Chiral effects in superfluid

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Plan

Recently, there were intense studies of hydrodynamics of chiral liquids. A crucial novel point is existence of new transport coefficients, overlooked in the text-book approaches.

$$\overrightarrow{j}_{CME} = \frac{\mu_5}{2\pi^2}\overrightarrow{B} \quad , \quad \overrightarrow{j}_{CVE} = \frac{\mu_5 \mu}{\pi^2}\overrightarrow{\omega},$$

The coefficients are considered to be completely fixed by the coefficient in front of the chiral anomaly. This result could be obtained in a lot of approaches.

Equation of motion for relativistic liquid in external EM field plus anomaly:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \quad , \quad \partial_\mu J^\mu = C E^\mu B_\mu ,$$

where

$$T^{\mu\nu} = w u^\mu u^\nu + P g^{\mu\nu} + \tau^{\mu\nu} \quad , \quad J^\mu = n u^\mu + \nu^\mu$$

For ideal liquid $\tau_{\mu\nu}$ are ν are absence but in presence of the anomaly situation is not the same!

Entropy non-decreasing for ideal liquid transform to entropy current conservation and in presence of anomaly it takes form

$$\partial_\mu s^\mu = -C\frac{\mu}{T}E\cdot B.$$

where $s^\mu = su^\mu$. To improve this relation one should introduce another definition of entropy current and turn on ν^μ then

$$s_\mu = su_\mu - \frac{\mu}{T}\nu^\mu + D\omega_\mu + D_B B_\mu \quad \text{and} \quad \partial_\mu s^\mu \geq 0,$$

where

$$\nu_\mu = \xi\omega_\mu + \xi_B B_\mu$$

$$\omega_\mu = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}u^\nu\partial^\alpha u^\beta \quad , \quad B_\mu = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}u^\nu F^{\alpha\beta}$$

From the modified entropy current conservation ($\partial_\mu s^\mu$) coefficients take following form (arXiv:0906.5044 [hep-th])

$$\xi = C(\mu^2 - \frac{2}{3}\frac{\mu^3 n}{\epsilon + p})$$

$$\xi_B = C(\mu - \frac{1}{2}\frac{\mu^2 n}{\epsilon + p})$$

The terms of higher order in chemical potential are not universal and depend on equation of state. It is interesting that in two current model only coefficient of chiral magnetic effect (coefficient in vector current for magnetic field) could be fixed.

Relation between the chiral anomaly and the coefficients could be studied in effective field theory (arXiv:1012.1958v1 [hep-th]). Lets consider the action for chiral fermions with chemical potential

$$S_{eff} = \int dx \left(i\bar{\psi}\gamma^{\rho} D_{\rho}\psi + \mu\bar{\psi}\gamma^{0}\psi + \mu_5\bar{\psi}\gamma^{0}\gamma_5\psi \right) + S_{int}.$$

One can consider that action after some modification as model of chiral liquid generated by some non-anomalous interaction. To modify action one should bring it to naively Lorentz invariant form

$$S_{eff} = \int dx \left(i\bar{\psi}\gamma^{\rho} D_{\rho}\psi + \mu u_{\nu}\bar{\psi}\gamma^{\nu}\psi + \mu_5 u_{\nu}\bar{\psi}\gamma^{\nu}\gamma_5\psi \right)$$

by introducing of liquid velocity u^{ν} as slowly varying external field.

After calculating of anomaly in the effective filed theory we get

$$\partial_\mu J^{5\mu} = -\frac{1}{4\pi^2}\epsilon_{\mu\nu\alpha\beta}(\partial^\mu(A^\nu+\mu u^\nu)\partial^\alpha(A^\beta+\mu u^\beta)+\partial^\mu(\mu_5 u^\nu)\partial^\alpha(\mu_5 u^\beta))$$

$$\partial_\mu J^\mu = -\frac{1}{2\pi^2}\epsilon_{\mu\nu\alpha\beta}\partial^\mu(A^\nu+\mu u^\nu)\partial^\alpha(\mu_5 u^\beta)$$

It should be noted that there are terms of higher orders in chemical potentials. However that contributions are not anomalous and moreover depend on details of IR cutoff

$$\delta\xi \sim \frac{\mu^3}{\epsilon_{IR}} \ , \ \epsilon_{IR} \sim (\epsilon+p)/n$$

One can take naive hydrodynamical limit by substituting

$$< J^\mu > = \ n u^\mu \ \ , \ \ < J^{5,\mu} > = \ n_5 u^\mu$$

and hydrodynamical currents could be redefined as

$$J^{5,\mu} = n_5 u^\mu + \frac{1}{2\pi^2}(\mu^2 + \mu_5^2)\omega^\mu + \frac{1}{2\pi^2}\mu B^\mu$$

$$J^\mu \ = \ n u^\mu + \frac{1}{\pi^2}(\mu\mu_5)\omega^\mu + \frac{1}{2\pi^2}\mu_5 B^\mu$$

This result coincides with the answer obtained through pure hydrodynamical consideration.

The other way to obtain CME and CVE is consideration of effective gravity. One can consider slowly moving chiral liquid as in the rest frame by performing following coordinate transformation

$$ds = -dt^2 + 2v_i dt dx^i + dx^2$$

In that frame Dirac action should be modified

$$S = \int dx \; i\bar{\psi}\gamma^a e_a^\rho \left(i\partial_\rho - \omega_\rho^{ab} + A_\rho \right) \psi,$$

where ω_ρ^{ab} is spin-connection and e_a^ρ is vierbien. One readily finds that in the low velocity limit this effective theory coincides with the previous one.

The chiral kinetic coefficients could be obtained in the theory with effective gravity as a linear response. Lets consider only CVE in the axial current here then one should calculate the following correlators:

$$\sigma = \lim_{k_c \to 0} \epsilon_{abc} \frac{-ik_c}{2k^2} \langle J^a T^{0b} \rangle|_{\omega=0}$$

and after some calculation (arXiv:1103.5006v2 [hep-ph]):

$$\sigma = \frac{\mu^2 + \mu_5^2}{4\pi^2} + \frac{T^2}{12}$$

It was shown that two terms are proportional to the chiral and gravitational anomalies respectively.

Despite of evaluation methods variety there is non-answered question of the chiral effects microscopic realization. To clarify that one could consider Dirac operator eigenstates. The Dirac Hamiltonian in external constant magnetic field uniform in the third direction is

$$H = -i(\partial_i - ieA_i)\gamma^0\gamma^i + m\gamma^0,$$

where we can remove chemical potential as an energy shift and Dirac equation takes form

$$-H_R\psi_L + m\psi_R = E\psi_L \quad , \quad H_R\psi_R + m\psi_L = E\psi_R,$$

where $H_R = (-i\partial_i + eA_i)\sigma^i$ and $\left(H_R^2 + m^2\right)\psi_R = E^2\psi_R$

We can go to momentum eigenstates $-i\partial_3\psi_R = p_3\psi_R$ and then

$$H_R = p_3\sigma^3 + H_\perp,$$

where $H_\perp = (-i\partial_a + eA_a)\sigma^a$, $a = 1, 2$. The eigenstates of H_R can be expressed in terms of eigenstates of $H_\perp$ ($[H_\perp, H_R^2] = 0$). So each eigenstates of $H_\perp$ generates two eigenstate of H_R and the zero modes of $H_\perp$ are simultaneously eigenstates of H_R with eigenvalue $\epsilon = p_3\sigma^3$.

The axial current at finite μ, T is $j_5^3 = \sum n(E)\bar{\psi}_E \gamma^3 \gamma^5 \psi_E$ where $n(E)$ is Fermi-Dirac distribution. It could be shown that one can rewrite it through only zero modes of $H_\perp$

$$J_5^3 = Index(H_\perp)\frac{1}{L}\sum_{p_3}\left(n(\sqrt{p_3^2+m^2}) + n(-\sqrt{p_3^2+m^2})\right),$$

where $Index(H_\perp) = N_+ - N_- = \frac{e\Phi}{2\pi}$ is index of $H_\perp$. For massless fermions at zero temperature the summation gives $\frac{\mu}{\pi}$ and we get

$$J_5^3 = \frac{e\mu\Phi}{2\pi^2} \quad , \quad j_5^3 = \frac{e\mu}{2\pi^2}B^3$$

One can see that this result coincides with the result obtained macroscopically.

It is well known that motion of superfluid is potential and $v_i^s = \partial_i \phi / \mu$, where ϕ is Goldstone field. One readily finds that $\mathbf{rot} v^s = 0$ and rotation is forbidden. However, it is known that for superfluid there is solution with non-zero angular momentum $\phi = \mu t + \varphi$. Reproducing the same consideration as for effective filed theory we obtain

$$j_5^3 = \frac{1}{4\pi^2} \epsilon_{\alpha\beta\mu\nu} \partial^\nu \phi \partial^\alpha \partial^\beta \phi = \frac{\mu}{2\pi} \delta(x, y),$$

since $[\partial_x, \partial_y] \varphi = 2\pi \delta(x, y)$ and $J_5^3 = \frac{\mu}{2\pi}$.

The consideration of zero modes is quite similar to the one considered above so we can write for current

$$J_5^3 = (N_+ - N_-)\frac{1}{L}\sum_{p_3}(\theta(\mu - p_3) + \theta(\mu + p_3))$$

One has to calculate zero modes number but it could be shown that $N_+ - N_- = 1$ for vortex with $n = 1$ and $J_5^3 = \frac{\mu}{\pi}$.

The factor of one half is absent from the counting the number of zero modes. The result for the vortical effect looks so as if there were no identical vertices. The reason is that the chemical potential μ plays two different roles.

Common derivation assumes that the liquid is single-component and is characterized by a single velocity u_μ while the physical picture behind counting zero modes is a two-component liquid.

This two-component picture does not affect the final answer in case of the chiral magnetic effect because there are no identical vertices in the triangle graph in this case.

Thanks for your attention!