [hep-lat].

$\beta_c = \frac{6}{g^2(T_c)}$ as a function of N_f

From our analysis

$$\langle r \rangle^3 n_c = const.$$

$$\langle r \rangle \sim \frac{1}{MN_f}$$

where n_c is critical density of dyons which $n_c \sim e^{-\frac{8\pi^2}{g^2}}$, which means that

$$\beta_c^2 = \beta_c^1 + (\dots) \ln \frac{\langle r_2 \rangle}{\langle r_1 \rangle} \approx$$

$$\approx \beta_c^1 + (\dots) \ln \frac{N_f^1}{N_f^2}$$

Some works and comments on confinement

Dyons in the vacuum present a potential explanation for confinement and they have been explored by: Diakonov & Petrov, Bruckmann et al, Unsal and Poppitz, ...
The main results are the following

- Diakonov & Petrov showed that a theory made out of only self-dual dyons in pure gluonic theory confines
- Bruckmann et. al showed that a random ensemble of dyons and antidyons of all kinds yields confinement
- Unsal, Poppitz et al explored the role of dyons in perturbative regime where they force Polyakov loop to be confining.

Large N

Why instantons cannot confine at large N!

$$Z \propto e^{-S_{inst}} = e^{-N\frac{8\pi}{\lambda}} \to 0$$

So pairs will be suppressed as pulling an instanton–anti-instanton pair out of the vacuum costs infinite action!

However dyons carry a fractional action, and at maximally nontrivial holonomy $S_{dyon} = \frac{1}{N} S_{inst}$. So pulling a dyon pair out of the vacuum costs $\sim 2 \times \frac{8\pi}{\lambda}$, i.e. they are suppressed as $e^{-2\times\frac{8\pi}{\lambda}}$, so at strong coupling they become quite common fluctuations!

Conclusion

- Dyons in the vacuum appear as pair fluctuations of dyon-antidyon pairs, suppressed by their action (smallness of coupling)
- The pairs by themselves do not break chiral symmetry, but generate Dirac spectra with two bumps around 0 eigenvalues
- As the density of dyons becomes larger the gap between the bumps closes and develops eigenvalue density at zero, breaking chiral symmetry
- The parameter which does this is the holonomy, and it has qualitatively different behaviour when fermions are fundamental and adjoint, as well as when they are periodic and anti periodic
- Qualitative behaviour of adjoint chiral transition being pushed to much larger temperature is natural from the point of view of adjoint zero modes

Outlook and some things not mentioned

- Back reaction of fermionic zero modes on the holonomy (preference of nontrivial holonomy)
- Large N contributions of dyons
- Role of dyons at finite chemical potential
- Detailed interactions of dyons (string interactions)
- Moduli space interactions (very important!)
- Magnetic field and the "hairs" of topological charge
- Magnetic field and additional zero modes (on top of instantons Basar, Kharazeev)
- Full scale simulations of the dyon ensemble (first simulations soon by E. Shuryak and ...)
- Correlation functions, masses of hadrons, etc.