A dual geometry of the hadron in dense matter

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Theory of Hadronic Matter Under Extreme Conditions

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Holography as a tool for a quantum gravity

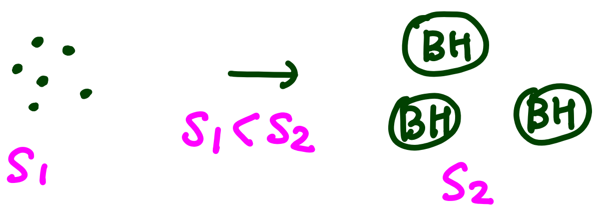
- $F \sim G\frac{mM}{r^2}$

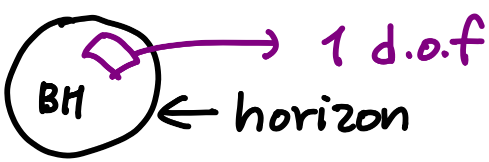
G is small so each particle can be interpreted as a free particle $\longrightarrow$ Thermodynamics

$N \to \infty$, $V \to \infty$, $\frac{N}{V} = \text{fixed}$.

Though G is small, they attract each other $\to$ Jean's instability

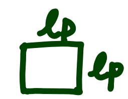
(Blackhole formulation)





In general, entropy $\sim$ # of d.o.f $\sim$ volume $/ l_p^3$

In gravity, entropy $\sim$ area $/ l_p^2$

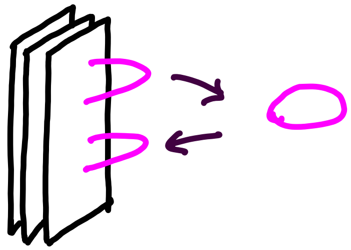


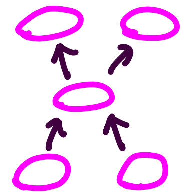
Holographic principle ['Hooft 93, Susskind 94]

: The effective degrees of freedom in gravity are those at the boundary of the system.

Gauge / Gravity duality

[Maldacena 97]





In the limit where the interaction between open / closed strings can be ignored

IIB closed string theory with N D3's $\longleftrightarrow$ Open string theory on D3-branes

closed / open string duality

$\downarrow$ low energy , near horizon limit

Gravity theory on $\underbrace{AdS_5}_{SO(4,2)} \times \underbrace{S^5}_{SO(6)}$ $\longleftrightarrow$ $\mathcal{N}=4$ SU(N) Super Yang–Mills

$\underbrace{SO(4,2)}_{\text{conformal } \mathbb{R}^{1,3}} \times \underbrace{SO(6)}_{\text{R-sym}}$

$\downarrow$ large N

Classical gravity (g_s, R) $\longleftrightarrow$ Strong coupling (g_{YM}^2, N)

AdS / CFT

$$\boxed{4\pi g_s N = \frac{R^4}{\alpha'^2} = \lambda = g_{YM}^2 N}$$

gauge / gravity : gravity $\longleftrightarrow$ non-conformal , less SUSY

AdS / CFT dictionary

[Witten 98][Gubser, Klebanov, Polyakov 98]

ϕ (p-form field in 5D) $\longleftrightarrow$ $\mathcal{O}$ in 4D

m_5^2 : mass squared. Δ : conformal dimension

$$\boxed{(\Delta-p)(\Delta+p+4)=m_5^2}$$

4D : $\mathcal{O}(x)$	5D : $\phi(x,z)$	p	Δ	m_5^2
$\bar{q}_L\gamma^\mu T^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R\gamma^\mu T^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}_R^\alpha q_L^\beta$	$\frac{2}{z}X^{\alpha\beta}$	0	3	-3
$\left.\begin{array}{l}\bar{q}_L\gamma^\mu q_L\\ \bar{q}_R\gamma^\mu q_R\end{array}\right\}$ baryon density	A_μ	1	3	0

Two approaches of AdS/QCD

1) Top down approach

: brane configuration for the gravity dual

ex) N_c D3 and N_f D7

[Kruczenski, Mateos, Myers, Winters 2004]

N_c D4 and N_f D8$-\overline{\text{D8}}$

[Sakai, Sugimoto 2004]

2) Bottom up approach

: field theory on the (asymptotic) AdS_5

ex) hard wall model

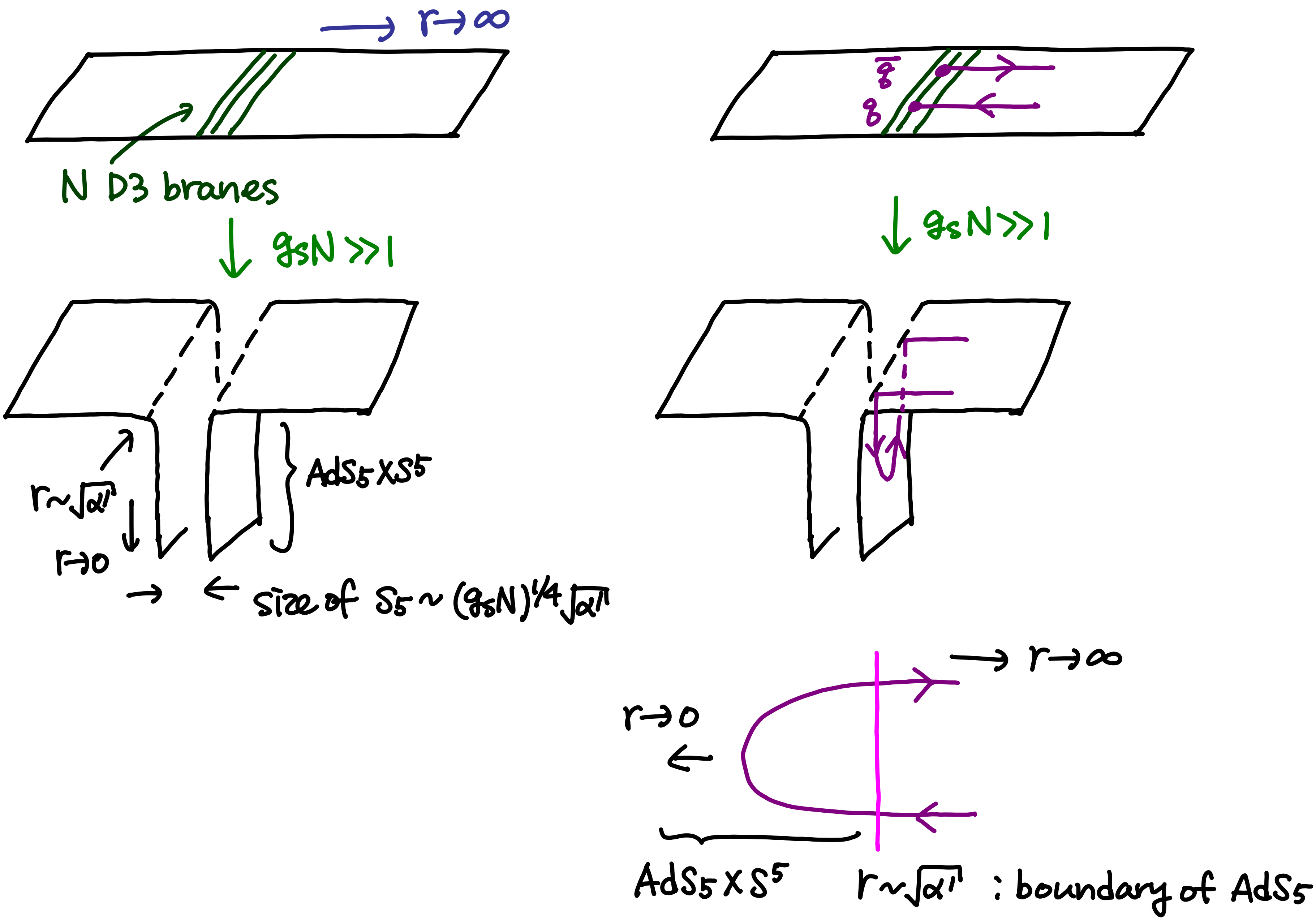
[Erlich, Katz, Son, Stephanov 2005]

[Da Rold, Pomarol 2005]

soft wall model

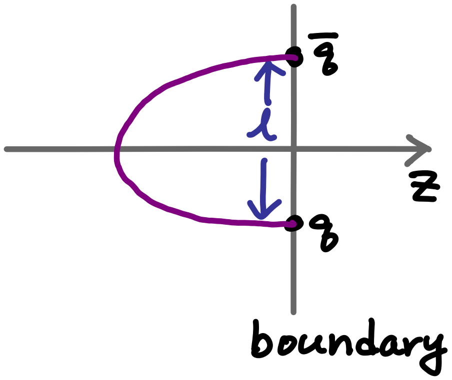
[Karch, Katz, Son, Stephanov 2006]

Geometry

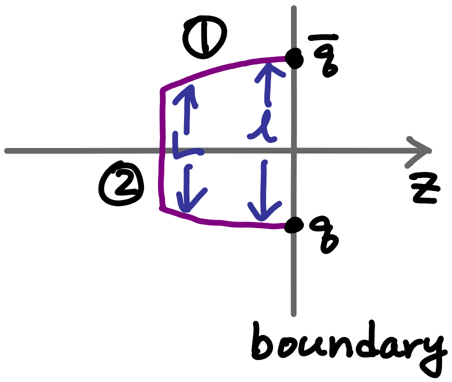


$r \to \infty$
N D3 branes
$g_s N \gg 1$
$AdS_5 \times S^5$
$r \sim \sqrt{\alpha'}$
$r \to 0$
size of $S_5 \sim (g_s N)^{1/4} \sqrt{\alpha'}$
$\bar{q}$
q
$g_s N \gg 1$
$r \to \infty$
$r \to 0$
$AdS_5 \times S^5$
$r \sim \sqrt{\alpha'}$: boundary of AdS_5

What we want...

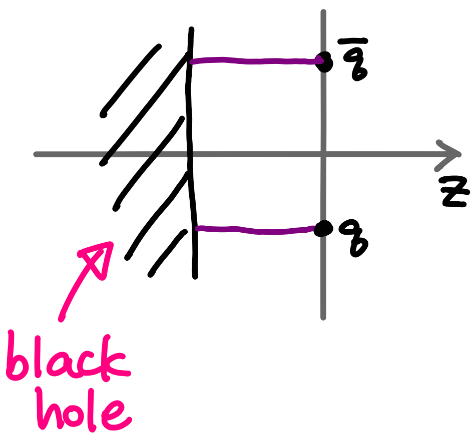


$E \sim \frac{1}{\ell}$: Coulomb potential



① $E \sim \frac{1}{\ell}$: Coulomb potential

② $E \sim T_s L$: confining potential



Black hole

$\leftrightarrow$ thermal gauge theory

Hawking temperature

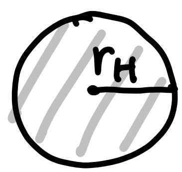
$S = \frac{A}{4\pi}$ (S : entropy A : surface)

$T \sim \kappa$ (κ : surface gravity)

$\uparrow$ Hawking temperature

$$\left(\begin{array}{l} dU = TdS + \cdots \\ dM = \kappa dA + \cdots \end{array} \right)$$

horizon



metric in Euclidean signature $ds^2 = f(r)d\tau^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega^2$

· $f(r_H) = 0$: definition of the horizon

$\left(e^{-S_E} = e^{-\beta H} \quad , \beta = \frac{1}{T} \right)$

Near the horizon $\qquad r \to r_H$

$$ds^2 = f(r)\,d\tau^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$f(r) = \underbrace{f(r_H)}_{=0} + (r-r_H)f'(r_H) + \cdots$$

$$= (r-r_H)f'(r_H) + \cdots$$

$$\to ds^2 = (r-r_H)f'(r_H)\,d\tau^2 + \frac{d(r-r_H)^2}{(r-r_H)f'(r_H)}$$

$$\xi = 2\sqrt{\frac{r-r_H}{f'(r_H)}}$$

$$\to ds^2 = \frac{f'(r_H)^2\xi^2}{4}d\tau^2 + d\xi^2 + \cdots$$

$$\frac{f'(r_H)}{2}d\tau = d\phi \quad \to \frac{f'(r_H)}{2}\Delta\tau = 2\pi \qquad \Delta\tau = \frac{4\pi}{f'(r_H)} = \frac{1}{T}$$

$$\therefore \quad \boxed{T = \frac{f'(r_H)}{4\pi}} \quad \to \text{Hawking temperature}$$

Black hole

: Hawking radiation

→ Hartle – Hawking state

: BH inside the thermal bath of $T = T_H$

→ Unruh state

: Hawking temperature defined
BH evaporates by Hawking radiation.

AdS Schwarzschild BH
$\xleftrightarrow[\text{dual to}]{}$ thermal SCFT on the boundary of AdS.

$\boxed{\text{Black hole}}$ (in Euclidean signature), in Global patch.

$$ds^2 = f(r)d\tau^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_{d-1}^2$$

$$\begin{cases} \omega_d = \dfrac{16\pi G}{(d-1)\,\mathrm{Vol}(S^{d-1})} \\ M : \text{ADM mass} \end{cases}$$

- Schwarzschild (flat) BH in $(d+1)$ dimensions

$$f(r) = 1 - \frac{\omega_d M}{r^{d-2}}$$

- AdS BH

$$f(r) = 1 + \frac{r^2}{R^2} - \frac{\omega_d M}{r^{d-2}}$$

$$\begin{cases} M \to 0 \quad : \quad f(r) \to 1 + \dfrac{r^2}{R^2} \quad : AdS_{d+1} \\ \Lambda \sim -\dfrac{1}{R^2} \to 0 \quad : \quad f(r) \to 1 - \dfrac{\omega_d M}{r^{d-2}} \quad : \text{flat BH} \end{cases}$$

Horizon : $f(r_H) = 0 = 1 + \dfrac{r_H^2}{R^2} - \dfrac{\omega_d M}{r_H^{d-2}}$

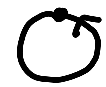
$\tau \sim \tau + \beta$, $\beta = \dfrac{4\pi R^2 r_H}{d \cdot r_H^2 + (d-2)R^2} \sim \dfrac{4\pi R^2}{d(\omega_d R^2)^{1/d} M^{1/d}} = \dfrac{1}{T_H}$

Confinement / deconfinement

[Witten 98]

temporal Wilson line

$$\langle W\rangle = \langle \mathrm{tr}\, e^{P\oint_{\tau}^{\tau+\beta} A_\tau d\tau} \rangle \sim \exp(-\beta F)$$



$\tau \sim \tau+\beta$

quark in thermal bath with temperature $T=\beta^{-1}$

$\langle W\rangle \neq 0$: finite free energy for external quark $\rightarrow$ deconfinement

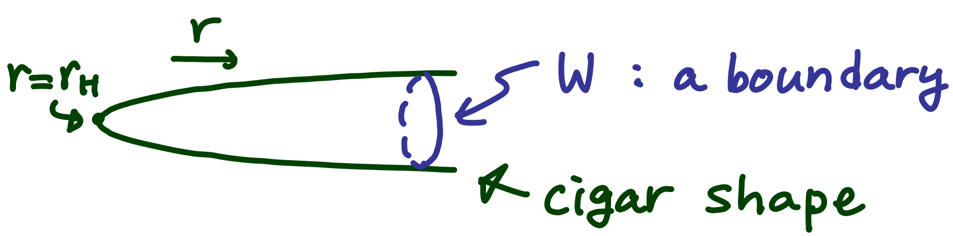
$\langle W\rangle = 0$: infinite free energy for external quark $\rightarrow$ confinement

- thermal AdS : AdS in Euclidean signature

$\langle W\rangle = 0$

confinement

- thermal AdS BH : AdS BH in Euclidean signature



$\langle W\rangle \neq 0$

deconfinement

Hawking – Page transition

[Hawking, Page 83]

Euclidean on-shell action

$\Delta S = S_E(\text{thermal AdS blackhole}) - S_E(\text{thermal AdS})$

$\Delta S = 0$ at $T = T_c$: Hawking-Page transition

$T < T_c$: thermal AdS stable
$T > T_c$: thermal AdS BH stable

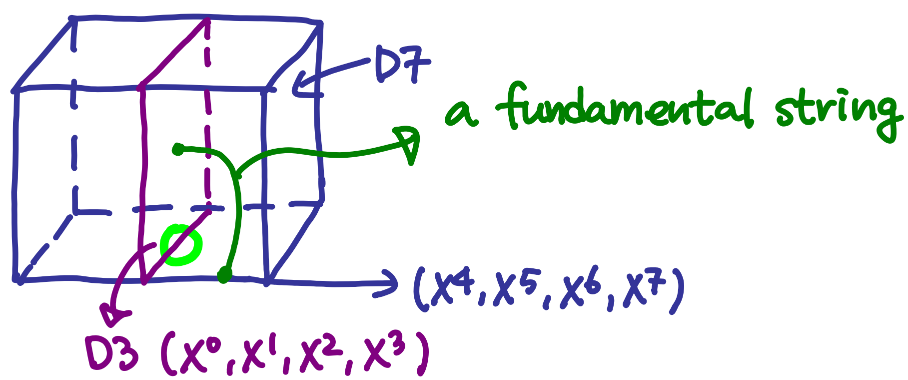
- parameter $\langle W \rangle \begin{cases} = 0 & \text{confined} \\ \neq 0 & \text{deconfined} \end{cases}$
- corresponding parameter : mass $\begin{cases} = 0 & \text{tAdS} \\ \neq 0 & \text{tAdS BH} \end{cases}$

But this argument is not always applicable.

For example arXiv : 1203.4883

D3-D7

	0	1	2	3	4	5	6	7	8	9
D3	x	x	x	x						
D7	x	x	x	x	x	x	x	x		



A fundamental string

- localized on O $\mathbb{R}^{3,1}$
- fundamental for $SU(N_{D3})$ and fundamental for $SU(N_{D7})$

RNS-formalism : D3-D7 nonchiral.

($\mathcal{N}=2$ in 4D)

black D3 branes + D-instanton

A single D7 brane as a probe

↳ probe approximation / quenched approximation.

10D supergravity action [Liu, Tseytlin 99]

$$S = \frac{1}{\kappa}\int d^{10}x\sqrt{g}\left(R - \frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}e^{2\Phi}(\partial\chi)^2 - \frac{1}{6}F_{(5)}^2\right)$$

$$\chi = -e^{-\Phi} + \chi_0$$

$$ds_{10}^2 = e^{\Phi/2}\left[\frac{r^2}{R^2}\left(f(r)^2 dt^2 + d\vec{x}^2\right) + \frac{1}{f(r)^2}\frac{R^2}{r^2}dr^2 + R^3 d\Omega_5^2\right] \qquad R^4 = 4\pi g_s N_c \alpha'^2$$

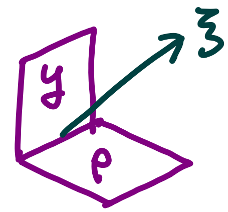
$$e^{\Phi} = 1 + \frac{q}{r_T^4}\log\frac{1}{f(r)^2}, \qquad \chi = -e^{-\Phi} + \chi_0, \qquad T = r_T/\pi R^2$$

$$f(r) = \sqrt{1 - \left(\frac{r_T}{r}\right)^4},$$

$$ds_{10}^2 = e^{\Phi/2}\left[\frac{r^2}{R^2}\left(f^2 dt^2 + d\vec{x}^2\right) + \frac{1}{f^2}\frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2\right]$$

$\downarrow$ introducing $\frac{d\xi^2}{\xi^2} = \frac{dr^2}{r^2 f^2}$

$$= e^{\Phi/2}\left[\frac{r^2}{R^2}\left(f^2 dt^2 + d\vec{x}^2\right) + \frac{R^2}{\xi^2}\underbrace{\left(d\xi^2 + \xi^2 d\Omega_5^2\right)}_{\mathbb{R}^6}\right]$$



$\xi^2 = y^2 + \rho^2$

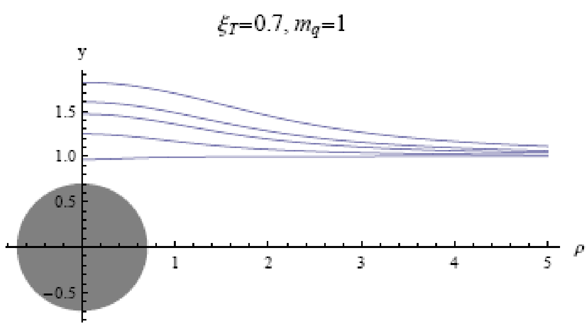
y: transverse

To find the induced metric on D7,

$\mathbb{R}^6 \to \mathbb{R}^4 \; \mathbb{R}^2$: $d\rho^2 + \rho^2 d\Omega_3^2 + dy^2 + y^2 d\phi^2$

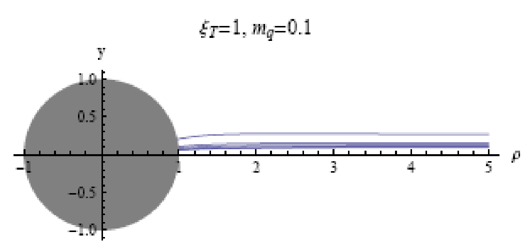
D7 { spans $(t, \vec{x}, \rho)$ and wraps S^3.
is orthogonal to (y, ϕ).

Set $\phi = 0$ using the SO(2) symmetry in (x^8, x^9).



(a)

Minkowski embedding



(b)

blackhole embedding

"A dual geometry of the hadron in dense matter"

B-H Lee, C Park, S-J Sin   JHEP 07 (2009) 087

⊙ Dual geometry for QCD with quark matters

- gravity in the Minkowskian signature

$$S_M = \int d^5x\sqrt{-G}\left[\frac{1}{2\kappa^2}(\mathcal{R}-2\Lambda)-\frac{1}{4g^2}F_{MN}F^{MN}\right]$$

$$\boxed{\begin{aligned}\frac{1}{2\kappa^2} &= \frac{N_c^2}{8\pi^2R^3}\\ \frac{1}{g^2} &= \frac{N_cN_f}{4\pi^2R}\end{aligned}}$$

- equations of motion

$$\mathcal{R}_{MN}-\frac{1}{2}G_{MN}\mathcal{R}+G_{MN}\Lambda = \frac{\kappa^2}{g^2}\left(F_{MP}F_N^P-\frac{1}{4}G_{MN}F_{PQ}F^{PQ}\right),$$
$$0 = \partial_M\sqrt{-G}G^{MP}G^{NQ}F_{PQ},$$

$\left(\begin{array}{l} M,N=0,1,\cdots,4 \\ x^0=t\ ,\ x^4=z \end{array}\right)$

- Ansatz

$$A_0 = A_0(z),$$
$$A_i = A_4 = 0 \quad (i=1,\ldots,3),$$
$$ds^2 = \frac{R^2}{z^2}\left(-f(z)dt^2+dx_i^2+\frac{1}{f(z)}dz^2\right)$$

$\left(z=\frac{R^2}{r}\right)$

- Solution

→ Reissner-Nördstrom AdS BH

$$f(z) = 1-mz^4+q^2z^6,$$
$$A_0 = \mu - Qz^2,$$
$$\left(q^2 = \frac{2\kappa^2}{3g^2R^2}Q^2.\right)$$

⊙ Dual geometry of the quark-gluon plasma

In the Euclidean signature

$$S = \int d^5x\sqrt{G}\left[\frac{1}{2\kappa^2}\left(-\mathcal{R}+2\Lambda\right)+\frac{1}{4g^2}F_{MN}F^{MN}\right]$$

$$ds^2 = \frac{R^2}{z^2}\left((1-mz^4+q^2z^6)d\tau^2+d\vec{x}^2+\frac{1}{1-mz^4+q^2z^6}dz^2\right)$$

$$\underset{\hat{z}}{A}(z) = i\left(\mu - Qz^2\right)$$

horizon z_+ $\quad 0 = f(z_+) = 1 - mz_+^4 + q^2 z_+^6$

$$m = \frac{1}{z_+^4} + q^2 z_+^2$$: BH mass

$$T_{RN} = \frac{1}{\pi z_+}\left(1 - \frac{1}{2}q^2 z_+^6\right)$$: Hawking temperature

Dirichlet boundary condition $A(z_+) = 0$ $\quad Q^2 = \frac{\mu^2}{z_+^4}$

Horizon z_+ as a function of μ and T_{RN}

$$z_+ = \frac{3g^2R^2}{2\kappa^2\mu^2}\left(\sqrt{\pi^2T_{RN}^2 + \frac{4\kappa^2\mu^2}{3g^2R^2}} - \pi T_{RN}\right)$$

A system having the fixed chemical potential

On shell action w/ the boundary condition $A(0) = i\mu$

$$S_{RN}^D = \frac{V_3R^3}{\kappa^2}\frac{1}{T_{RN}}\left(\frac{1}{\epsilon^4} - \frac{1}{z_+^4} - \frac{2\kappa^2}{3g^2R^2}\frac{\mu^2}{z_+^2}\right),$$

→ diverges !

V_3 : spatial volume of the boundary.

superscript D : Dirichlet b.c.

Subscript RN : RNAdS BH

Regularize the action using the background subtraction method.

$$\bar{S}_{RN}^D = S_{RN}^D - S_{AdS}$$

- grand potential (in grand canonical ensemble)

$$\Omega_{RN} = \bar{S}^{D}_{RN} T_{RN}$$
$$= -\frac{V_3 R^3}{\kappa^2}\left(\frac{1}{2z_+^4} + \frac{\kappa^2}{3g^2R^2}\frac{\mu^2}{z_+^2}\right)$$

- free energy (in canonical ensemble)

$$F = \Omega + \mu N \qquad N = -\frac{\partial\Omega}{\partial\mu}$$

- Imposing the Neumann B.C. at the UV cut-off

$$\bar{S}^{N}_{RN} = \bar{S}^{D}_{RN} + S_b$$

$$\begin{cases} S_b = \frac{1}{g^2}\int_{\partial M} d^4x \sqrt{G^{(4)}}\, n^M A^N F_{MN} \\ G^{(4)} = \frac{R^8}{z^8} f(z) \\ n^M = \{0, 0, 0, 0, -\frac{z}{R}\sqrt{f(z)}\} \end{cases}$$

$$F = \bar{S}^{N}_{RN} T_{RN} = \Omega + \frac{2R}{g^2}\mu Q V_3$$

Q : quark # density.

⊙ Dual geometry of the hadronic phase

$$ds^2 = \frac{R^2}{z^2}\left(f(z)\,d\tau^2 + d\vec{x}^2 + \frac{1}{f(z)}\,dz^2\right)$$

In the absence of quark matters

confinement

thermal AdS

$f(z) = 1$

deconfinement

Schwarzschild AdS BH

$f(z) = 1 - mz^4$

In the presence of quark matters

Confinement

thermal charged AdS

$f(z) = 1 + q^2 z^6$

deconfinement

Reisner-Nordstrom AdS BH

$f(z) = 1 - mz^4 + q^2 z^6$

satisfies the Einstein and Maxwell eq.

asymptotically AdS.

$\therefore$ Dual geometry of the hadronic phase

$$\boxed{ds^2 = \frac{R^2}{z^2}\left((1+q^2z^6)d\tau^2 + d\vec{x}^2 + \frac{1}{1+q^2z^6}dz^2\right)}$$

- Grand canonical ensemble $\rightarrow$ Dirichlet boundary condition

$A(z_{IR}) = i\alpha\mu$ (α : constant)

$A(z) = i(\mu - Qz^2) \rightarrow Q = \frac{(1-\alpha)}{z_{IR}^2}\mu$

Regularized on-shell action of the tcAdS

$$\bar{S}^D_{tc} = S^D_{tc} - S^D_{tAdS} = -\frac{V_3R^3}{\kappa^2}\frac{1}{T_{tc}}\left(\frac{1}{z_{IR}^4} + \frac{2\kappa^2}{3g^2R^2}\frac{(1-\alpha)^2\mu^2}{z_{IR}^2}\right)$$

grand potential $\Omega = T_{tc}\bar{S}^D_{tc}$

$$N = -\frac{\partial\Omega}{\partial\mu} = \frac{2}{3}(1-\alpha)\frac{2R}{g^2}QV_3$$

- Canonical ensemble → Neumann boundary condition

$\mu N = S_b T_{tc}$

$S_b = \frac{\mu}{T_{tc}} \frac{2R}{g^2} Q V_3$

"$\alpha = -\frac{1}{2}$"

Renormalized action

$$\bar{S}_{tc}^{D} = -\frac{V_3 R^3}{\kappa^2} \frac{1}{T_{tc}} \left(\frac{1}{z_{IR}^4} + \frac{3\kappa^2}{2g^2 R^2} \frac{\mu^2}{z_{IR}^2} \right)$$

$$\mu = \frac{2}{3} Q z_{IR}^2.$$

μ : chemical potential

Q : quark # density

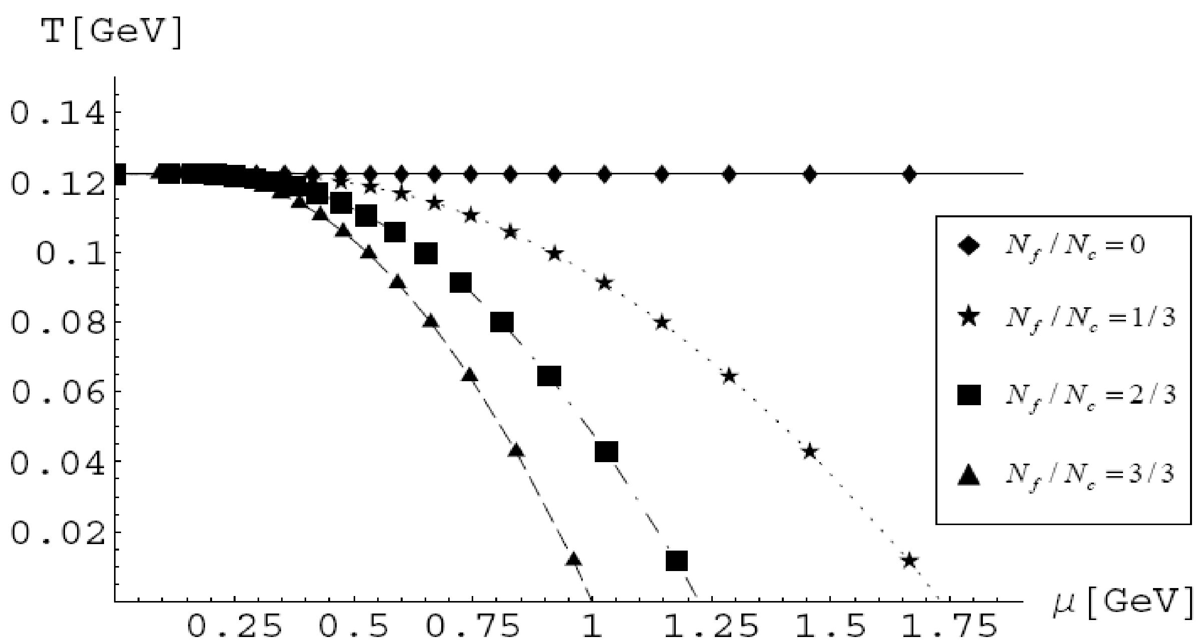
⊙ Confinement / Deconfinement phase transition

In grand canonical ensemble

$$\Delta S = S^D_{RN} - S^D_{tc}$$

$$S^D_{RN} = \frac{V_3 R^3}{\kappa^2}\frac{1}{T_{RN}}\left(\frac{1}{\epsilon^4} - \frac{1}{z_+^4} - \frac{2\kappa^2}{3g^2R^2}\frac{\mu^2}{z_+^2}\right)$$

$$S^D_{tc} = \frac{V_3 R^3}{\kappa^2}\frac{1}{T_{tc}}\left(\frac{1}{\epsilon^4} - \frac{1}{z_{IR}^4} - \frac{3\kappa^2}{2g^2R^2}\frac{\mu^2}{z_{IR}^2}\right)$$

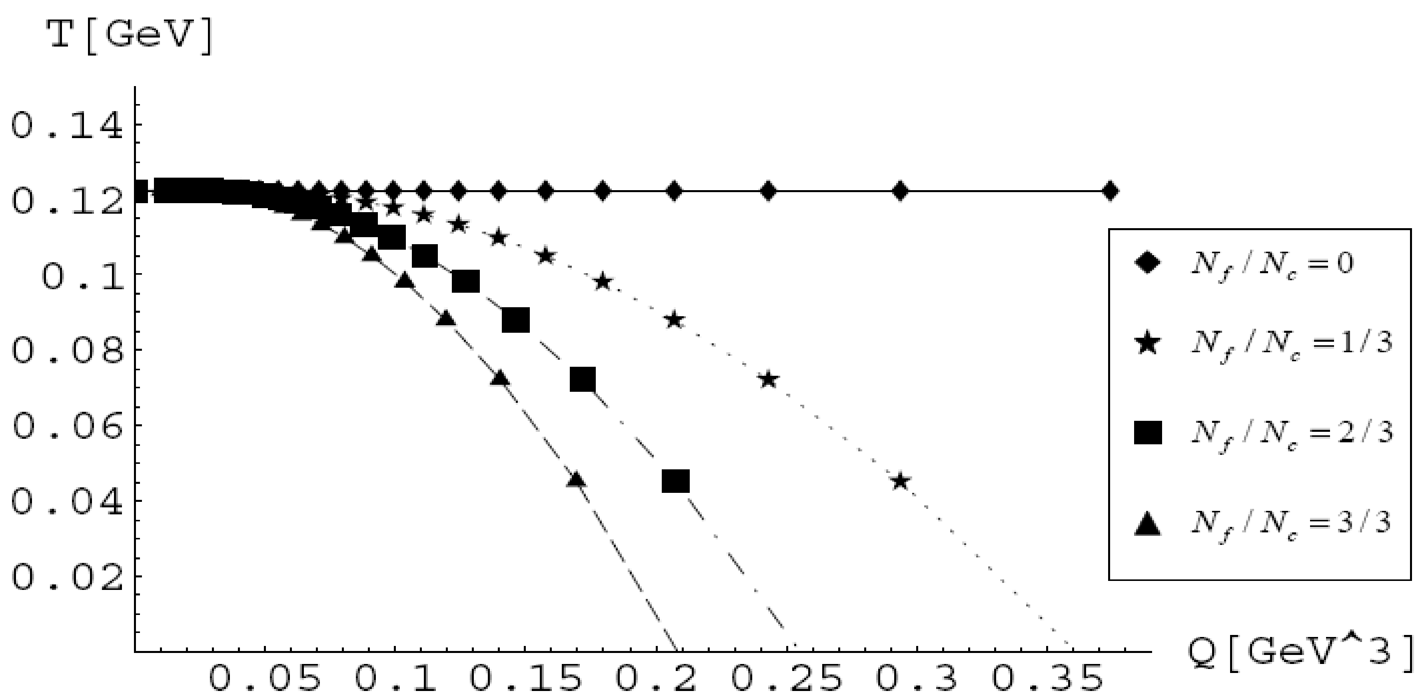


In canonical ensemble

$$\Delta S \equiv S^N_{RN} - S^N_{tc}$$

$$S^N_{RN} = \frac{V_3 R^3}{\kappa^2}\frac{1}{T_{RN}}\left(\frac{1}{\epsilon^4} - \frac{1}{z_+^4} + \frac{4\kappa^2 Q^2}{3g^2R^2}z_+^2\right)$$

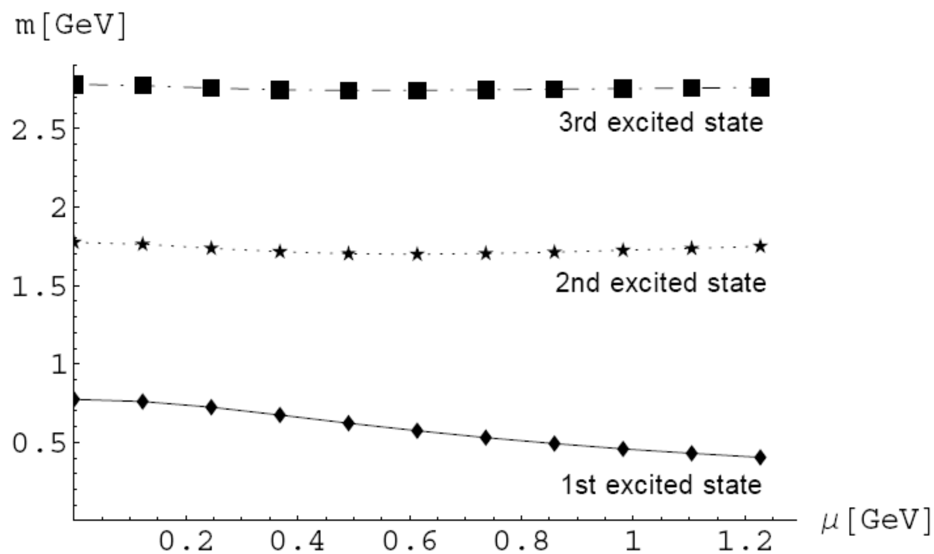
$$S^N_{tc} = \frac{V_3 R^3}{\kappa^2}\frac{1}{T_{tc}}\left(\frac{1}{\epsilon^4} - \frac{1}{z_{IR}^4} + \frac{2\kappa^2 Q^2}{3g^2R^2}z_{IR}^2\right)$$



⊙ Mass of the excited vector mesons

$\delta A_\mu = V_\mu(z,p)\, e^{ip\cdot x}$ in thermal charged AdS.

$$0 = \partial_z^2 V_i - \frac{1}{z}\frac{(1-5q^2z^6)}{(1+q^2z^6)}\,\partial_z V_i + m_m^2 V_i,$$



	$\mu=0$	$\mu=0.245$	$\mu=0.491$	$\mu=0.736$	$\mu=0.982$	$\mu=1.227$
mass of the 1st	0.774	0.724	0.622	0.530	0.458	0.404
mass of the 2nd	1.775	1.737	1.702	1.704	1.724	1.750
mass of the 3rd	2.782	2.758	2.743	2.747	2.755	2.762

Summary

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- Phase diagrams close.
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Thank you for your attention.