

Space-time picture and bulk observables in relativistic  
heavy ion collisions  
(HydroKinetic approach)



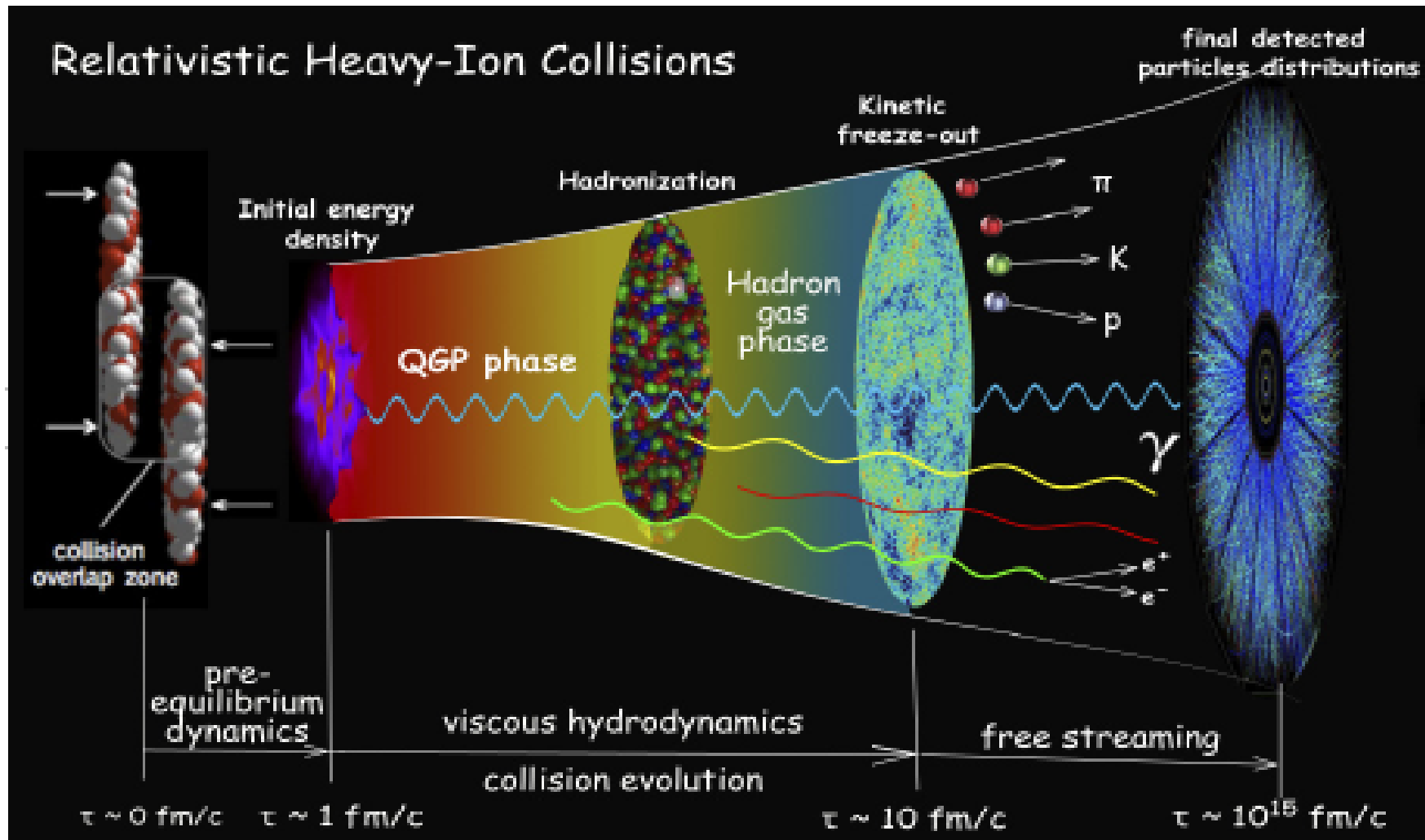
---

Yu.M. Sinyukov (BITP, Kiev)

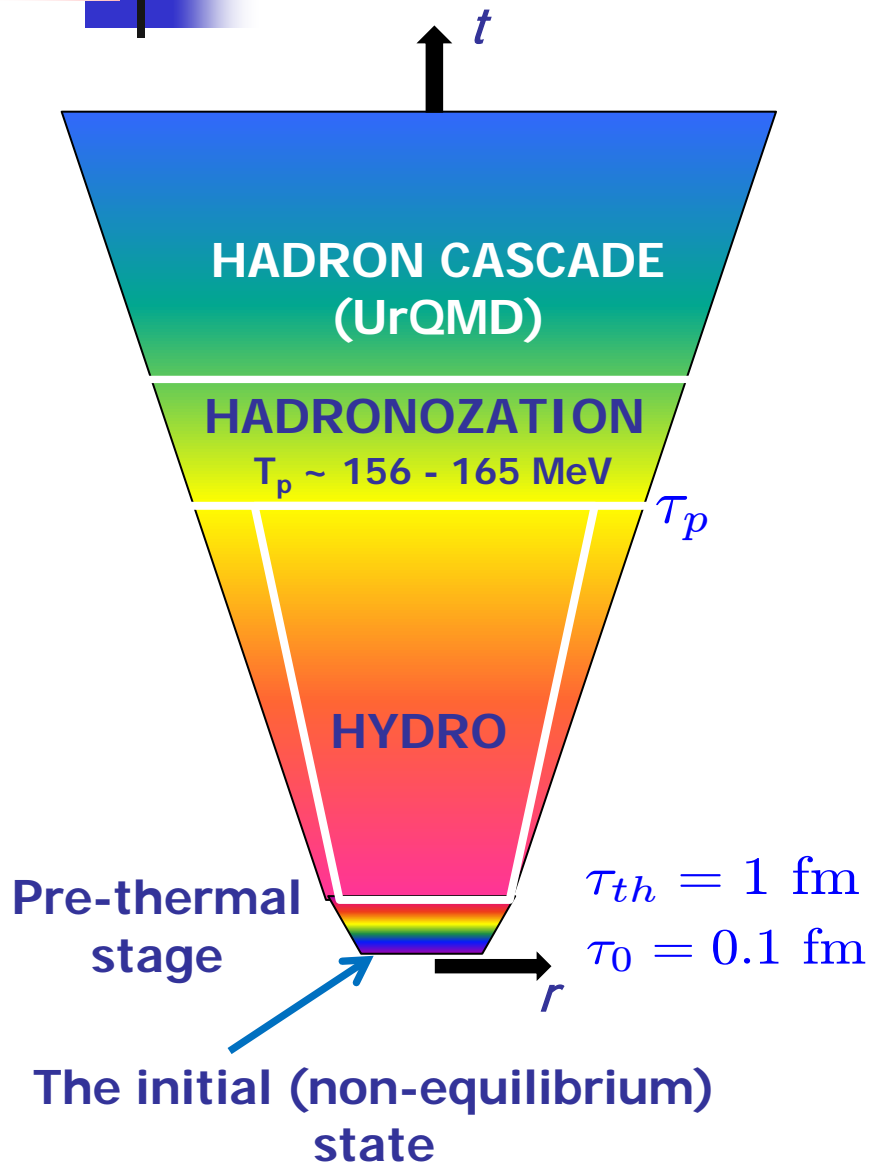
Seminar BLTP  
*6 June , 2019*

# The stages of the matter evolution in A+A collisions

The initial huge kinetic energy of colliding nuclei converts into masses of the final observed particles (several tens of thousands) + the energy of collective flow



# Integrated HydroKinetic Model: HKM $\rightarrow$ iHKM



**Complete algorithm incorporates the stages:**

- generation of the initial states: (MC Glaub & CGC)
- thermalization of initially non-thermal matter;
- viscous chemically equilibrated hydrodynamic expansion;
- particlization of expanding medium in the hadronization area ;
- a switch to UrQMD cascade with near equilibrium hadron gas as input;
- simulation of observables.

Yu.S., Akkelin, Hama: PRL 89 (2002) 052301;

... + Karpenko: PRC 78 (2008) 034906;

Karpenko, Yu.S. : PRC 81 (2010) 054903;

... PLB 688 (2010) 50;

Akkelin, Yu.S. : PRC 81 (2010) 064901;

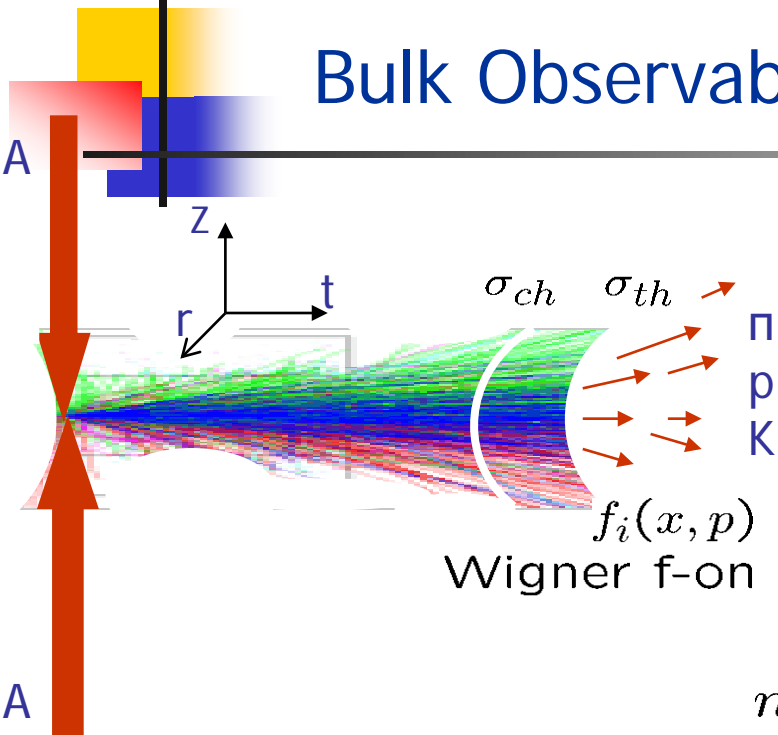
Karpenko, Yu.S., Werner: PRC 87 (2013) 024914;

Naboka, Akkelin, Karpenko, Yu.S. : PRC 91 (2015) 014906;

Naboka, Karpenko, Yu.S. PRC 93 (2016) 024902.

# Bulk Observables, "Soft Physics" measurements

$T_{f.o.}$   
**Landau, 1953**  
 $\sigma_{f.o.}$



$$N_i = \int_{\sigma_{ch}} \frac{d^3 p}{p^0} d\sigma_{\mu} p^{\mu} f_i(x, p)$$

$$n_i(p) \equiv p^0 \frac{d^3 N_i}{d^3 p} = \int_{\sigma_{th}} d\sigma_{\mu} p^{\mu} f_i(x, p) \sim e^{-\sqrt{m_i^2 + p_T^2} / T_{eff,i}}$$

$$n_i(p_1, p_2) \equiv p_1^0 p_2^0 \frac{d^6 N_i}{d^3 p_1 d^3 p_2} = C(p, q) n(p_1) n(p_2)$$

$$p = (p_1 + p_2) / 2$$

$$q = p_1 - p_2$$

$\left\{ \frac{N_i}{N_j} \right\} \rightarrow T_{ch}$  and  $\mu_{ch}$  soon after hadronization (chemical f.o.)

$$\frac{d^3 N}{dp, dy d\varphi} = \frac{d^2 N}{dp, dy} \frac{1}{2\pi} (1 + 2v_1 \cos(\varphi) + 2v_2 \cos(2\varphi) + \dots)$$

Directed flow

Elliptic flow

Radial flow

$$T_{eff,i} \approx T_{f.o.} + m_i \frac{\langle v_T^2 \rangle}{2}$$

Inverse of spectra slope

QS correlation function

$$1 + \lambda \exp(-R_L(p)^2 q_L^2 - R_S(p)^2 q_S^2 - R_O(p)^2 q_O^2)$$

Radii  $R_i$ ,  $i=Long, Side, Out$

3D geometrical system sizes

# The iHKM parameters (at Laine-Shroeder EoS example)

The  $\frac{dN_{ch}}{d\eta}(c)$  is OK at fixed relative contribution of binary collision  $\alpha = 0.24$ .

but at different max initial energy densities when other parameters change:

The two values of the shear viscosity to entropy is used for comparison:

$$\eta/s = 0.08 \approx \frac{1}{4\pi} \text{ and } \eta/s = 0.2$$

The basic result (selected by red) is compared with results at other parameters, including viscous and ideal pure thermodynamic scenarios (starting at  $\tau_0$  without pre-thermal stage but with subsequent hadronic cascade).

Model	$\Lambda$	$\tau_{rel}$	$\eta/S$	$\tau_0$	$\langle \frac{\chi^2}{ndf} \rangle$	$\epsilon_0$ (GeV/fm <sup>3</sup> )
Hydro			0	0.1	5.16	1076.5
Hydro			0.08	0.1	6.93	738.8
iHKM	1	0.25	0.08	0.1	3.35	799.5
iHKM	100	0.25	0.08	0.1	3.28	678.8
iHKM	100	0.75	0.08	0.1	3.52	616.5
iHKM	100	0.25	0.2	0.1	6.61	596.9
iHKM	100	0.25	0.08	0.5	5.36	126.7

$\tau_{ol} \approx 0.07, 0.005, 0.003$  fm/c for energies RHIC  $\sqrt{s_{NN}} = 200$  GeV,

LHC  $\sqrt{s_{NN}} = 2.76$  TeV and  $\sqrt{s_{NN}} = 5.02$  TeV correspondingly.

Slope of the pion spectra  $\longrightarrow \tau_0 = 0.1$  fm/c  $> \tau_{ol}$ <sup>5</sup>

# The iHKM parameters (at Laine-Shroeder EoS example)

The  $\frac{dN_{ch}}{d\eta}(c)$  is OK at fixed relative contribution of binary collision  $\alpha = 0.24$ .

but at different max initial energy densities when other parameters change:

The two values of the shear viscosity to entropy is used for comparison:

$$\eta/s = 0.08 \approx \frac{1}{4\pi} \text{ and } \eta/s = 0.2$$

The basic result (selected by red) is compared with results at other parameters, including viscous and ideal pure thermodynamic scenarios (starting at  $\tau_0$  without pre-thermal stage but with subsequent hadronic cascade).

Model	$\Lambda$	$\tau_{rel}$	$\eta/S$	$\tau_0$	$\langle \frac{\chi^2}{ndf} \rangle$	$\epsilon_0$ (GeV/fm <sup>3</sup> )
Hydro			0	0.1	5.16	1076.5
Hydro			0.08	0.1	6.93	738.8
iHKM	1	0.25	0.08	0.1	3.35	799.5
iHKM	100	0.25	0.08	0.1	3.28	678.8
iHKM	100	0.75	0.08	0.1	3.52	616.5
iHKM	100	0.25	0.2	0.1	6.61	596.9
iHKM	100	0.25	0.08	0.5	5.36	126.7

$\tau_{ol} \approx 0.07, 0.005, 0.003$  fm/c for energies RHIC  $\sqrt{s_{NN}} = 200$  GeV,

LHC  $\sqrt{s_{NN}} = 2.76$  TeV and  $\sqrt{s_{NN}} = 5.02$  TeV correspondingly.

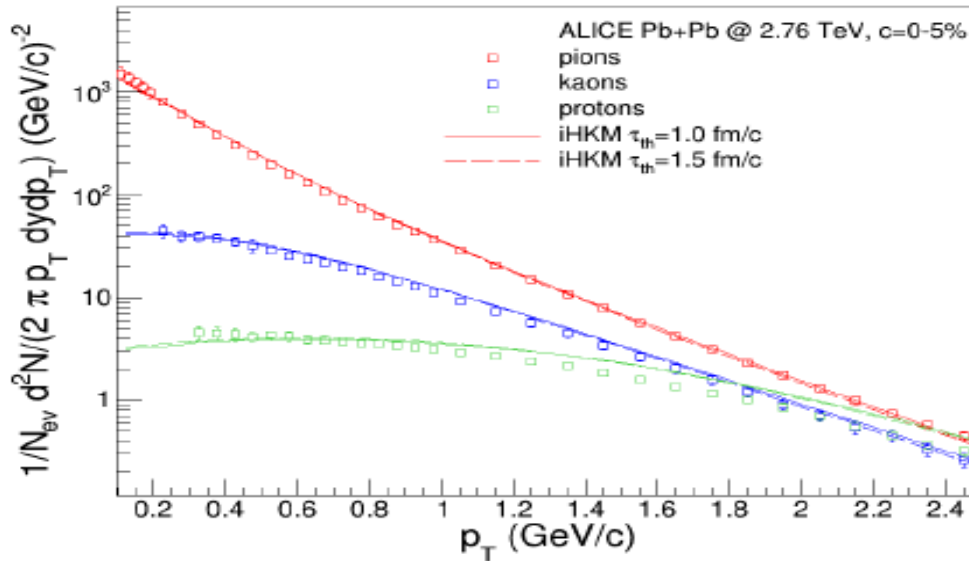
Slope of the pion spectra  $\rightarrow \tau_0 = 0.1$  fm/c  $> \tau_{ol}$

No dramatic worsening of the results happens if simultaneously with changing of parameters/scenarios renormalize maximal initial energy density  $\epsilon_0(\tau_0)$ .

# The sensitivity of the results to the model parameters

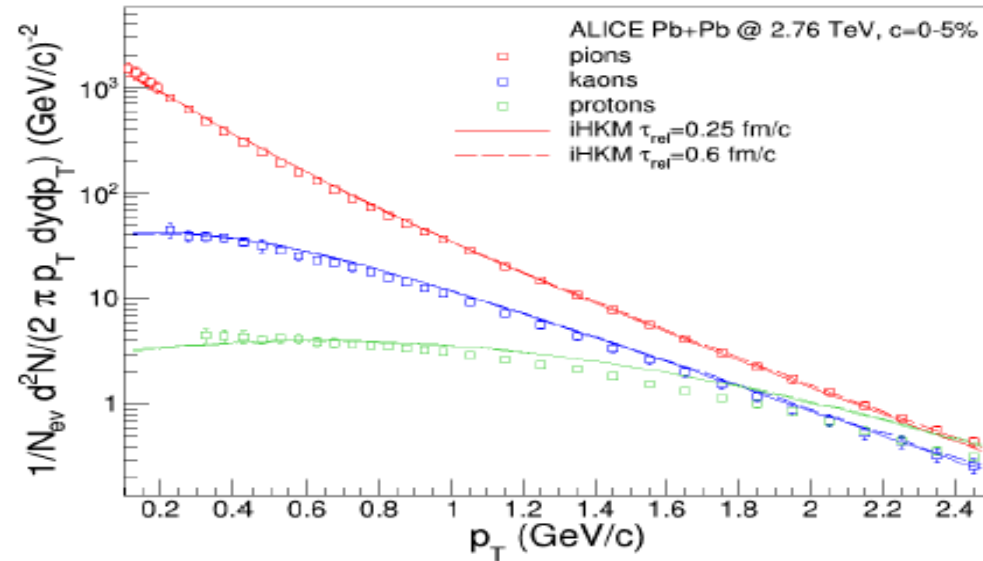
## Pre-thermal stage parameters

The observables behavior depends strongly on initial time  $\tau_0$



Two thermalization times  $\tau_{th}$  at fixed  
 $\tau_{rel} = 0.25 \text{ fm}/c$ :

$$\begin{aligned} \tau_{th} = 1.0 \text{ fm}/c, & \quad \epsilon_0 = 834 \text{ GeV}/\text{fm}^3 \\ \tau_{th} = 1.5 \text{ fm}/c, & \quad \epsilon_0 = 681 \text{ GeV}/\text{fm}^3 \end{aligned}$$



Two relaxation times  $\tau_{rel}$  at fixed  
 $\tau_{th} = 1.5 \text{ fm}/c$ :

$$\begin{aligned} \tau_{rel} = 0.25 \text{ fm}/c, & \quad \epsilon_0 = 681 \text{ GeV}/\text{fm}^3 \\ \tau_{rel} = 0.6 \text{ fm}/c, & \quad \epsilon_0 = 630 \text{ GeV}/\text{fm}^3 \end{aligned}$$

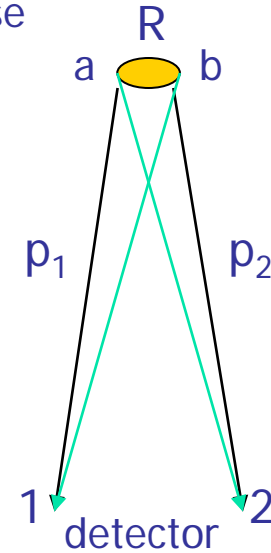
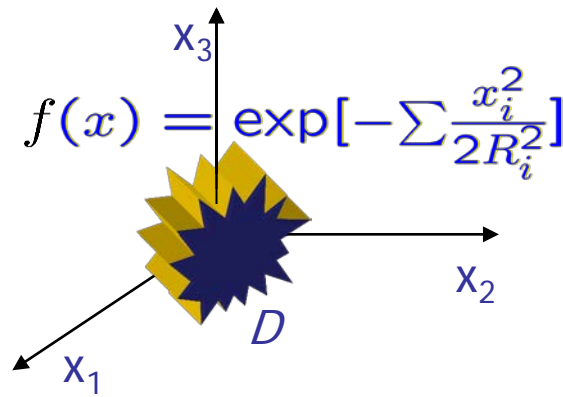
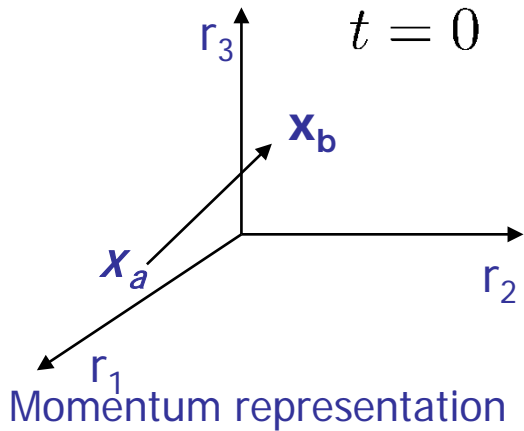
B. Abelev *et al.* (ALICE Collaboration), *Phys. Rev. C* **88** (2013) 044910.

+

The variation EoS (and the corresponding hadronization temperature) can be compensated to get the same bulk results by  $\epsilon(\tau_0)$ . Initial time and  $\epsilon_0$  are main param.

# Interferometry microscope (Kopylov, Podgoretcky: 1971-1973)

The idea of the correlation femtoscopy is based on an impossibility to distinguish between registered particles emitted from different points because of particle identity.



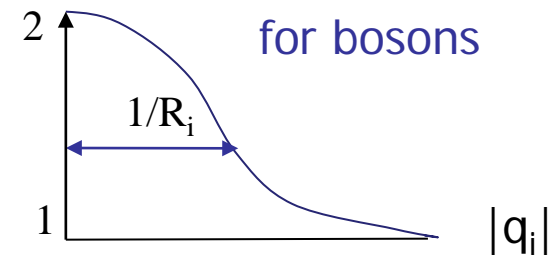
$$\Psi_{x_a, x_b}(p_1, p_2) = \frac{1}{\sqrt{2}} \left[ e^{-i\mathbf{p}_1 \cdot \mathbf{x}_a} e^{-i\mathbf{p}_2 \cdot \mathbf{x}_b} \pm e^{-i\mathbf{p}_2 \cdot \mathbf{x}_a} e^{-i\mathbf{p}_1 \cdot \mathbf{x}_b} \right]$$

Probabilities:

$$W_{x_a, x_b}(p_1, p_2) = |\Psi_{x_a, x_b}(p_1, p_2)|^2 = 1 \pm \cos[\underbrace{(\mathbf{p}_1 - \mathbf{p}_2)}_{\mathbf{q}} \cdot (\mathbf{x}_a - \mathbf{x}_b)]$$

The model of independent particle emission

$$W_D(p_1, p_2) = \int_D d^3x_1 d^3x_2 f(x_1) f(x_2) W_{x_1, x_2}(p_1, p_2) = 1 \pm \left| \int d^3x f(x) e^{i\mathbf{q}\mathbf{x}} \right|^2 = 1 \pm \exp \left[ -\sum q_i^2 R_i^2 \right]$$





# THE DEVELOPMENT OF THE FEMTOSCOPY (Yu.S.1986 – 1995)

To provide calculations analytically one should use the saddle point method and Boltzmann approximation to Bose-Einstein distribution function. Then the single particle spectra are proportional to homogeneity volume:

$$p^0 \frac{d^3 N}{d^3 p} \propto \prod_i \lambda_i(p)$$

and just these homogeneity lengths forms exponent in Bose-Einstein correlation function

$$C = 1 + \exp \left[ -\sum q_i^2 R_i(p)^2 \right]$$

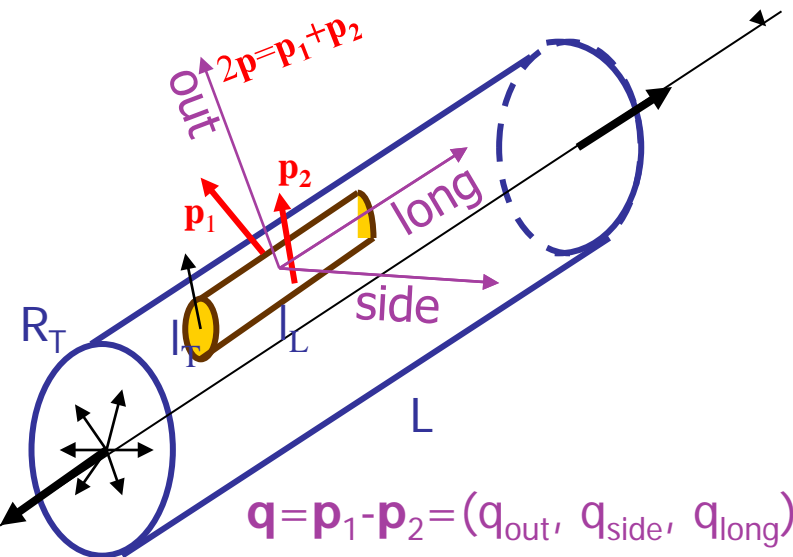
Interferometry radii:

$$R_L(p_T) \approx \lambda_L = \tau \sqrt{\frac{T_{f.o.}}{m_T}} / \cosh(y), \quad m_T = \sqrt{m^2 + p_T^2}$$

$$R_S \approx \lambda_T = R_T / \sqrt{1 + I m_T / T_{f.o.}}, \quad I \propto \langle v_T^2 \rangle$$

$$R_O^2 \approx \lambda_T^2 + v^2 \langle \Delta t^2 \rangle_p - 2v \langle \Delta x_o \Delta t \rangle_p, \quad v = \frac{p_{out}}{p_o}$$

$$C(p, q) = \frac{d^6 N / d^3 p_1 d^3 p_2}{d^3 N / d^3 p_1 d^3 N / d^3 p_2} \approx 1 + e^{R_L^2(p) q_L^2 + R_S^2(p) q_S^2 + R_O^2(p) q_O^2}$$



QGP  $\longrightarrow$   $R_{out}/R_{side} \gg 1$  Exp :  $R_{out}/R_{side} \approx 1$  RHIC HBT PUZZLE

# Correlation femtoscopy of nucleus-nucleus collisions

The theory, method and interpretation of the correlation femtoscopy measurements that are utilized by all the collaborations dealing with such kind of analysis in A+A and p+p collisions at the SPS, RHIC та LHC, are developed. It allows to study the homogeneity lengths in extremely inhomogeneous fast expanding hadron and quark-gluon systems, with accuracy  $10^{-15}$  m and  $10^{-23}$  s.

«Sinyukov-Makhlin formula» that allow to measure the life-time of the hot matter at “Little bang”

1987

$$R_L \approx \tau \sqrt{\frac{T_{f.o.}}{m_T}}$$

$$R_i^2(k_T) = \tau^2 \lambda^2 \left( 1 + \frac{3}{2} \lambda^2 \right)$$

2015

$$\lambda^2 = \frac{T}{m_T} (1 - \bar{v}_T^2)^{1/2}$$

$\bar{v}_T^2$  - mean transv. velocity

Femto “homogeneity lengths”.

The general interpretation of the femtoscopy scales as the spatio-temporal homogeneity length has been formulated

$$R_i^2 = \frac{f(x_0, p)}{\lambda_i^2} = \frac{f(x_0, p)}{\left| f''_{x_i}(x_0, p) \right|}$$

“Bowler–Sinyukov treatment”

The method that allow to separate the quantum-statistical (QS) correlations from Coulomb ones and long-lived (l-l) resonance contributions is proposed

$$C_{tot}(q) = (1 - \alpha) + \alpha K(q_{inv})(1 + C_{QS}(\mathbf{q}))$$

$\alpha$  - fraction of l-l resonances

$K$  - Coulomb wave function.

$C_{QS}$  - QS-кореляційна ф.<sup>10</sup>

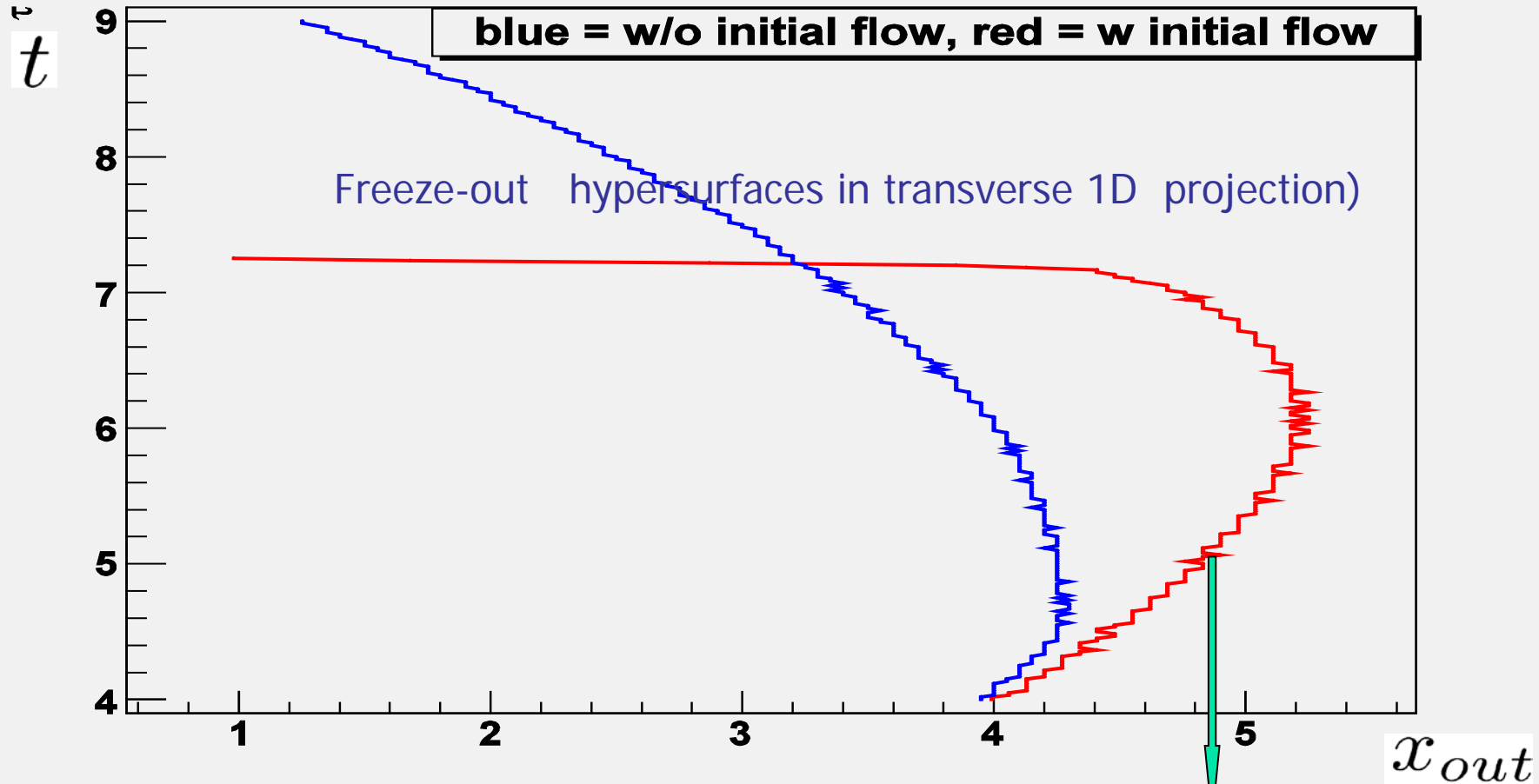


HKM → iHKM

---

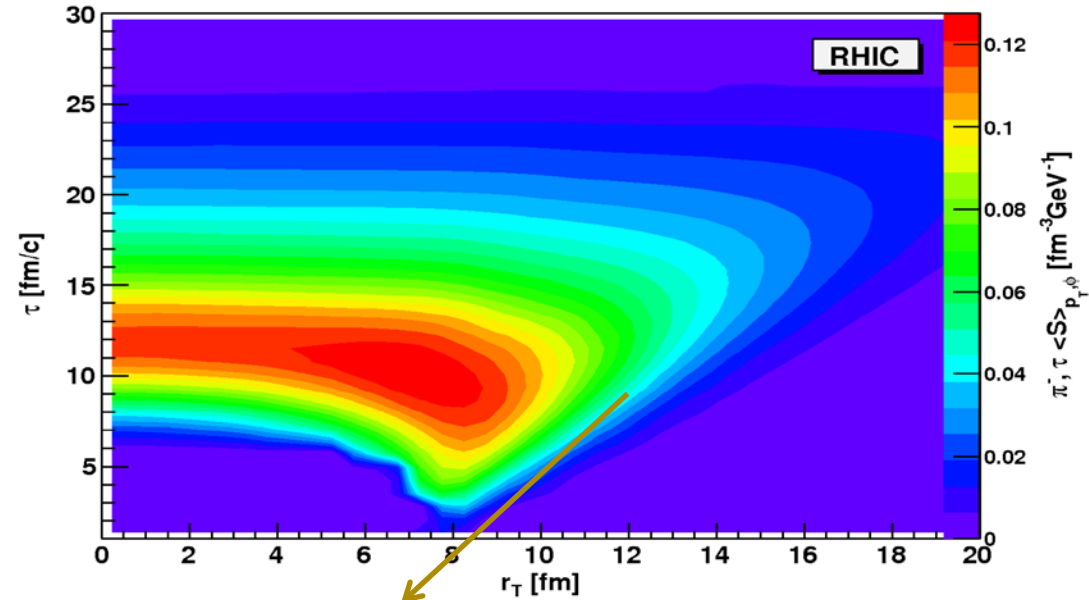
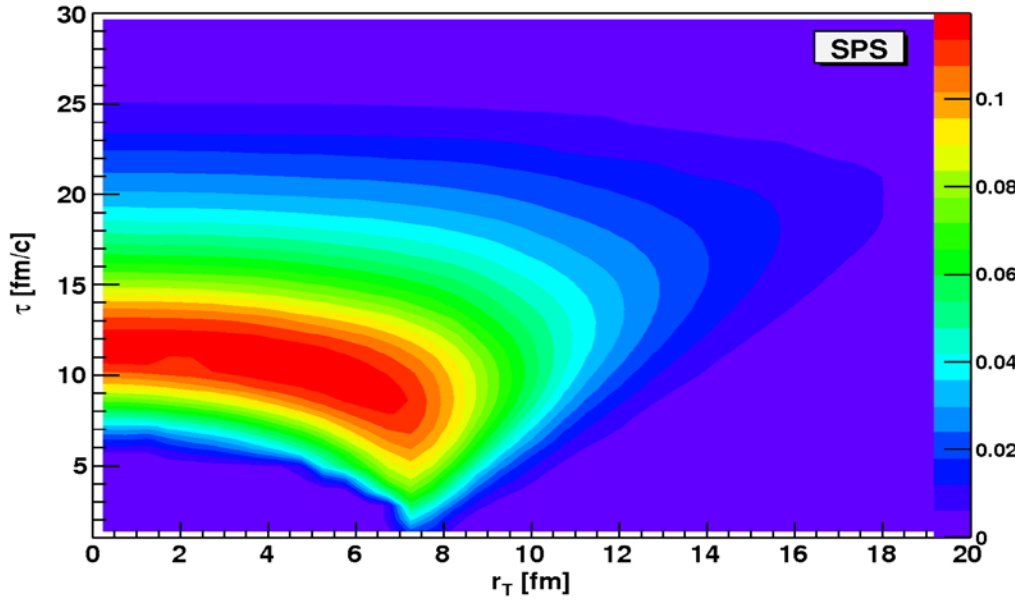
Evolution of ideas and main femtoscopy results

# Initial flows and Ro/Rs ratio ( $t_0=1-2$ fm/c)

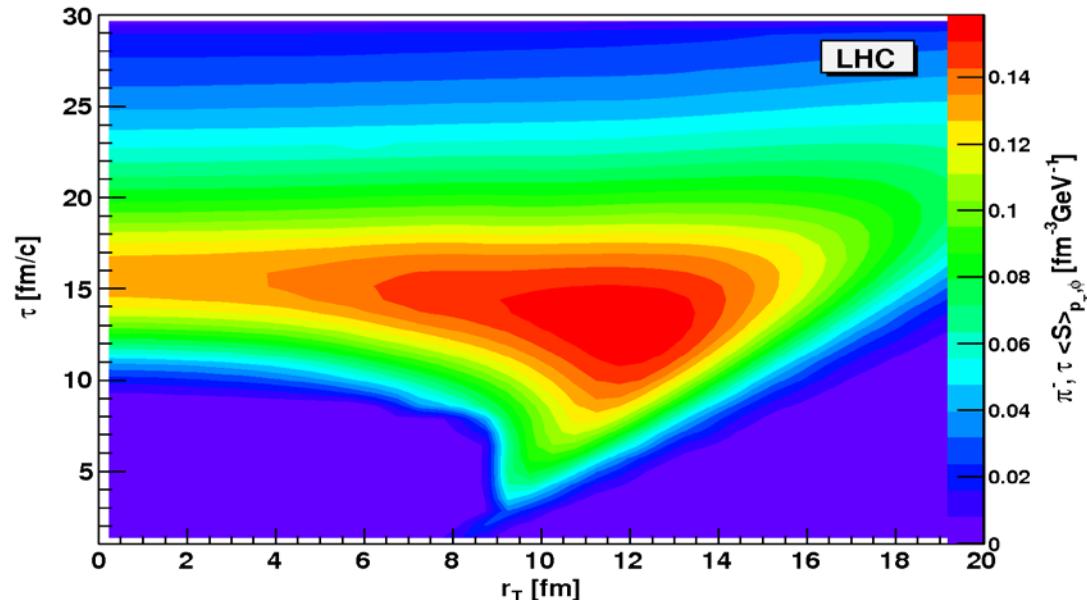


$$R_{out}^2 \approx R_{side}^2 + v^2 \langle \Delta t^2 \rangle_p - 2v \langle \Delta x_{out} \Delta t \rangle_p, v = \frac{p_T}{p_0}$$

# Emission functions in HKM for top SPS, RHIC and LHC energies



$$R_{out}^2 \approx R_{side}^2 + v^2 \langle \Delta t^2 \rangle_p - 2v \langle \Delta x_{out} \Delta t \rangle_p, \quad v = \frac{p_T}{p^0}$$



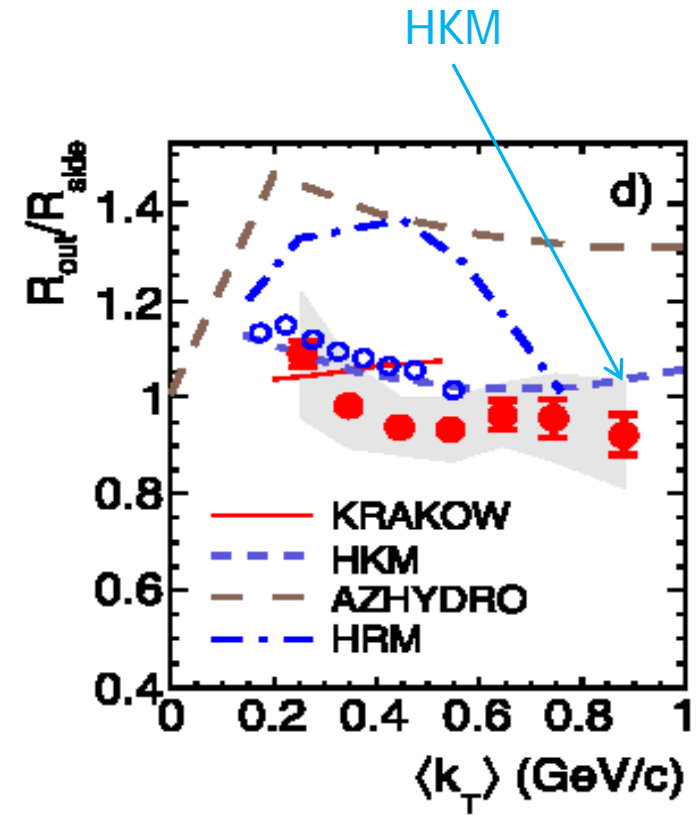
# HKM prediction: solution of the HBT Puzzle

Two-pion Bose–Einstein correlations in central Pb–Pb collisions  
at  $\sqrt{s_{NN}} = 2.76$  TeV<sup>☆</sup> ALICE Collaboration Physics Letters B 696 (2011) 328.



## Quotations:

Available model predictions are compared to the experimental data in Figs. 2-d and 3. Calculations from three models incorporating a hydrodynamic approach, AZHYDRO [45], KRAKOW [46,47], and HKM [48,49], and from the hadronic-kinematics-based model HRM [50,51] are shown. An in-depth discussion is beyond the scope of this Letter but we notice that, while the increase of the radii between RHIC and the LHC is roughly reproduced by all four calculations, only two of them (KRAKOW and HKM) are able to describe the experimental  $R_{out}/R_{side}$  ratio.



[48] I.A. Karpenko, Y.M. Sinyukov, Phys. Lett. B 688 (2010) 50.

[49] N. Armesto, et al. (Eds.), J. Phys. G 35 (2008) 054001.

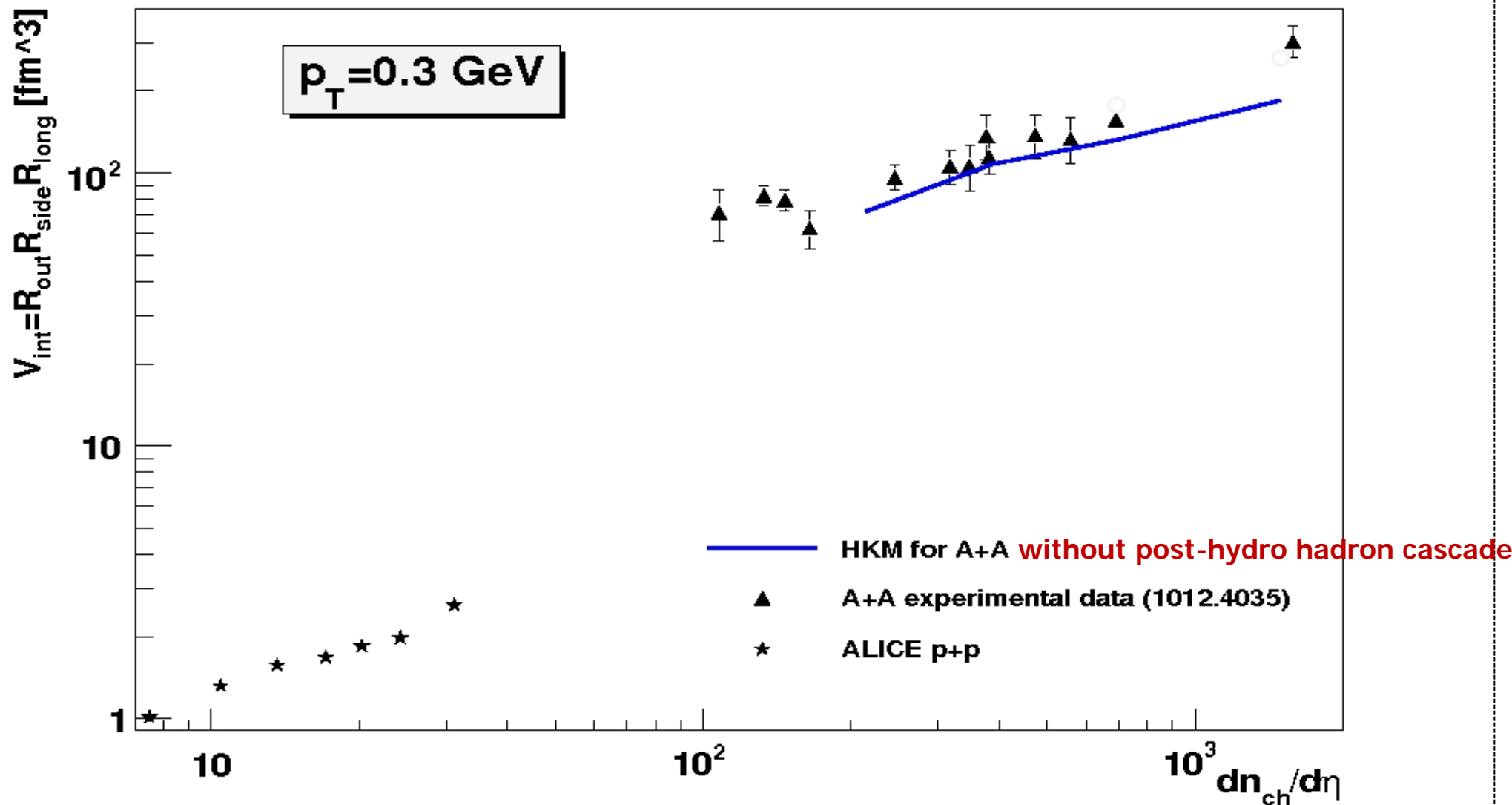


HKM → iHKM

---

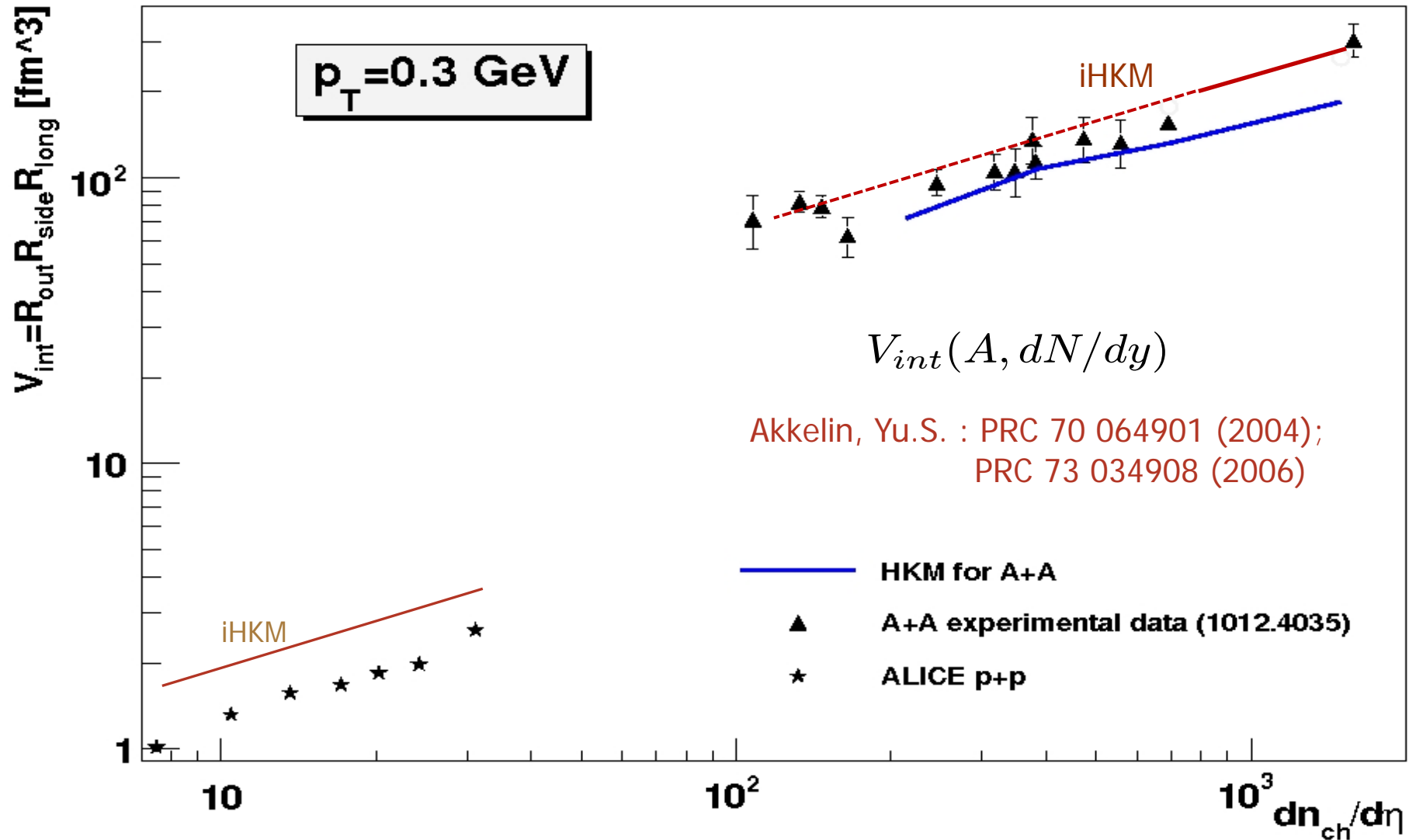
Femtoscopes scales vs multiplicity and initial system size

# Interferometry volume $V_{\text{int}}$ in LHC p-p and **central** Au-Au, Pb-Pb collisions

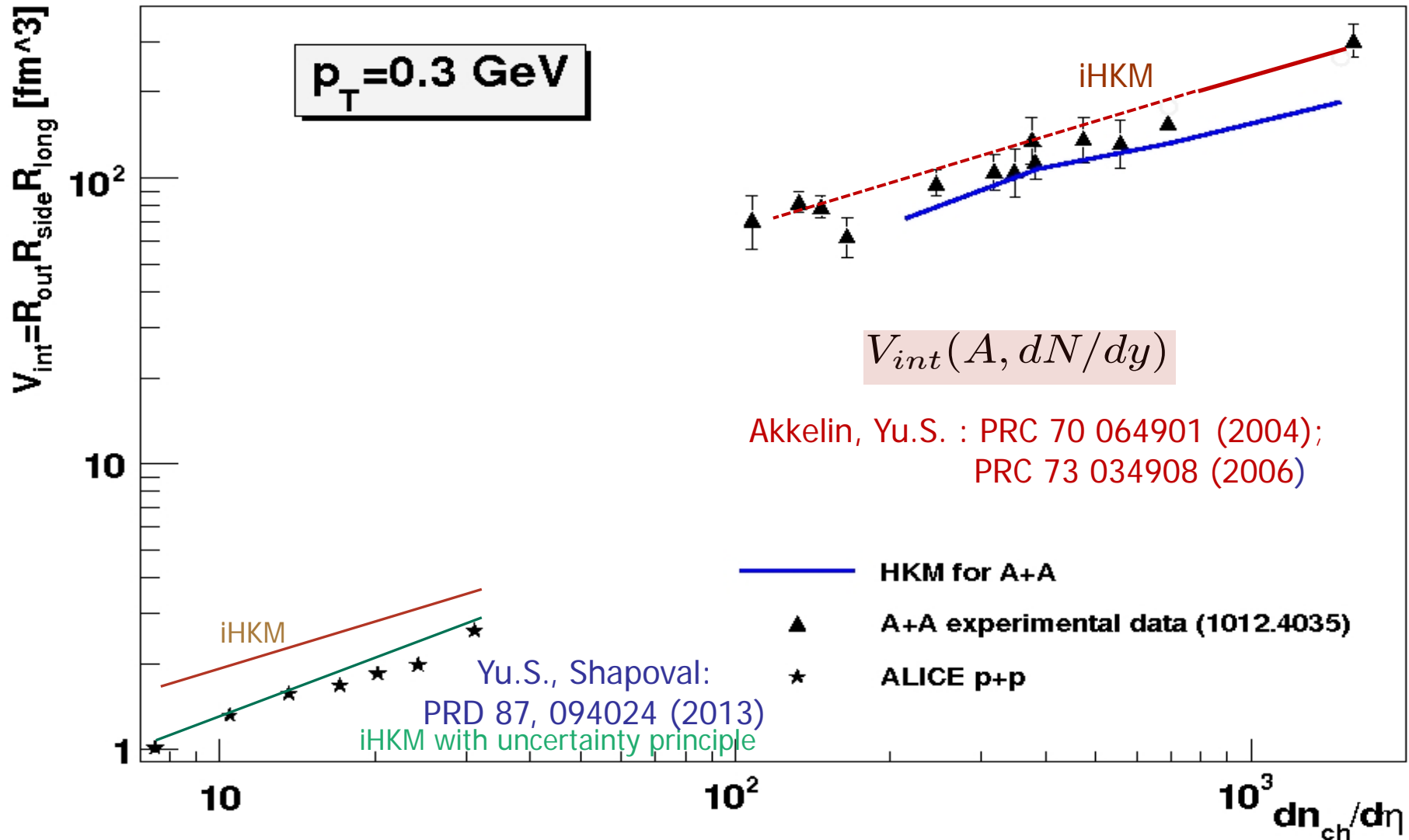




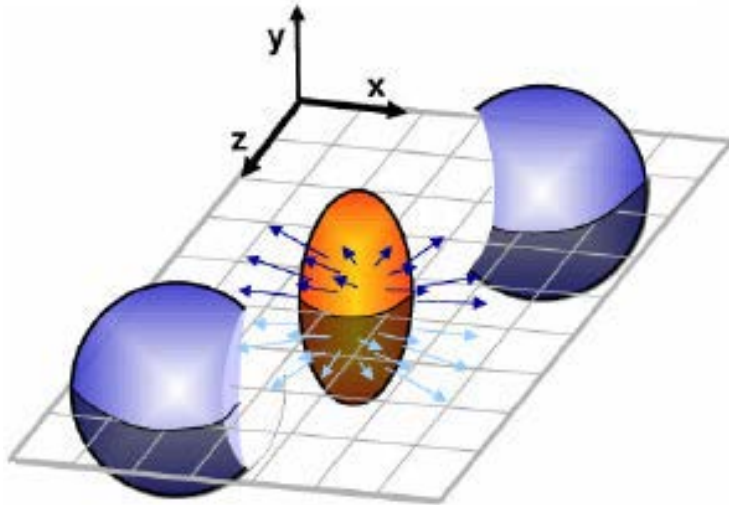
# Interferometry volume $V_{int}$ in LHC p-p and **central** Au-Au, Pb-Pb collisions



# Interferometry volume $V_{int}$ in LHC p-p and **central** Au-Au, Pb-Pb collisions



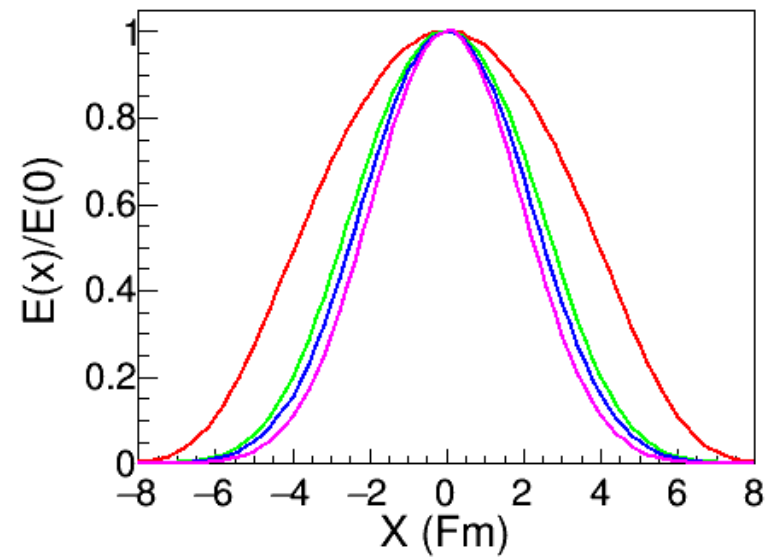
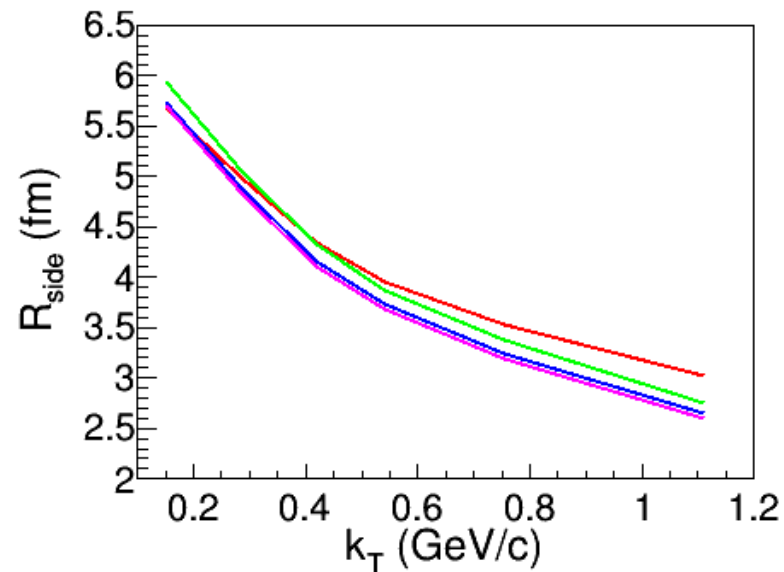
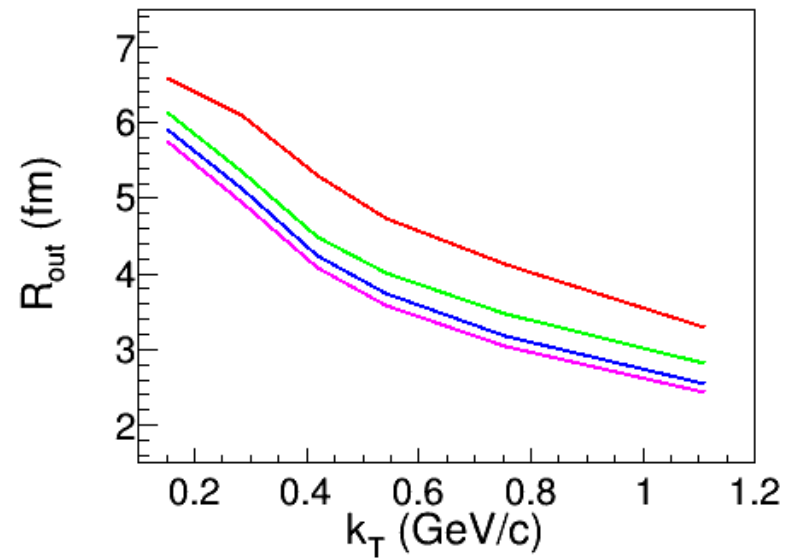
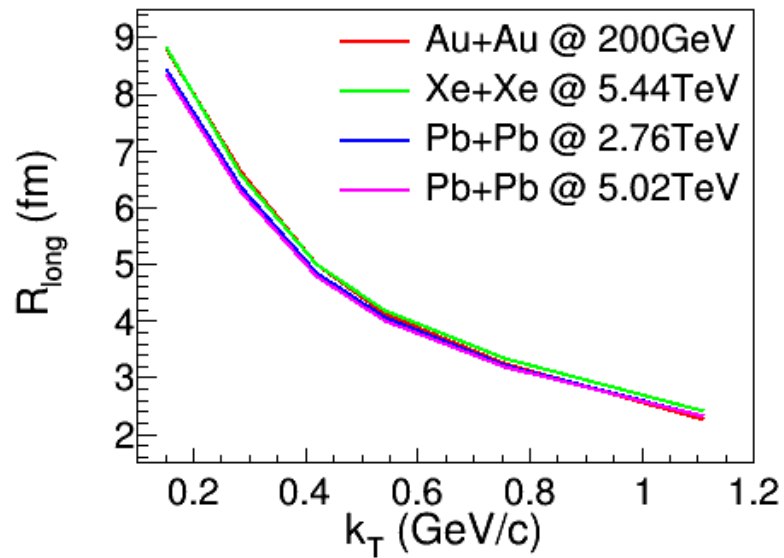
# Interferometry volume vs initial overlapping area at the *fixed* multiplicity



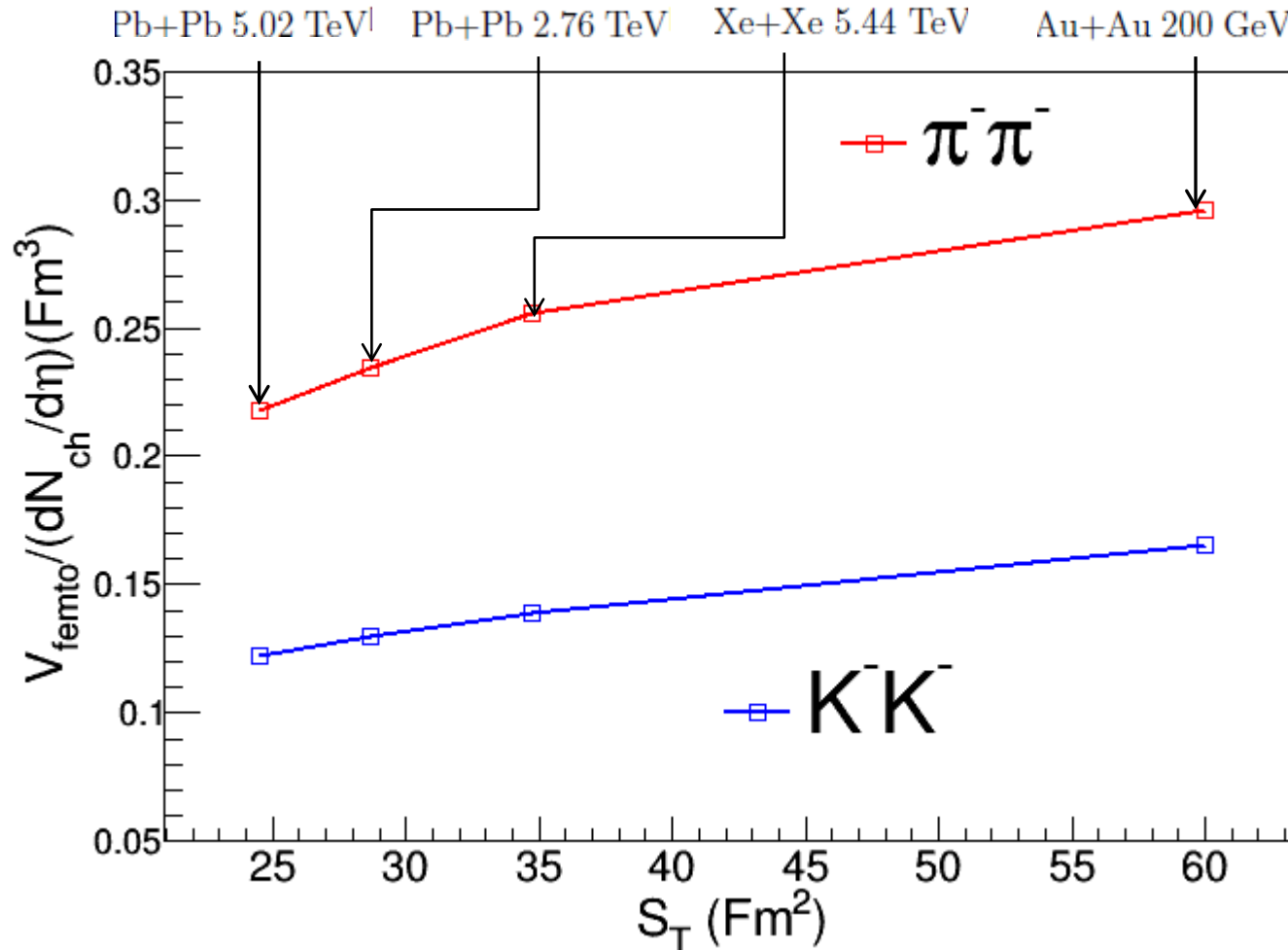
M. Adzhimambetov, Yu.S. , 2019  
in preparation

experiment	centrality	$\frac{dN}{d\eta_{ch}}$	$\alpha$	$\epsilon_0$	EoS	$\tau_0$
Au+Au 200 GeV <sup>[1]</sup>	0-5%	688	0.18	235 GeV/fm <sup>3</sup>	LS	0.1 fm
Xe+Xe 5.44 TeV	10-19%	680	0.44	445 GeV/fm <sup>3</sup>	HQCD	0.1 fm
Pb+Pb 2.76 TeV <sup>[2]</sup>	19-28%	693	0.24	679 GeV/fm <sup>3</sup>	LS	0.1 fm
Pb+Pb 5.02 TeV <sup>[3]</sup>	23-33%	677	0.24	1067 GeV/fm <sup>3</sup>	LS	0.1 fm

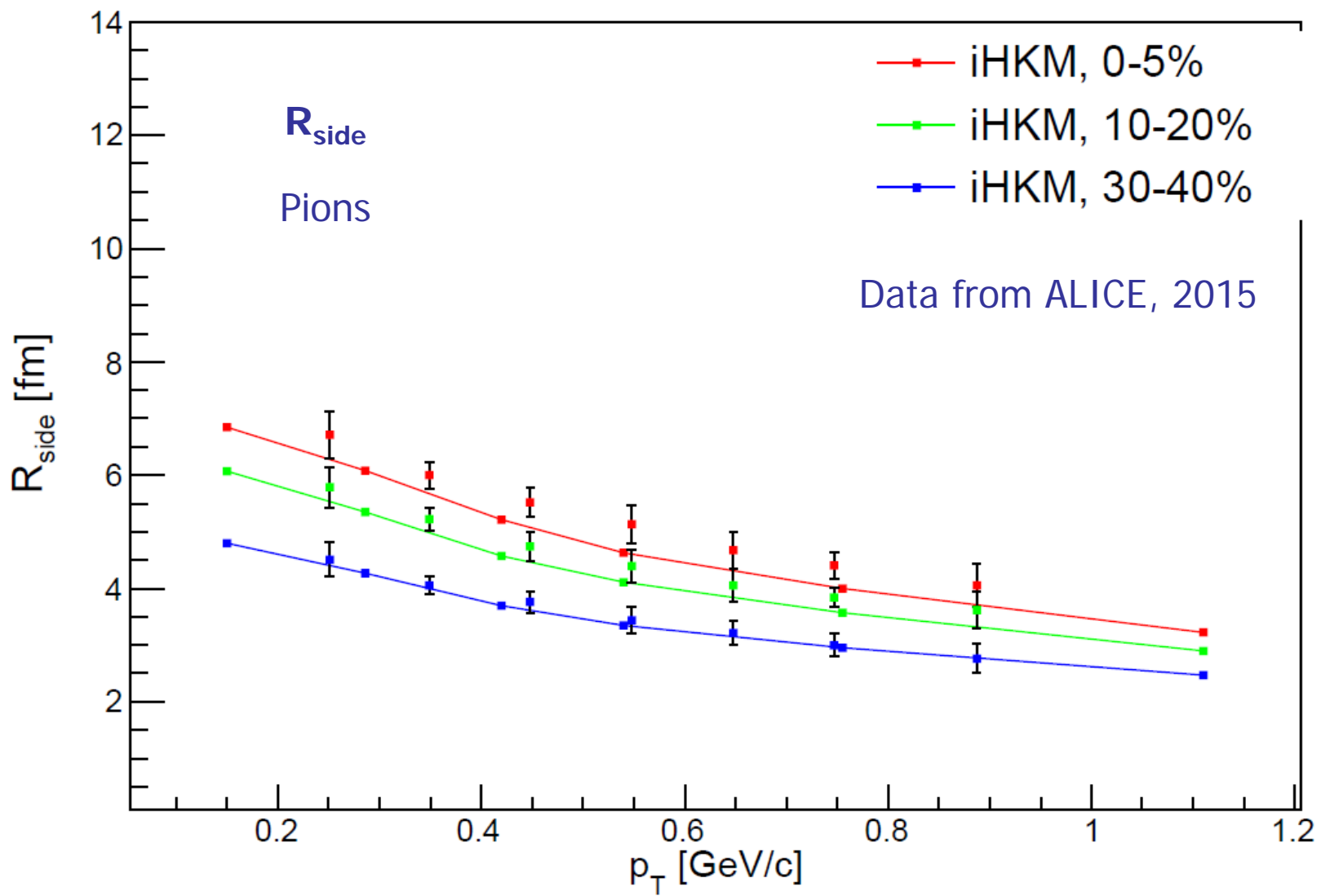
# The femtoscopy radii at different energies and the same multiplicity



# Femtoscscopy volume vs initial transverse overlapping area of creating systems

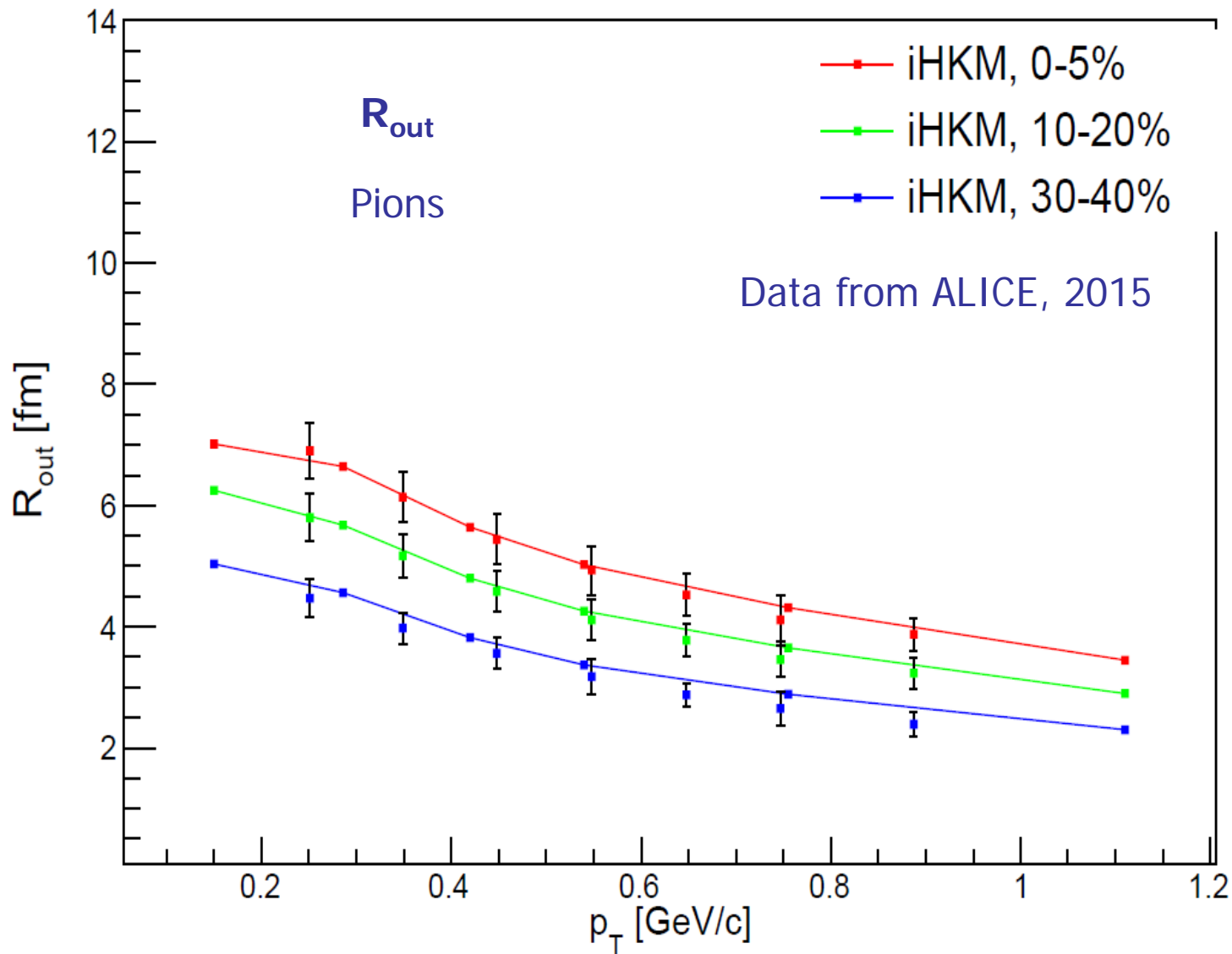


Initial transverse size  $S_T$  = effective transverse area of overlapping nuclei at the initial stage of collision process

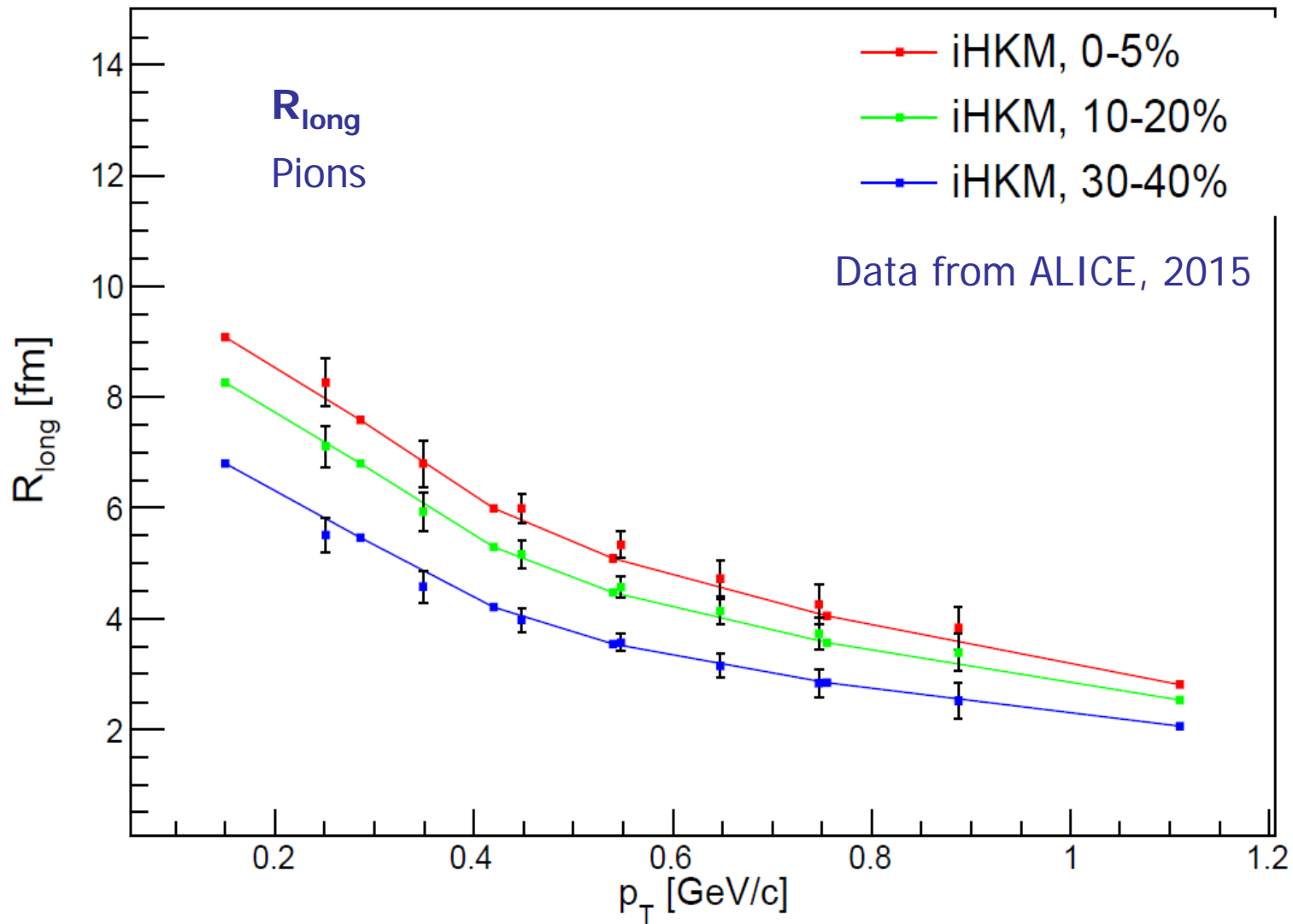


The  $R_{side}$  dependence on transverse momentum for different centralities in the iHKM

scenario under the same conditions as in Fig. 1. The experimental data are from [33].



The  $R_{out}$  dependence on transverse momentum for different centralities in the iHKM basic scenario under the same conditions as in Fig. 1.



The  $R_{long}$  dependence on transverse momentum for different centralities in the iHKM

basic scenario - the same conditions as in Fig. 1. The experimental data are from [33].



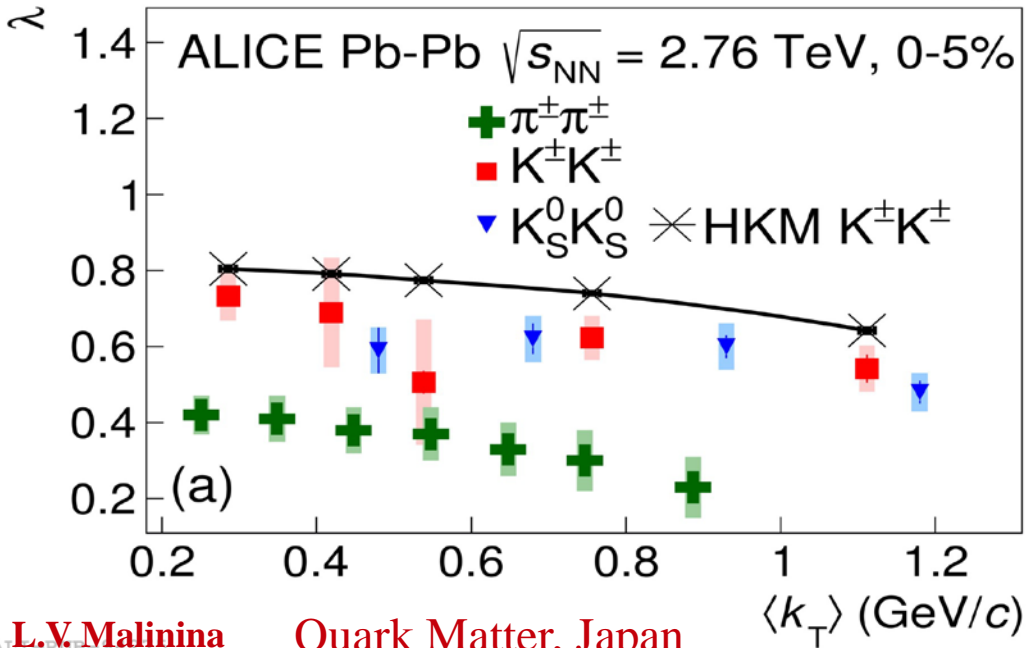
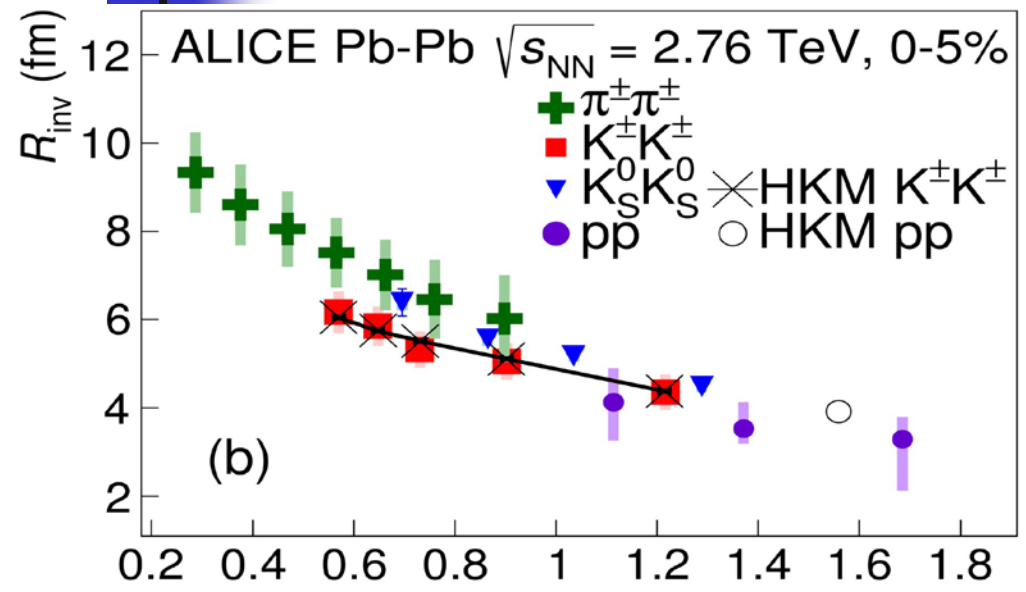


iHKM

---

Femtoscropy scales: pions vs kaons

# $K^\pm K^\pm$ and $K^0 K^0$ in Pb-Pb: HKM model

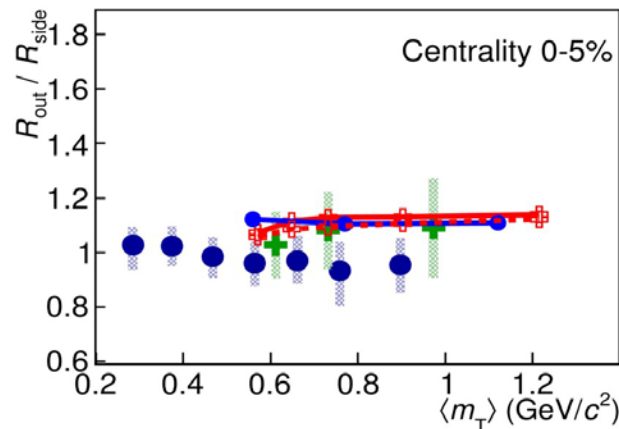
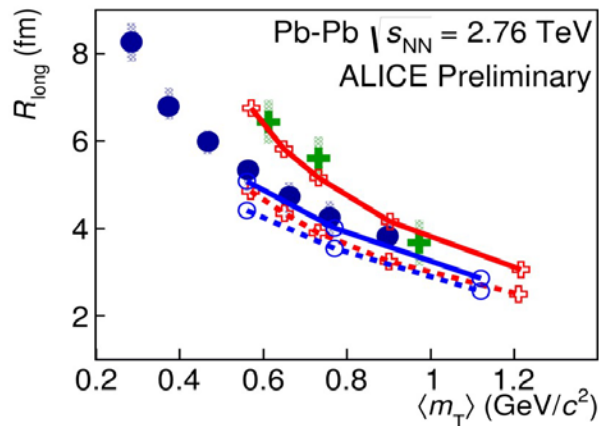
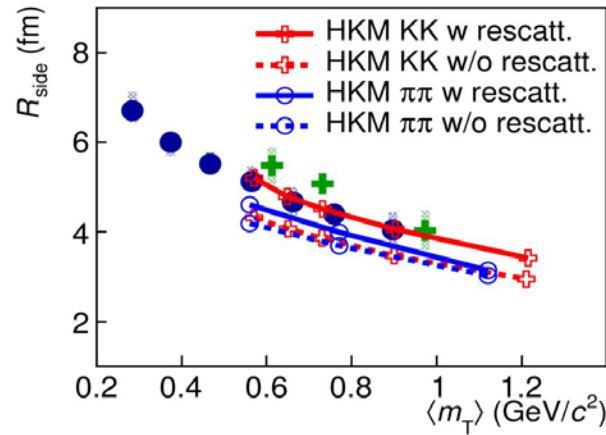
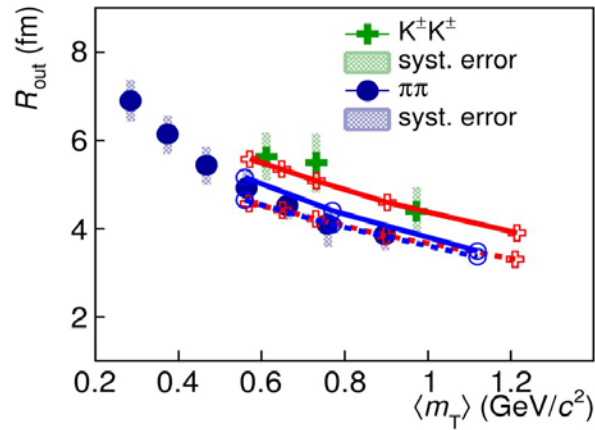


*New results from ArXiv.org:1506.07884*

- R and  $\lambda$  for  $\pi^\pm \pi^\pm$ ,  $K^\pm K^\pm$ ,  $K^0 K^0$ , pp for 0-5% centrality
- Radii for kaons show good agreement with HKM predictions for  $K^\pm K^\pm$  (V. Shapoval, P. Braun-Munzinger, I. Karpenko, Yu. Sinyukov Nucl.Phys.A929 (2014))
- $\lambda$  decrease with  $k_T$ , both data and HKM
- HKM prediction for  $\lambda$  slightly overpredicts the data
- $\Lambda_\pi$  are lower  $\Lambda_K$  due to the stronger influence of resonances

ALICE Coll. Phys. Rev. C 96 ... (2017)

# Comparison with HKM for 0-5% centrality



- HKM model with re-scatterings (M. Shapoval, P. Braun-Munzinger, Iu.A. Karpenko, Yu.M. Sinyukov, Nucl.Phys. A 929 (2014) 1.) describes well ALICE  $\pi$  & K data.

HKM model w/o re-scatterings demonstrates approximate  $m_T$  scaling

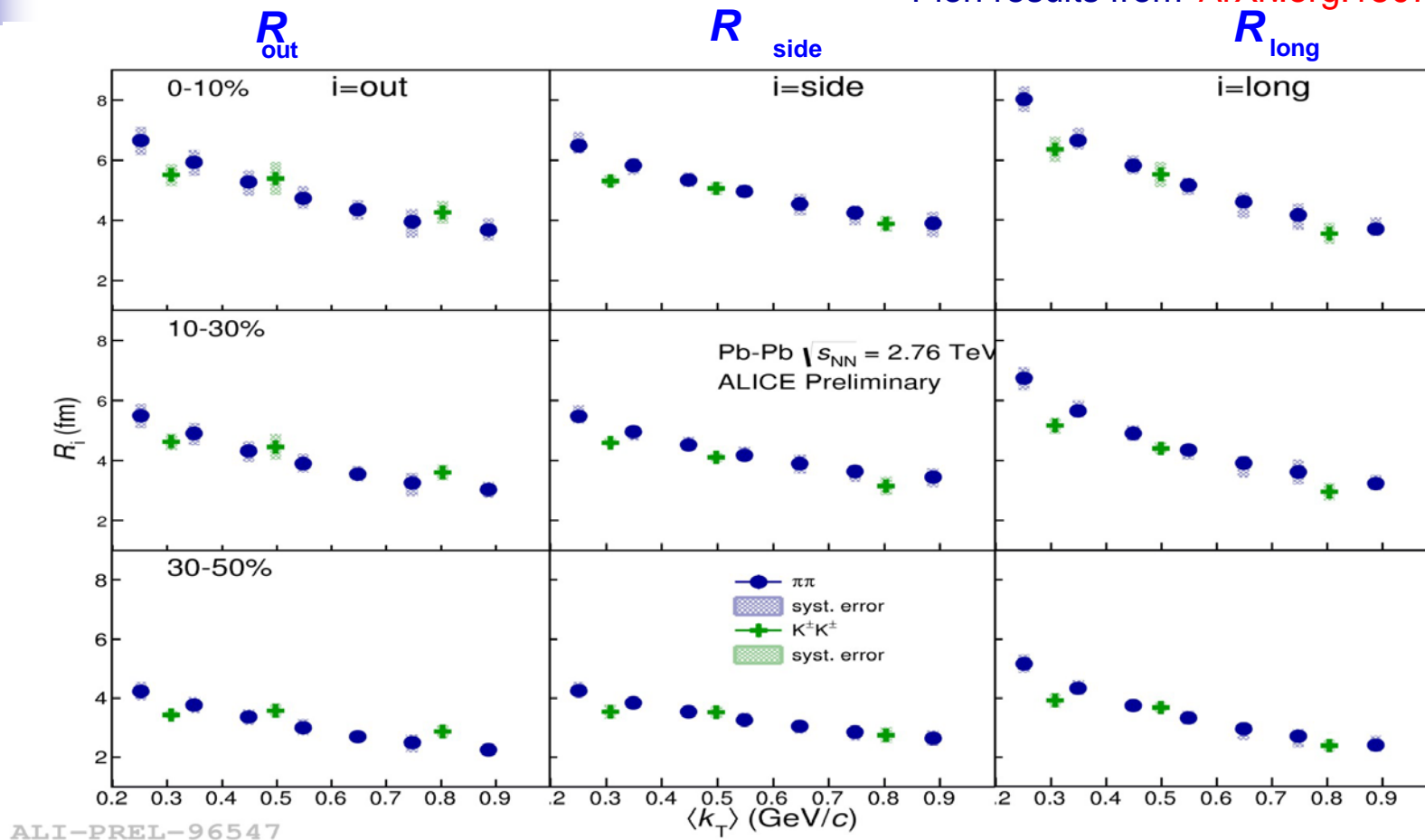
for  $\pi$  & K, but does not describe ALICE  $\pi$  & K data

- The observed deviation from  $m_T$  scaling is explained in (M. Shapoval, P. Braun-Munzinger, Iu.A. Karpenko, Yu.M. Sinyukov, Nucl.Phys. A 929 (2014) by essential transverse flow & re-scattering phase.

● HKM model slightly underestimates  $R_{side}$   
overestimates  $R_{side} / R_{out}$

# 3D $K^\pm K^\pm$ & $\pi\pi$ radii versus $k_T$

Pion results from [ArXiv.org:1507.06842](https://arxiv.org/abs/1507.06842)



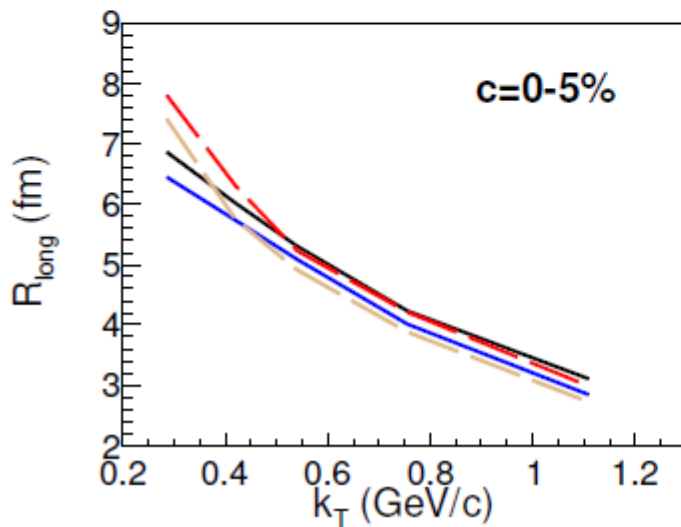
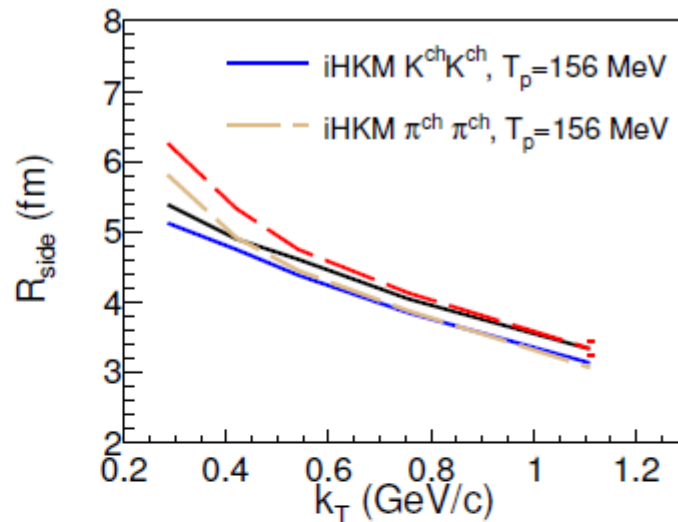
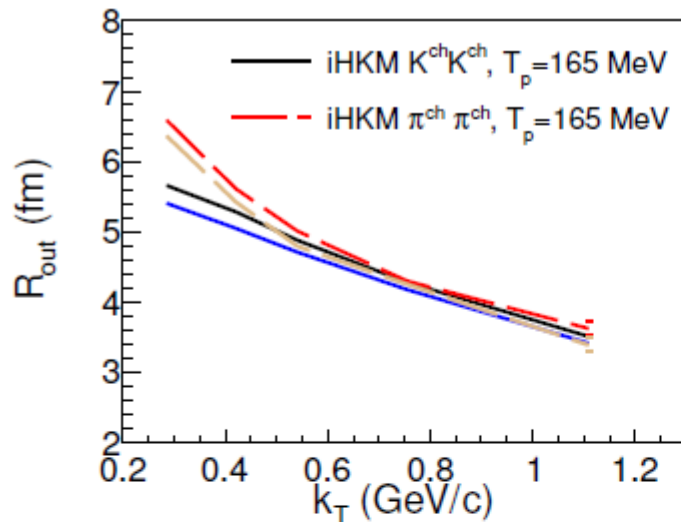
ALI-PREL-96547

● Radii scale better with  $k_T$  than  $m_T$  according to HKM

(V. Shapoval, P. Braun-Munzinger, Iu.A. Karpenko, Yu.M. Sinyukov, Nucl.Phys. A 929 (2014) 1);

● Similar observations were reported by PHENIX at RHIC ([arxiv:1504.05168](https://arxiv.org/abs/1504.05168)).

# Predictions for the pion and kaon femtoscopy scales for LHC energy per nucleon pair 5.02 TeV



The iHKM prediction of the charged pion and kaon interferometry radii  $k_T$  dependence for the centrality  $c=0-5\%$ . The calculations were performed at the two hadronization temperatures: 165 MeV and 156 MeV.

# Space-time picture of the pion and kaon emission

$$R_l^2(k_T) = \underbrace{\tau^2 \lambda^2 \left(1 + \frac{3}{2} \lambda^2\right)}_{2015} \approx \text{w/o transv. expansion} \approx \underbrace{\tau^2 \frac{T}{m_T}}_{1987} \underbrace{\frac{K_2\left(\frac{m_T}{T}\right)}{K_1\left(\frac{m_T}{T}\right)}}_{1995}$$

where

$$\lambda^2 = \frac{T}{m_T} \left(1 - \frac{\overbrace{k_T^2}^{\bar{v}_T^2}}{(m_T + \alpha T)^2}\right)^{1/2}$$

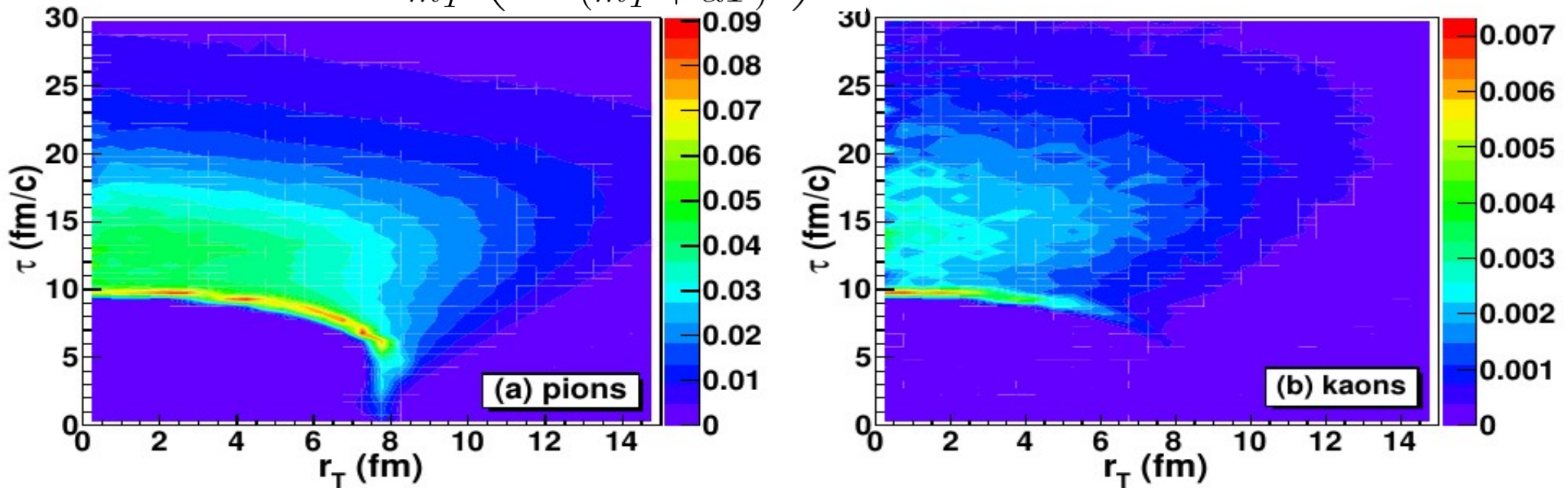


FIG. 4. The momentum angle averaged emission functions per units of space-time and momentum rapidities  $g(\tau, r_T, p_T)$  [fm<sup>-3</sup>] (see body text) for pions (a) and kaons (b) obtained from the HKM simulations of Pb+Pb collisions at the LHC  $\sqrt{s_{NN}} = 2.76$  GeV,  $0.2 < p_T < 0.3$  GeV/c,  $|y| < 0.5$ ,  $e = 0 - 5\%$ . From Yu.S., Shapoval, Naboka, Nucl. Phys. A 946 (2016) 247 ([arXiv:1508.01812](https://arxiv.org/abs/1508.01812))

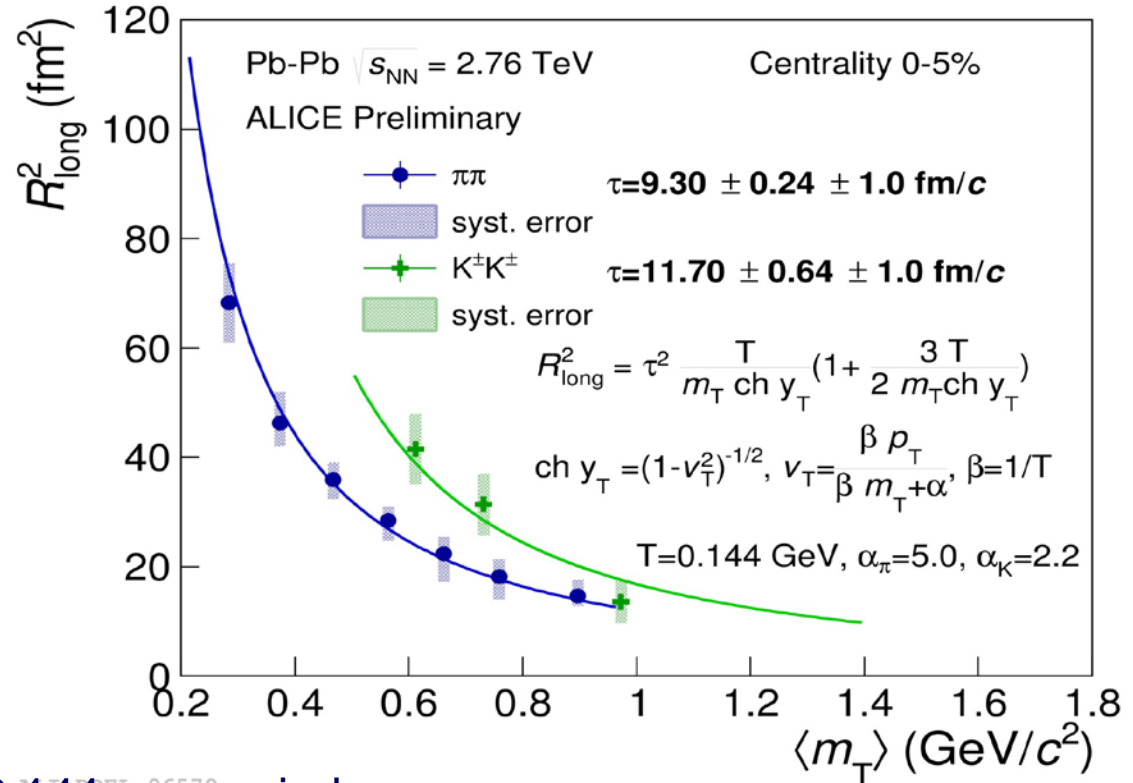
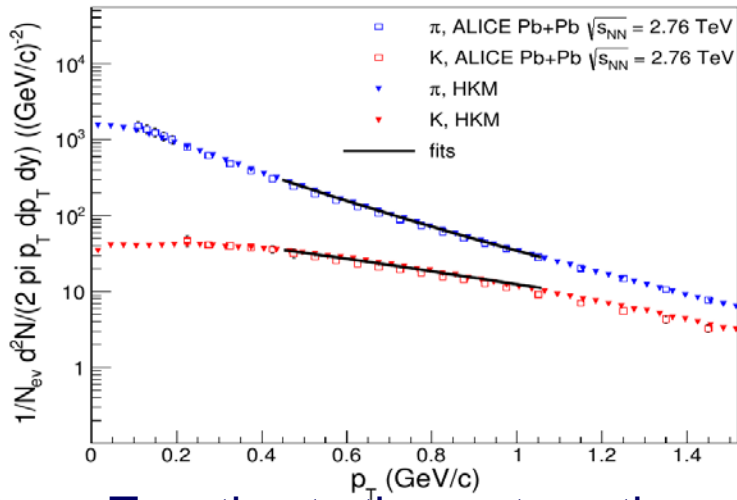


# Extraction of emission time from fit $R_{\text{long}}$




- The new formula for extraction of the maximal emission time for the case of strong transverse flow was used ( Yu. S., Shapoval, Naboka, Nucl. Phys. A 946 (2016) 227 )

- The parameters of freeze-out:  $T$  and “intensity of transverse flow”  $\alpha$  were fixed by fitting  $\pi$  and  $K$  spectra ( arxiv:1508.01812 )



- To estimate the systematic errors:  $T = 0.144$  was varied on  $\pm 0.03$  GeV & free  $\alpha_{\pi}$ ,  $\alpha_K$ , were used; systematic errors  $\sim 1$  fm/c
- Indication:  $\tau_{\pi} < \tau_K$ . Possible explanations ( arxiv:1508.01812 ): HKM includes re-scatterings (UrQMD cascade): e.g.  $K\pi \rightarrow K^*(892) \rightarrow K\pi$ ,  $KN \rightarrow K^*(892)X$ ; ( $K^*(892)$  lifetime 4-5 fm/c) [ $\pi N \rightarrow N^*(\Delta)X$ ,  $N^*(\Delta) \rightarrow \pi X$  ( $N^*$ 's( $\Delta$ s)- short lifetime)]



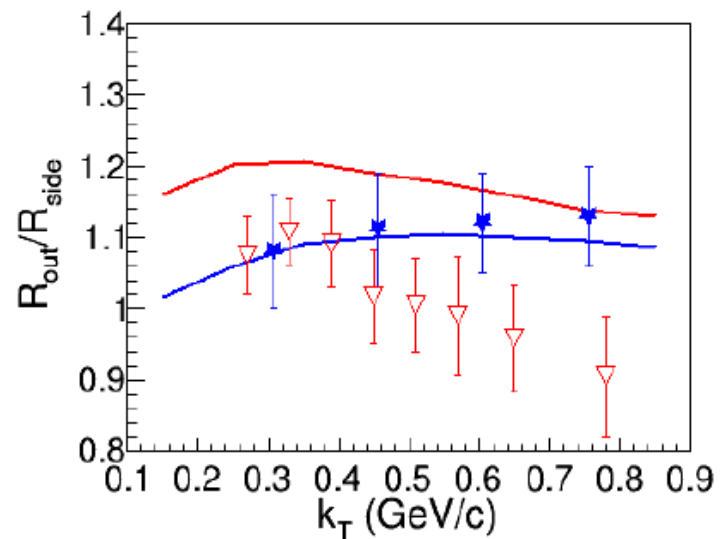
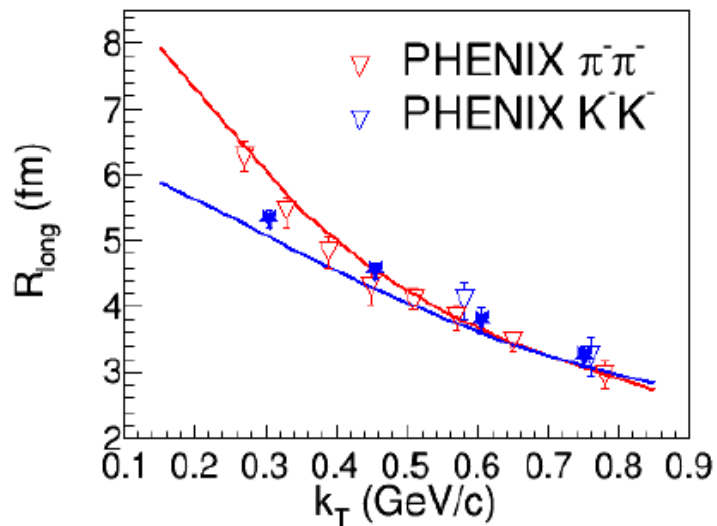
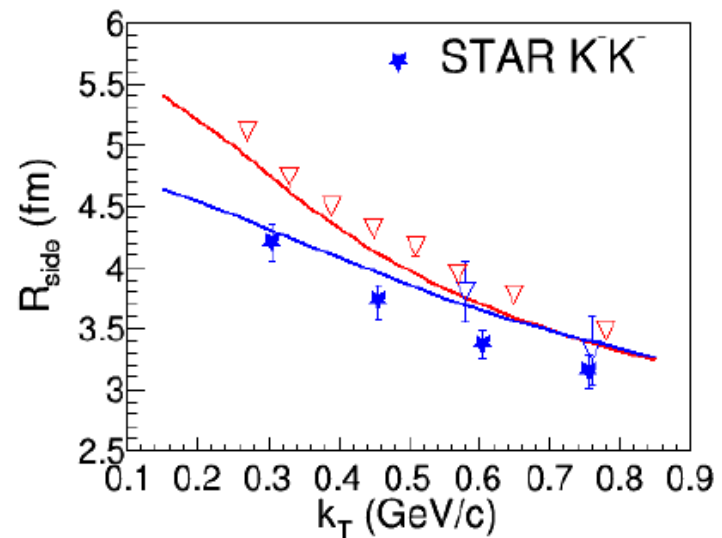
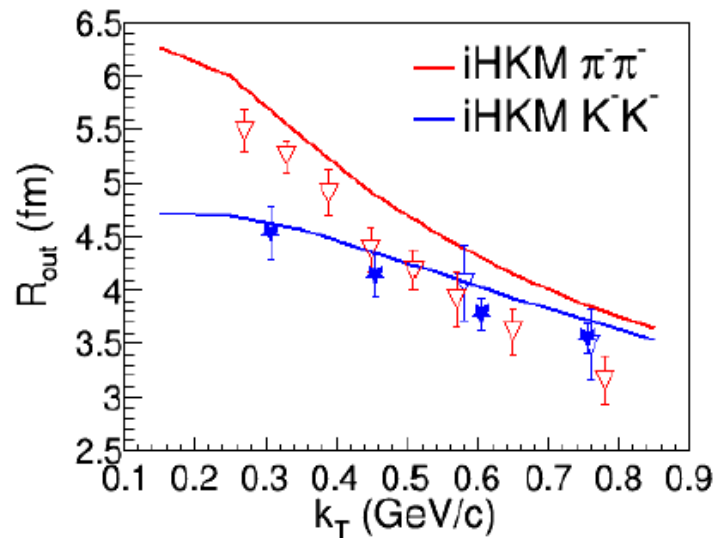
iHKM results for the top RHIC energy

$$\sqrt{s_{NN}} = 200 \text{ GeV}$$

M. Adzhimambetov, V. Shapoval, Yu.S.,  
Nucl.Phys. A **987** (2019) 321–336.

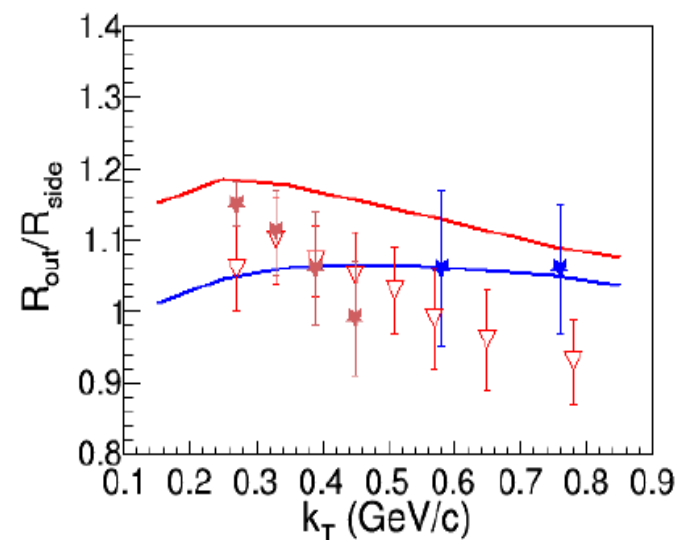
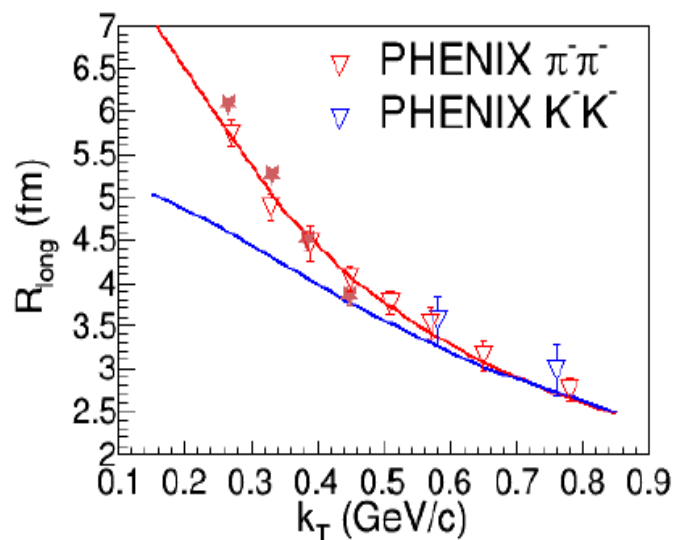
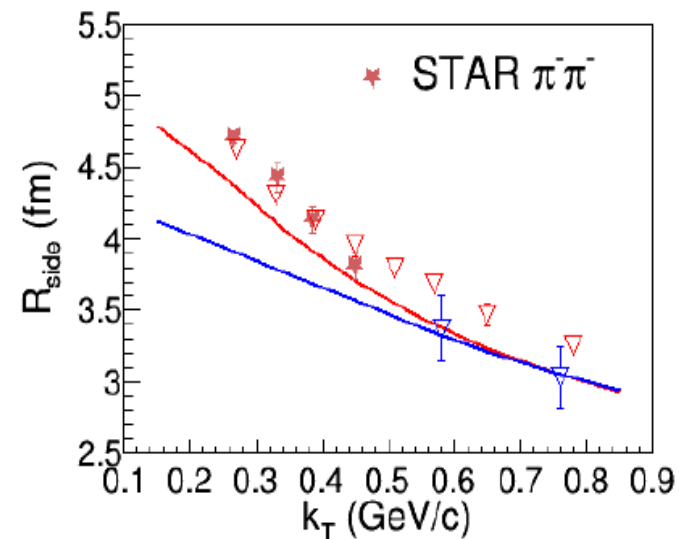
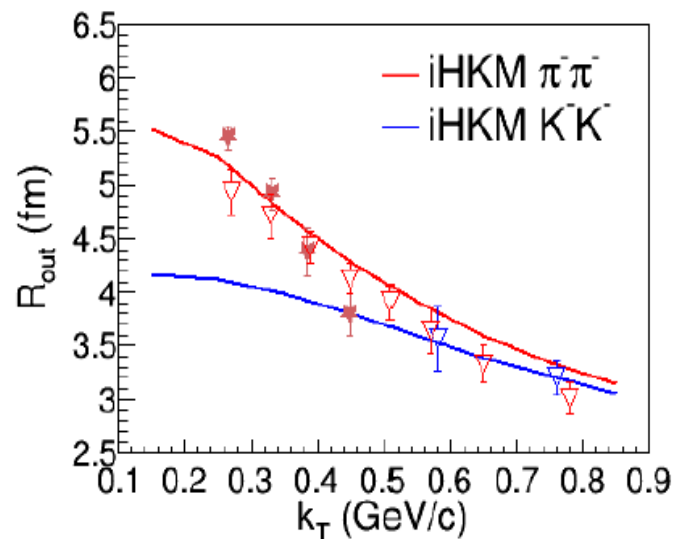


# Interferometry radii, $c = 0 - 10\%$



$0.15 < p_T < 1.55$  GeV/c,  $|\eta| < 1$

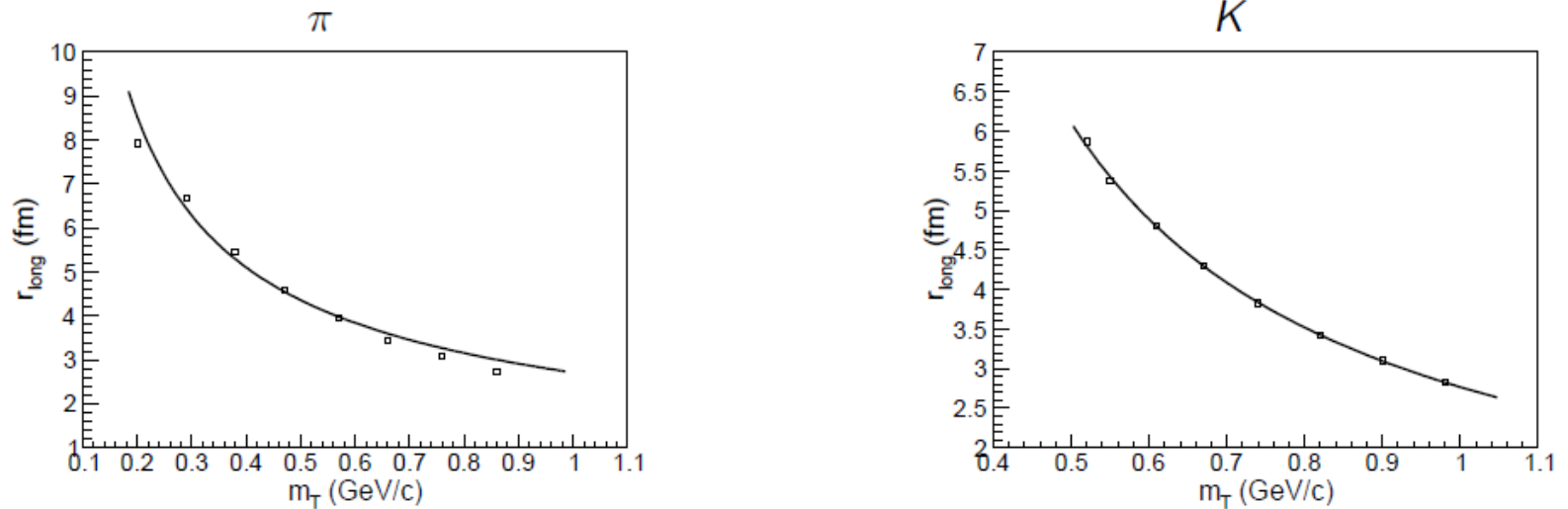
# Interferometry radii, $c = 10 - 20\%$



$$0.15 < p_T < 1.55 \text{ GeV}/c, |\eta| < 1$$

G. Nigmatkulov, arXiv:1712.09964v1; J. Adams *et al.* (STAR Collaboration), Phys. Rev. C 71 (2005), 044906; A. Adare *et al.* (PHENIX Collaboration), Phys. Rev. C 92 (2015), 034914.

## Time of maximal emission extraction



Combined  $\pi$  and  $K$  spectra fit for  $0.45 < p_T < 1.0$  GeV/c with the formula

$$p_0 \frac{d^3 N}{d^3 p} \propto \exp [-(m_T/T + \alpha)(1 - \bar{v}_T^2)^{1/2}]$$

gives us the temperature  $T = 141$  MeV,  $\alpha_\pi = 7.86 \pm 2.11$  and  $\alpha_K = 5.54 \pm 2.61$ .

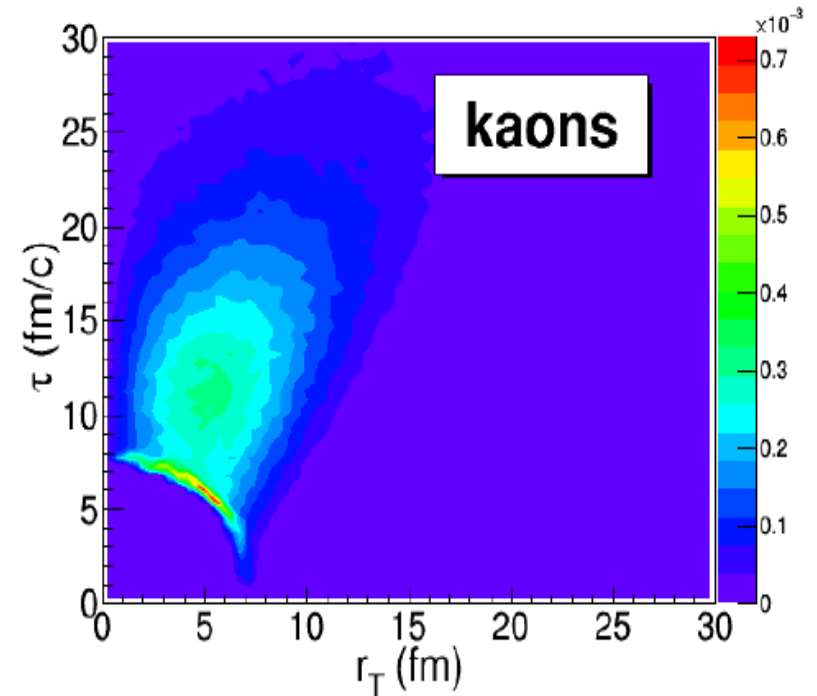
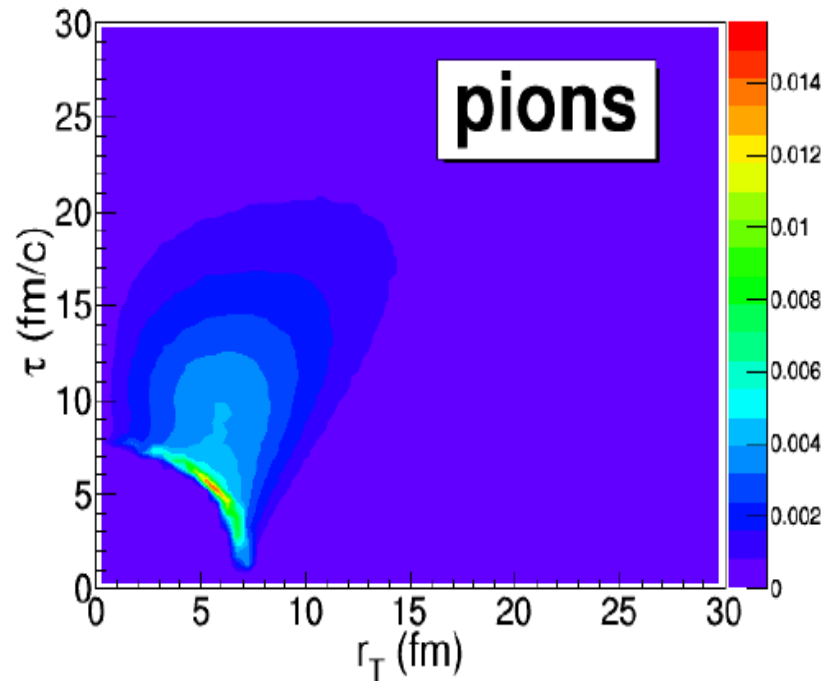
Fitting the  $R_{long}$  dependence on  $m_T$  with the formula

$$R_l^2(k_T) = \tau^2 \lambda^2 \left( 1 + \frac{3}{2} \lambda^2 \right)$$

gives the maximal emission times:  $\tau_\pi = 7.12 \pm 0.01$  fm/c and  $\tau_K = 9.71 \pm 0.02$  fm/c.

As for  $\alpha$ 's, the pion value is in agreement with the spectra fit,  $\alpha_\pi = 7.05 \pm 0.17$ , while for kaons it is reduced,  $\alpha_K = 0.12 \pm 0.02$ .

## Time of maximal emission extraction

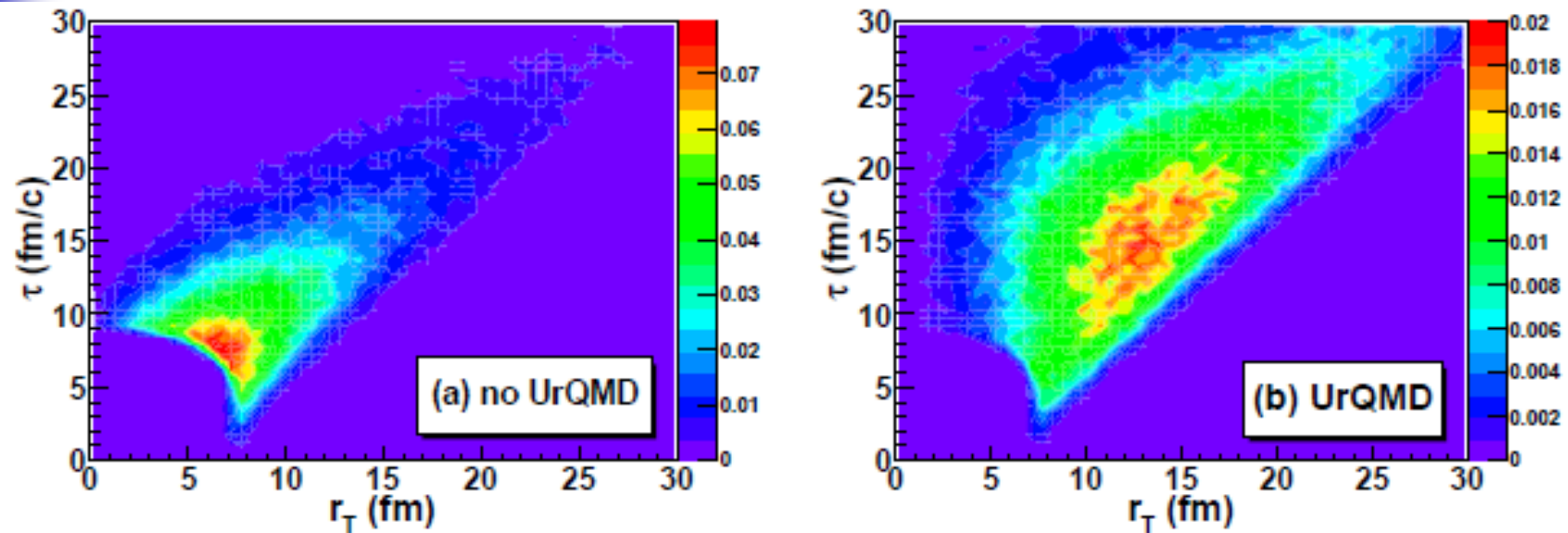


The maximal emission times are extracted following the procedure, suggested in Yu.M. Sinyukov, V.M. Shapoval, V.Yu. Naboka, Nucl. Phys. A **946** (2016), 227 and later used by the ALICE Collaboration in their study

S. Acharya *et al.* (ALICE Collaboration), Phys. Rev. C **96** (2017), 064613

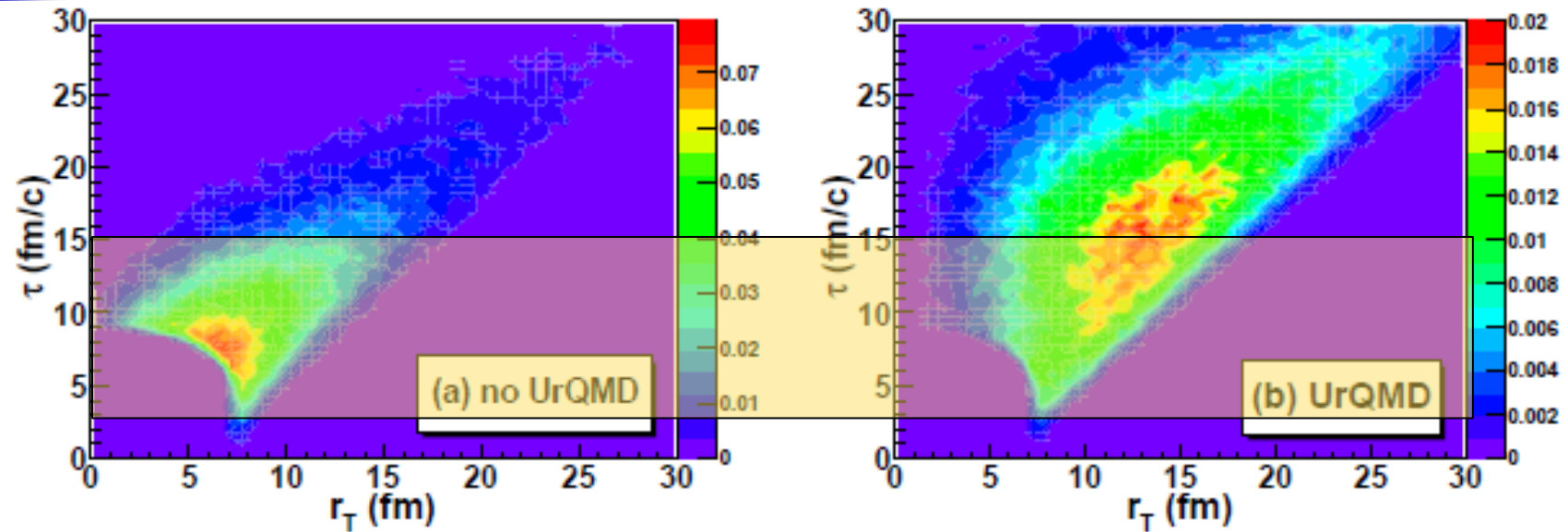
# $K^*$ probes

$K^*(892)$  life time is 4.2 fm/c



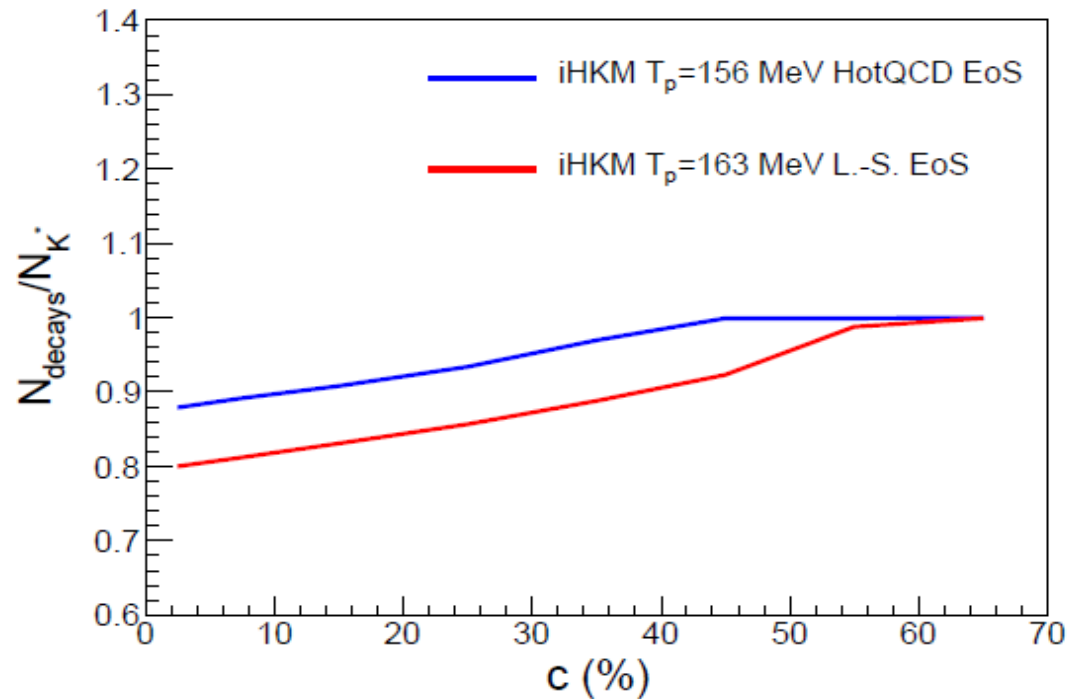
The comparison of the emission functions  $g(\tau, r_T)$ , averaged over complementary space and momentum components, of  $K^+\pi^-$  pairs, associated with  $K(892)^{*0}$  decay products, for two cases: (a) free-streaming of the particles and resonances, and (b) UrQMD hadron cascade. The plots are obtained using iHKM simulations of Pb+Pb collisions at the LHC  $\sqrt{s_{NN}} = 2.76$  GeV,  $0.3 < k_T < 5$  GeV/c,  $|y| < 0.5$ ,  $c = 5 - 10\%$ .

$K^{*0} \rightarrow K^+\pi^-$  radiation picture in iHKM.  
Sudden vs continuous thermal freeze-out at the LHC.



Less than 30% of direct  $K^*$  can be seen till 15 fm/c

# Suppression of $K^{*0}$ due to continuous thermal freeze-out (LHC)



70% - 20% = 50%  
Therefore  
at least 50% of direct  $K^{*0}$  are  
recreated in reactions:

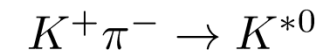
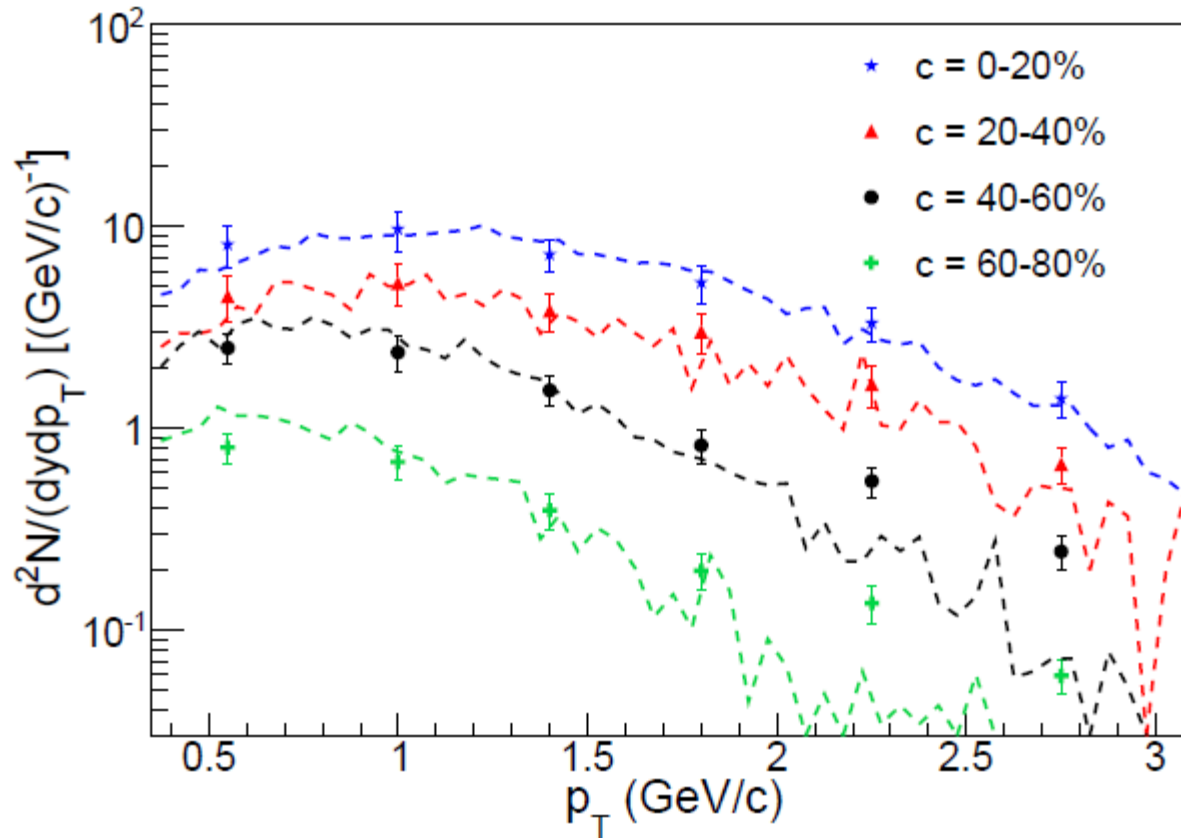


FIG. 3. The fraction of  $K^+\pi^-$  pairs coming from  $K(892)^*$  decay, which can be identified as daughters of  $K^*$  in iHKM simulations after the particle rescattering stage modeled within UrQMD hadron cascade. The simulations correspond to LHC Pb+Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV with different centralities. The iHKM results are presented for two cases: the Laine-Shroeder equation of state with particlization temperature  $T_p = 163$  MeV (red line) and the HotQCD equation of state with  $T_p = 156$  MeV (blue line).



## Spectra of $K^{*0}$ (LHC)



The  $K(892)^*$  resonance  $p_T$  spectra for Pb+Pb collision events with different centralities at the LHC energy  $\sqrt{s_{NN}} = 2.76 \text{ TeV}$  obtained in iHKM simulations (lines) in comparison with the experimental data [6] (markers).



# Summary for space-time structure of ultrarelativistic A+A collisions

---

- It seems that we understood in detail the femtoscopic picture of ultrarelativistic A+A collisions at the top RHIC and available LHC energies.
- The dependence of the interferometry volume on both main parameters, namely, multiplicity and initial size of the system formed, is obviously demonstrated.
- As for the complete space-time picture of collision process, the femoscopy analysis altogether with  $K^{*0}(892)$  probes demonstrate that even at the first 4-5 fm/c (proper time!) after hadronization **at least** 70% of decay products are re-scattered. The intensive re-generation
- of  $K^*$  takes place. **At least** 50% of direct  $K^{*0}(892)$  are re-combine.
- Quite intensive “afterburner life” at the last hadron evolution stage, leads not only to violation of kaon-pion femtoscopy  $m_T$  – scaling, but also to continuous “chemical freeze-out”

## Thermal and evolutionary approaches

Yu.S., V. Shapoval,

**Phys. Rev. C 97 064901 (2018)**

# Continuous freeze-out vs sudden freeze-out

- Thermal models of particle production vs dynamic/evolutionary approaches

## Kinetic/thermal freeze-out

### Sudden freeze-out

Cooper-Frye prescription

$$p^0 \frac{d^3 N_i}{d^3 p} = \int_{\sigma_{th}} d\sigma_\mu p^\mu f_i(x, p)$$

The  $\sigma_{th}$  is typically isotherm.

### Continuous freeze-out

$$p^0 \frac{d^3 N_i}{d^3 p} = \int d^4 x S_i(x, p) \approx \int_{\sigma(p)} d\sigma_\mu p^\mu f_i(x, p)$$

The  $\sigma(p)$  is piece of hypersurface where the particles with momentum near  $p$  has a maximal emission rate.  
Yu.S. *Phys. Rev. C78*,

## Chemical freeze-out

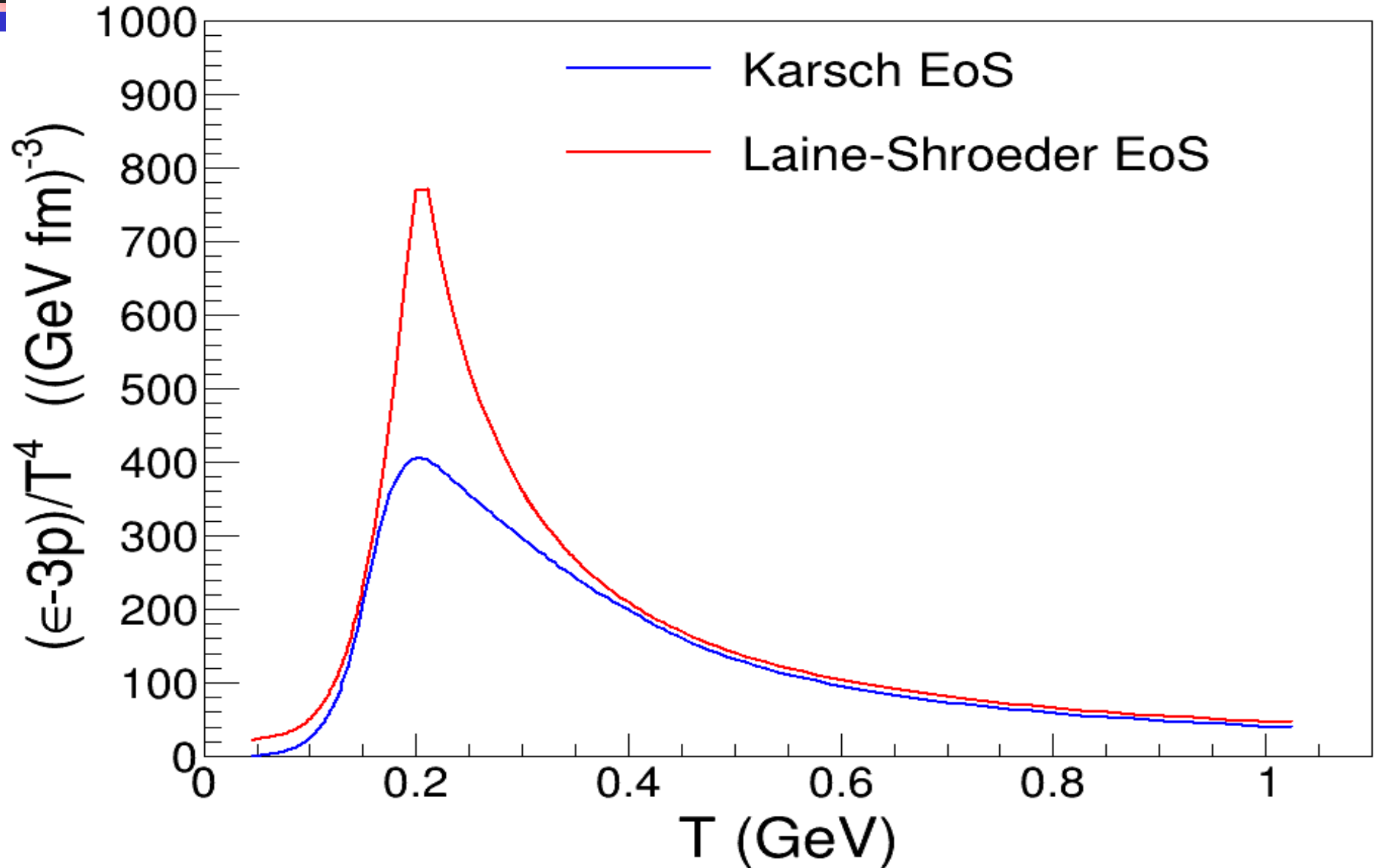
$$N_i = \int_p \int_{\sigma_{ch}} \frac{d^3 p}{p^0} d\sigma_\mu p^\mu f_i\left(\frac{p^\mu u_\mu(x)}{T_{ch}}, \frac{\mu_{i, ch}}{T_{ch}}\right)$$

$$= n_i(T, \mu) V_{eff} \quad V_{eff} = \int_{\sigma_{ch}} u^\mu d\sigma_\mu$$

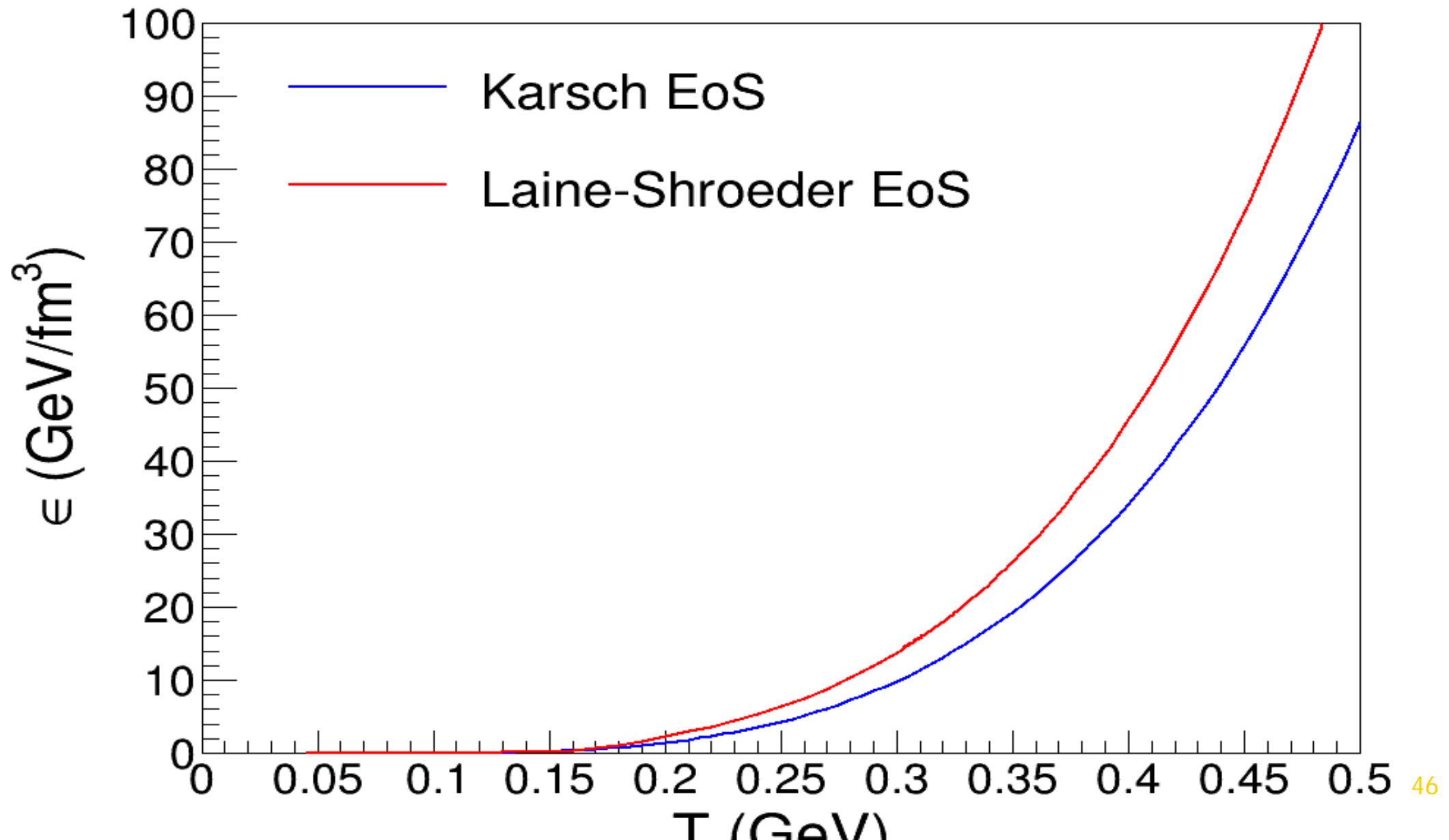
The numbers of quasi-stable particles is defined from  $N_i$  with taking into account the resonance decays but **not** inelastic re-scattering.

The  $T_{ch}$  is the minimal temperature when the expanding system is still (near) in local thermal and chemical equilibrium. Below the hadronic cascade takes place:  $T_{ch} \rightarrow T_{part}$ . The inelastic reactions, annihilation processes in hadron-resonance gas change the quasi-particle yields in comparison with sudden chem. freeze-out.

# Equation of State - 1



# Equation of state -2



# Thermal models vs evolutionary approach

Basic matter properties:  
thermodynamic **EoS**

**Thermal models**

Chemical freeze-out at

$$T_{ch} \approx T_h$$

Particle number ratios

$$\left\{ \frac{N_i}{N_j} \right\}$$

L.-S.  $\longrightarrow$  Karsch (lattice QCD)

**Evolutionary models**

$$\frac{dN_{charge}}{d\eta} (c)$$

$$\frac{dN_\pi}{p_T dp_T}$$

High dense matter formation time  $\tau_0$

Max. energy density  $\epsilon(\tau_0) \equiv \epsilon_0$

At the particlization temperature  $T_{part} \approx T_h$  hydrodynamic evolution transforms (suddenly or continuously) into interact. hadron gas evolution

**EoS:**

$$T_h = 165 MeV \longrightarrow 156 MeV$$

$$\tau_0 = 0.1 \text{ fm/c} \longrightarrow 0.15 \text{ fm/c}$$

$$\epsilon_0 = 679 \text{ GeV/fm}^3 \longrightarrow 495 \text{ GeV/fm}^3$$

**iHKM**

**Kinetic freeze-out**

«Blast-wave» parametrization of freeze-out hypersurface and transverse flows on it. Spectra

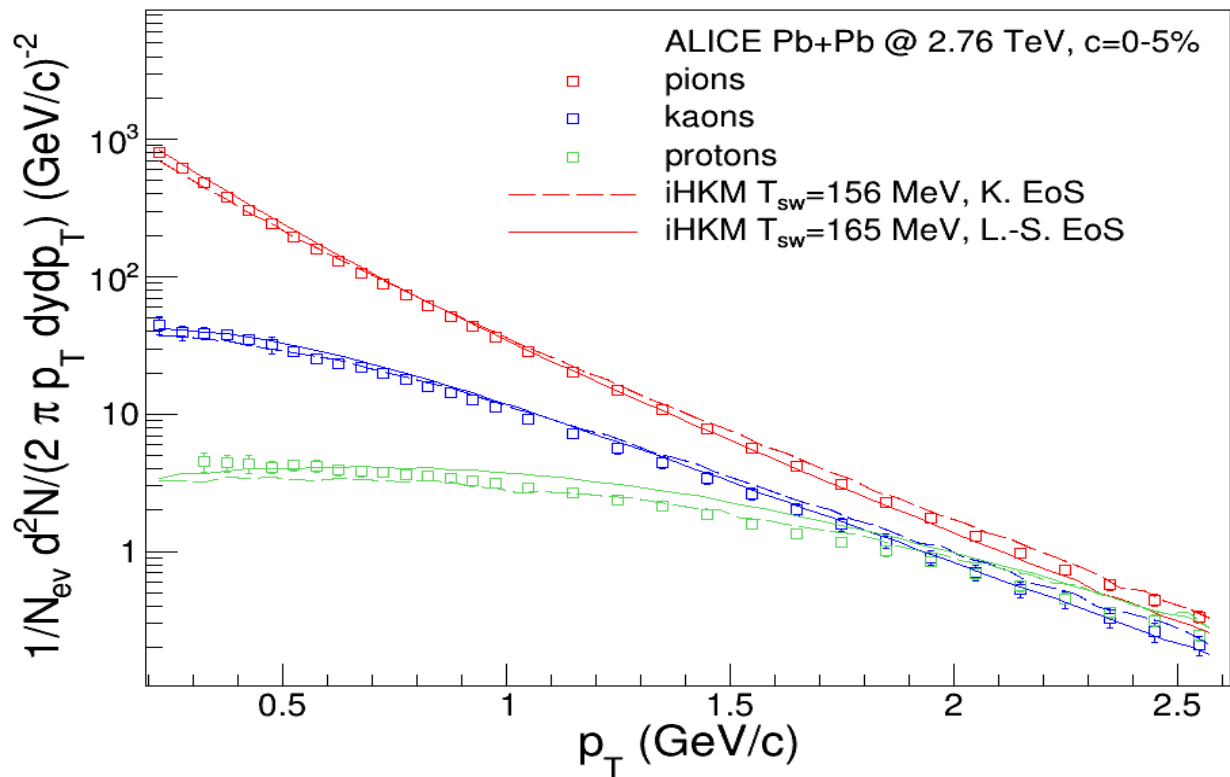
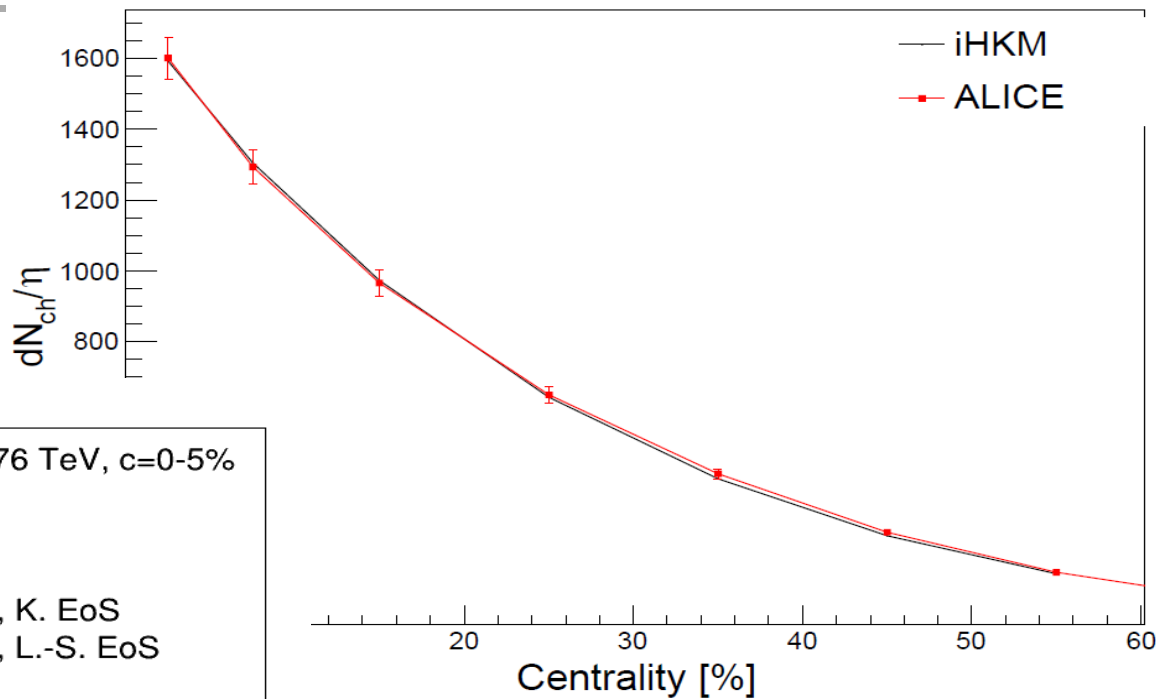
$$\frac{dN_i}{p_T dp_T} \longrightarrow T_{th}$$

Kinetic freeze-out is continuous, lasts more than 5 fm/c. «Effective temperature» of maximal emission:

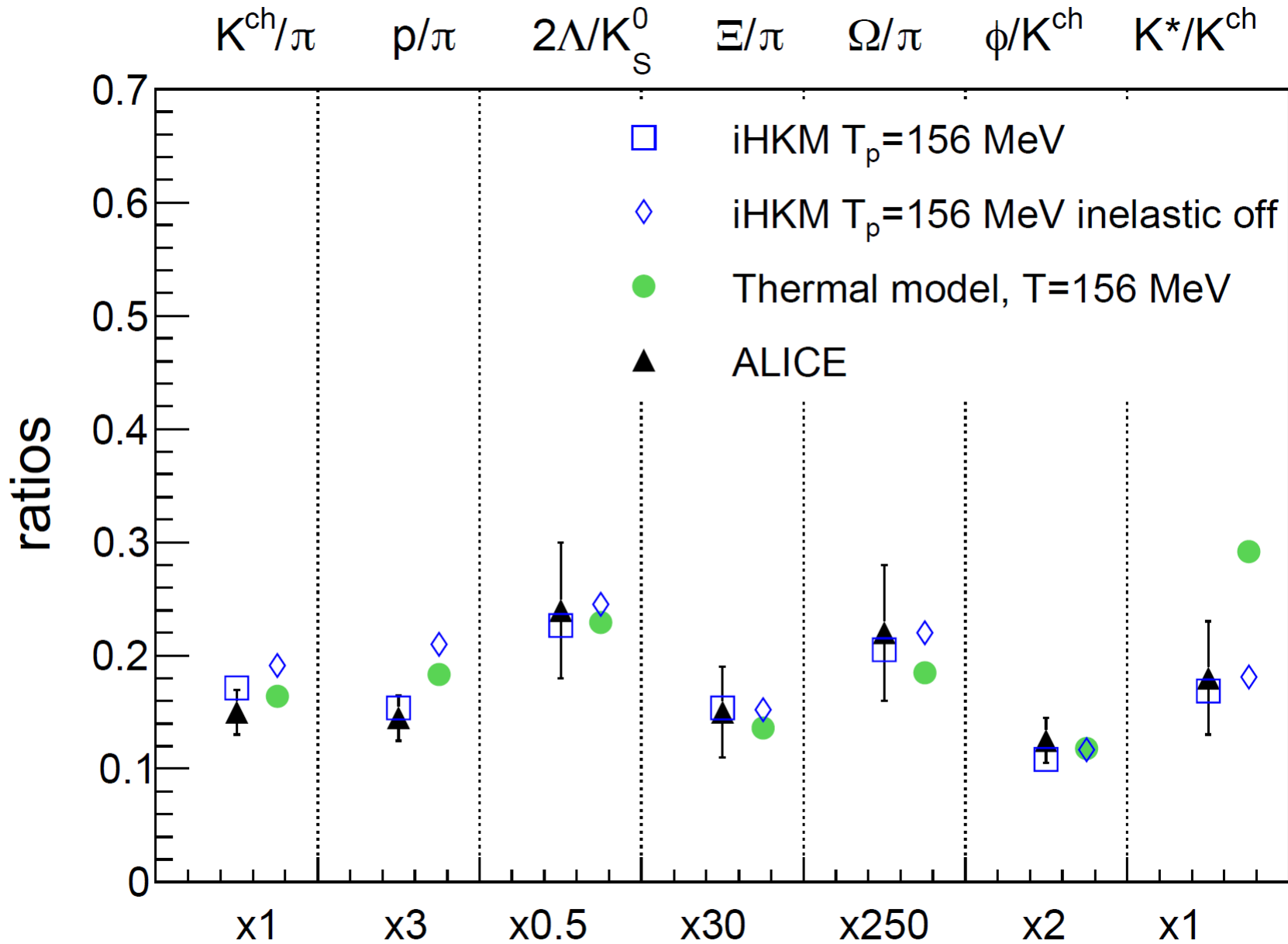
$$T_{th}(p)$$

# Multiplicity dependence of all charged particles on centrality and spectra for LHC energy

$\sqrt{s_{NN}} = 2.76 \text{ TeV}$

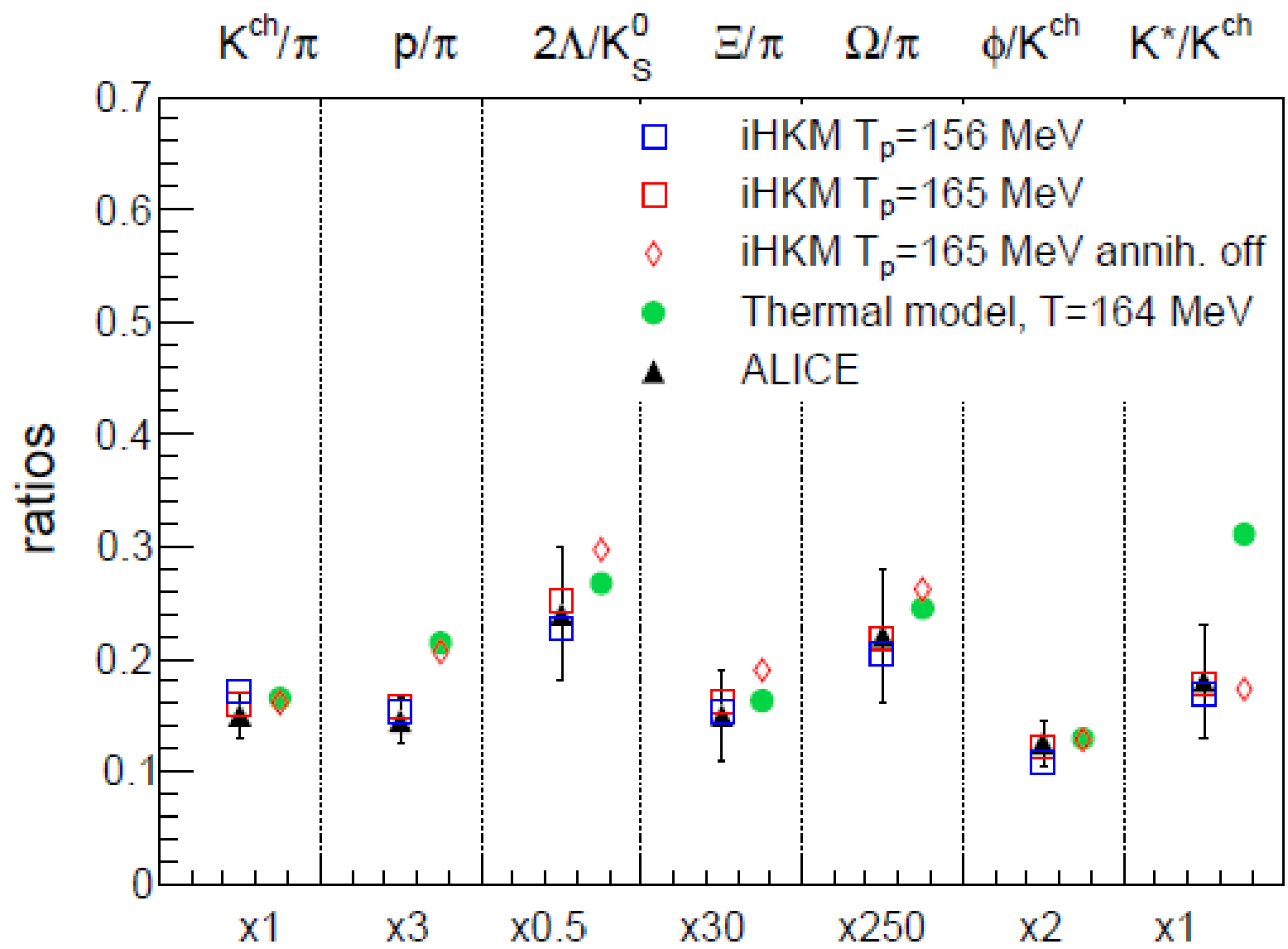




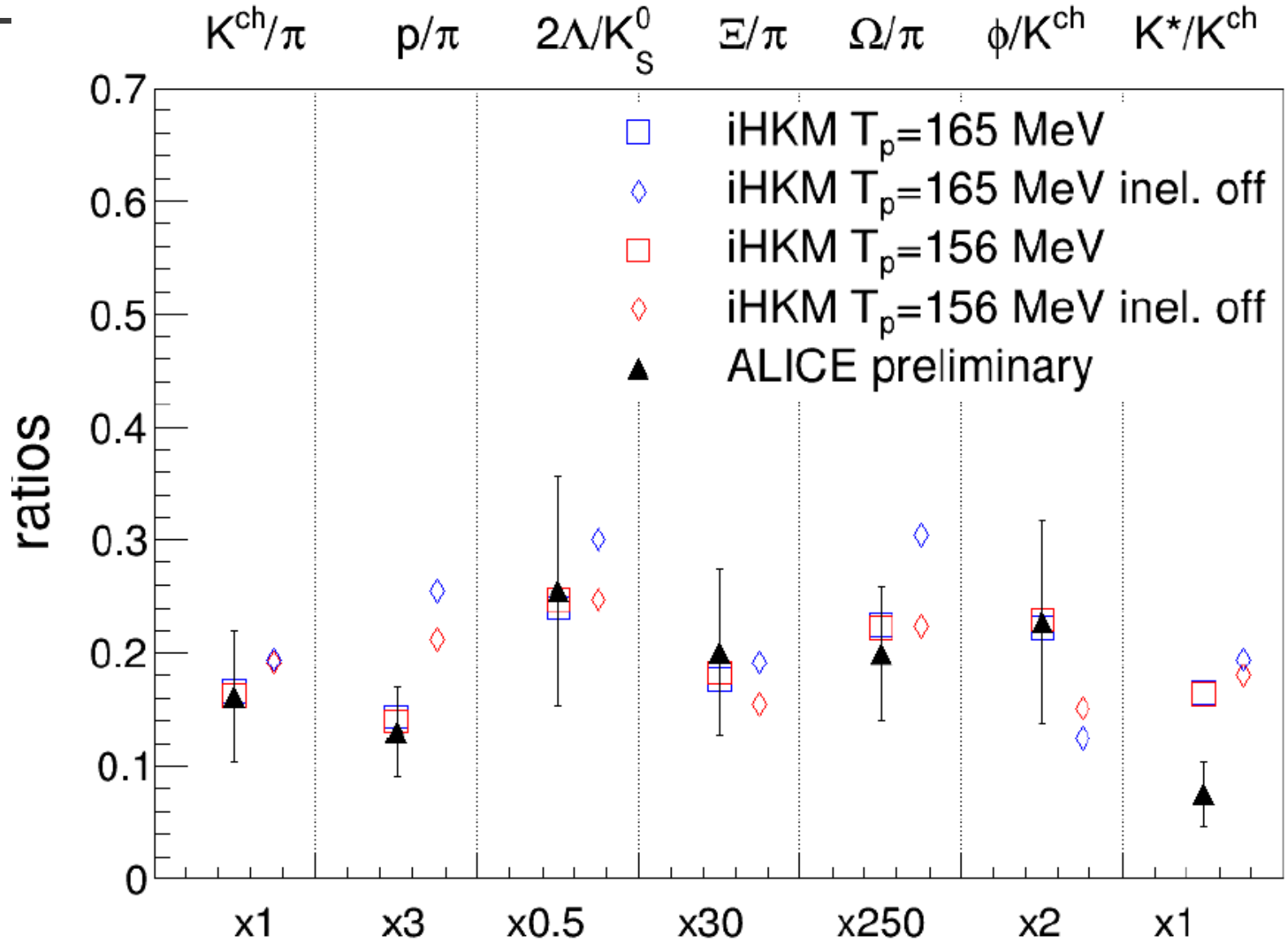


Yu.S. , Shapoval,  
 Phys. Rev. C 97 064901 (2018)

# Particle number ratios at the LHC, 2.76 TeV, L-S EoS

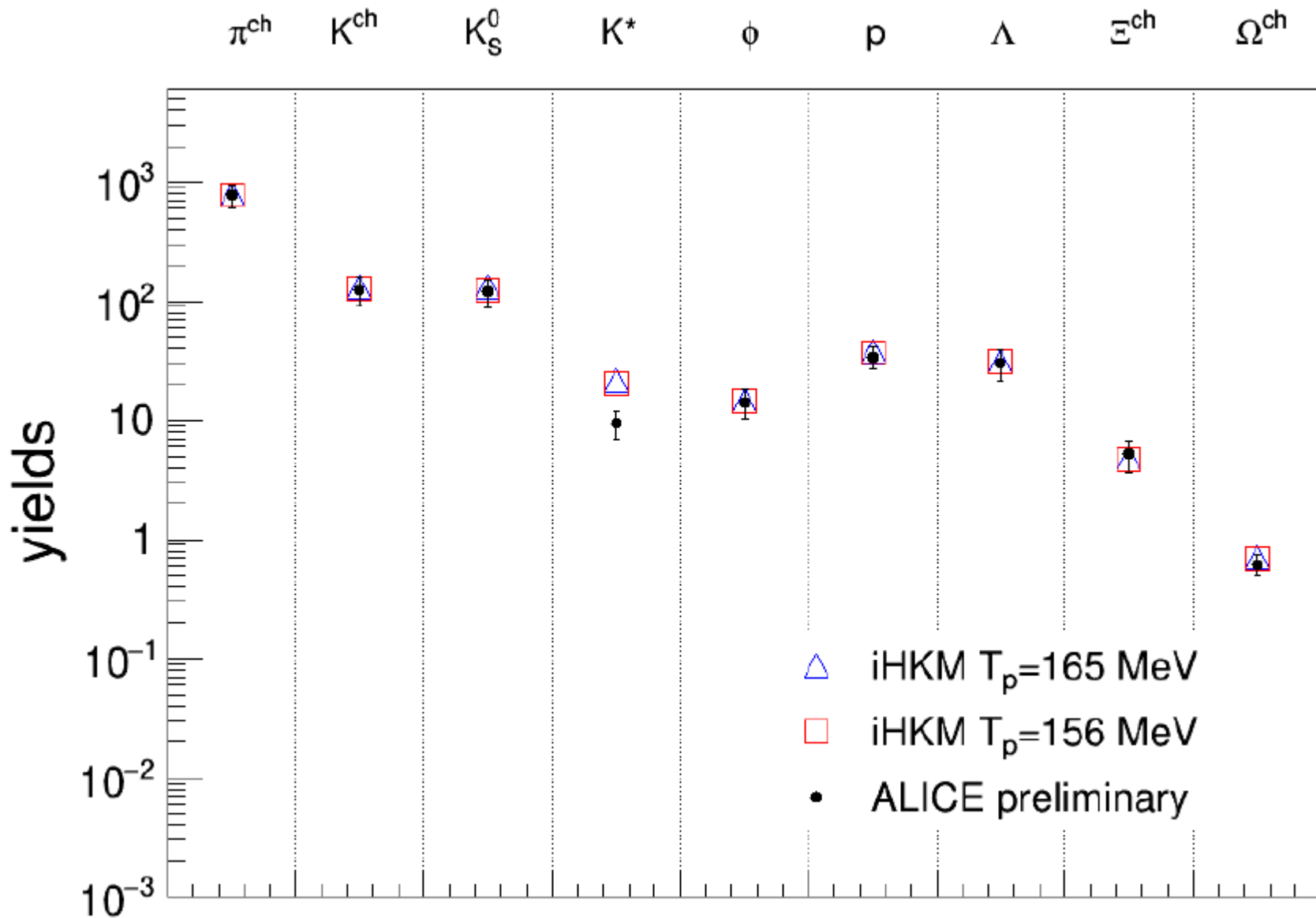


# Particle number ratios at the LHC, 5.02 TeV/n.p.



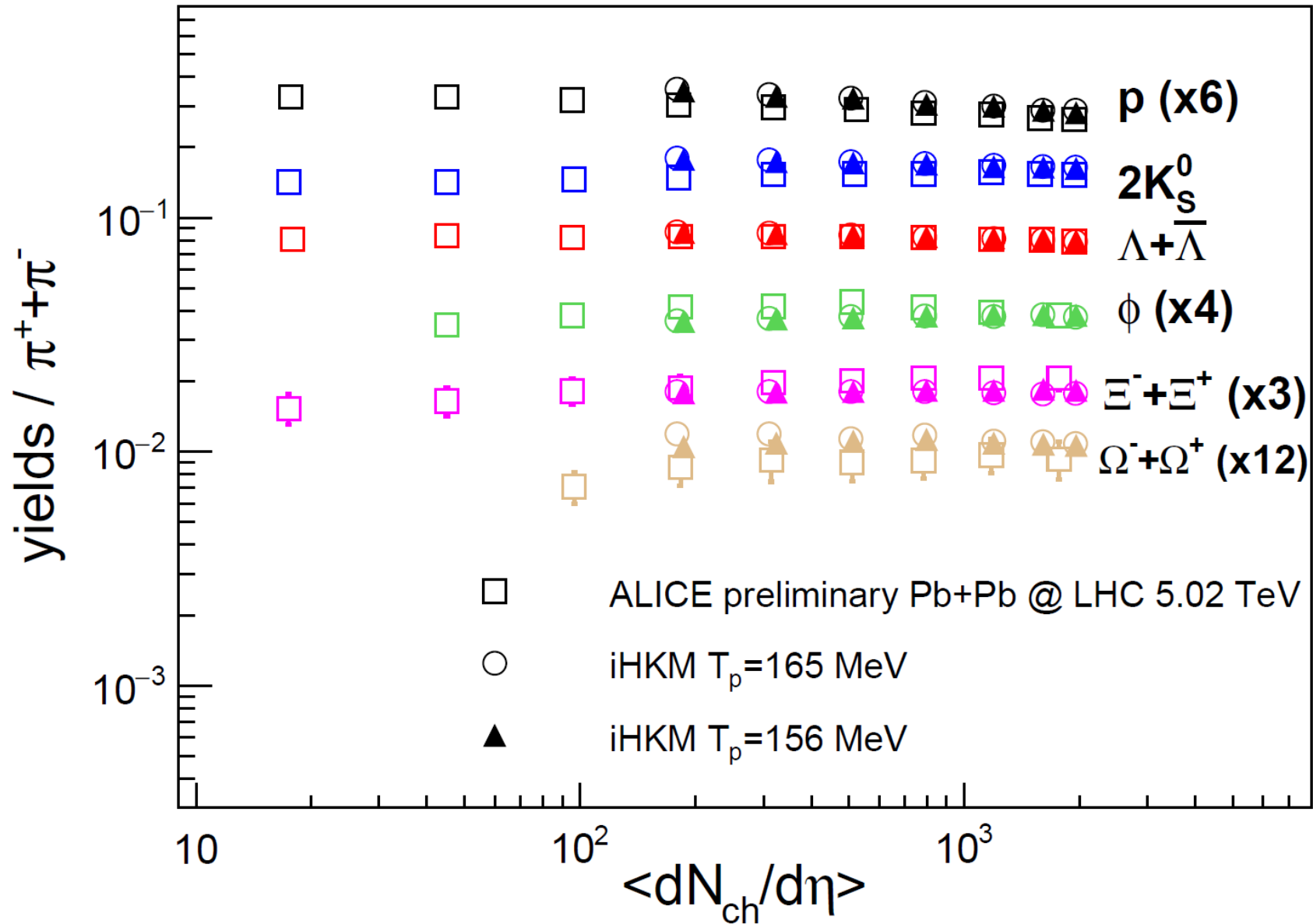
Yu.S. , Shapoval, [arXiv:1809.07400](https://arxiv.org/abs/1809.07400) (will be published Phys.Rev. C (2019)).

# Particle yields at LHC, 5.02 TeV/n.p.



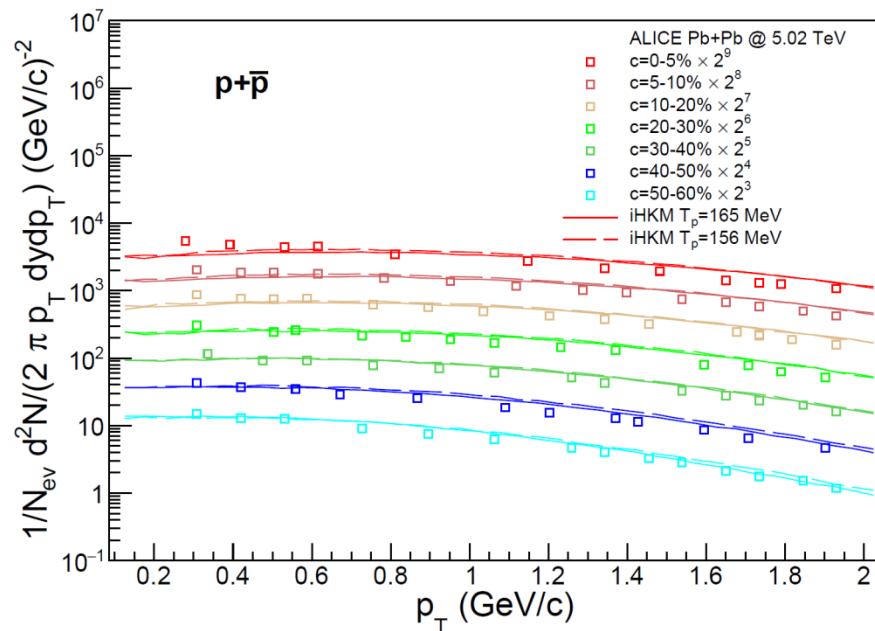
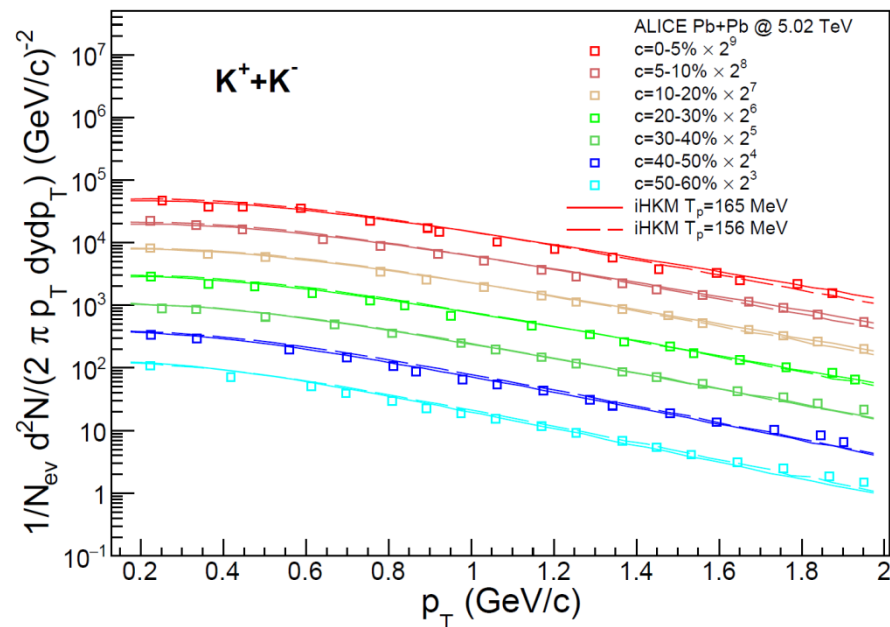
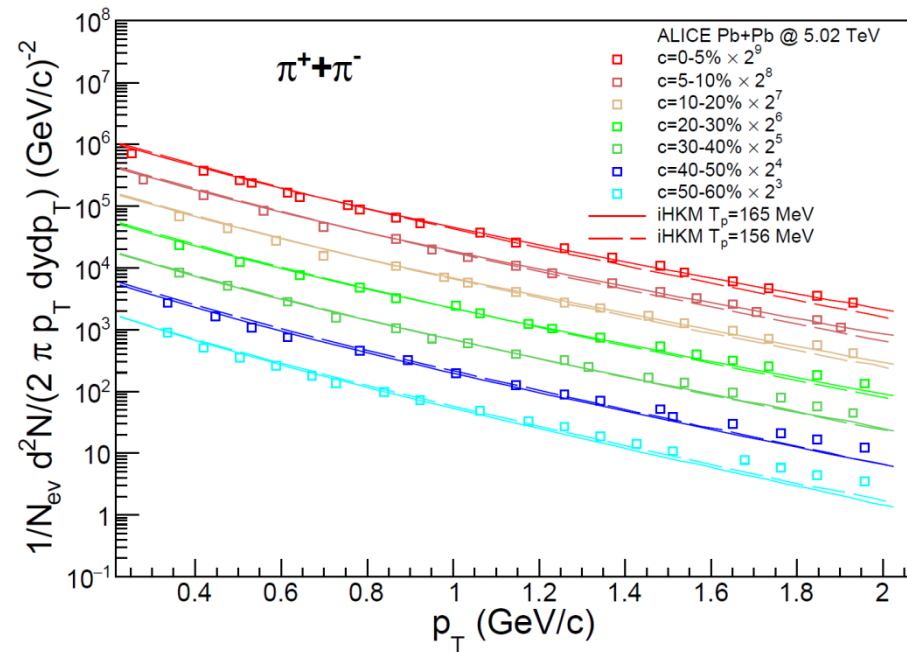
Yu.S. , Shapoval, [arXiv:1809.07400](https://arxiv.org/abs/1809.07400) (will be published in Phys.Rev. C (2019)).

# Particle ratios at LHC, 5.02 TeV/n.p.



Yu.S. , Shapoval, [arXiv:1809.07400](https://arxiv.org/abs/1809.07400) (will be published in Phys.Rev. C (2019)).

# Spectra at LHC 5.02 TeV/n.p



# Elliptic flow LHC 5.02 TeV/n.p.

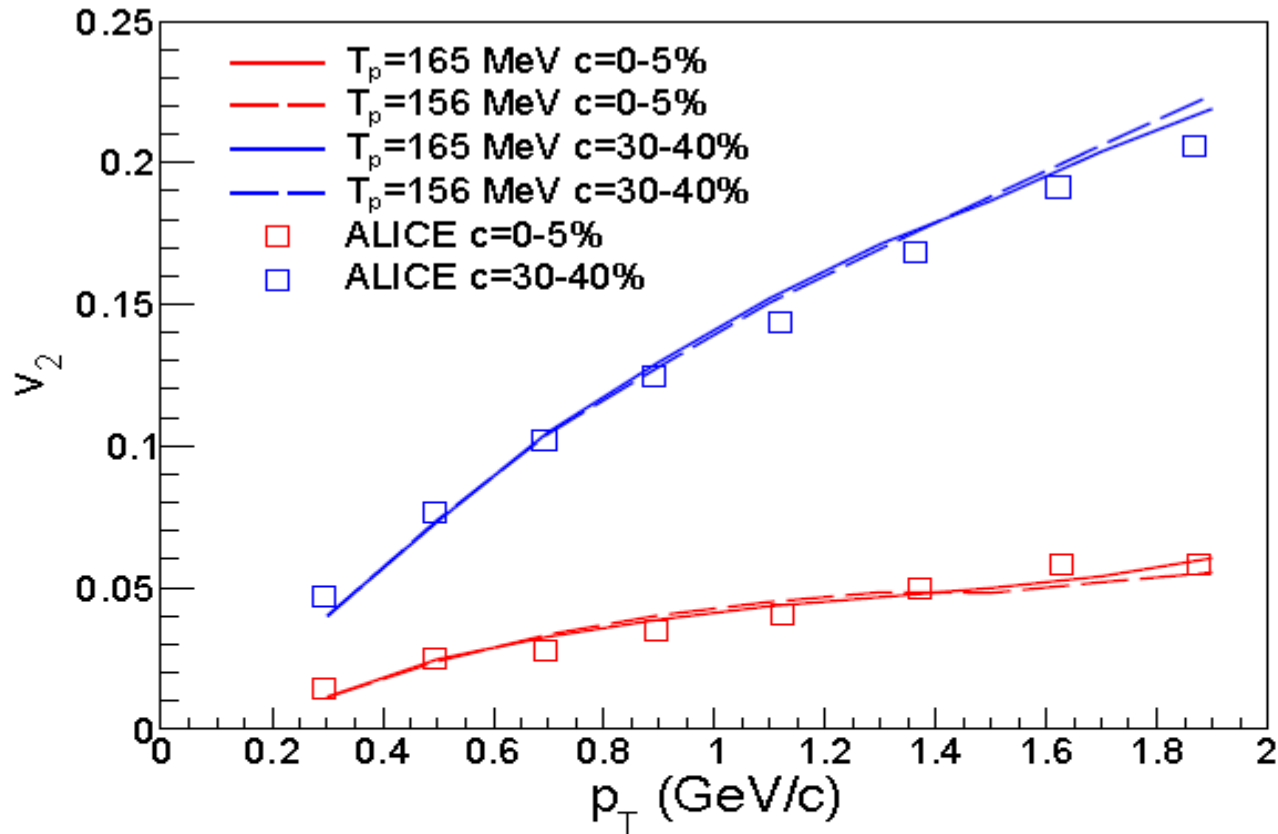
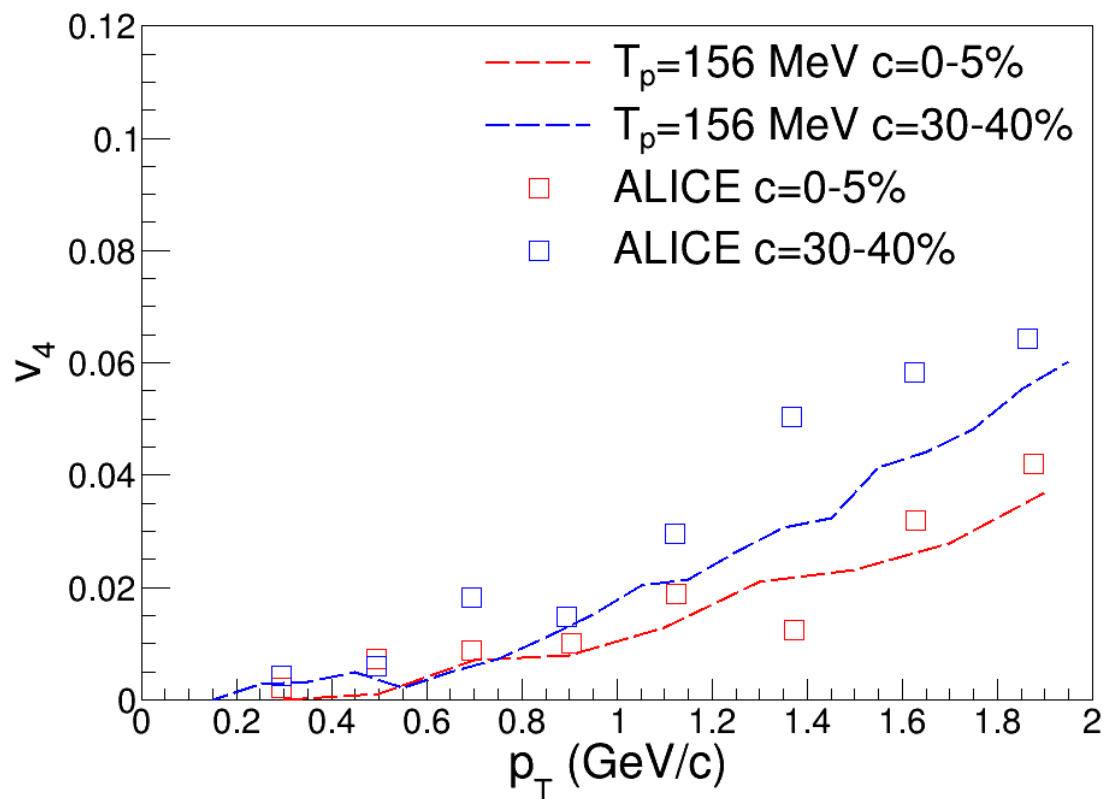
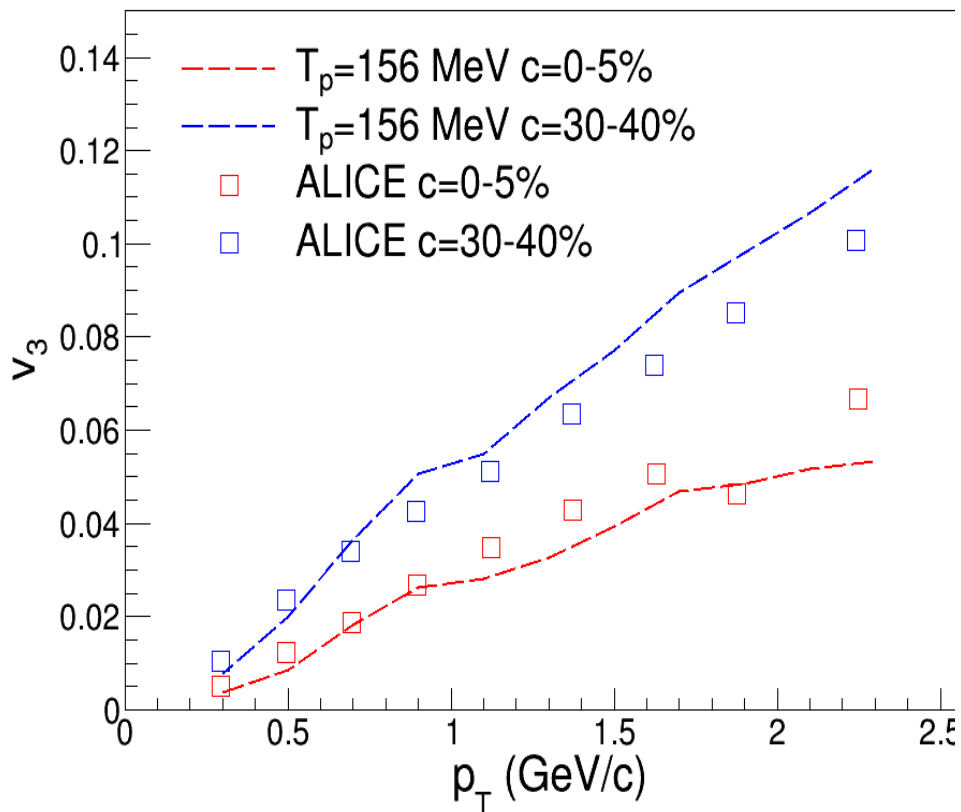



FIG. 13. The iHKM results on the elliptic flow  $v_2$  dependence on  $p_T$  for all charged particles together with the corresponding ALICE data [16] for the centrality classes  $c = 0 - 5\%$  and  $c = 30 - 40\%$ . The model curves for the two particleization temperatures,  $T_p = 165$  MeV and  $T_p = 156$  MeV, are presented.

Yu.S. , Shapoval, [arXiv:1809.07400](https://arxiv.org/abs/1809.07400) (will be published in Phys.Rev. C (2019)).

# Asymmetric n-flow LHC 5.02 TeV/n.p.







iHKM results for the top RHIC energy

$$\sqrt{s_{NN}} = 200 \text{ GeV}$$

M. Adzhimambetov, V. Shapoval, Yu.S.,  
Nucl.Phys. A **987** (2019) 321–336.

## The equation of state Au+Au, top RHIC energy

We utilize Laine-Schröder equation of state for quark-gluon phase, modified for non-zero baryon and strange chemical potentials:

$$\frac{p(T, \mu_B, \mu_S)}{T^4} = \frac{p(T, 0, 0)}{T^4} + \frac{1}{2} \frac{\chi_B}{T^2} \left( \frac{\mu_B}{T} \right)^2 + \frac{1}{2} \frac{\chi_S}{T^2} \left( \frac{\mu_S}{T} \right)^2. \quad (1)$$

We put  $\mu_B = 21 \text{ MeV}$  to obtain the best agreement with the experimental ratio of protons yield to that of antiprotons in central ( $c = 0 - 5\%$ ) collisions.

We also put  $\mu_S = 5 \text{ MeV}$  to obtain vanishing strangeness at the hadronization hypersurface:

$$S|_{\sigma_p} = \sum_i (N(i) - \bar{N}(i)) \mu_{S,i} = 0,$$

where  $N(i)$  and  $\bar{N}(i)$  are the numbers of particles and corresponding antiparticles of species  $i$  at the hadronization hypersurface, and  $\mu_{S,i}$  is the strange chemical potential of the particle species  $i$ .

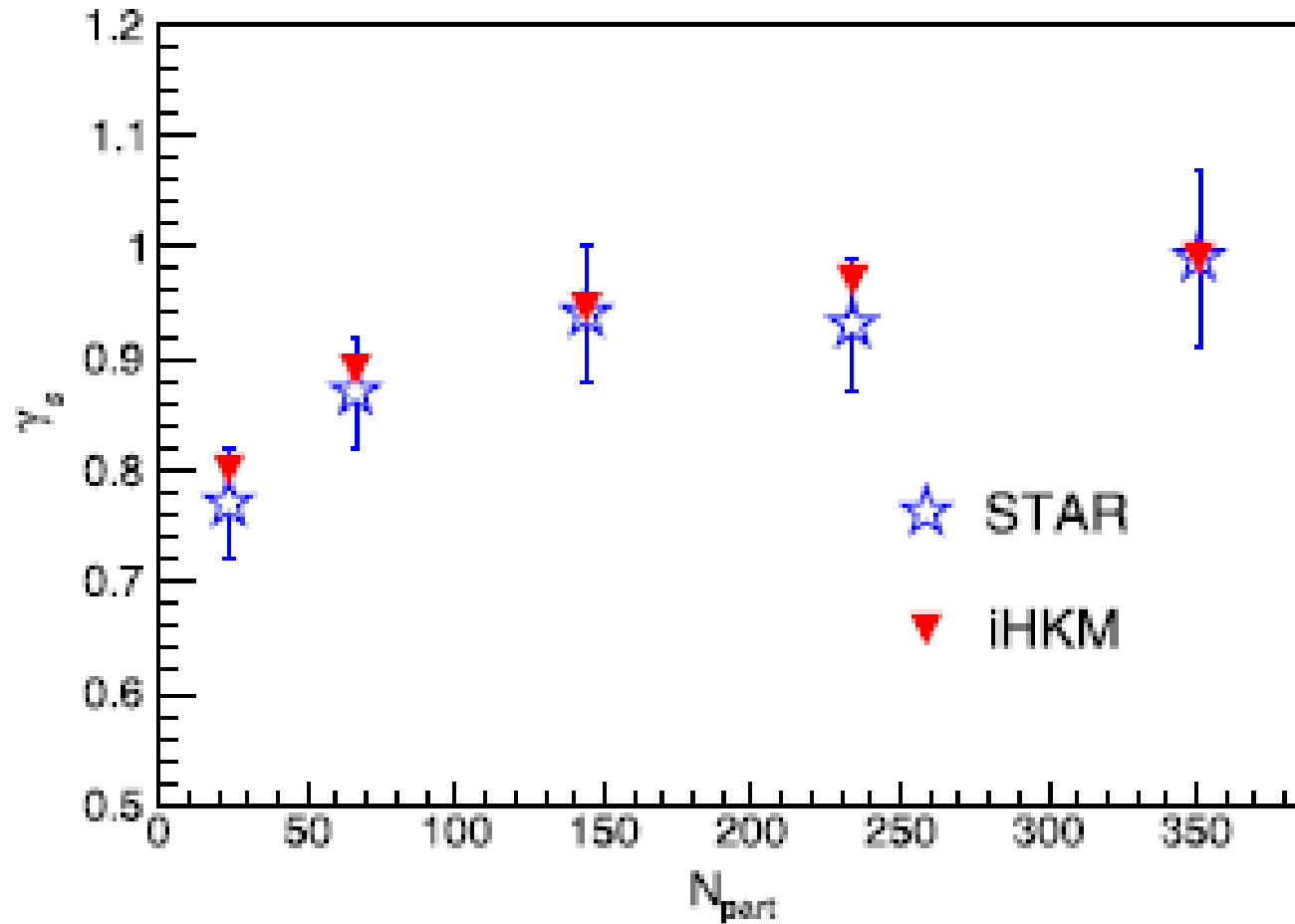
We assume that at considered energy strange quarks do not have enough time to reach the chemical equilibrium in non-central collisions, so that the kaon spectra get down. The same concerns about a half of produced protons, coming from the decays of strange resonances (such as  $\Lambda$ ,  $\Sigma$ ,  $\Xi$ ).

To take this into account we introduce an effective downscaling factor  $\gamma_S(\tau_p)$ , depending on the characteristic particle production time for each given centrality.

We assume the dependence  $\gamma_S(\tau_p) = A \exp(-b/\tau_p)$ , with  $A = 1.1$  and  $b = 0.8 \text{ fm}/c$ . This choice guarantees  $\gamma_S = 1$  for the most central events and a good description of kaon and proton spectra, together with  $K/\pi$  ratio.

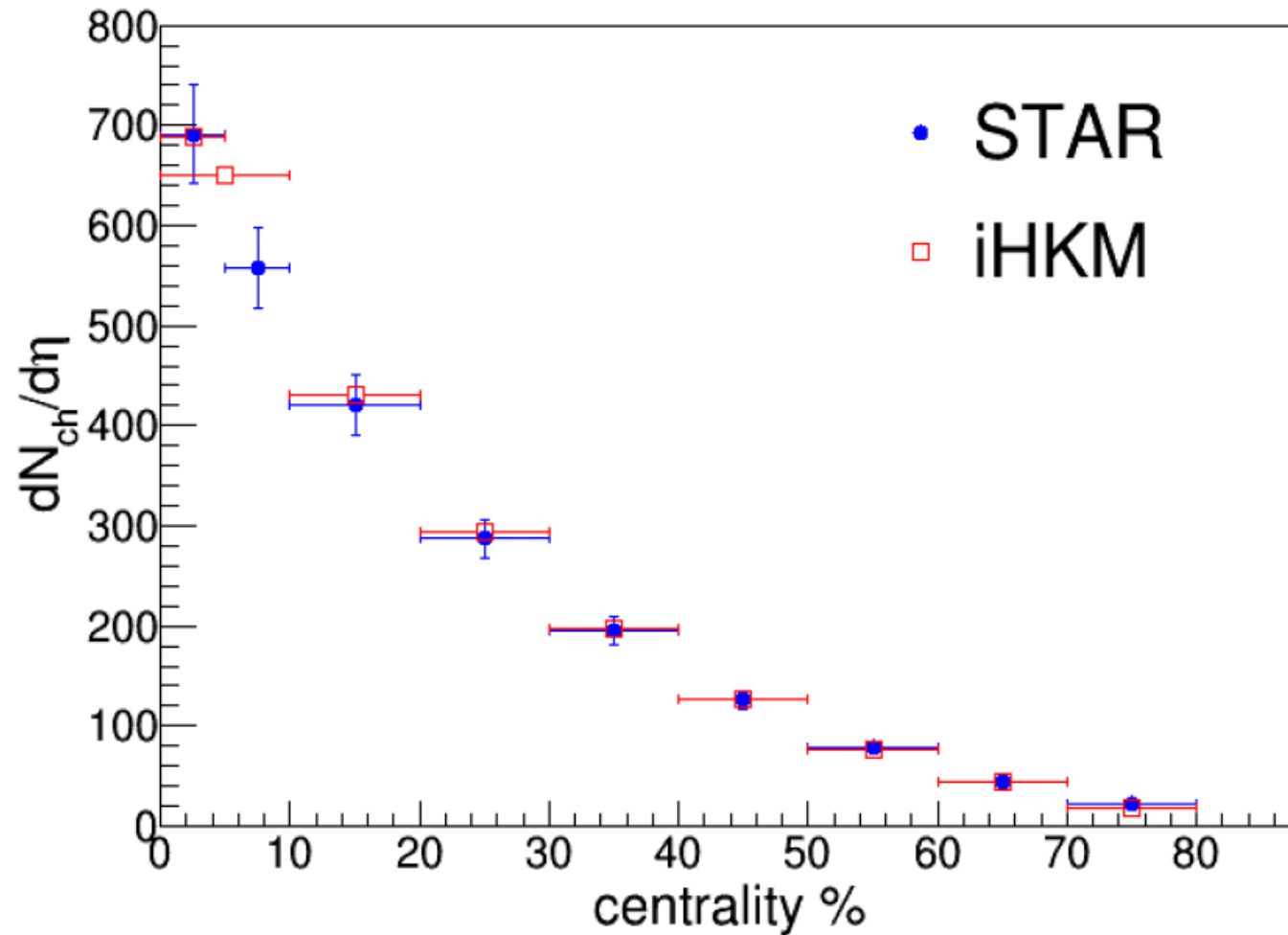
$\gamma_S$ 

Au+Au, top RHIC energy



$$\gamma_S(\tau_p) = A \exp(-b/\tau_p) \quad \text{with } A = 1.1 \text{ and } b = 0.8 \text{ fm}/c.$$

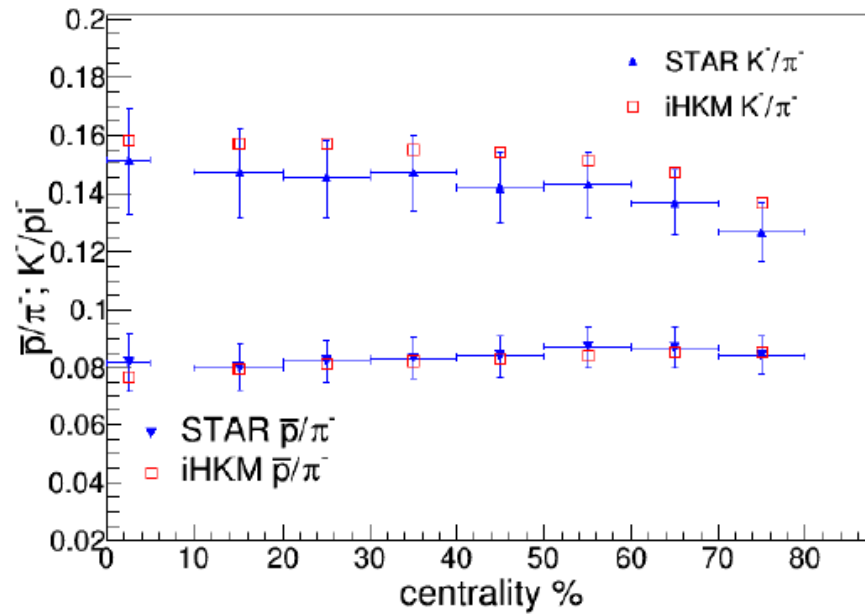
# Main model parameters adjustment Au+Au, top RHIC energy



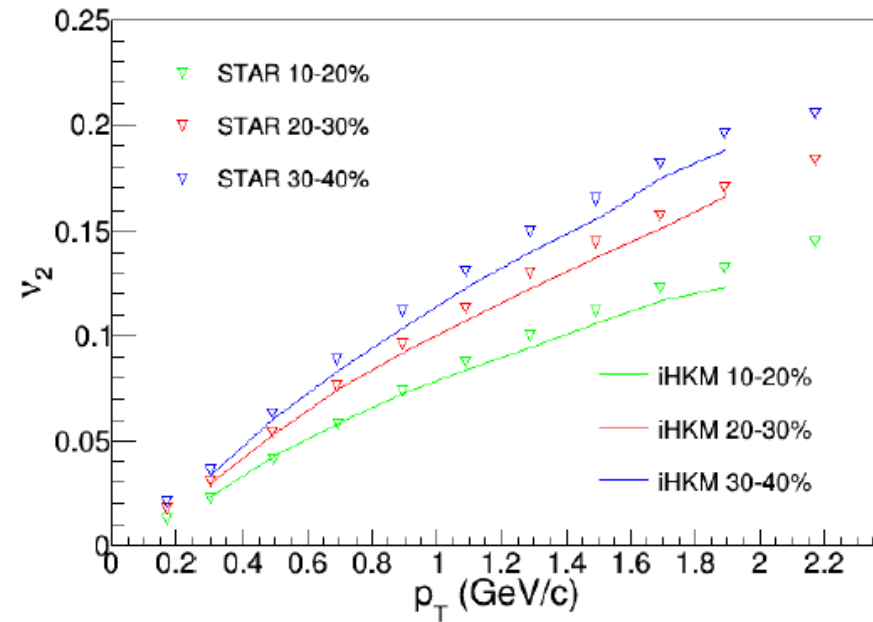
$$\epsilon_0 = 235 \text{ GeV}/\text{fm}^3 \text{ at } \tau_0 = 0.1 \text{ fm}/c, \alpha = 0.18$$

B.I. Abelev *et al.* (STAR Collaboration), Phys. Rev. C **79** (2009), 034909

# Particle number ratios and $v_2$ Au+Au, top RHIC energy

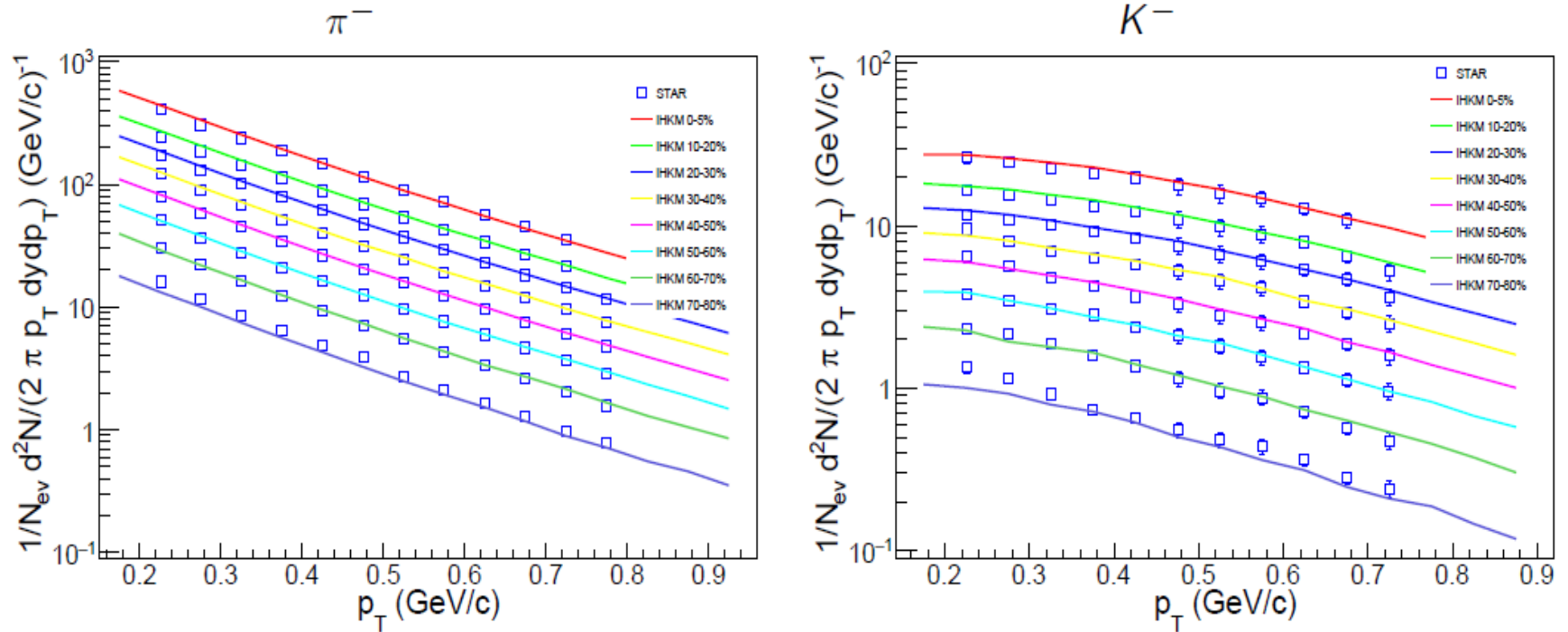


J. Adams *et al.* (STAR Collaboration),  
Phys. Rev. Lett. **92** (2004) 112301



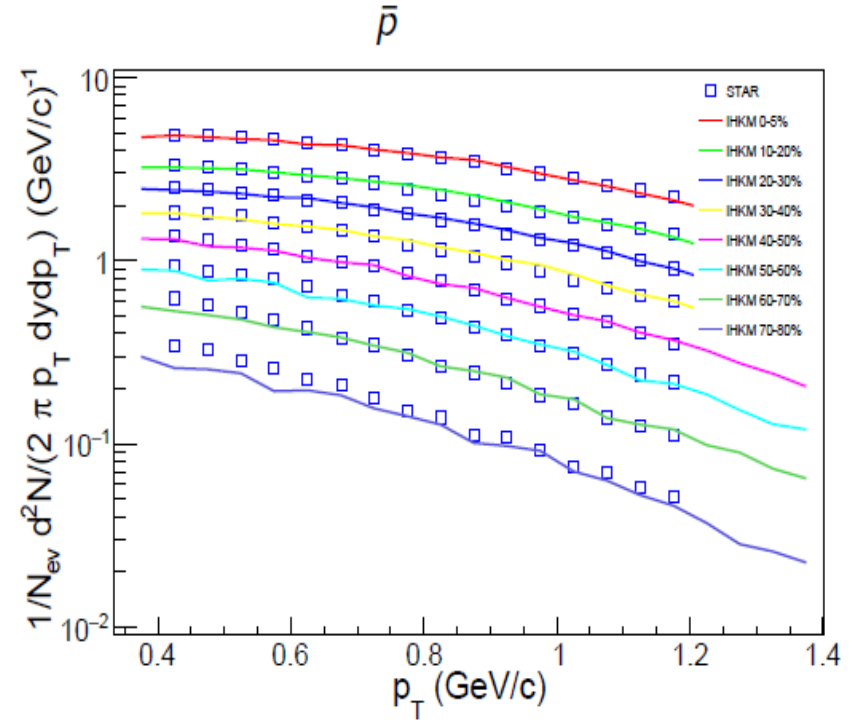
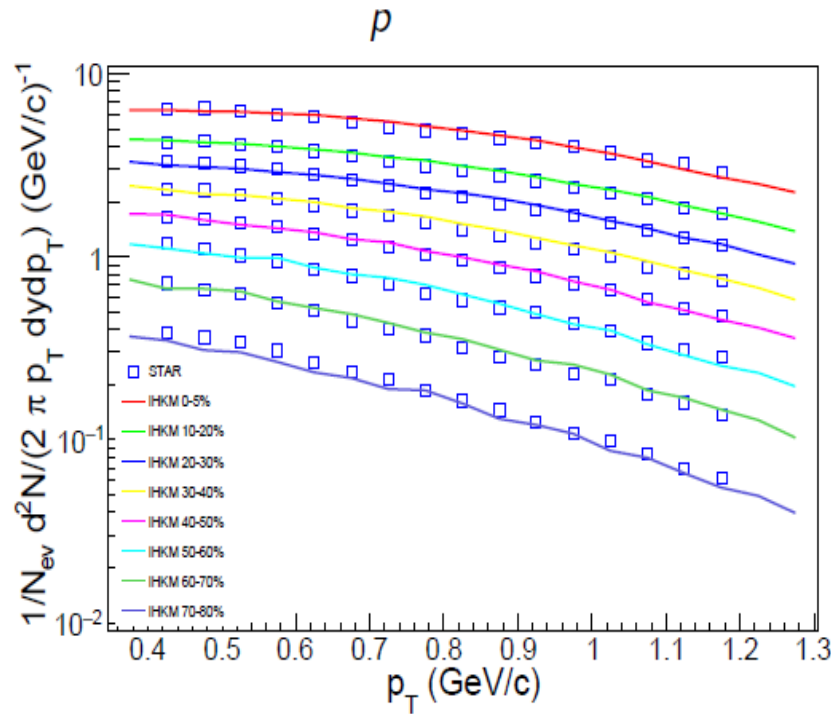
J. Adams *et al.* (STAR Collaboration),  
Phys. Rev. C **72** (2005), 014904

$p_T$  spectra description,  $|y| < 0.1$  Au+Au, top RHIC energy



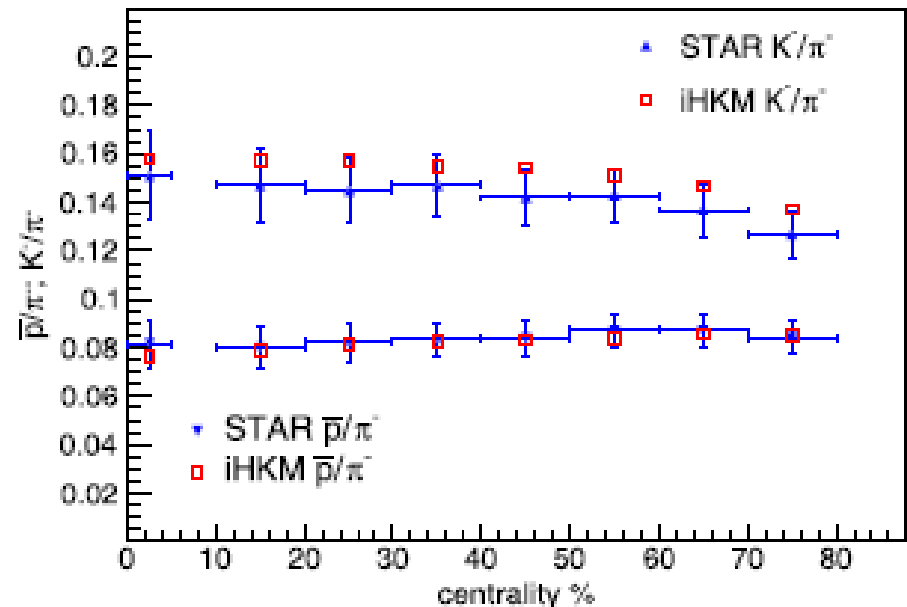
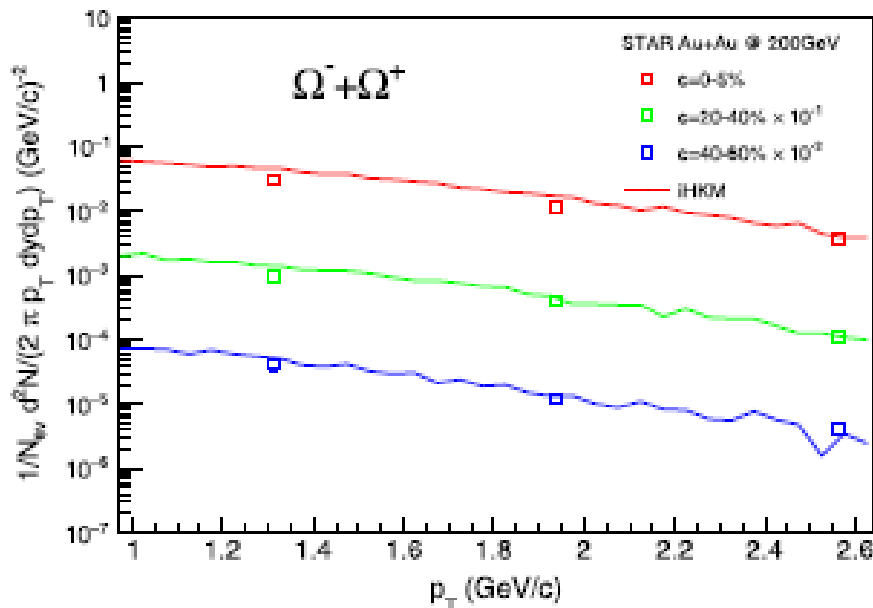
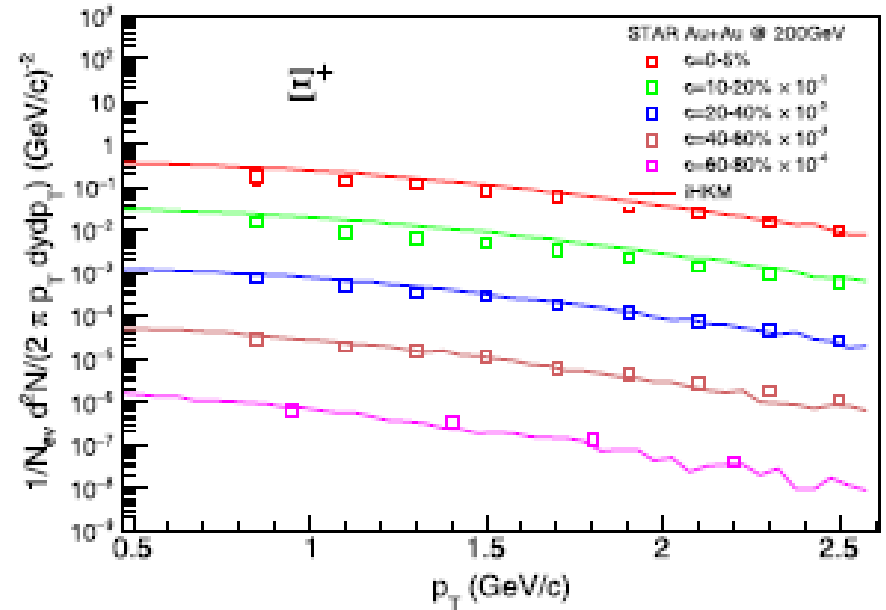
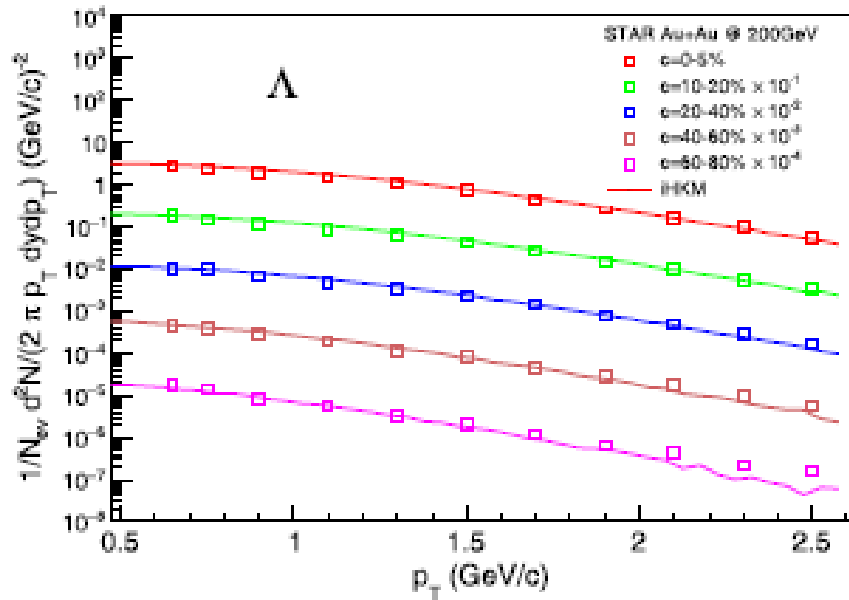
J. Adams *et al.* (STAR Collaboration), Phys. Rev. Lett. **92** (2004) 112301

$p_T$  spectra description,  $|y| < 0.1$  Au+Au, top RHIC energy



J. Adams *et al.* (STAR Collaboration), Phys. Rev. Lett. **92** (2004) 112301

# Spectra of strange baryons, $\left\{ \frac{\bar{p}}{\pi^-}, \frac{K^-}{\pi^-} \right\}$ on centrality





# Summary on the particle production

- Neither thermal nor chemical freeze-out cannot be considered as sudden at some corresponding temperatures.
- Particle yield probe  $\frac{dN_i}{d\eta} / \frac{dN_j}{d\eta}$  as well as absolute values  $\frac{dN_i}{d\eta}$  (!) demonstrate that even at the minimal hadronization temperature  $T_{ch} = T_h = 156$  MeV, the annihilation and other non-elastic scattering reactions play role in formation particle number ratios, especially.
- It happens that the results for small and relatively large  $T_h$  are quite similar. It seems that inelastic processes (other than the resonance decays), that happen at the matter evolution below  $T_h$ , play a role of the compensatory mechanism in formation of  $\frac{dN_i}{d\eta} / \frac{dN_j}{d\eta}$ .  
Chemical freeze-out is continuous.
- The iHKM works perfectly not only at LHC but also at RHIC energies and describes well spectra of pion, kaon, proton, antiprotons, Lambdas, Omega, Cascade, different particle number ratios? Elliptic flow and femtoscopy scales. For this aim non-zero baryon chemical potential is introduced as well as  $\gamma_s$ . The latter depends on the proper life-time of QGP, which is calculated in the iHKM.



iHKM

---

**DIRECT PHOTONS**

# Direct-photon spectrum and elliptic flow produced from Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV at the CERN Large Hadron Collider within an integrated hydrokinetic model

V. Yu. Naboka,<sup>1</sup> Yu. M. Sinyukov,<sup>1</sup> and G. M. Zinovjev<sup>1</sup>

<sup>1</sup>*Bogolyubov Institute for Theoretical Physics, 03680 Kiev, Ukraine*



(Received 22 October 2017; revised manuscript received 6 March 2018; published 16 May 2018)

The photon transverse momentum spectrum and its anisotropy from Pb+Pb collisions at the CERN Large Hadron Collider energy  $\sqrt{s_{NN}} = 2.76$  TeV are investigated within the integrated hydrokinetic model (iHKM). Photon production is accumulated from the different processes at the various stages of relativistic heavy ion collisions: from the primary hard photons of very early stage of parton collisions to the thermal photons from equilibrated quark-gluon and hadron gas stages. Along the way a hadronic medium evolution is treated in two distinct, in a sense opposite, approaches: chemically equilibrated and chemically frozen system expansion. Studying the centrality dependence of the results obtained allows us to conclude that a relatively strong transverse momentum anisotropy of thermal radiation is suppressed by prompt photon emission which is an isotropic. We find out that this effect is getting stronger as centrality increases because of the simultaneous increase in the relative contribution of prompt photons in the soft part of the spectra. The substantial results obtained in iHKM with nonzero viscosity ( $\eta/s = 0.08$ ) for photon spectra and  $v_2$  coefficients are mostly within the error bars of experimental data, but there is some systematic underestimation of both observables for the near central events. We claim that a situation could be significantly improved if an additional photon radiation that accompanies the presence of a deconfined environment is included. Since a matter of a space-time layer where hadronization takes place is actively involved in anisotropic transverse flow, both positive contributions to the spectra and  $v_2$  are considerable, albeit such an argument needs further research and elaboration.

## Direct-photon spectrum and elliptic flow produced from Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV at the CERN Large Hadron Collider within an integrated hydrokinetic model

V. Yu. Naboka,<sup>1</sup> Yu. M. Sinyukov,<sup>1</sup> and G. M. Zinovjev<sup>1</sup>

<sup>1</sup>*Bogolyubov Institute for Theoretical Physics, 03680 Kiev, Ukraine*



(Received 22 October 2017; revised manuscript received 6 March 2018; published 16 May 2018)

The photon transverse momentum spectrum and its anisotropy from Pb+Pb collisions at the CERN Large Hadron Collider energy  $\sqrt{s_{NN}} = 2.76$  TeV are investigated within the integrated hydrokinetic model (iHKM). Photon production is accumulated from the different processes at the various stages of relativistic heavy ion collisions: from the primary hard photons of very early stage of parton collisions to the thermal photons from equilibrated quark-gluon and hadron gas stages. Along the way a hadronic medium evolution is treated in two distinct, in a sense opposite, approaches: chemically equilibrated and chemically frozen system expansion. Studying the centrality dependence of the results obtained allows us to conclude that a relatively strong transverse momentum anisotropy of thermal radiation is suppressed by prompt photon emission which is an isotropic. We find out that this effect is getting stronger as centrality increases because of the simultaneous increase in the relative contribution of prompt photons in the soft part of the spectra. The substantial results obtained in iHKM with nonzero viscosity ( $\eta/s = 0.08$ ) for photon spectra and  $v_2$  coefficients are mostly within the error bars of experimental data, but there is some systematic underestimation of both observables for the near central events.

We claim that a situation could be significantly improved if an additional photon radiation that accompanies the presence of a deconfined environment is included. Since a matter of a space-time layer where hadronization takes place is actively involved in anisotropic transverse flow, both positive contributions to the spectra and  $v_2$  are considerable, albeit such an argument needs further research and elaboration.

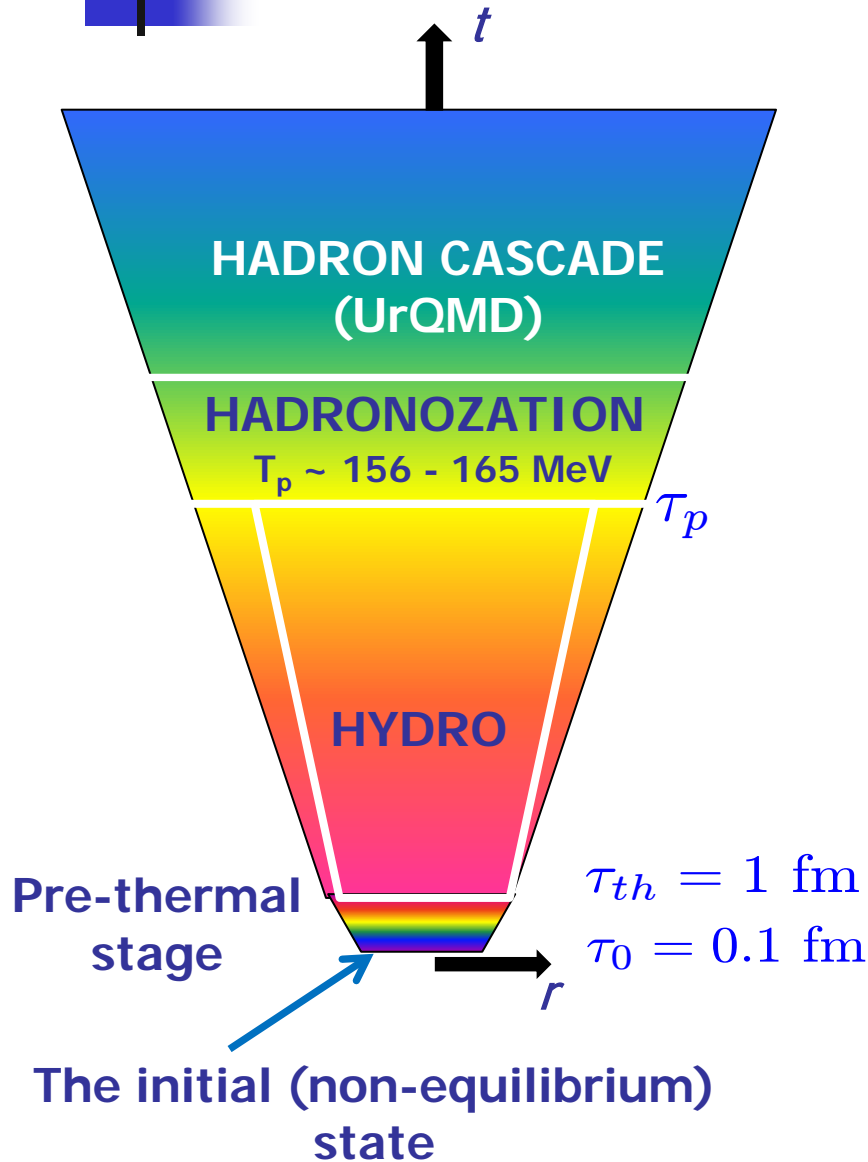


# Blow up text

---

We claim that a situation could be significantly improved if an additional photon radiation that accompanies the presence of a deconfined environment is included. Since a matter of a space-time layer where hadronization takes place is actively involved in anisotropic transverse flow, both positive contributions to the spectra and  $v_2$  are considerable, albeit such an argument needs further research and elaboration.

# Photon radiation in iHKM



- generation of the initial states: (MC Glaub & CGC)

## PROMPT PHOTONS

- thermalization of initially non-thermal matter;

## PRE-THERMAL PHOTONS

- viscous chemically equilibrated hydrodynamic expansion;

## THERMAL PHOTONS FROM QGP

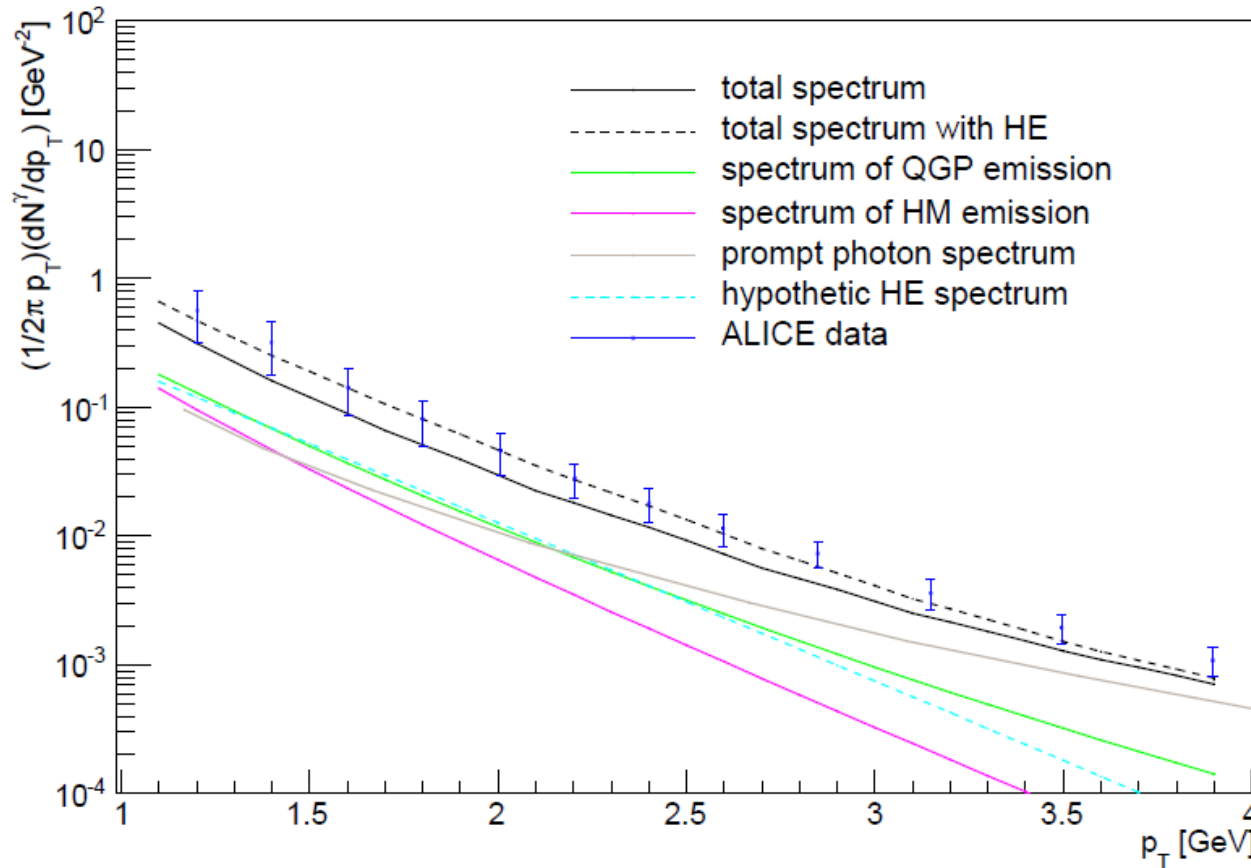
- hadronization of expanding medium

## HADRONIZATION EMISSION

- hadron matter expansion

## THERMAL PHOTONS FROM HADRONIC STAGE

# Direct photons. Transverse Spectra



We claim that a description of photon spectra and its anisotropy could be significantly improved if an additional photon radiation, that accompanies the presence of deconfined environment, is included.

FIG. 1. Total direct photon spectra in iHKM: thermal QGP + thermal HM + prompt + hadronization emission (HE). Centrality is 0-40%. Experimental results are taken from [7].

# Photon puzzle: Anisotropy of spectra, large $v_2$ coefficients.

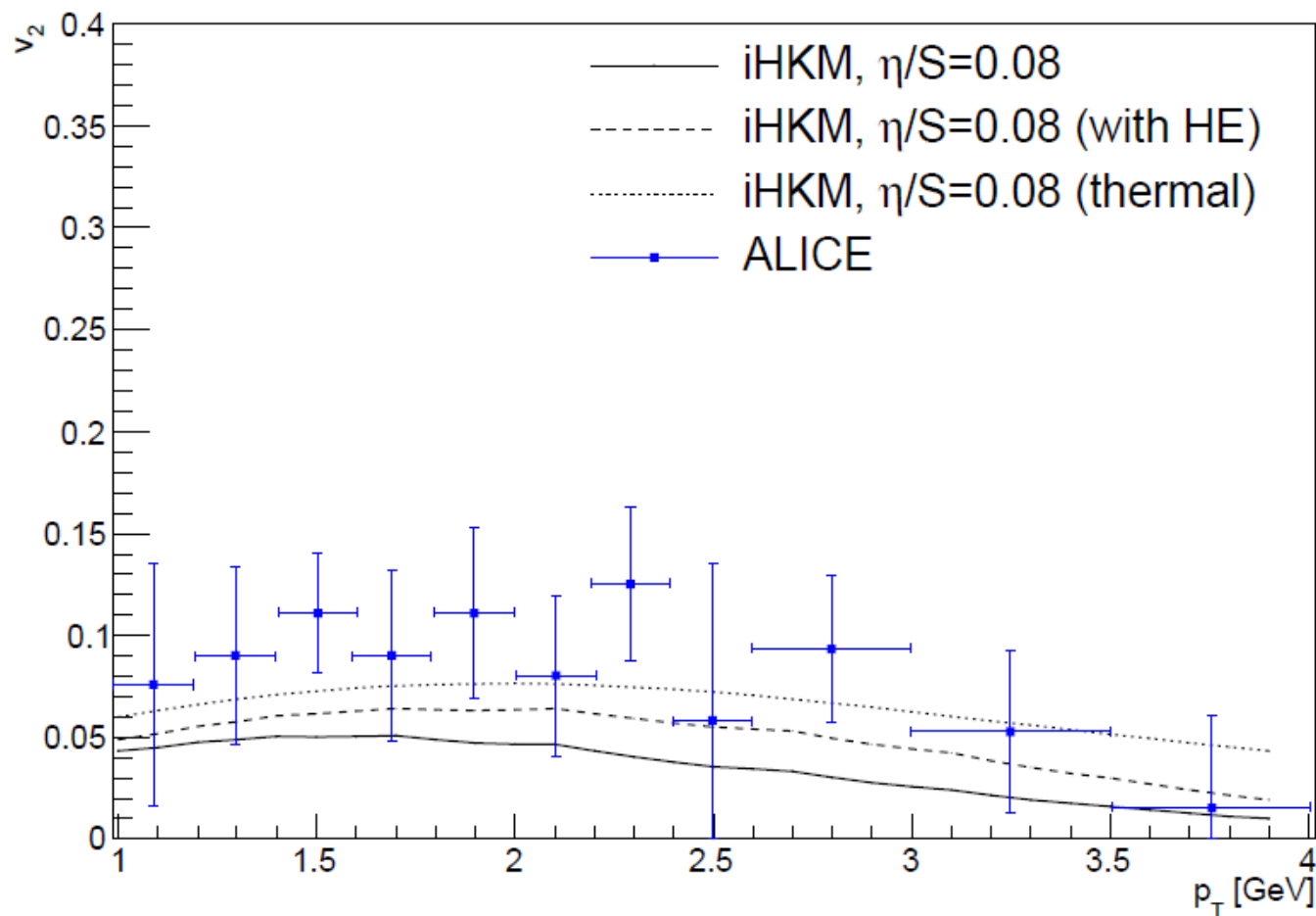


FIG. 2. Photon momentum anisotropy  $v_2$ -coefficient for 0-40% centrality. The results including the synchrotron radiation (HE) and results for prompt photons only (without HE) are also presented.





# Photons at RHIC

---

Photon spectra and anisotropic flow in heavy ion collisions at the top RHIC energy within the integrated hydrokinetic model with photon hadronization emission

V. Yu. Naboka<sup>1</sup>, Yu. M. Sinyukov<sup>1</sup>, and G. M. Zinovjev<sup>1</sup>

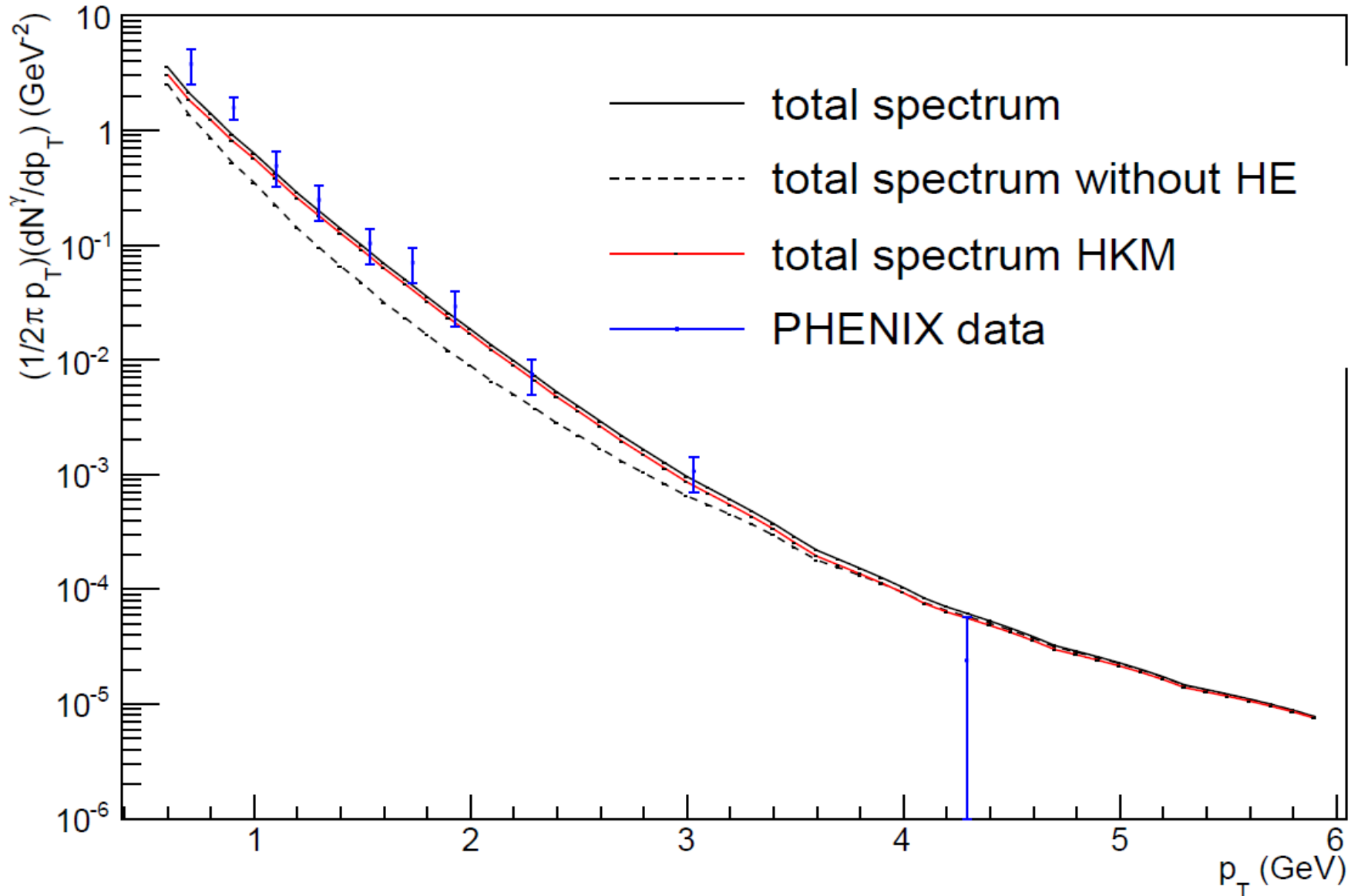
<sup>1</sup>*Bogolyubov Institute for Theoretical Physics, 03680 Kiev, Ukraine*

*European Physical Journal A - Manuscript ID*

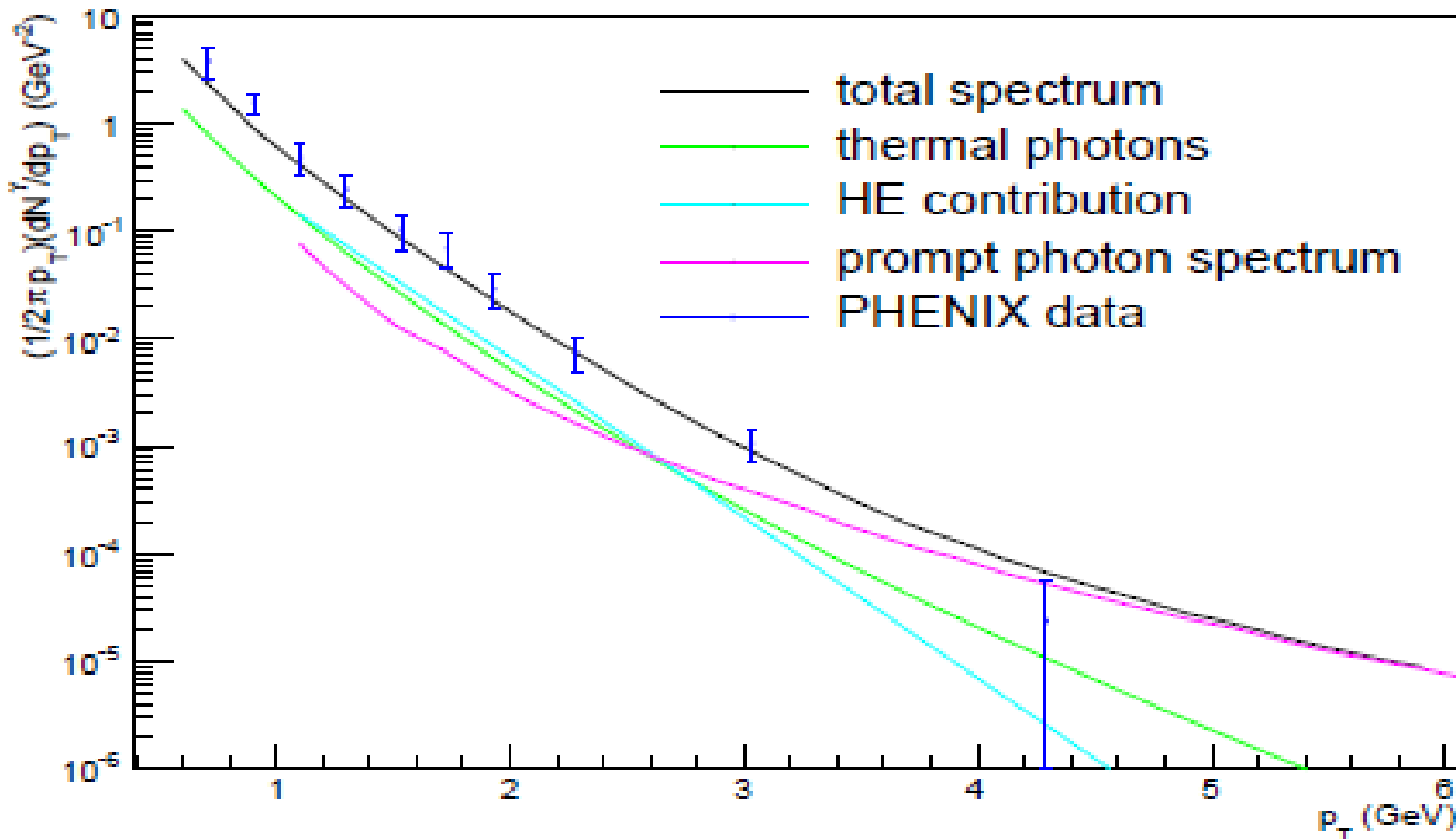
*EPJA-104987.R1*

[arXiv:1812.02763](https://arxiv.org/abs/1812.02763)

# Spectra: Photons at RHIC, c. 10-20 %

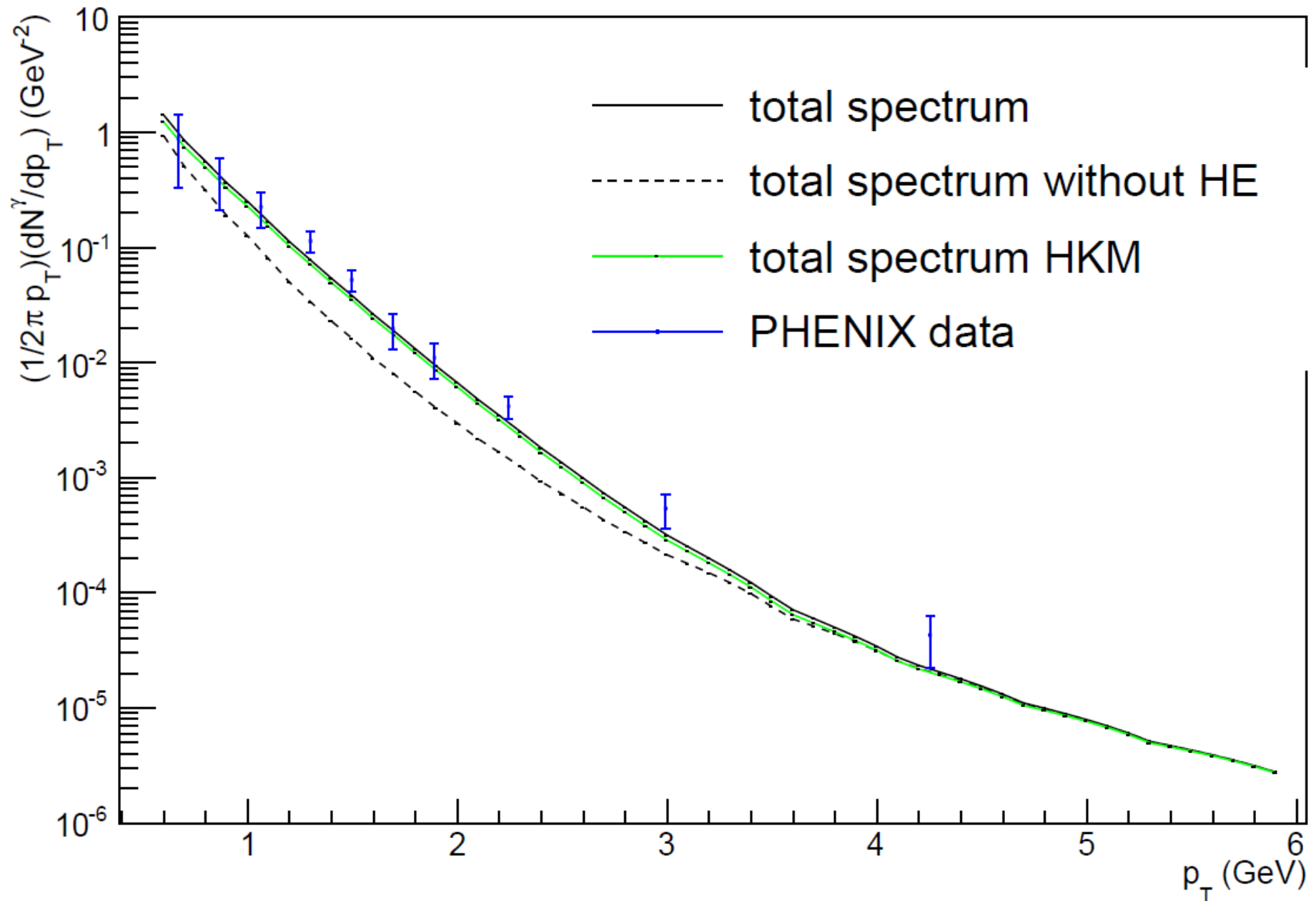


# Contributions to photon spectra, c. 0-20%

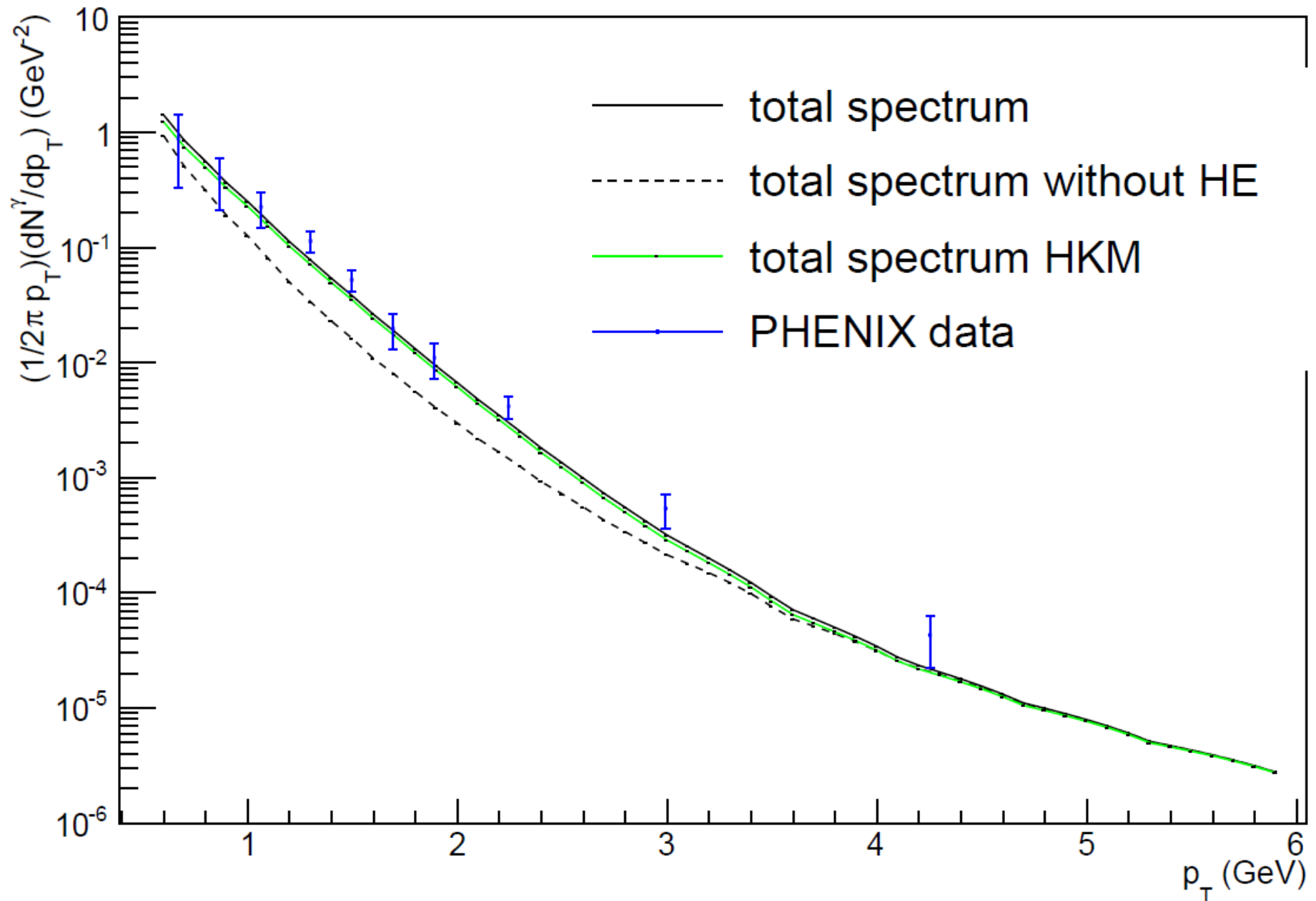


! Total photon spectra calculated within chemically equilibrated iHKM for 0-20% centrality along with its constituents: thermal (including prethermal) photons, prompt photons, and hadronization emission (HE) contribution. Experimental results are taken from [51].

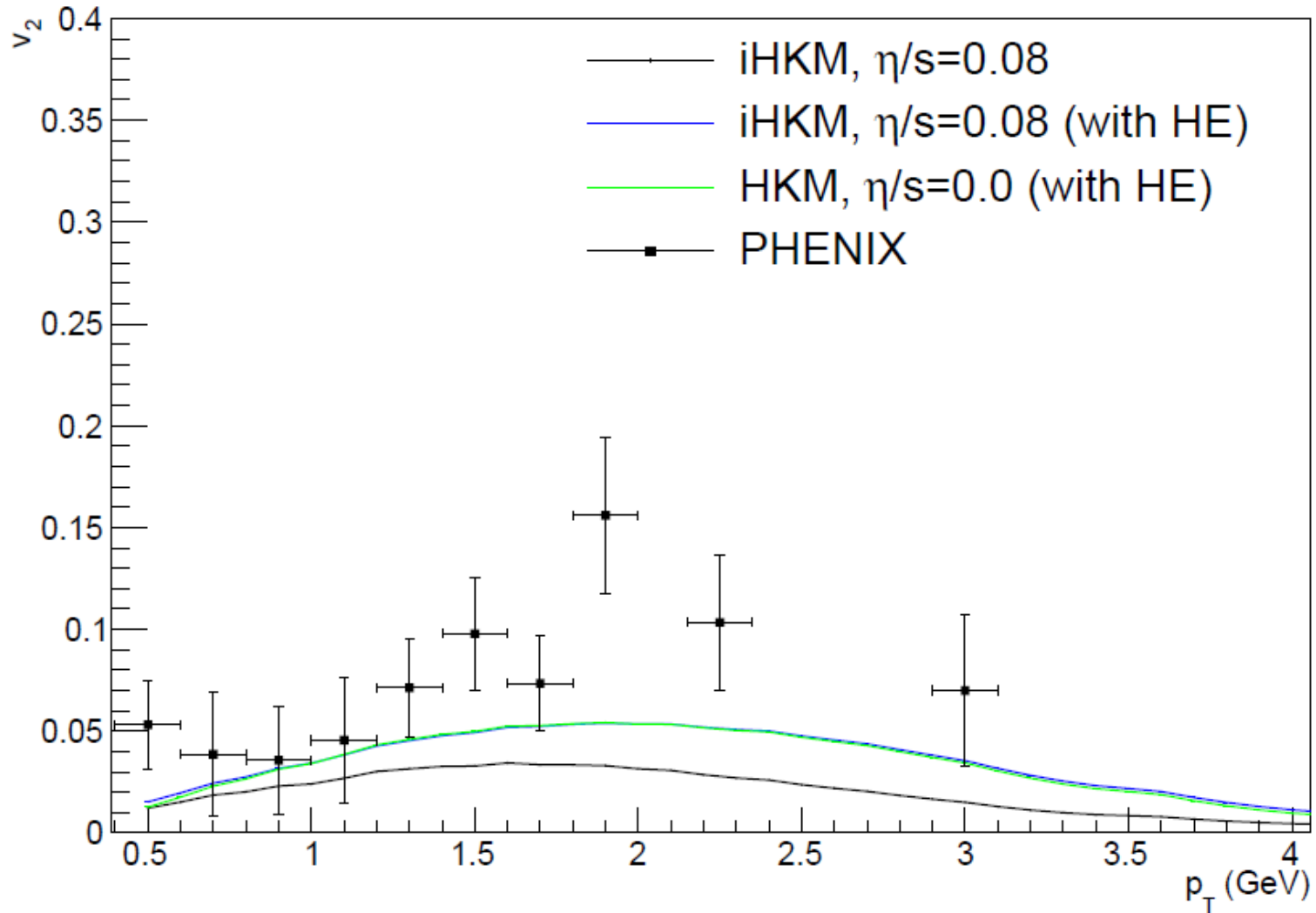
# Spectra: Photons at RHIC, c. 20-40 %



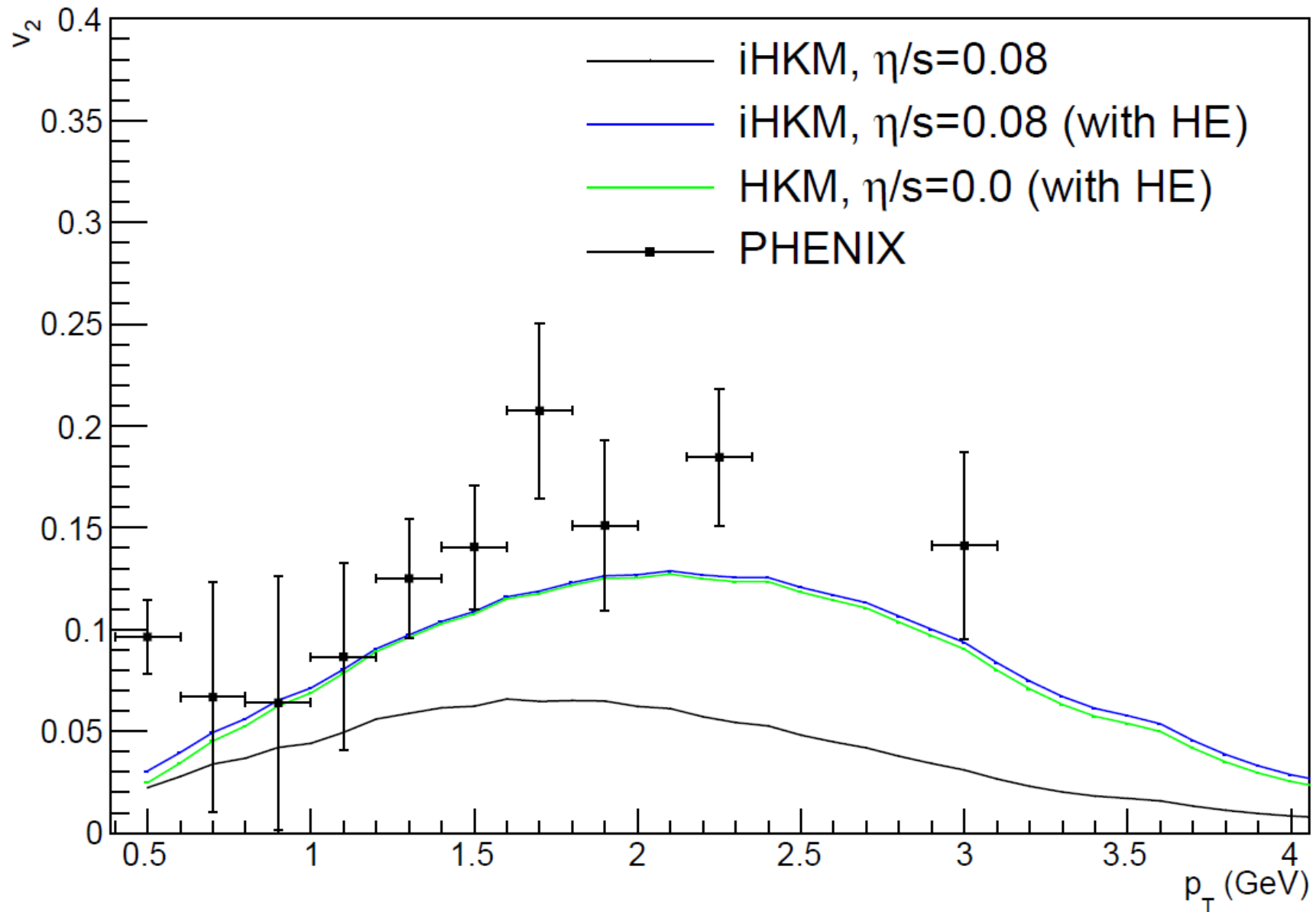
# Spectra: Photons at RHIC, c. 40-60 %



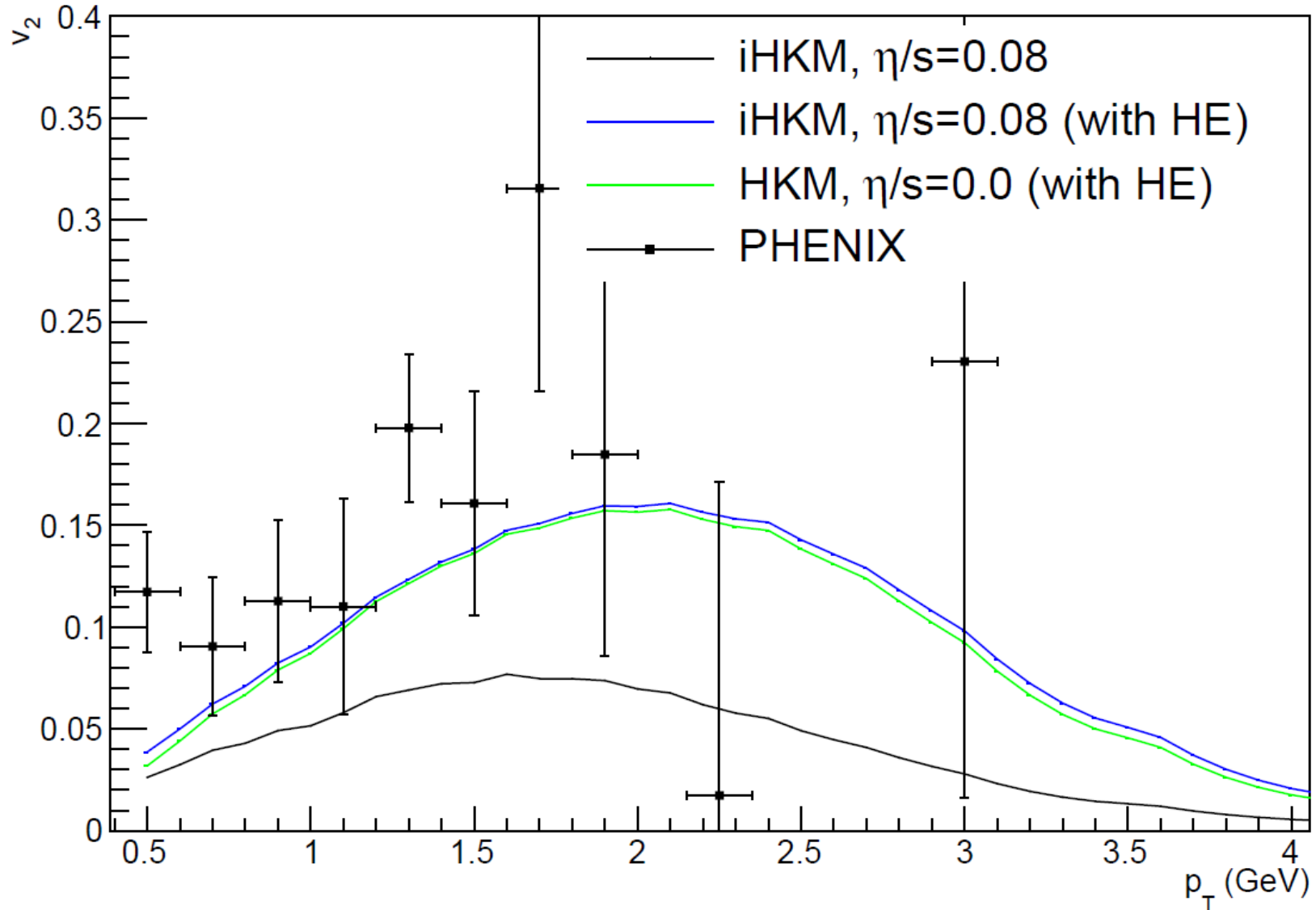
# Photons at RHIC, $v_2$ , c. 10 – 20 %



# Photons at RHIC, $v_2$ , c. 20 – 40 %



# Photons at RHIC, $v_2$ , c. 40 – 60 %





# Triangular flow, $v_3$ coefficients, c. 0-20%

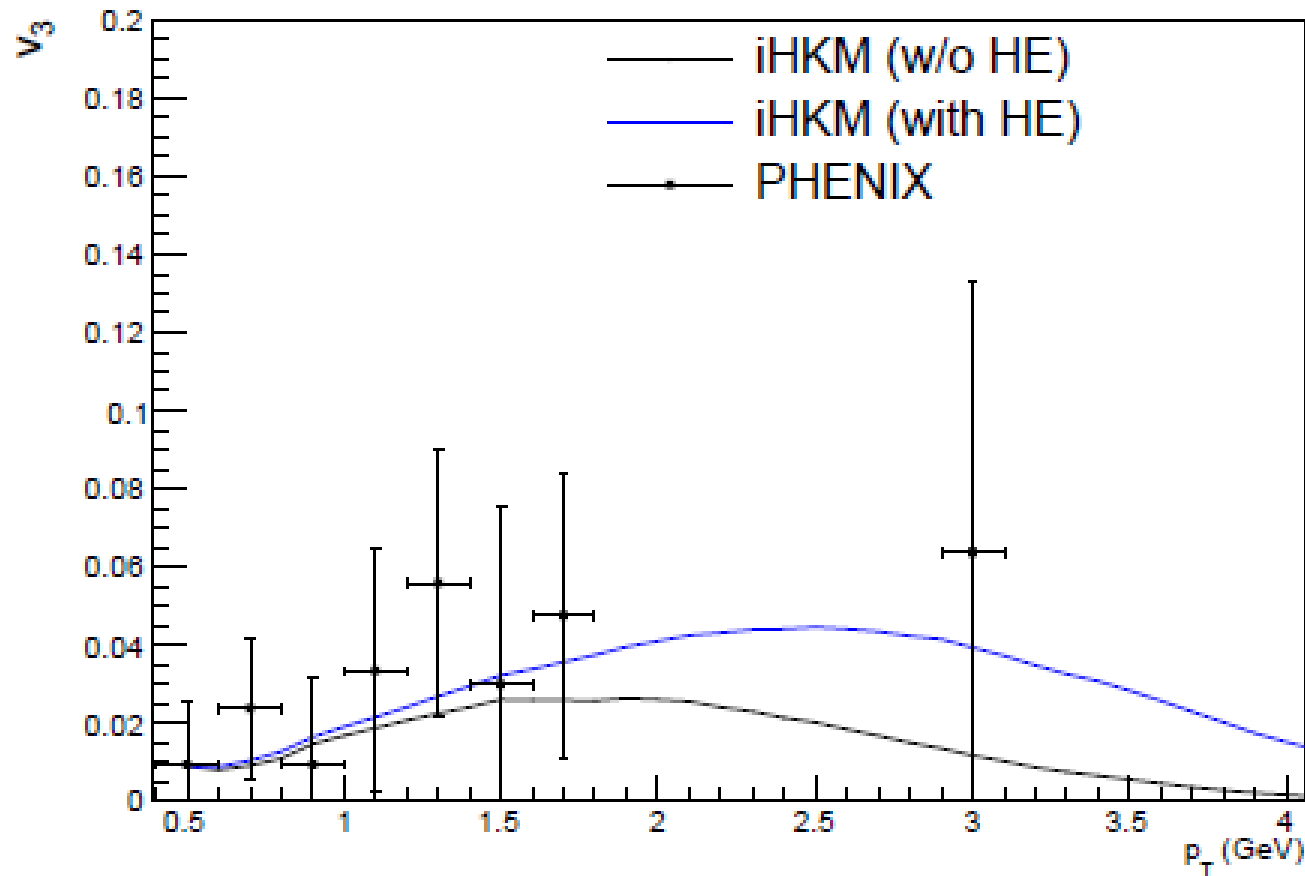
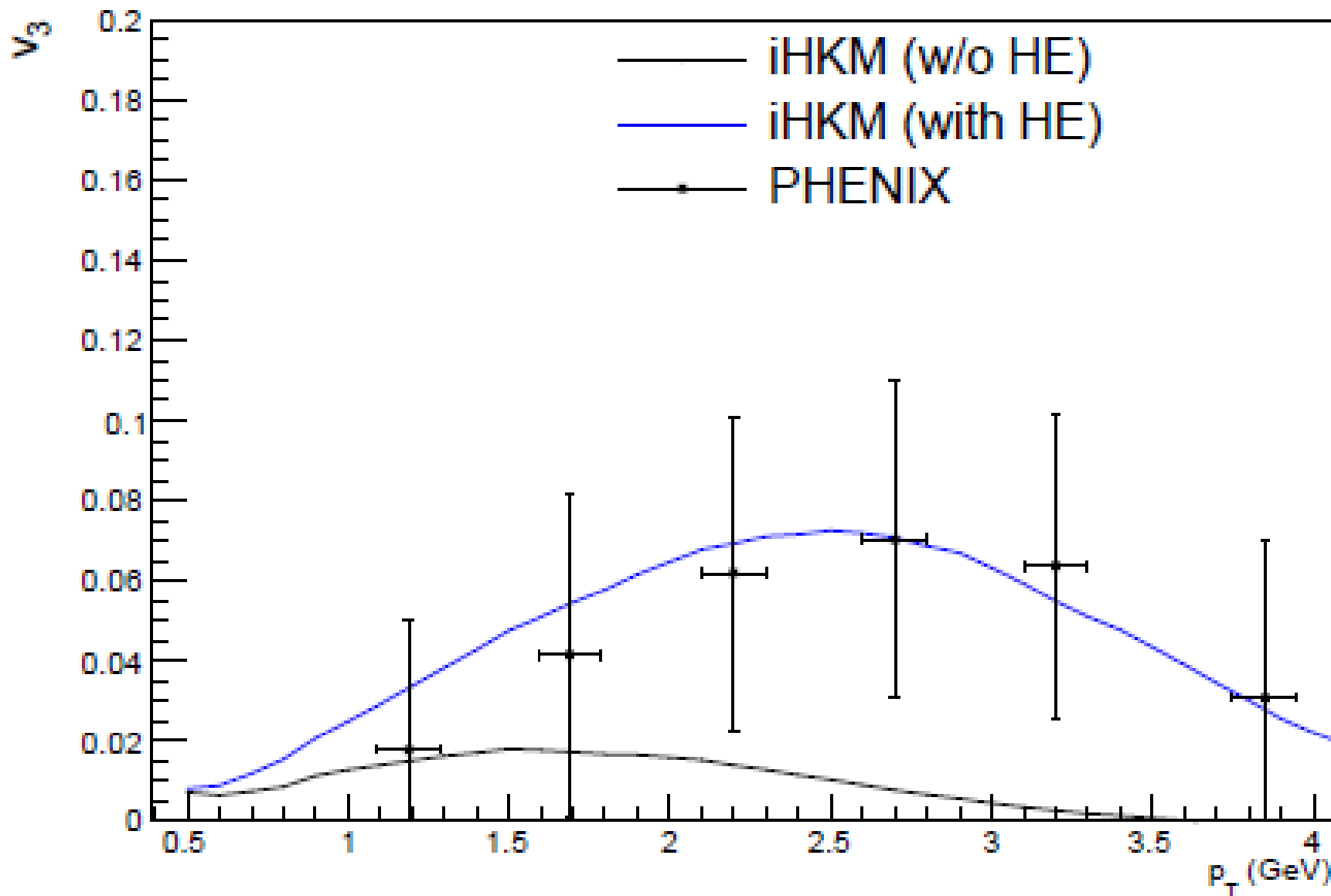
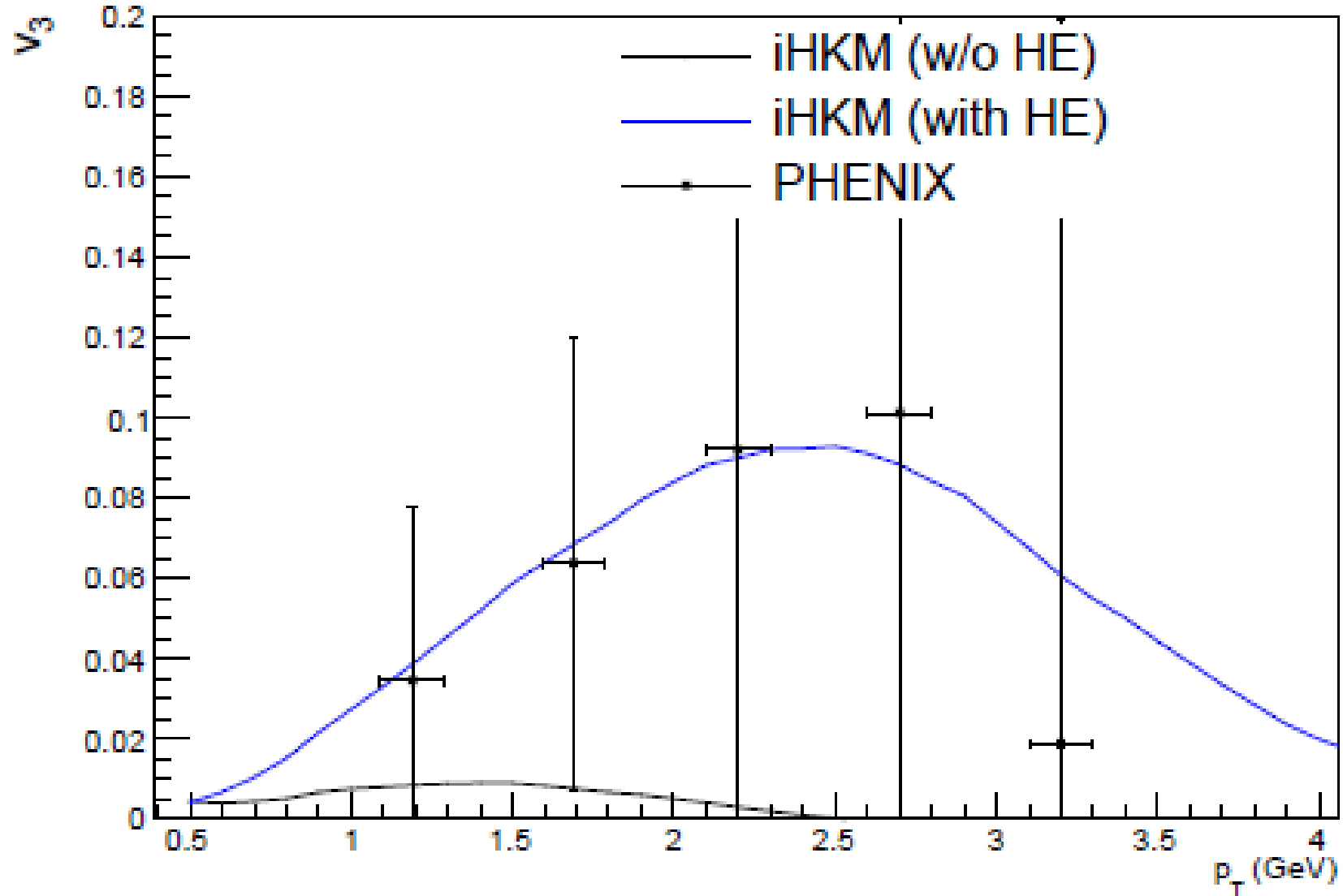


FIG. 6. Triangular flow for 0-20% centrality for the different models: iHKM chemically equilibrated without hadronization emission (thermal and prompt photons only) contribution and iHKM chemically equilibrated with HE contribution. Experimental results are taken from [52].

# Triangular flow, $v_3$ coefficients, c. 20-40%



# Triangular flow, $v_3$ coefficients, c. 40-60%





# Summary for photons

---

The iHKM contains all the stages of the nucleus collision process, has natural zero initial transverse velocity and continuous freeze-out. Being applied here to the photon business, it needs the only additional parameter to describe photon spectra, elliptic and triangular flow at three centralities classes at RHIC. This parameter for the photon rate  $\beta = 0.04$  is related to the specific processes of photon radiation that is connected to confining interactions at the hadronization transition. It could probably include photons from additional reaction channels for hadrons with modified properties that just created in hadronizing medium.



# Acknowledgement

---

Thank you for your attention