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Anisotropy of thermal photons and dileptons

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PHENIX Collaboration (Phys. Rev. Lett. 104, 132301 (2010)):

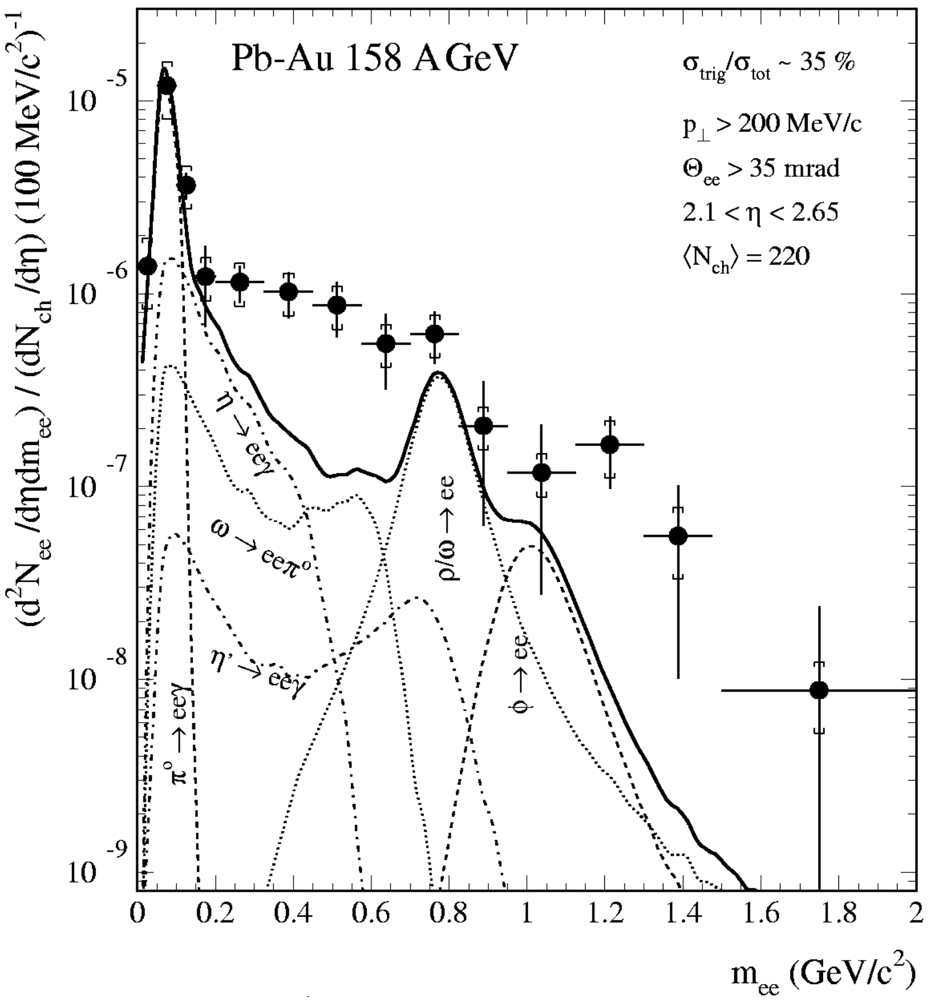
In central Au+Au collisions, the excess of direct photon yield over p+p is exponential in transverse momentum, with inverse slope **T = 221** +/- 19 (stat) +/- 19 (syst) MeV. Hydrodynamical models with initial temperatures ranging from **300–600 MeV** at times of 0.6 - 0.15 fm/c after the collision are in qualitative agreement with the data.

PHENIX Collaboration (Phys. Rev. Lett. 109, 122302 (2012)):

The second Fourier component v(2) of the azimuthal anisotropy with respect to the reaction plane is measured for direct photons at midrapidity and transverse momentum (p(T)) of 1-12 GeV/c in Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV. .... in the p(T) $\leq$ 4 GeV/c region dominated by thermal photons, we find **a substantial direct-photon v(2) comparable to that of hadrons**, whereas model calculations for thermal photons in this kinematic region underpredict the observed v(2).

A serious contradiction with expected dominance of photon production from QGP

The excess has been observed for the first time in CERN (special seminar: 10 February, 2000 —— QGP (?)) (CERES, Phys. Lett. B 422, 405 (1998)).



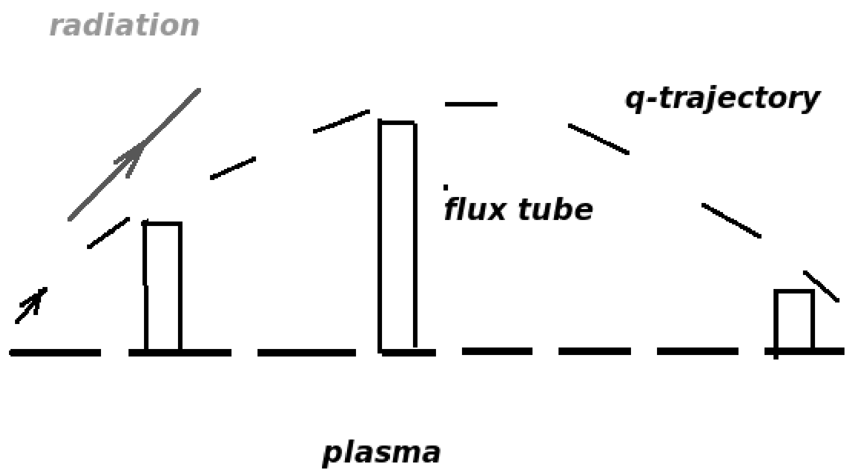
Our explanation of this PHENIX (+ ALICE now) puzzle :

Intensive radiation of magnetic bremsstrahlung type (synchrotron radiation) resulting from the interaction of escaping quarks with the collective confining colour field is discussed as a new possible mechanism of observed direct photon anisotropy.

Theoretically, the basic conditions to have such a radiation available are easily realized as:
1 — the presence of relativistic light quarks (u and d quarks) in QGP;
2 — the semiclassical nature of their motion;
3 — confinement.

Then as a result, each quark (antiquark) at the boundary of the system volume moves along a curve trajectory and (as any classical charge undergoes an acceleration) emits photons.

The interaction of escaping quarks with the collective confining color field (in the chromo-electric flux tube model):



Confinement $\rightarrow$ a constant restoring force $\sigma \simeq 0.2$ Gev^2 directed along the normal to the QGP surface.

Model calculations (Yad. Fiz.; Z. Phys. C; Phys. Lett. B (1988)):

A large value of σ results in the large magnitude of characteristic parameter

$$\chi = ((3/2)\sigma E/m^3)^{1/3}$$

(where E and m are the energy and mass of the emitting particle, respectively) for u and d quarks (the strong-field case). In this regime the probability of emitted photons is independent of the mass of the emitting particle and

$$\frac{dN}{d\omega dt} = 0.52 e_q^2 \alpha \omega^{-2/3} (\sigma \sin\varphi / E)^{2/3}, \qquad 0 \le \omega < E,$$

where $\alpha = 1/137$ is the fine structure constant, e_q is the quark charge in units of electron charge and φ is the angle between the quark velocity and the direction of quark confining force (the normal to the QGP surface in our case).

We assume that at each instant of time the direction of the emitted photons coincides with the direction of the quark velocity (since an ultrarelativistic particle emits photons at small (m/E) angles around the instantaneous direction of the velocity)

$$\frac{dN_\gamma}{d\omega d\Omega} = \int\limits_{-p_{z0}/\sigma}^{p_{z0}/\sigma} dt \frac{0.52 e_q^2 \alpha \sigma^{2/3}}{\omega^{2/3}} \frac{\sin^{2/3}\varphi(t)}{p^{2/3}(t)} \delta(\mathrm{n} - \mathrm{v}(t)) \theta[\omega < p(t)],$$

$\mathrm{v}(t)$ is the quark velocity,
n is the unit vector along the photon momentum and

$$p(t) = (p_x^2 + p_y^2 + p_z^2)^{1/2}, \quad \sin\varphi(t) = (p_x^2 + p_y^2)^{1/2}/p(t)$$

Knowing the law of motion

$$p_z = \sigma t, \quad p_y = p_{y0}, \quad p_x = p_{x0}, \quad -p_{z0}/\sigma \le t \le p_{z0}/\sigma$$

and multiplying by the flux of quarks reaching the surface and integrating over all quarks initial momenta, we obtain

$$\frac{dN_\gamma}{dS dt \omega^2 d\omega d\Omega} = \frac{1.04 g \langle e_q^2 \rangle \alpha}{(2\pi)^3 \sigma^{1/3}} \ \frac{3}{7} \omega^{2/3} \sin^{2/3}\varphi_0 \int\limits_1^\infty d\xi \exp\Big(-\frac{\omega}{T} \xi \Big) (\xi^{7/3} - 1),$$

where $\langle e_q^2 \rangle = e_u^2 + e_d^2$**,** e_u **and** e_d **are the** u**- and** d**-quark charges,**
$g =$ **spin** $\times$ **color** $=$ **6 is the number of quark degrees of freedom,**
T **is the plasma temperature.**
φ_0 **is the angle between the normal to QGP surface and the direction of emitted photons.**

Evaluating the integrals over $d\omega$ **and** $d\Omega$**, we obtain the number of photons emitted per unit time from unit surface area:**

$$\boldsymbol{dN_\gamma/dSdt = A\langle e_q^2\rangle \alpha T^{11/3}\sigma^{-1/3},}$$

$A = 3.12g \cdot 2^{5/3}\Gamma^2(4/3)/(2\pi)^2 \simeq 1.2$**,** Γ **is the gamma function.**

In the simplest case, if the plasma occupies a spherical volume of radius R and does exist during the time τ, then the total number of photons is easy estimated as

$$N_\gamma = \frac{dN_\gamma}{dSdt} 4\pi R^2 \tau = \frac{4}{3}\pi R^3 \tau \alpha T^4 B' \frac{1}{RT^{1/3}\sigma^{1/3}}, \quad B' = 3A\langle e_q^2 \rangle.$$

In this case the "standard"("Compton scattering of gluons", $gq \to \gamma q$ and annihilation of quark-antiquark pairs, $q\bar{q} \to \gamma g$) mechanism for photon emission gives:

$$N_{st}^\gamma = \frac{4}{3}\pi R^3 \tau \alpha T^4 B, \quad B \simeq \frac{5}{144}\pi \alpha_s \ln\frac{1}{\alpha_s} \, .$$

Then the relevant quantity is the ratio

$$\frac{N_{\text{surface}}^\gamma}{N_{\text{volume}}^\gamma} = \frac{B'/B}{RT^{1/3}\sigma^{1/3}} \, .$$

This result is still valid when the space-time plasma evolution (Bjorken) has been included

$$\frac{\boldsymbol{N^{\gamma}_{\text{surface}}}}{\boldsymbol{N^{\gamma}_{\text{volume}}}} = \frac{\textbf{const}}{\boldsymbol{r T_c^{1/3} \sigma^{1/3}}},$$

where T_c is the phase-transition temperature, r is the transverse size of cylindrically symetric plasma volume with the longitudinal expansion, $\sigma \simeq 0.2$ **Gev**^{2} is the quark confining force. Volume photons come from the channels $gq \to \gamma q$, $q\bar{q} \to \gamma g$.

Taking into account the value of constant we find

$$N^{\gamma}_{\text{surface}}/N^{\gamma}_{\text{volume}} \approx 2 \textbf{ at } r = 10 \textbf{ fm} \ .$$

The similar estimation can be obtained for hard enough photons also in analytical form.

Obviously, the photon emission from the surface mechanism of non-central ion collisions is nonisotropic. Indeed, photons are emitted mainly around the direction determined by the normal to the ellipsoid-like surface.

In the transverse (x-y) plane (the beam is running along (z)-axis) the direction of this normal (emitted photons) is determined by the spatial azimuthal angle $\phi_s = \tan^{-1}(y/x)$ as

$$\tan(\phi_\gamma) = (R_x/R_y)^2 \tan(\phi_s).$$

The shape of quark-gluon system surface in transverse plane is controlled by the radii $R_x = R\sqrt{1-\epsilon}$ and $R_y = R\sqrt{1+\epsilon}$ with the eccentricity $\epsilon = b/2R_A$ (b is the impact parameter, R_A is the radius of the colliding (identical) nuclei).

The photon azimuthal anisotropy can be characterized by the second Fourier component

$$v_2^\gamma = \frac{\int d\phi_\gamma \cos(2\phi_\gamma)(dN^\gamma/d\phi_\gamma)}{\int d\phi_\gamma(dN^\gamma/d\phi_\gamma)}$$

and is proportional to the "mean normal"

$$v_2^\gamma \propto \frac{\int d\phi_s \cos(2\phi_\gamma)}{2\pi} = \epsilon.$$

Summarizing we would like to maintain positively that the surface mechanism of photon production is intensive enough, develops the azimuthal anisotropy and is capable of resolving the PHENIX direct photons puzzle still without appealing to the non-equilibrium dynamics of heavy ion collision process.

One of the most distinctive features of the proposed mechanism is a large degree of photon polarization:

$$\frac{dN_1}{d\omega dt} = \frac{1}{4}\frac{dN_0}{d\omega dt}, \qquad \frac{dN_2}{d\omega dt} = \frac{3}{4}\frac{dN_0}{d\omega dt}, \qquad \frac{dN_l}{d\omega dt} = \frac{1}{2}\frac{dN_0}{d\omega dt},$$

$$\frac{dN_0}{d\omega dt} = 0.52 e_q^2 \alpha \omega^{-2/3} (\sigma \sin\varphi / E)^{2/3}.$$

$l = 1$ describes a right-handed circularly polarized photons,
$l = -1$ describes a left-handed circularly polarized photons,
N_1 corresponds to linear polarization of photons along the vector e_1,
N_2 corresponds to linear polarization of photons along the vector e_2,

$$\mathbf{e}_1 = \frac{[\boldsymbol{\sigma}\mathbf{k}]}{|[\boldsymbol{\sigma}\mathbf{k}]|}, \qquad \mathbf{e}_2 = \frac{[\mathbf{k}\mathbf{e}_1]}{|[\mathbf{k}\mathbf{e}_1]|},$$

k is the photon momentum.

After integration over the surface these photons are dominantly polarized along the normal to the plane spanned by the collision axis and the momentum of registerted photons with high enough degree of polarization:

$$\delta = 50\%(initial) \qquad \rightarrow \delta \simeq 20\%.$$

The appearance of such a polarization is closely connected with the direction of the collective confining color field where quarks are moving and its value is virtually insensitive to the parameter regulating an intensity of bremsstahlung.

Many problems for experimental search for this effect, but observing lepton-pair spectra resulting from the polarization of intermediate photon could be a potentially efficient probe of the collective confining colour field.

Lepton-pair radiation

Again in the regime of strong-field the probability of emitting a "massive" photon is independent of the mass of the emitting particle and in the first order in inverse powers of the parameter χ can be written as

$$dW_\gamma(M^2)/dt = 1.56 e_q^2 \alpha (\sigma \sin\varphi)^{2/3} E^{-1/3}.$$

Using the well-known relation between the cross sections for virtual-photon and lepton-pair production, we easily find the lepton-pair distribution in the invariant mass:

$$\frac{dN}{dt dM^2} = \frac{\alpha}{3\pi} f(M) \frac{dW_\gamma(M^2)}{dt},$$

$$f(M) = \frac{1}{M^2}\left(1 + \frac{2\mu^2}{M^2}\right)\left(1 - \frac{4\mu^2}{M^2}\right)^{1/2}, 2\mu \leq M \leq E.$$

Further, in order to obtain the number of lepton pairs radiated per unit surface area of QGP per unit time in invariant mass interval M^2, $M^2 + dM^2$, it is necessary to average Eq. above over the quark paths and to convolute it with the flux of quarks reaching the boundary of the QGP volume from within. This procedure does not differ from the analogous one performed in detail for photons spectra, so we present only the final result here:

$$\frac{dN}{dSdtdM^2} = A\alpha^2\sigma^{-1/3} f(M) M^{11/3} \int_1^\infty d\xi(\xi^{8/3} - 1)\exp\left(-\frac{M\xi}{T}\right),$$

where

$$A = \frac{1.56}{2(2\pi)^3} \frac{\Gamma(4/3)\Gamma(1/2)}{\Gamma(11/6)} g(e_u^2 + e_d^2).$$

The total number of lepton pairs emitted per unit time from unit surface area of QGP is estimated as (at $\beta \ll 1$)

$$\frac{dN}{dSdt} \simeq 2A\Gamma\left(\frac{11}{3}\right)\alpha^2\sigma^{-1/3} T^{11/3}[\ln(T/2\mu) + a + O(\beta^2)],$$

$$a = \ln 2 - 5/6 + \Gamma'(8/3)/\Gamma(8/3), \quad \beta = 2\mu/T.$$

This is a reasonable estimate of the total number of electron-positron pairs, since $\mu_e \simeq$ **0.5 MeV** is considerably less than the minimal plasma temperature $T \simeq$ **200 MeV**.

In the simplest case, if the plasma occupies a spherical volume of radius R and does exist during the time τ, then the total number of electron-positron pairs is easy estimated as

$$\boldsymbol{N = 4\pi R^2 \tau dN/dSdt.}$$

Of course, it is interesting to compare this result with the total number of electron-positron pairs produced by "standard" quark-antiquark annihilation processes in the QGP volume

$$\boldsymbol{N_{ann} = \frac{4}{3}\pi R^3 \tau B \alpha^2 T^4, \qquad B = 10/9\pi^3.}$$

Then the relevant quantity is the ratio

$$\boldsymbol{\frac{N}{N_{ann}} = \frac{C}{RT^{1/3}\sigma^{1/3}}[\ln(T/2\mu) + a],}$$

where $C = 6\Gamma(11/3)A/B \approx 11.8$. Numerically $N/N_{ann} \simeq$**40** on setting $R = 1$ fm , and $N/N_{ann} \simeq$**4** on setting $R = 10$ fm at $T \simeq 200$ MeV.

This result is still valid when the space-time plasma evolution has been included

$$\frac{N^{l^+l^-}_{\rm surface}}{N^{l^+l^-}_{\rm volume}} = \frac{\rm const}{rT_c^{1/3}\sigma^{1/3}}[\ln(T_c/2\mu)+a],$$

where T_c **is the phase-transition temperature,** r **is the transverse size of cylindrically symetric plasma volume with the longitudinal expansion.**
Volume leptons come from the quark-antiquark annihilation processes.

Taking into account the value of constant we find

$$N^{e^+e^-}_{\rm surface}/N^{e^+e^-}_{\rm volume} \approx 4 \textbf{ at } r = 10 \textbf{ fm .}$$

The ratio of the invariant mass spectra can be also calculated with similar estimation.
For muon pairs we have the similar estimation also.

Angular distribution

Considering the decay of massive photons with the four-momentum k into a lepton pair, the following expression gives the squared matrix element of this process:

$$|M|^2 = 4\pi\alpha Sp[(\hat{p}_1 + \mu)\gamma_\mu(\hat{p}_2 - \mu)\gamma_\nu]e_\mu e^*_\nu$$

$$= 16\pi\alpha[k^2/2 + (p_1 e)(p_2 e^*) + (p_1 e^*)(p_2 e)],$$

where e is the polarization four-vector of the photon and $(ee^*) = -1$; p_1 and p_2 are the four-momenta of the lepton and antilepton, respectively.

Drawing the relevant phase space of the pair and taking into account the transversality condition $(ek) = 0$, the lepton distribution per unit time in the radiation angle reads as

$$\frac{dW}{dt d\Omega_1} = \frac{\alpha}{2\pi k^0}\int \frac{p^2 dp}{p_1^0(k^0 - p_1^0)}\delta[f(p)][k^2/2 - 2(p_1 e)(p_1 e^*)],$$

where

$$f(p) = k^0 - p_1^0 - (\mu^2 + \mathbf{k}^2 + p^2 - 2|\mathbf{k}|p\cos\theta_1)^{1/2}.$$

If the initial photons are unpolarized, Eq. above has to be averaged over polarization and then it results to the lepton distribution independent of the radiation azimuthal angle ϕ_1. This dependence exists at decay of the polarized photons. Defining $n(1+\delta)/3$ as the photon number of the states with polarization vector e_1, $n(1-\delta)/3$ as the photon number of the states with polarization vector e_2 and $n/3$ as the same with polarization vector e_3, and choosing the reference frame with the z axis directed along the three-vector k and the x and y axes tallying with the directions of $\mathbf{e_1}$ and $\mathbf{e_2}$, we have then

$$e_1 = \{0,\ 1,\ 0,\ 0\},\ e_2 = \{0,\ 0,\ 1,\ 0\},$$

$$e_3 = \{|\mathbf{k}|/\sqrt{k^2},\ 0,\ 0,\ k^0/\sqrt{k^2}\},\ k = \{k^0,\ 0,\ 0,\ |\mathbf{k}|\},$$
$$p_1 = \{\sqrt{p^2+\mu^2},\ p\sin\theta_1\cos\phi_1,\ p\sin\theta_1\sin\phi_1,\ p\cos\theta_1\}.$$

Finally, the lepton distribution in the radiation angle takes the form

$$\frac{dN}{dtd\Omega_1} = \frac{\alpha n}{2\pi k^0}\int\frac{p^2dp}{p_1^0(k^0-p_1^0)}\delta[f(p)][\frac{k^2+2\mu^2}{3} - \frac{2}{3}\delta p^2\sin^2\theta_1\cos2\phi_1].$$

For a lepton pair with momenta p_1 and p_2 one should measure the angle distribution of leptons with respect the spatial momentum $\vec{k} = \vec{p}_1 + \vec{p}_2$
(or the orientation of vector $\vec{q} = \vec{p}_1 - \vec{p}_2$ with respect to $\vec{k}$).

The degree of deviation from symmetric angular distribution is regulated by the degree of polarization of intermediate photons which is not small

$$\boldsymbol{\delta \simeq 20\%}$$

and is closely related to the geometry of the plasma volume.

In our case, the intermediate photons (as we have a strong field regime) could be considered up to the masses $\sqrt{k^2} \simeq \sqrt{\sigma} = 0.45$ GeV as having a small virtuality and their properties are quite close to real photons

Conclusions

The interaction of quarks with the collective color field confining them results in an intensive radiation of the magnetic bremsstrahlung type (synchrotron radiation). The intensity of such a radiation for the hot medium of size 1-10 fm that is expected in ultrarelativistic collisions of heavy ions is comparable with that of the volume mechanism of photon and di-lepton production in the temperature range of $T = 200 - 500$ **MeV.**

Quantitavelly an effect is regulated by the three basic parameters: the characteristic medium (QGP) size R**, the QGP temperature** T**, and the confining force** σ**, which are firmly fixed.**

$$(\boldsymbol{R T_c^{1/3} \sigma^{1/3}})^{-1}$$

Possible uncertainties come mainly from the simple modeling of confinement and simplification of the QGP geometry what allow us to obtain estimates in transparent analytical form.

The most striking feature of magnetic bremsstrahlung is the high degree ($\sim 20\%$) of polarization of both real and "massive" (virtual) photons that is mainly determined by the medium (QGP) geometry.

The virtual photons develop the noticeable specific anisotropy in the angle distribution of leptons with respect to the three-momentum of pair. The origin of this anisotropy is rooted in the existence of a characteristic direction in the field where the quarks are moving.

Besides the synchrotron radiation will be nonisotropic for the non-central collisions because the photons are dominantly emitted around the direction fixed by a surface normal. As result the coefficient of elliptic anisotropy for di-lepton pairs will be also proportional to the eccentricity of QGP system as it takes place for the bremsstrahlung real photons and can be experimentally measured.