Chiral Drag Force

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As it was argued a long time ago[1] one could calculate drag force for a heavy quark moving through strongly coupled holographic plasma by considering a probe string, described by NG action

$$S = -\frac{1}{2\pi\alpha'}\int d^2\sigma\sqrt{-det\ g}\ ,\ g_{\alpha\beta} = G_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu$$

where the bulk AdS-BH metric describing the plasma is

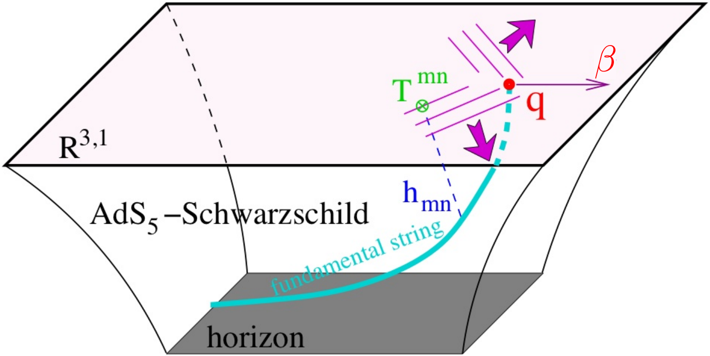
$$ds^2 = H^{-1/2}(-f(r)dt^2 + dx^2) + H^{1/2}\left(\frac{dr^2}{f(r)} + d\Omega_5^2\right)$$

and $f(r) = 1 - \frac{M}{r^4}\ ,\ H(r) = 1 + \frac{1}{r^4}$.

EOMs are

$$\nabla_\alpha P^\alpha_\mu = 0\quad ,\quad P^\alpha_\mu = -\frac{1}{2\pi\alpha'}G_{\mu\nu}\partial^\alpha X^\nu$$

[1] C. Herzog et al, JHEP 0607, 013 (2006); S. Gubser, Phys.Rev.D 74, 126005(2006)



T^{mn}
q
β
$R^{3,1}$
h_{mn}
AdS_5 –Schwarzschild
fundamental string
horizon

For a trailing string the action is reduced in the static gauge to

$$S = -\frac{1}{2\pi\alpha'}\int \; dtdr\sqrt{1+\frac{f(r)}{H(r)}x'^2-\frac{\dot{x}^2}{f(r)}}$$

and the corresponding ansatz for the string profile is

$$x(t,r) = vt + \xi(r) + o(t)$$

where v is a late-time velocity of a quark and for the solution we have

$$\xi' = \pm\pi_\xi \frac{H(r)}{f(r)}\sqrt{\frac{f(r)-v^2}{f(r)-\pi_\xi^2 H(r)}} \;\; , \;\; \pi_\xi = \frac{v\ r_h^2}{\sqrt{1-v^2}}$$

and finally

$$\xi = -\frac{v}{2r_h}\left(\tan^{-1}\frac{r}{r_h}+\log\sqrt{\frac{r+r_h}{r-r_h}}\right) \;\; , \;\; \frac{dp_x}{dt} = \sqrt{-g}P_x^r = -\frac{r_h^2}{2\pi\alpha'}\frac{p_x}{m}$$

To conserve energy momentum tensor one has to exert an external force upon the heavy quark to keep constant its velocity:

$$\partial_\nu T^{\nu\mu} = -f^\mu(t)\delta^{(3)}(\vec{x}-\vec{v}t)\ ,\ \ f^\mu(t) = \lim_{r\to\infty} n_M \int d^3x\sqrt{-g}\,T^{M\mu}$$

where $f^\mu(t)$ is the drag force on the quark (the endpoint of the string) and n_M is the unit normal to the boundary.

$$f^\mu(t) = -\frac{dp^\mu}{dt}(t) = -\lim_{r\to\infty}\eta^{\mu\nu}\pi^r_\nu(t,r)$$

and

$$\pi^r_\mu(t,r) = -\frac{\sqrt{\lambda}}{2\pi}G_{\mu N}\frac{1}{\sqrt{-g}}\left(g_{tr}\partial_t X^N - g_{tt}\partial_r X^N\right)$$

Finally, from the string solution the drag force is

$$f^\mu_{(0)} = -\frac{\sqrt{\lambda}}{2\pi}\frac{\pi^2 T^2}{\gamma}(sw^\mu + u^\mu)$$

where $w^\mu = \gamma(1,\vec{v})$ and $s = u\cdot w = -\gamma$

Now let's turn to the simplest generaliztion of the setup. The plasma above was static but it is well known how to include hydrodynamic perturbations to the theory[2]. Here the starting point is the five dimensional Einstein-Maxwell action:

$$S = -\frac{1}{16\pi G_5}\int \left(\sqrt{-g}\left(R + 12 - \frac{1}{4}F^2\right) + \frac{\kappa}{3}\epsilon^{MNOPQ}A_M F_{NO} F_{PQ}\right) d^5x$$

It was shown that the theory at the boundary could be described by relativistic hydrodynamics:

$$\partial_\mu T^{\mu\nu} = 0\ ,\ \ T^{\mu\nu} = wu^\mu u^\nu + Pg^{\mu\nu} + \tau^{(1)\mu\nu}$$
$$\partial_\mu J^\mu = 0 \quad , \quad J^\mu = nu^\mu + \nu^{(1)\mu},$$

where $\nu^{(1)}$ and $\tau^{(1)\mu\nu}$ are corrections of the first order in spatial gradients.

[2] J. Erdmenger et al, JHEP 0901, 055 (2009)

Solving Einstein-Maxwell equations by the gradient expansion we find the bulk metric as a perturbative series:

$$ds^2 = r^2 k(r) u_\mu u_\nu dx^m u dx^\nu + r^2 h(r) P_{\mu\nu} dx^\mu dx^\nu + r^2 \pi_{\mu\nu}(r) dx^\mu dx^\nu$$
$$+ r^2 j_\sigma(r) \left(P^\sigma_\mu u_\nu + P^\sigma_\nu u_\mu \right) dx^\mu dx^\nu - 2S(r) u_\mu dx^\mu dr$$

and up to the first order in gradients it is (in $h(r) = 1$ gauge)

$$S(r) = 1 \ , \ k(r) = -f(r) + \frac{2}{3r} \partial \cdot u \ , \ \pi_{\mu\nu}(r) = F(r) \sigma_{\mu\nu}$$
$$j_\sigma = -\frac{1}{r} (u \cdot \partial) u_\sigma + \frac{3\sqrt{3} Q^3 \kappa}{2\sqrt{2} M r^6} l_\sigma + J(r) \partial_\sigma \frac{\mu}{T}$$

where $f(r) = 1 - \frac{M}{r^4} + \frac{Q^2}{r^6}$ and we should add also solution for the gauge field which however is not required for our consideration now.

In the same manner one could use the gradient expansion procedure to find corrections to the drag force on the heavy quark running through a perturbed plasma[3] at $\mu = 0$, then

$$\vec{x}(t,r) = \vec{x}_0(t,r) + \vec{x}_1(t,r) \quad , \quad \vec{x}_0(t,r) = \vec{v}\left(t - \frac{1}{\pi T}\left(\tan^{-1}\frac{r}{\pi T} - \frac{\pi}{2}\right)\right)$$

and one can check that $\vec{x}^{(1)} = t\ D_t \vec{x}_0(t,r)|_{t=0} + \vec{g}(r)$ solves EOMs.

After some algebra the correction to the instantaneous drag force reads

$$f_\mu^{(1)} = -\frac{\sqrt{\lambda}}{2\pi}\frac{\pi T}{\gamma}\left(c_1(s)(u_\mu(w\cdot\partial)s - s\partial_\mu s - s(su_\alpha + w_\alpha)\partial^\alpha U_\mu)\right.$$
$$\left. + c_2(s)U_\mu(\partial\cdot u) - \sqrt{-s}(u\cdot\partial)U_\mu\right)$$

where $c_1(s) = \pi/2 - \tan^{-1}\left(\sqrt{-s}\right) - \pi T F(\pi T\sqrt{-s})$ and $c_2(s) = \frac{1}{3}(\sqrt{-s} + (1+s^2)c_1(s))$

[3] M. Lekaveckas et al, JHEP 1402, 068 (2014)

We can now turn on chemical potential and expand in powers of $\frac{\mu}{T}$. The first non-zero contribution to the drag force appears at the second order in this expansion and it is

$$f_\mu^{(0)} = -\frac{\sqrt{\lambda}}{2\pi}\frac{\pi^2 T^2}{\gamma}(sw^\mu + u^\mu)\left(1 + \frac{(1+3s)}{6\pi^2 s}\left(\frac{\mu}{T}\right)^2\right)$$

$$f_\mu^{(1,2)} = \frac{\mu^2\sqrt{\lambda}}{48\gamma\pi^2 T}\left(\frac{2c_5(s)}{s}\left((w\partial)\log\frac{\mu}{T}\right)u_\mu + c_3(s)\left(u_\mu(w\partial)s + s\partial_\mu s\right)\right)$$
$$-\frac{\mu^2\sqrt{\lambda}}{48\gamma\pi^2 T}U_\mu\left(c_6(s)(w\partial)\log\frac{\mu}{T} - c_4(s)(\partial u) + c_7(s)(su^\alpha + w^\alpha)\partial_\alpha s + c_{10}(s)(w\partial)s\right)$$
$$-\frac{\mu^2\sqrt{\lambda}}{48\gamma\pi^2 T}\left(c_8(s)(u\partial)U_\mu + c_9(s)(w\partial)U_\mu + 2c_5(s)\partial_\mu\log\frac{\mu}{T}\right)$$

where $c_i(s)$ describe kinematic properties and we won't bring them here.

More interesting physics appears if we concentrate on the anomalous contributions to the bulk metric. In the presence of the external magnetic field and vorticity the anomalous contribution takes form

$$j_\sigma = \frac{\kappa}{\pi^2 T^2}\left(\frac{\mu}{T}\right)^2 C_B(r) B_\sigma + \frac{2\sqrt{3}\kappa Q^3}{M r^6} l_\sigma$$

and for the drag force it means

$$f_\mu = -\frac{\sqrt{\lambda}}{2\pi^3\gamma}\left(\frac{\mu}{T}\right)^2 s^2\kappa\; C_B(\pi T\sqrt{-s})\left(B_\mu + (B\cdot w)w_\mu\right) - \\ -\frac{\kappa\sqrt{\lambda}\mu^3}{3\gamma\pi^3 T^2}\frac{l_\mu + (l\cdot w)w_\mu}{s}$$

- New chiral effect that follows directly from the anomaly
- Anomalous effect for heavy particles
- New contribution to the 'drag' force, in the direction of (B, Ω), same direction for all quarks/antiquarks independent of their charge (even for heavy quarks at rest)
- Phenomenological consequence in correlation between CME current and heavy quark momenta. In an event in which CME pushes positive light quarks up, all heavy quarks get a kick downward.
- Corrections to CME and CVE in presence of non-zero heavy particle density

Adding gravitational CS term to our consideration

$$S_{gCS} = -\frac{1}{16\pi G_5}\int \sqrt{-g}d^5x\ \kappa_g \epsilon^{MNPQR} A_M R^A_{BNP} R^B_{AQR}$$

we expect to gain T^2 term in the axial current along vorticity and similarly one finds contribution to the vortical part of the chiral drag force

$$\vec{f} = -\frac{\kappa_g \sqrt{\lambda}\mu}{\gamma\pi^3} C_V(s)\vec{l}$$

and for instance in the case of a single $U(1)_L$ chiral fermion $\frac{\kappa}{\kappa_g} = 24$

In the presence of external B and Ω the drag force for $\vec{w} = \vec{u} = 0$ has form

$$\vec{f} = -\frac{\kappa\sqrt{\lambda}}{2\pi^3}\frac{\mu^2}{T^2}\vec{B} - \frac{2\kappa\sqrt{\lambda}}{3\pi^3}\frac{\mu^3}{T^2}\vec{\Omega}$$

and comparing with the usual drag force

$$\vec{f} = \frac{\sqrt{\lambda}}{2\pi}\pi^2 T^2 \vec{v}$$

we get

$$\vec{v}_{terminal} = \kappa\frac{\mu^2\vec{B}}{(\pi T)^4} + \frac{4\kappa}{3}\frac{\mu^3\vec{\Omega}}{(\pi T)^4}$$

Thus anomalous transports gain corrections by **heavy** quarks (at finite heavy quarks density) and it is surprising!

But we worked in the Landau rest frame and chiral effects do have corrections there as well as the entropy current does:

$$J_\mu = nu_\mu + C\left(\mu - \frac{1}{2}\frac{n\mu^2}{\varepsilon + P}\right)B_\mu + \frac{1}{2}C\left(\mu^2 - \frac{2}{3}\frac{n\mu^3}{\varepsilon + P}\right)\ell_\mu\,,$$

$$s_\mu = su_\mu - \frac{1}{2}\frac{C\mu^2 s}{\varepsilon + P}B_\mu - \frac{1}{3}\frac{C\mu^3 s}{\varepsilon + P}\ell_\mu\,,$$

By a direct calculation one may check that the boost to the entropy rest frame is

$$\vec{v}_{boost} = -\frac{1}{2}\frac{C}{\epsilon + P}\left(\mu^2\vec{B} + \frac{4}{3}\mu^3\vec{\Omega}\right)$$

and we can readily see considerable similarity

For SYM $\mathcal{N} = 4$ we have

$$C = -\frac{N_c^2 \kappa}{\pi^2} = \frac{N_c^2}{4\pi^2\sqrt{3}}\ ,\ \epsilon + P = \frac{N_c^2 \pi^2 T^4}{2}$$

and hence $\vec{v}_{boost} = \vec{v}_{terminal}$, this statement is shown to be correct to all orders in powers of μ/T.

- In the entropy rest frame the chiral drag force for a heavy quark at rest is zero
- In the entropy rest frame there is non-zero charge and momentum transports caused by chiral effects so we may conclude that they do not dissipate.

A heavy ($m \to \infty$) quark may be considered as a defect in the fluid flow and the absence of the drag force indicates non-dissipativity of the anomalous transport. It can be shown that the absence of the drag force holds in the presence of other gradients.

Let's consider a quark at rest in the local fluid rest frame it feels

$$\vec{f}^B = -\kappa\sqrt{\lambda}\left(1 - \frac{16\kappa_g}{3\kappa}\right)\frac{\mu^2}{2\pi^3 T^2}\vec{B}$$

and it accelerates up to the terminal velocity

$$\vec{v}_{\rm terminal} = \kappa\left(1 - \frac{16\kappa_g}{3\kappa}\right)\frac{\mu^2}{(\pi T)^4}\vec{B}$$

on the timescale of order $m|\vec{v}_{\rm terminal}|/|\vec{f}^B| \sim 2\pi m/(\sqrt{\lambda}(\pi T)^2) \sim 1fm/c$. Then the characteristic momentum gained by quarks is

$$\vec{p}_{\rm terminal} = m_b\,\kappa\left(1 - \frac{16\kappa_g}{3\kappa}\right)\frac{\mu^2\vec{B}}{(\pi T)^4} = -0.449\,m_b\frac{\mu_V\mu_A\vec{B}}{(\pi T)^4}$$

$$\simeq -3\ \mathrm{MeV}\,\frac{m_b}{4.2\ \mathrm{GeV}}\,\frac{\mu_V}{0.1\ \mathrm{GeV}}\,\frac{\mu_A}{0.1\ \mathrm{GeV}}\,\frac{\vec{B}}{(0.1\ \mathrm{GeV})^2}\left(\frac{0.5\ \mathrm{GeV}}{\pi T}\right)^4$$

Thus the chiral magnetic drag force on b(c)-quarks and antiquarks gives them a common momentum of order ~ 3 MeV (~ 1 MeV).

- New contribution into the drag force with anomalous nature tied with CME and CVE
- The same sign of the effect for quarks and antiquarks despite of their charge
- The effect of the chiral drag force is in principle observable despite of smallness
- One may try to look for a correlation between heavy quark momenta and the CME current direction in a single event
- The picture can be turned around to show that a heavy defect doesn't dissipate the momentum flow of chiral effects