

# Event structure of multiparticle production in nucleus-nucleus collisions

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*LHEP*

*BLTP*

*21.04.2012*

# Multiparticle production at HEC

The statistical and hydrodynamical models predict an approximately exponential form of particle transverse mass spectra, provided collective flow of matter developed in the course of expansion is small.

## Pioneering ideas/models:

-1950: E. Fermi

**statistical hadron production:  $T = T_i \sim s_{NN}^{1/4}$**

-1951: I. Pomeranchuk

**freeze-out at  $T = T_{FO} \approx m_n$**

-1953: L. D. Landau

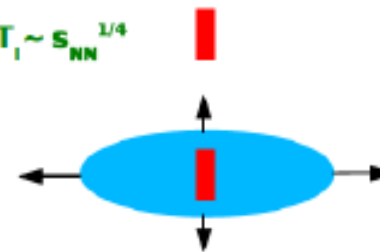
**hydrodynamical expansion from  $T_i$  to  $T_{FO}$**

**$T = f(m, v_T, T_{FO})$**

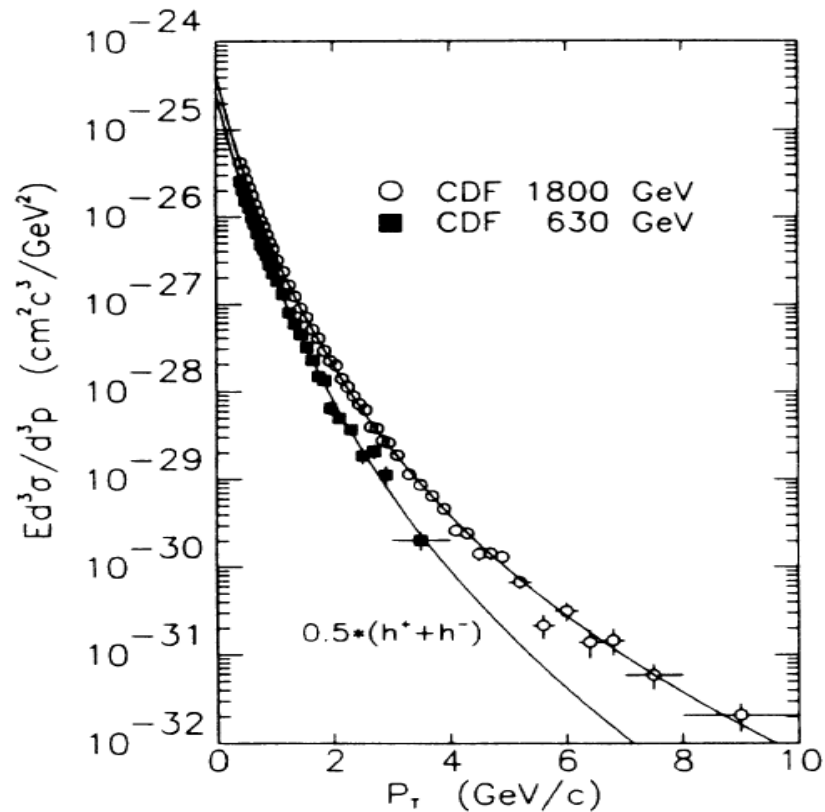
-1965: R. Hagedorn

**statistical hadron production at  $T = T_H \approx 150 \text{ MeV}$**

$$f(m_T) \sim e^{-m_T/T}$$



# PP collisions



Abe F. et al 1988 Phys. Rev. Lett. 61 1819

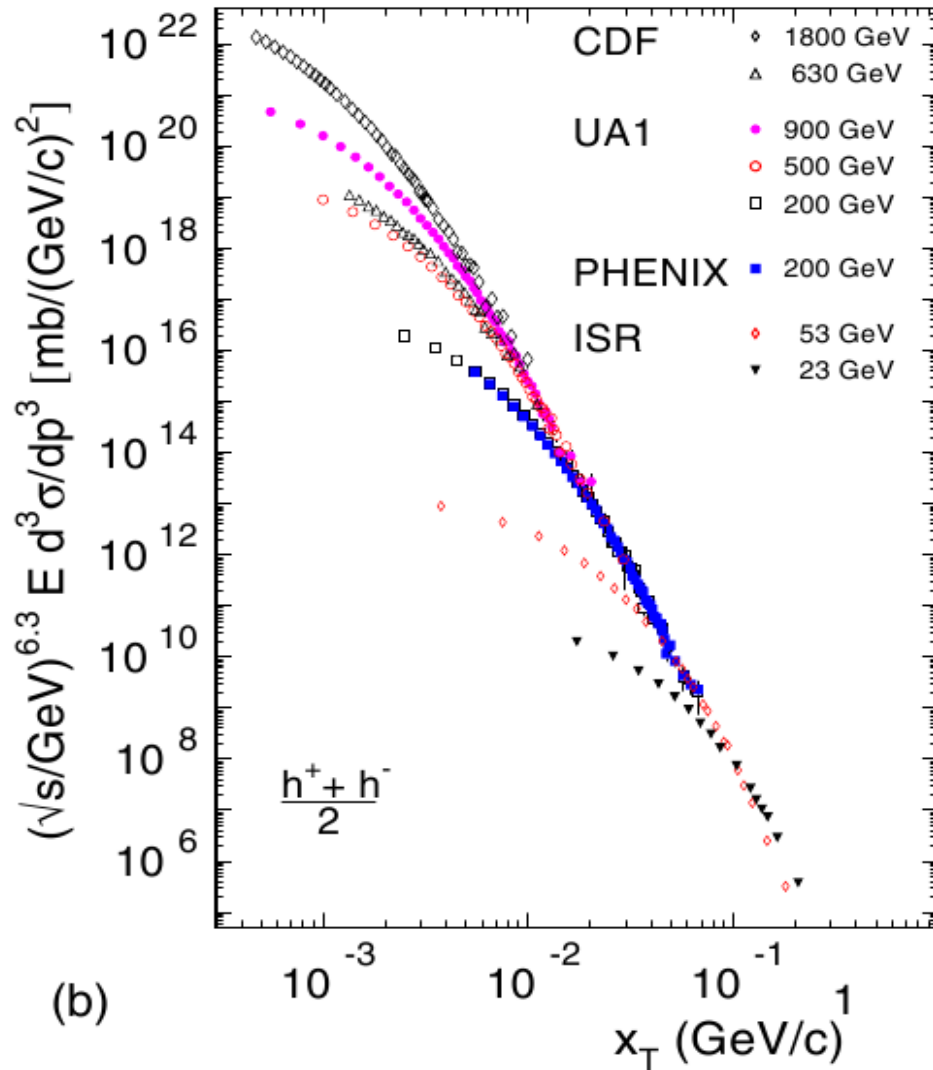
$pp\bar{p} \rightarrow h + X$

$$E \frac{d^3 \sigma}{d^3 p} = \frac{A p_0^n}{(p_T + p_0)^n}$$

Inclusive cross sections for rapidity  $|y| < 1.0$   
and fitted curves with  $p_0$  fixed at 1.3  $\text{GeV}/c$ .

# $x_T$ scaling

Adcox K et al PHENIX Collaboration 2005 Nucl. Phys. A 757 184–283



$$\sqrt{s}(\text{GeV})^{6.3} \times E d^3\sigma/d^3p \text{ vs. } x_T = 2p_T/\sqrt{s}$$

(b)

# Z scaling

## First LHC data on charged hadron production

pp collisions at low  $p_T$

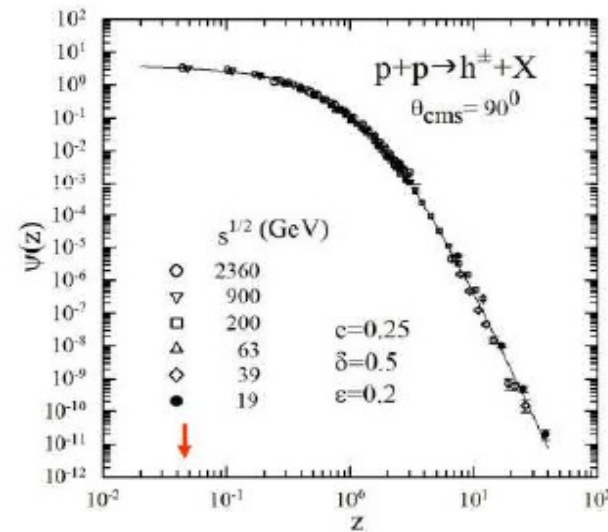
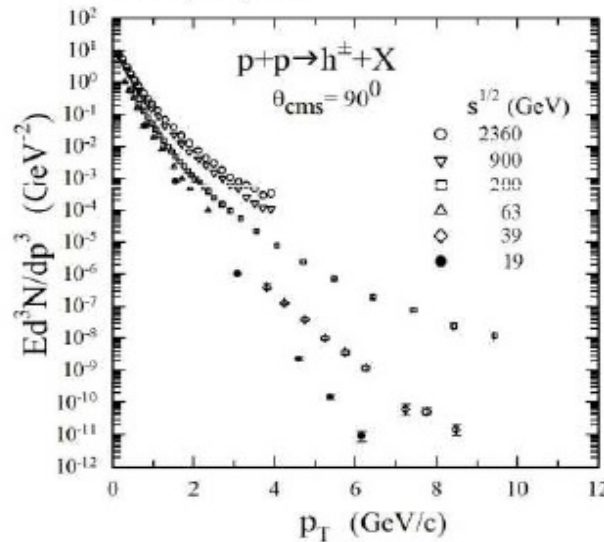
MT & I.Zborovsky  
J. Phys.G: Nucl.Part.Phys.  
37,085008(2010)

FNAL (fixed target)  
PRD 19 (1979) 764  
PRD 40 (1989) 2777

ISR: BS Coll.  
Nucl.Phys. B 100 (2007) 237

RHIC: STAR Coll.  
PRL 91 (2003) 172302

LHC: CMS Coll.  
JHEP02(2010) 041



- Energy independence at low  $p_T$
- CMS data confirm onset of saturation for  $z < 0.1$



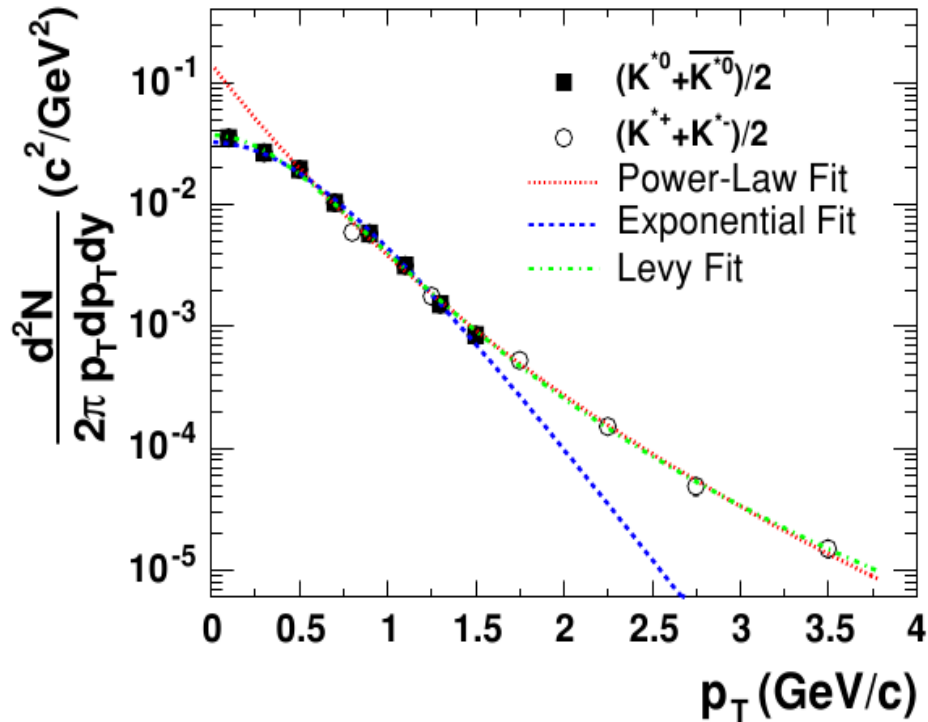
M.Tokarev



122

# Levy function

Phys.Rev. C 71, 064902 (2005) STAR



exp

$$\frac{1}{2\pi m_T} \frac{d^2N}{dy dm_T} = \frac{dN}{dy} \frac{1}{2\pi T(m_0 + T)} \exp\left(\frac{-(m_T - m_0)}{T}\right)$$

Power law

$$\frac{1}{2\pi p_T} \frac{d^2N}{dy dp_T} = \frac{dN}{dy} \frac{2(n-1)(n-2)}{\pi(n-3)^2 \langle p_T \rangle^2} \times \left(1 + \frac{p_T}{\langle p_T \rangle (n-3)/2}\right)^{-n}$$

Levy

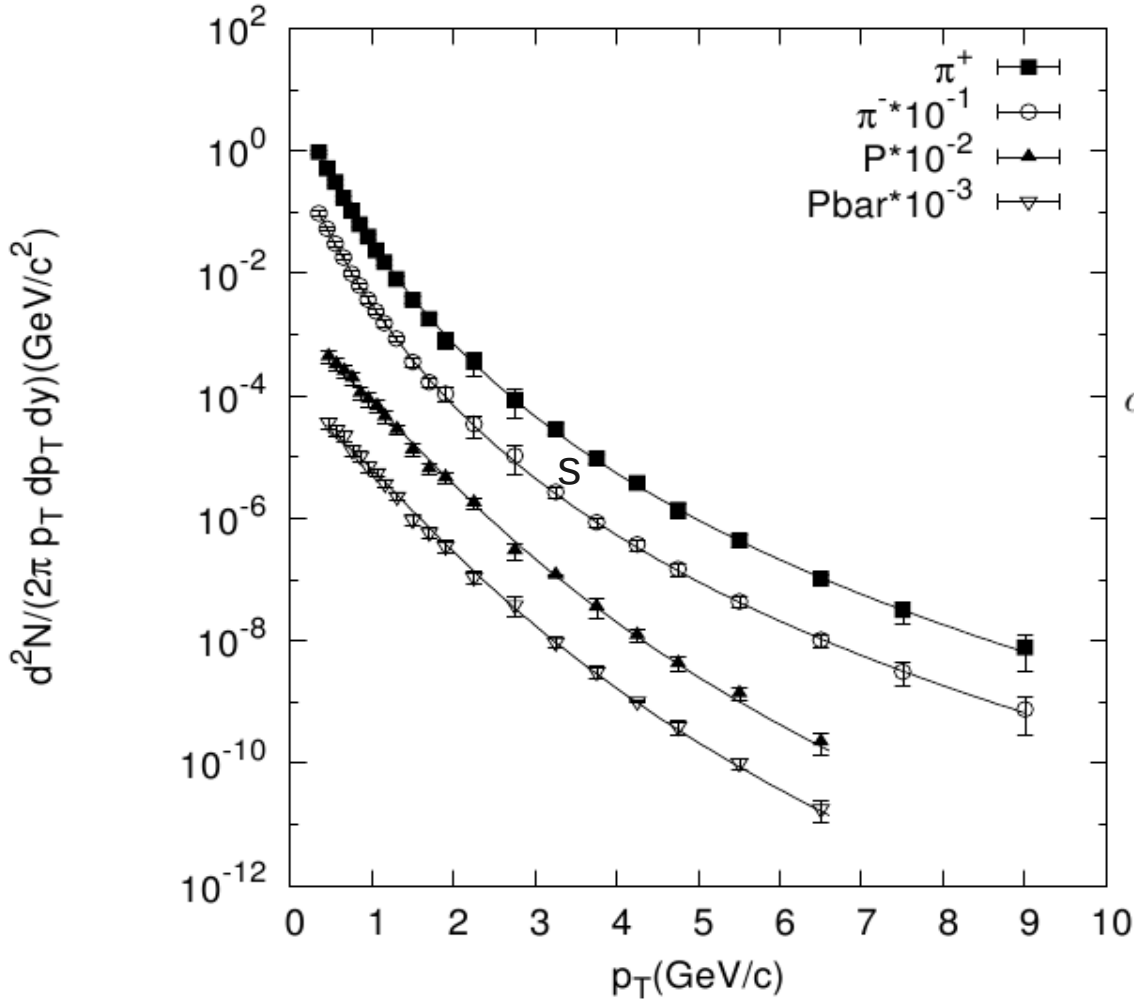
$$\frac{1}{2\pi p_T} \frac{d^2N}{dy dp_T} = \frac{dN}{dy} \frac{(n-1)(n-2)}{2\pi nT[nT + m_0(n-2)]} \times \left(1 + \frac{\sqrt{p_T^2 + m_0^2} - m_0}{nT}\right)^{-n}$$

The invariant yields for both  $(K^0 + \bar{K}^0)/2$  and  $(K^+ + K^-)/2$  as a function of  $p_T$  for  $|y| < 0.5$  in minimum bias  $p + p$  interactions. The dotted curve is the fit to the power-law function for  $p_T > 0.5$  GeV/c and extended to lower values of  $p_T$ . The dashed curve is the  $K^0$  spectrum fit to the exponential function and extended to higher values of  $p_T$ . The dashed-dotted curve is the fit to the Levy function for  $p_T < 4$  GeV/c. Errors are statistical only.

# Tsallis statistics

IJMP E Vol. 16, No. 6 (2007) 1687  
**STAR & PHENIX data**

P+P



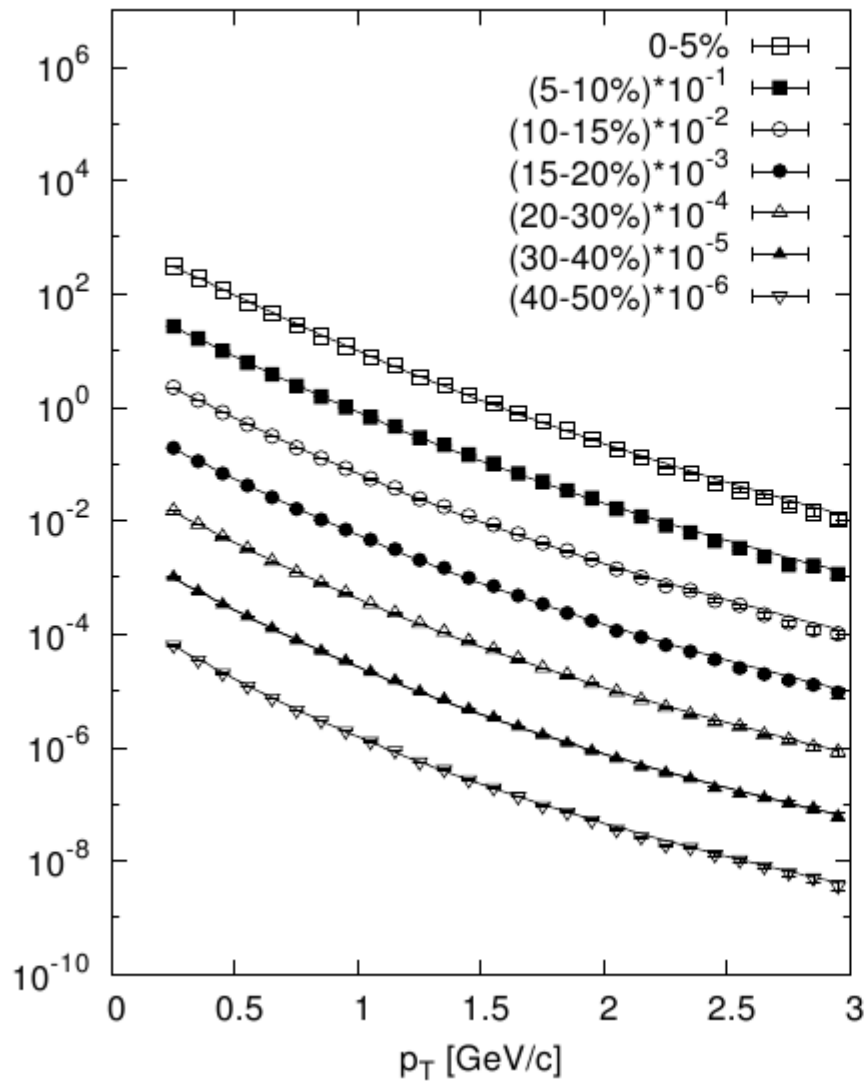
$$\frac{1}{\sigma} \frac{d\sigma}{dp_T} \approx$$

$$cp_T \int_0^\infty dp_L (1 + (q-1)\beta \sqrt{p_T^2 + p_L^2 + m_0^2})^{-q/(q-1)}$$

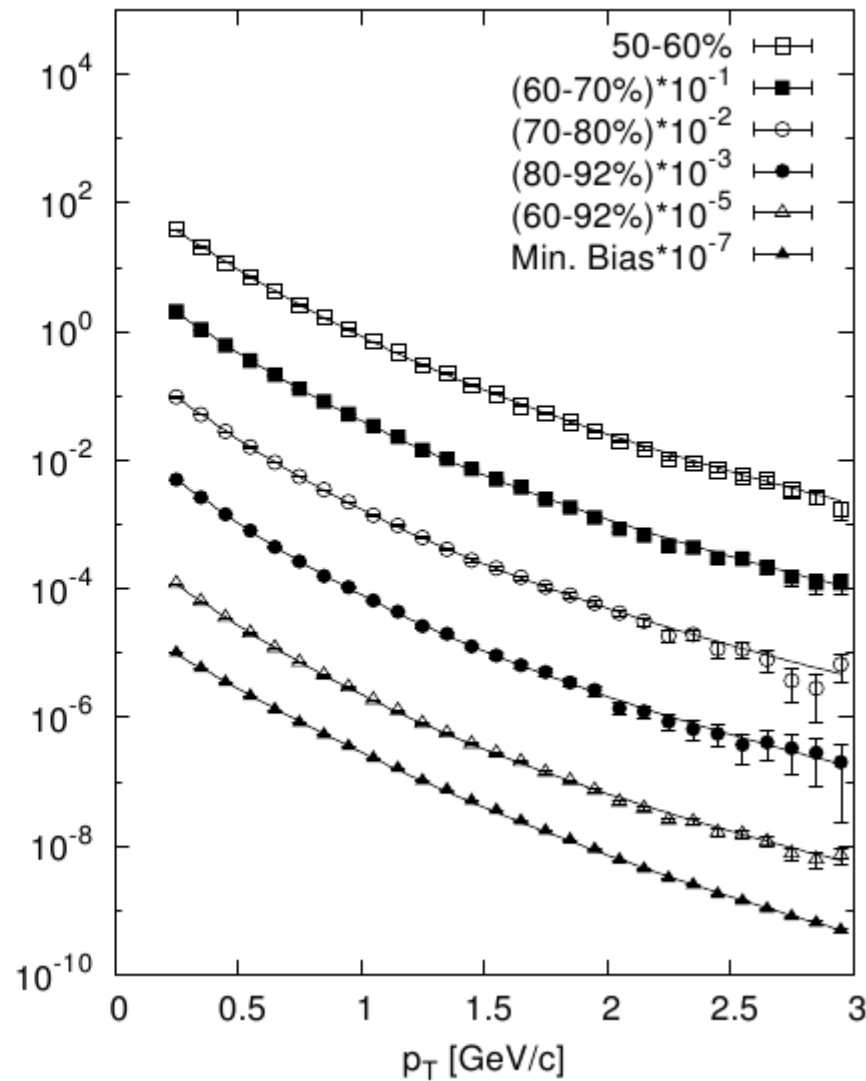
# Tsallis statistics (2)

Ibid.

Au+Au( $\pi^-$ )



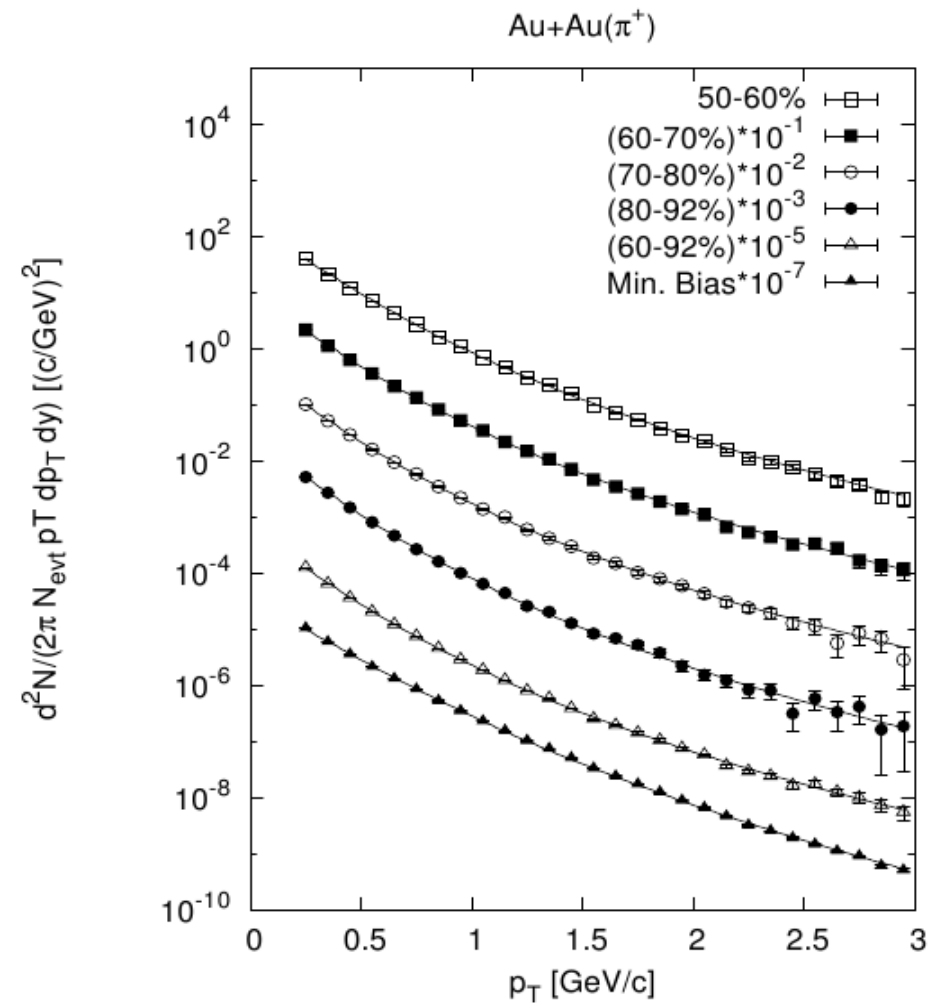
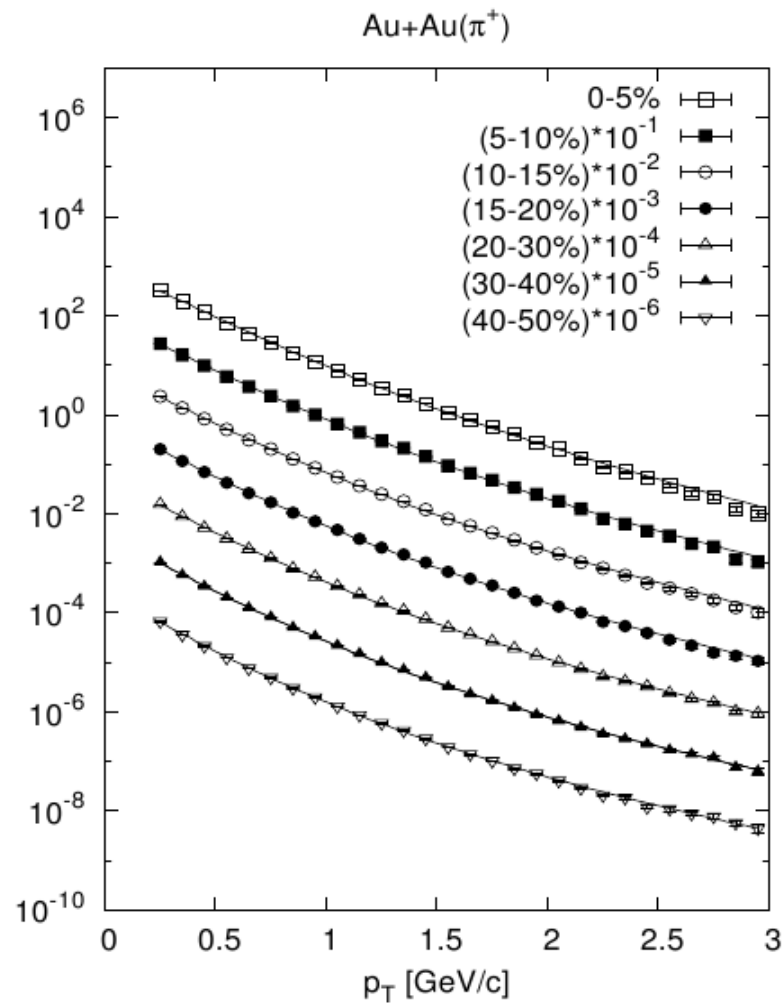
Au+Au( $\pi^-$ )





# Tsallis statistics (3)

Ibid.



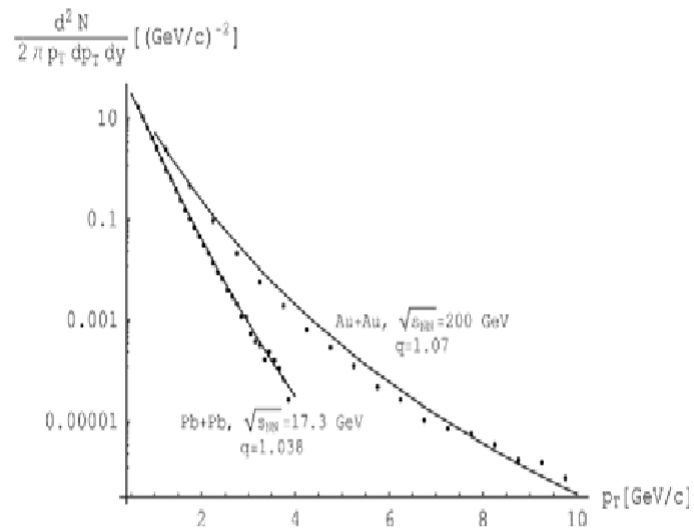
# Tsallis statistics (4)

Table 1. Values of fitted parameters with respect to experimental data on  $\pi^+$ -spectra at different centralities of  $Au + Au$ ,  $D + Au$  and  $P + P$  collisions at RHIC.

	Centrality	$C$	$q$	$T_0(\text{GeV})$	$\chi^2/ndf$
$Au + Au$	0–5	$7963 \pm 198$	$1.080 \pm 0.002$	$0.137 \pm 0.001$	35.180/25
	5–10	$7000 \pm 180$	$1.084 \pm 0.002$	$0.134 \pm 0.001$	33.758/25
	10–15	$6500 \pm 170$	$1.090 \pm 0.002$	$0.130 \pm 0.001$	28.543/25
	15–20	$6010 \pm 160$	$1.095 \pm 0.002$	$0.126 \pm 0.001$	27.543/25
	20–30	$4915 \pm 123$	$1.099 \pm 0.002$	$0.123 \pm 0.001$	28.704/25
	30–40	$3873 \pm 114$	$1.107 \pm 0.002$	$0.115 \pm 0.001$	31.913/25
	40–50	$2918 \pm 53$	$1.114 \pm 0.001$	$0.108 \pm 0.001$	21.123/25
	50–60	$2089 \pm 45$	$1.118 \pm 0.001$	$0.102 \pm 0.001$	22.208/25
	60–70	$1324 \pm 40$	$1.122 \pm 0.002$	$0.096 \pm 0.001$	24.902/25
	70–80	$769 \pm 32$	$1.125 \pm 0.003$	$0.090 \pm 0.001$	19.718/25
	80–92	$408 \pm 23$	$1.119 \pm 0.003$	$0.089 \pm 0.002$	14.443/25
	60–92	$815 \pm 24$	$1.121 \pm 0.001$	$0.094 \pm 0.001$	12.717/25
	Min. Bias	$3003 \pm 36$	$1.091 \pm 0.001$	$0.128 \pm 0.001$	24.140/25
$D + Au$	0–20	$259 \pm 51$	$1.092 \pm 0.003$	$0.135 \pm 0.006$	5.682/21
	20–40	$196 \pm 4$	$1.096 \pm 0.001$	$0.130 \pm 0.001$	7.505/21
	40–100	$107 \pm 13$	$1.099 \pm 0.002$	$0.120 \pm 0.004$	5.022/21
		Min. Bias	$145 \pm 21$	$1.099 \pm 0.002$	$0.126 \pm 0.004$
$P + P$	—	$81 \pm 6$	$1.102 \pm 0.002$	$0.106 \pm 0.002$	2.813/20

# Tsallis statistics PbPb AuAu

Eur. Phys. J. A 40, 313 (2009)



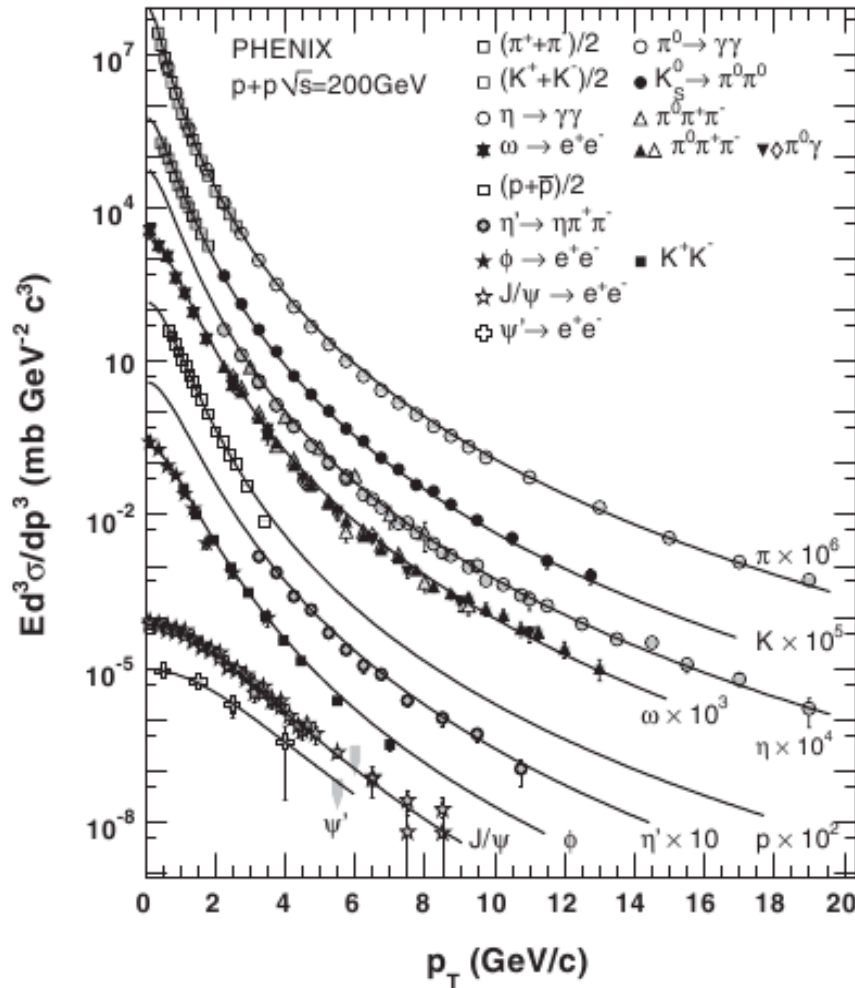
Experimental neutral pion invariant yields in central Pb+Pb collisions at  $\sqrt{s_{NN}} = 17.3$  GeV and in central Au+Au collisions at  $\sqrt{s_{NN}} = 200$  GeV compared with the modified thermal distribution shape by using non-extensive statistics

( $q = 1.038$  for Pb+Pb and  $q = 1.07$  for Au+Au collisions.)

$$\frac{d^2 N}{2\pi p_{\perp} dp_{\perp} dy} = C m_{\perp} \left[ 1 - (1 - q) \frac{m_{\perp}}{T} \right]^{1/(1-q)}$$

# PP PHENIX

PRD 83, 052004 (2011)



Tsallis distribution

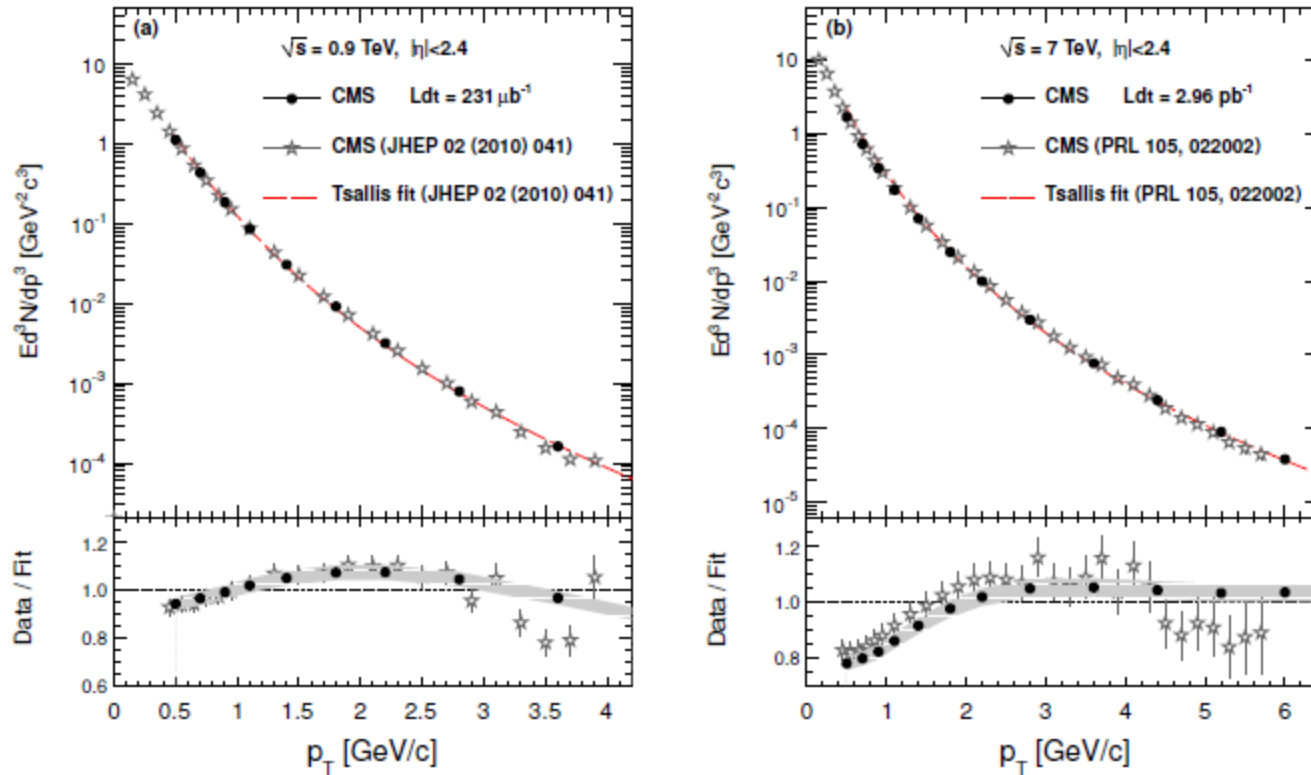
$$G_q(E) = C_q \left( 1 - (1 - q) \frac{E}{T} \right)^{1/(1-q)},$$

$$n = -\frac{1}{1 - q}.$$

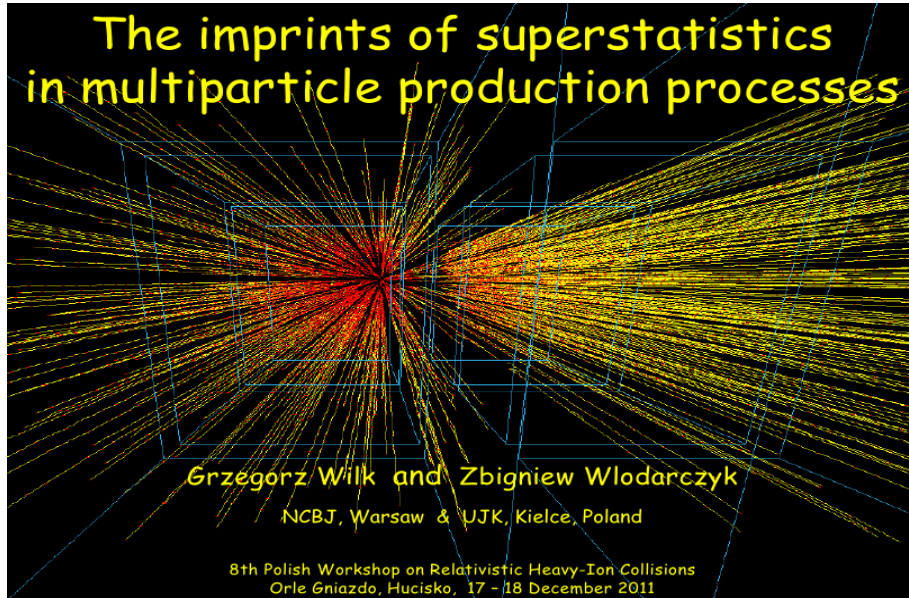
$$E \frac{d^3\sigma}{dp^3} = \frac{1}{2\pi} \frac{d\sigma}{dy} \frac{(n-1)(n-2)}{(nT + m_0(n-1))(nT + m_0)} \left( \frac{nT + m_T}{nT + m_0} \right)^{-n}$$

Invariant differential cross sections of different particles measured in p + p collisions at  $\sqrt{s} = 200$  GeV in various decay modes.

## Charged particle transverse momentum spectra in pp



**Figure 4.** (a) Upper panel: the invariant charged particle differential yield from the present analysis (solid circles) and the previous CMS measurements at  $\sqrt{s} = 0.9$  TeV (stars) over the limited  $p_T$  range of the earlier result. Lower panel: the ratio of the new (solid circles) and previous (stars) CMS results to a Tsallis fit of the earlier measurement. Error bars on the earlier measurement are the statistical plus systematic uncertainties added in quadrature. The systematic uncertainty band around the new measurement consists of all contributions, except for the common event selection uncertainty. (b) The same for  $\sqrt{s} = 7$  TeV.



## Tsallis statistics as Superstatistics

C. Beck et al., Physica A322 (2003) 267

**Superstatistics** is a superposition of two different statistics relevant to system under consideration with a **stationary state** and **intensive parameter fluctuations**

$$h(E/T) = \int_0^{\infty} f(E/T) g(1/T) d(1/T)$$

G. Wilk, Z. Włodarczyk, *Phys. Rev. Lett.* **84**, 2770 (2000); *Physica A* **376**(2007)279 *PRC* **79**(2009)054903; *EPJA* **40**(2009)299; *JPG* **38**(2011)065101; *Physica A* **390**(2011)3566 G. Wilk, Z. Włodarczyk, W. Wolak, *APPB*(2011)1277

M. Biyajima et al., *EPJC* **40**(2005)243 and *C48*(2006)593 (p<sub>T</sub> fits).  
T. Osada et al., *PRC* **77**(2008)044903; *PTPSuppl.* **174**(2008)168 (2008); *CEJP7*(2009)432; *IJP85*(2011)825 (q-hydrodynamics)

## Tsallis distribution

C. Tsallis, *J.Stat.Phys.* **52** (1988) 479

$$f(E) = \frac{2-q}{T} \left[ 1 - (1-q) \frac{E}{T} \right]^{1-q}$$

$q \rightarrow 1$  meaning of  $q$  ?

BG

$$f(E) = \frac{1}{T} \exp\left(-\frac{E}{T}\right)$$

R. Hagedorn (1965)

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In full phase space  $q$  measures dynamical fluctuations in  $P(N)$

(\*) Experiment:  $P(N)$  is adequately described by NBD depending on  $\langle N \rangle$  and  $k$  ( $k \geq 1$ ) affecting its width:

$$\frac{1}{k} = \frac{\sigma^2(N)}{\langle N \rangle^2} - \frac{1}{\langle N \rangle}$$

(\*) If  $1/k$  is understood as a measure of fluctuations of  $\langle N \rangle$ , then

$$P(N) = \int_0^{\infty} d\bar{n} \frac{\bar{n}^N \exp(-\bar{n})}{N!} \frac{\gamma^k \bar{n}^{k-1} \exp(-\gamma \bar{n})}{\Gamma(k)}$$

$$= \frac{\Gamma(k+N)}{\Gamma(1+N)\Gamma(k)} \frac{\gamma^k}{(\gamma+1)^{k+N}} \quad \text{with} \quad \gamma = \frac{k}{\langle \bar{n} \rangle}$$



(P. Carruthers, C.C. Shih, *Int.J.Phys.* **A4** (1989)5587)

$$\frac{1}{k} = D(\bar{n}) = \frac{\sigma^2(\bar{n})}{\langle \bar{n} \rangle^2} = q-1$$

(\*)  $\rightarrow$  one expects:  $q=1+1/k$  what indeed is observed

G.Wilk, Z.Włodarczyk, *EPJA* **40**(2009)299; F.Navarra, O.Utyuzh, WW, *PRD* **67**(2003)114002

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## Tsallis Distribution in High-Energy Collisions

(arXiv:1101.3023, accepted in EPL)

Gergely Gábor Barnaföldi

KFKI RMKI of the HAS

in collaboration

T.S. Biró, G. Kalmár, K. Ürmössy, P. Ván

High-pT Physics for the LHC 2011, Utrecht 4-7 April 2011

## MOTIVATION

- New LHC pp data (CMS)

CMS: JHEP 1002:041(2010)

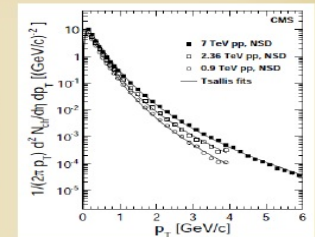
fitted Tsallis distribution for  $p_T$  spectra:

$$E \frac{d^3 N_{ch}}{dp^3} = \frac{1}{2\pi p_T} \frac{E}{p} \frac{d^2 N_{ch}}{d\eta dp_T} = C(n, T, m) \frac{dN_{ch}}{dy} \left(1 + \frac{E_T}{nT}\right)^{-n}$$

Parameters:

0.9 TeV T = 130 MeV, q = 1.13

2.36 TeV T = 140 MeV, q = 1.15



$n := (q-1)^{-1}$

- RHIC analysis on AuAu data ( $y=0$ )

Cooper-Frye model: K. Ürmössy, T.S. Biró: PL B689 14 (2010)

Parameters:  $f(E) = A[1 + (q-1)E/T]^{-1/(q-1)}$

200 GeV T = 51 MeV, q = 1.062 (fit for  $p_T < 6$  GeV/c)

G.G. Barnaföldi: Tsallis Distribution in High-Energy Collisions

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## Basics of non-extensive thermodynamics

Non-extensive thermodynamics (Based on: T.S. Biró: EPL84, 56003,2008) associative composition rule, (non-additive) :

$$h(h(x, y), z) = h(x, h(y, z))$$

Then should exist a strict monotonic function,  $X(x)$  'generalised logarithm' (an entropy-like quantity), for which:

$$h(x, y) = X^{-1}(X(x) + X(y))$$

$$X(h(x, y)) = X(x) + X(y).$$

Examples: (i) Classical Boltzmann-Gibbs thermodynamics:

$$f(E) = e^{-\beta E} / Z$$

$$h(x, y) = x + y.$$

(ii) Tsallis-Pareto-like distribution with  $a = q - 1$ :

$$f(E) = \frac{1}{Z} e^{-\frac{E}{a} \ln(1+aE)} = \frac{1}{Z} (1+aE)^{-\beta/a}$$

$$h(x, y) = x + y + axy$$

$$S = \int f \frac{e^{-\alpha \ln(f)} - 1}{a} = \frac{1}{a} \int (f^{1-a} - f).$$

G.G. Barnaföldi: Tsallis Distribution in High-Energy Collisions

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## Hadronization via non-extensive way

Our program:

- i) Search and fit Tsallis distribution to data from AA, pp, ee.

- ii) Test: can a BFKL / DGLAP-like evolution equation be obtained?

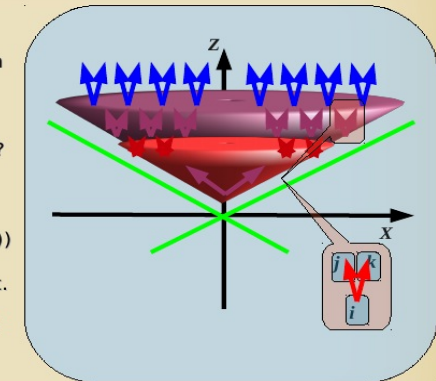
$$D(x, Q^2) \sim f(E, T, q) * f(\ln(Q^2))$$

$$D(x, Q^2) \sim f(E, T(\ln(Q^2)), q(\ln(Q^2)))$$

- iii) Build up a simple theory to test.

- iv) Search for physical meaning of T and q parameters.

→ This is a hard thing...



G.G. Barnaföldi: Tsallis Distribution in High-Energy Collisions

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# Tsallis distribution

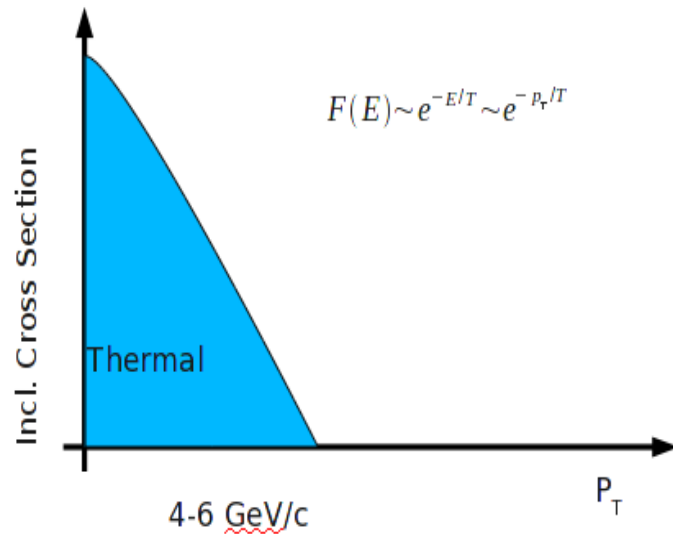
- ◆ Transverse-Momentum and Pseudorapidity Distributions of Charged Hadrons in Collisions at  $\sqrt{s}=7$  TeV  
Physical Review Letters 105, 022002 (2010)
- ◆ Transverse momentum spectra of charged particles in proton–proton collisions at  $\sqrt{s} = 900$  GeV with ALICE at the LHC  
Physics Letters B 693 53–68 (2010)
- ◆ Measurement of neutral mesons in p+p collisions at  $\sqrt{s} = 200$  GeV and scaling properties of hadron production  
Physical Review D 83, 052004 (2011)
- ◆ Nuclear modification factors of  $\phi$  mesons in d + Au, Cu + Cu, and Au + Au collisions at  $\sqrt{s_{NN}} = 200$  GeV  
Physical Review C 83, 024909 (2011)
- ◆ Strange particle production in proton–proton collisions at  $\sqrt{s} = 0.9$  TeV with ALICE at the LHC  
The European Physical Journal C 71, 1594(2011)
- ◆ Production of pions, kaons and protons in pp collisions at  $\sqrt{s} = 900$  GeV with ALICE at the LHC  
The European Physical Journal C 71, 1655(2011)
- ◆ Charged-particle multiplicities in pp interactions measured with the ATLAS detector at the LHC  
New Journal of Physics 13, 053033 (2011)



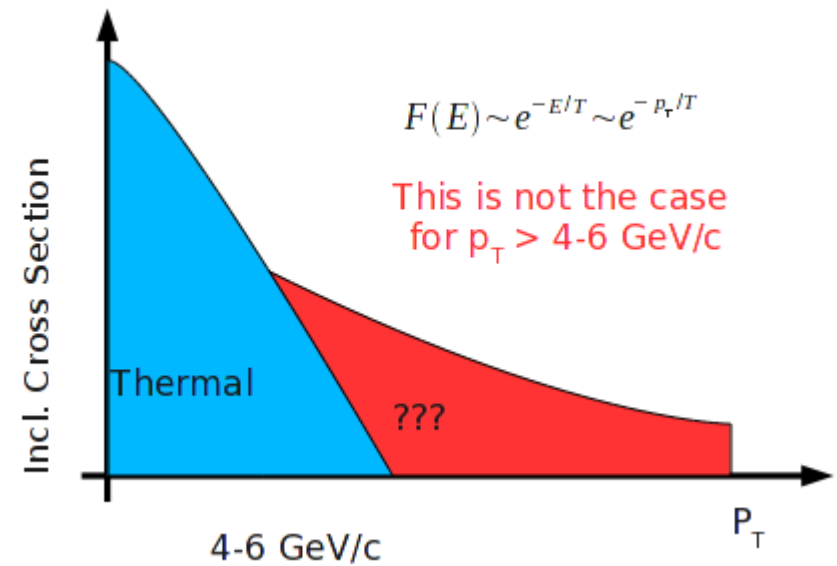
# Thermal hadron spectra

G.G. Barnaföldi: Tsallis Distribution in High-Energy Heavy Ion Collisions

- Thermalised system: spectra follow Boltzmann-Gibbs

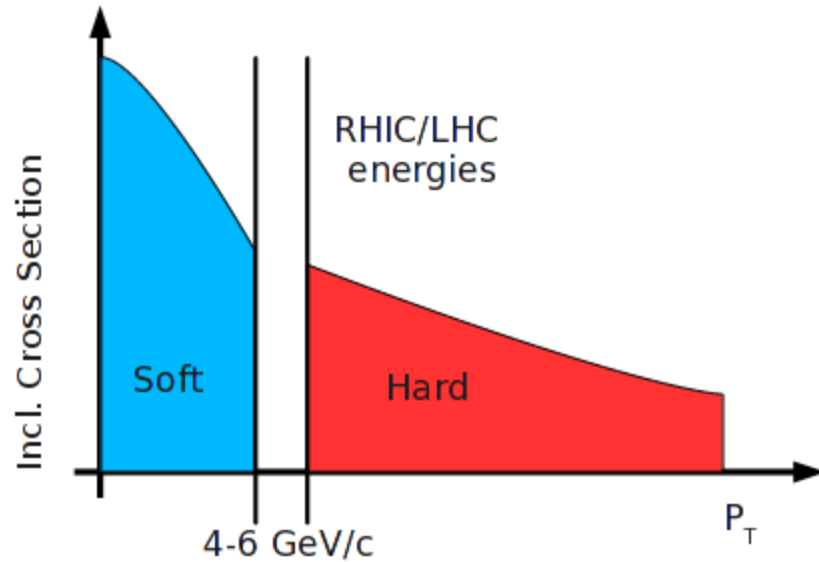


- But in HIC this is quite different...

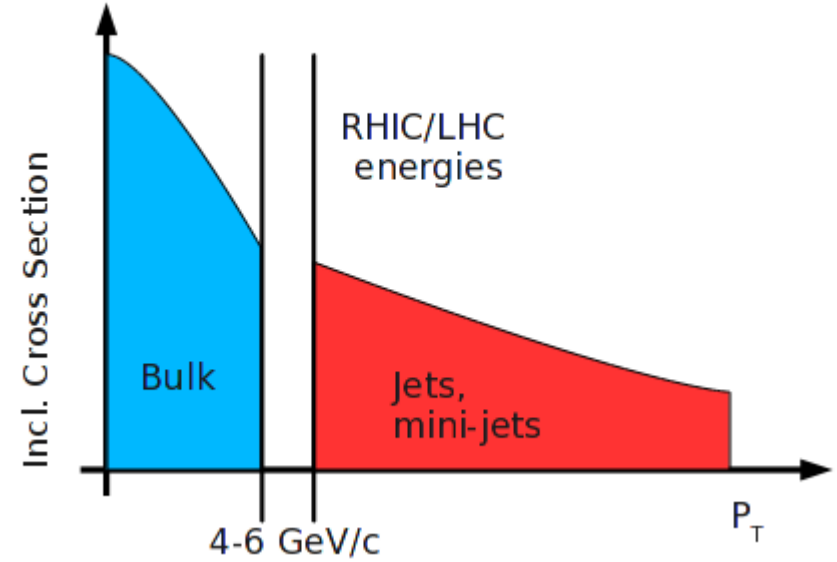


# High & low $p_T$ hadron spectra

- 1. interpretation 'soft' & 'hard' processes

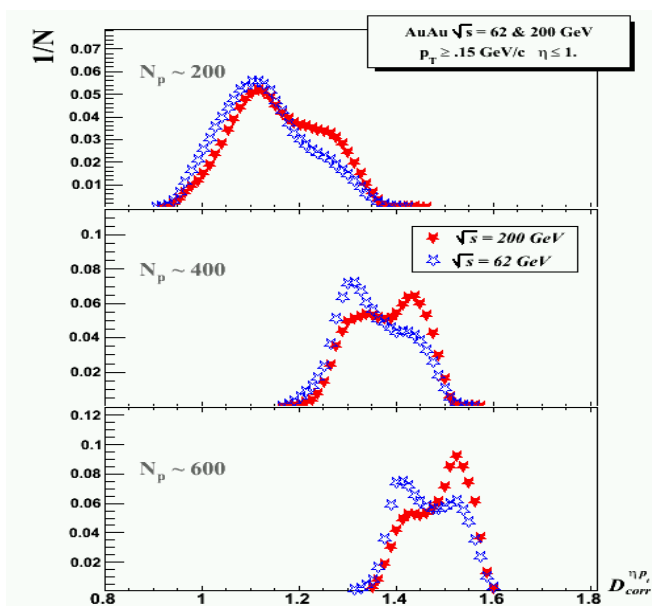


- 2. interpretation 'bulk' & 'jet-like' processes



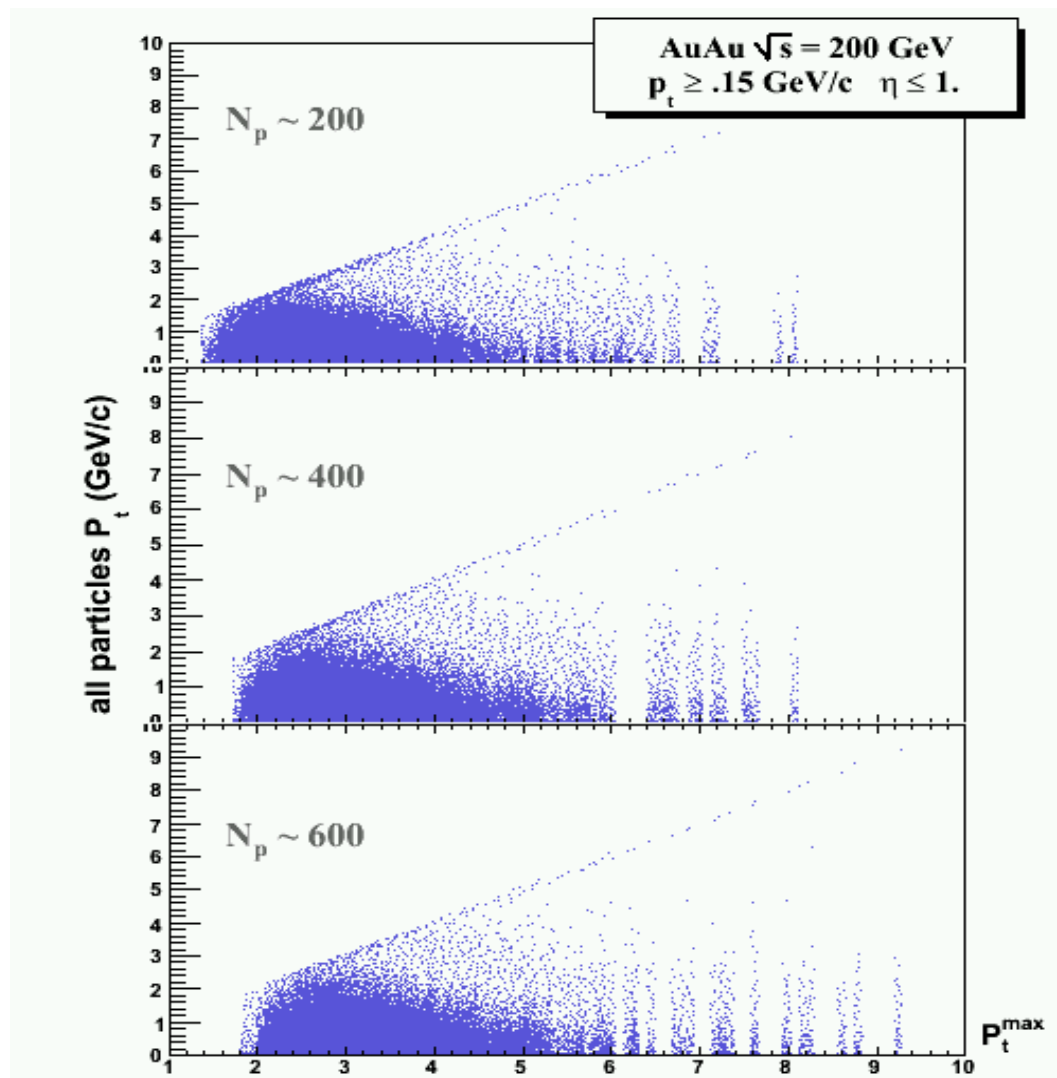
# Event fractal dimensions

R.O. ICHEP 2006



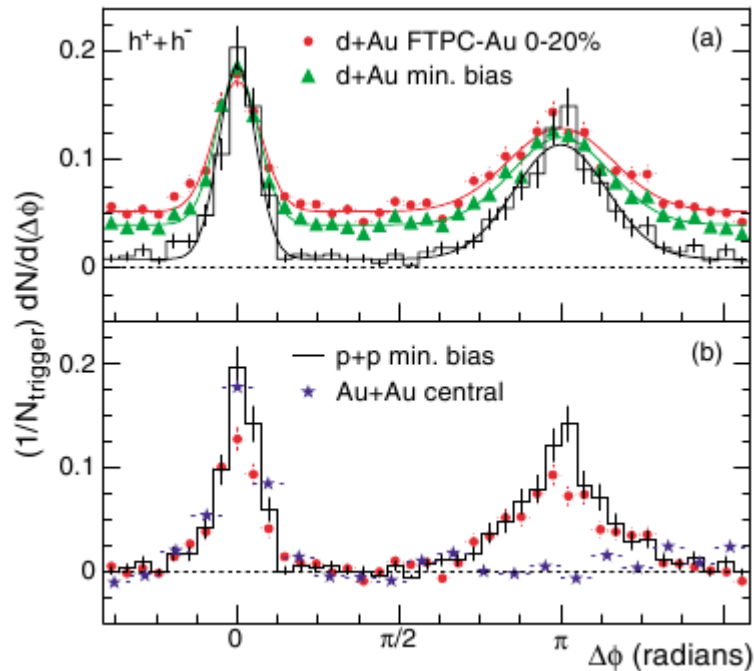
Fractal dimensions of events in rapidity-transverse momentum space

$p_T^{\max}$  – maximum transverse momentum of a particle in the event



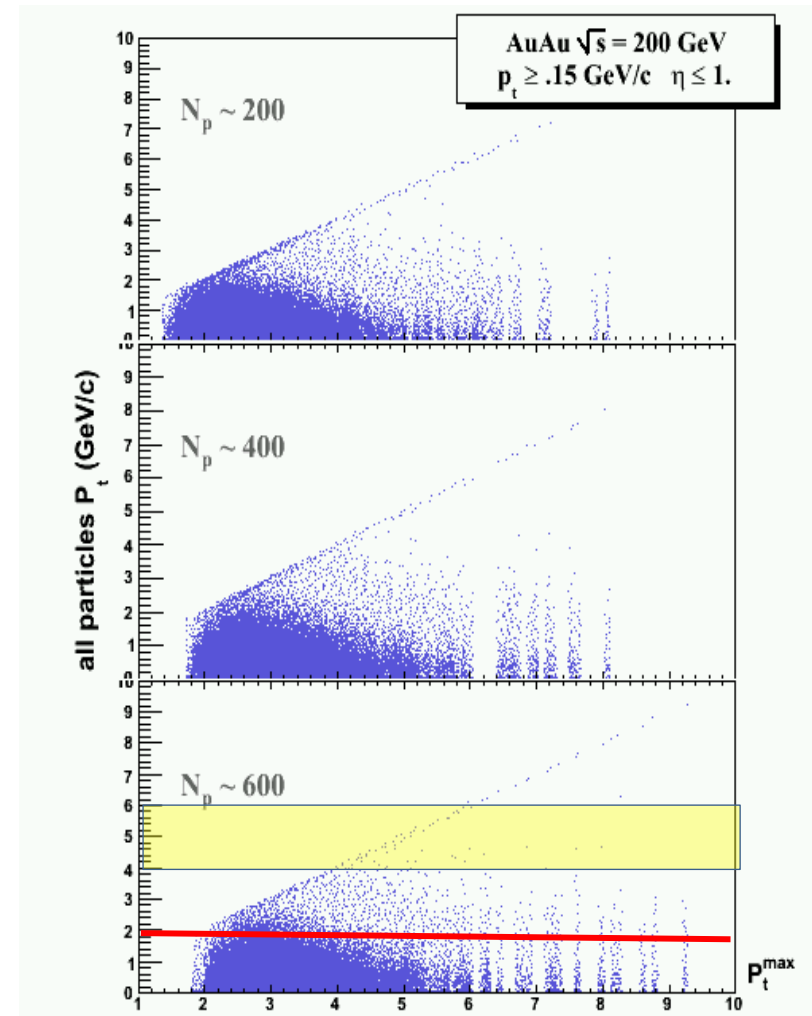
# Disappearance of away side jet

Phys. Rev. Lett. 91, 072304 (2003).



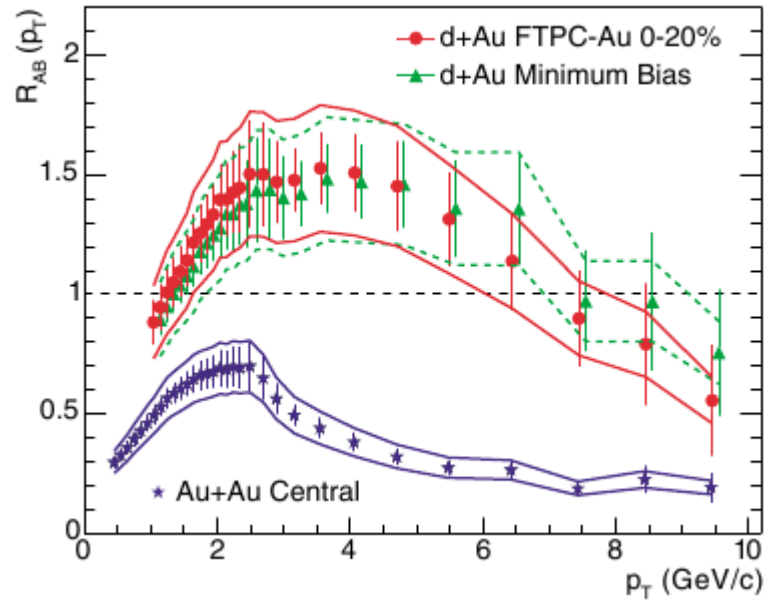
$$D(\Delta\phi) \equiv \frac{1}{N_{\text{trigger}}} \frac{1}{\epsilon} \frac{dN}{d(\Delta\phi)}$$

. Only particles within  $|\eta| < 0.7$  are included in the analysis.  $N_{\text{triggers}}$  is the number of particles within  $4 < p_T(\text{trig}) < 6$  GeV/c, referred to as trigger particles. The distribution results from the correlation of each trigger particle with all associated particles in the same event having  $2 < p_T < p_T(\text{trig})$ , where  $\epsilon$  is the tracking efficiency of the associated particles. The normalization uncertainties are less than 5%.

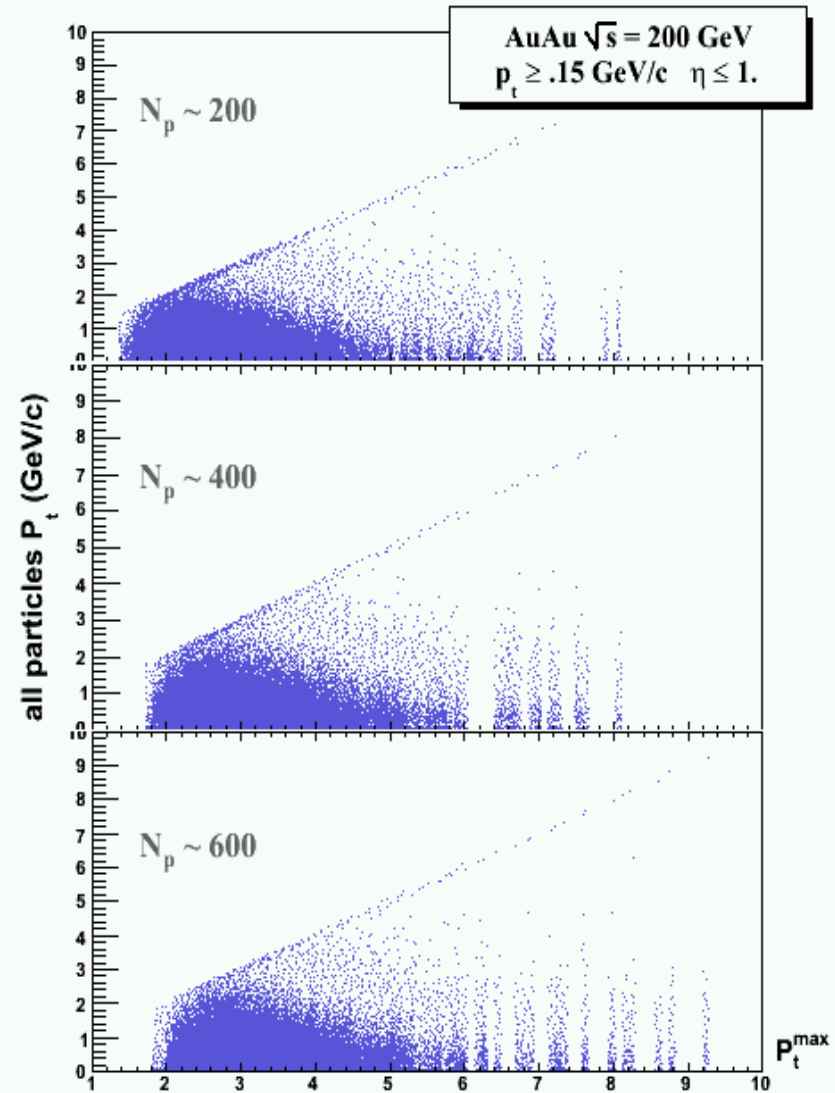


# High $p_T$ suppression

Phys. Rev. Lett. 91, 072304 (2003).

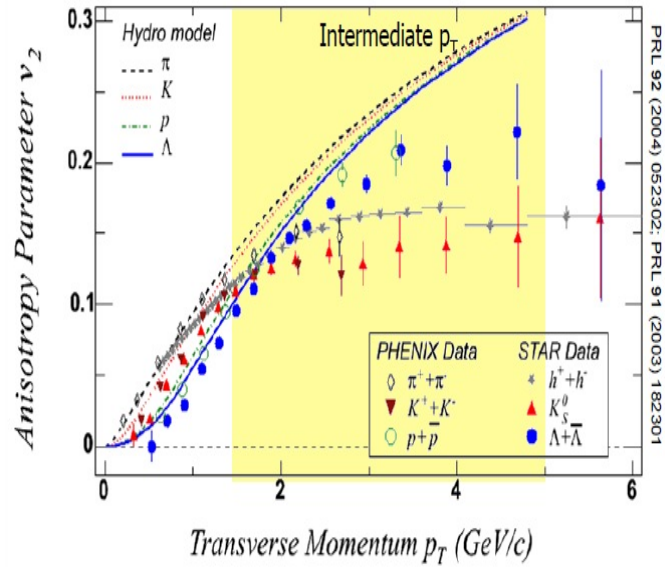


$$R_{AB}(p_T) = \frac{d^2N/dp_T d\eta}{T_{AB} d^2\sigma^{pp}/dp_T d\eta}$$

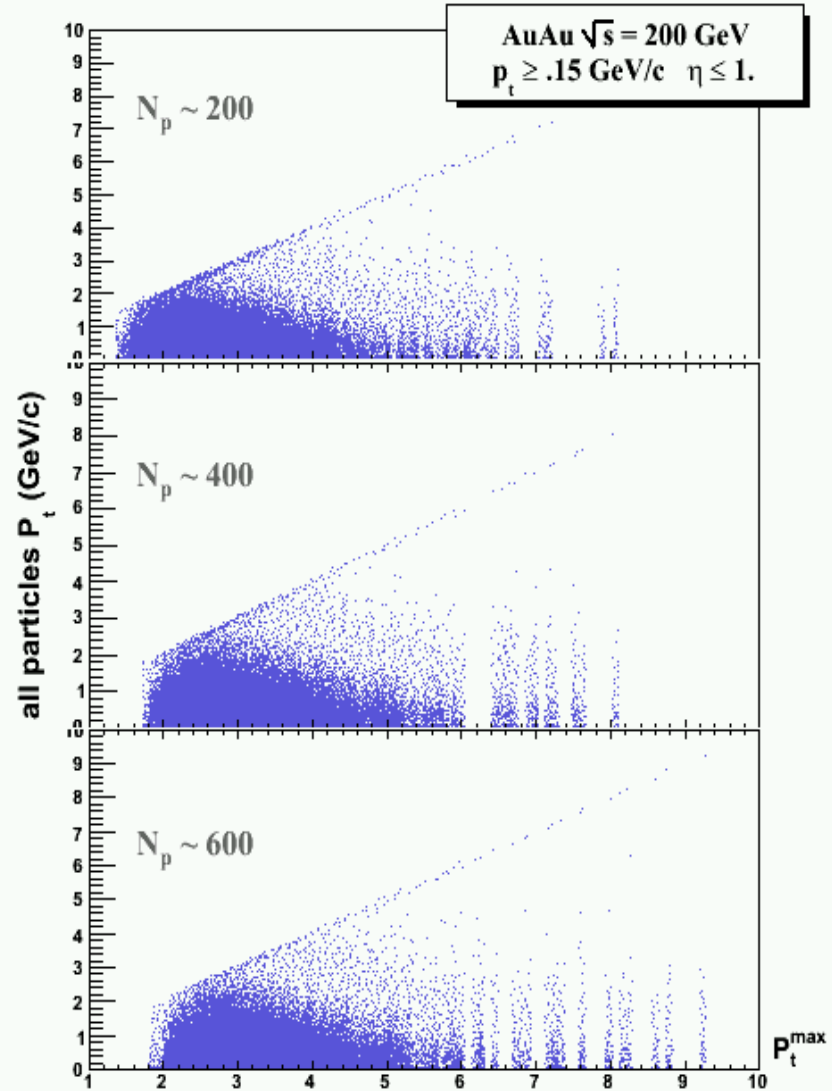
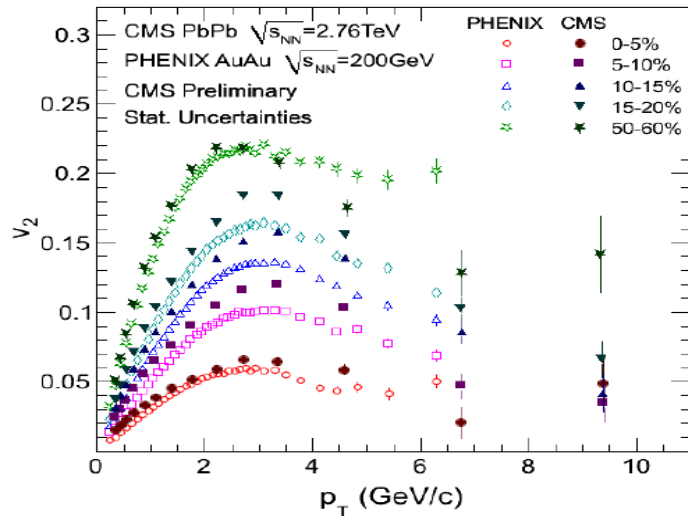


# Azimutal anisotropy

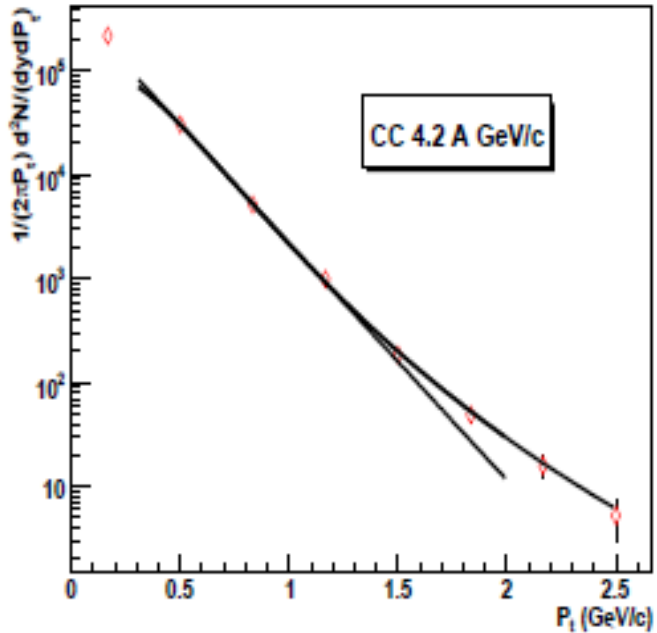
RHIC



LHC

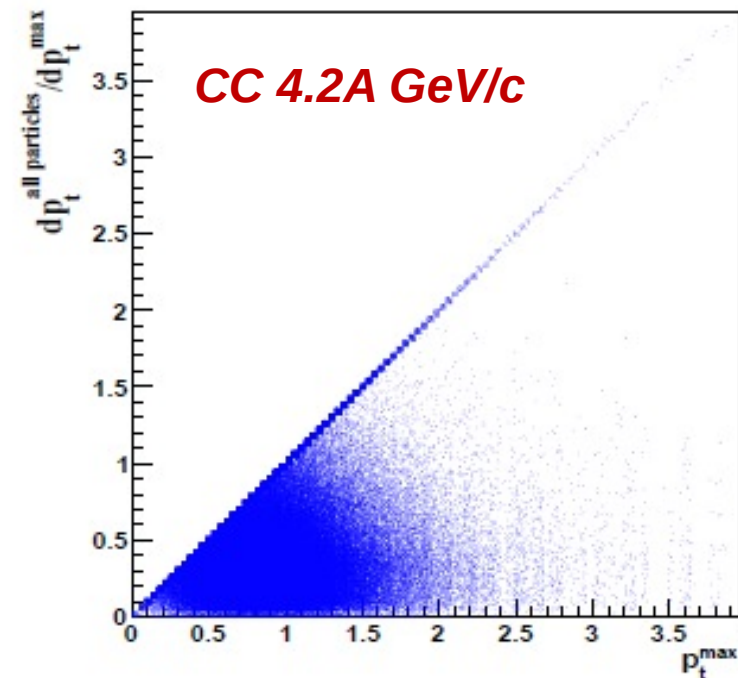


# pC dC aC CC @ 4.2A GeV/c



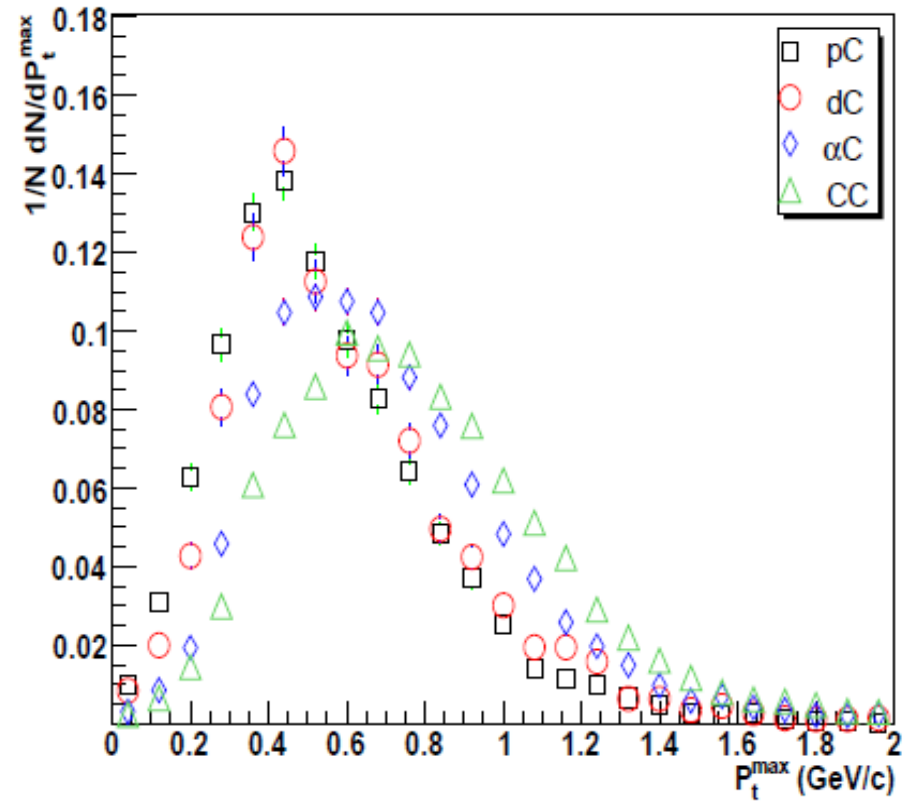
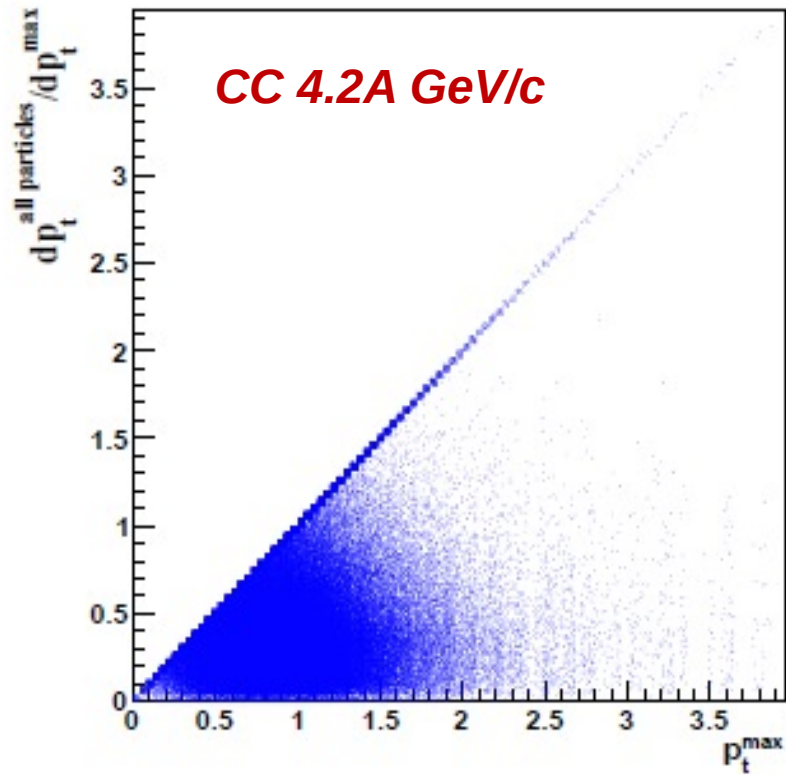
Levy distribution fit

Interaction	Number of events
<i>pC</i>	5722
<i>dC</i>	3826
<i>αC</i>	9643
<i>CC</i>	15842



	parameter value			
	n	T	B	$\chi^2/ndf$
<i>pC</i>	$7.45 \pm 1.22$	$6.86e-02 \pm 6.04e-03$	$2.29e+04 \pm 2.40e+03$	0.19
<i>dC</i>	$6.68 \pm 1.10$	$6.63e-02 \pm 6.78e-03$	$1.85e+04 \pm 2.31e+03$	0.19
<i>αC</i>	$6.59 \pm 0.57$	$7.08e-02 \pm 3.66e-03$	$6.10e+04 \pm 3.70e+03$	1.20
<i>CC</i>	$6.84 \pm 0.39$	$7.69e-02 \pm 2.39e-03$	$1.30e+05 \pm 4.55e+03$	3.20

# 4.2 GeV/c



- Shifts with A
- Broadening with A

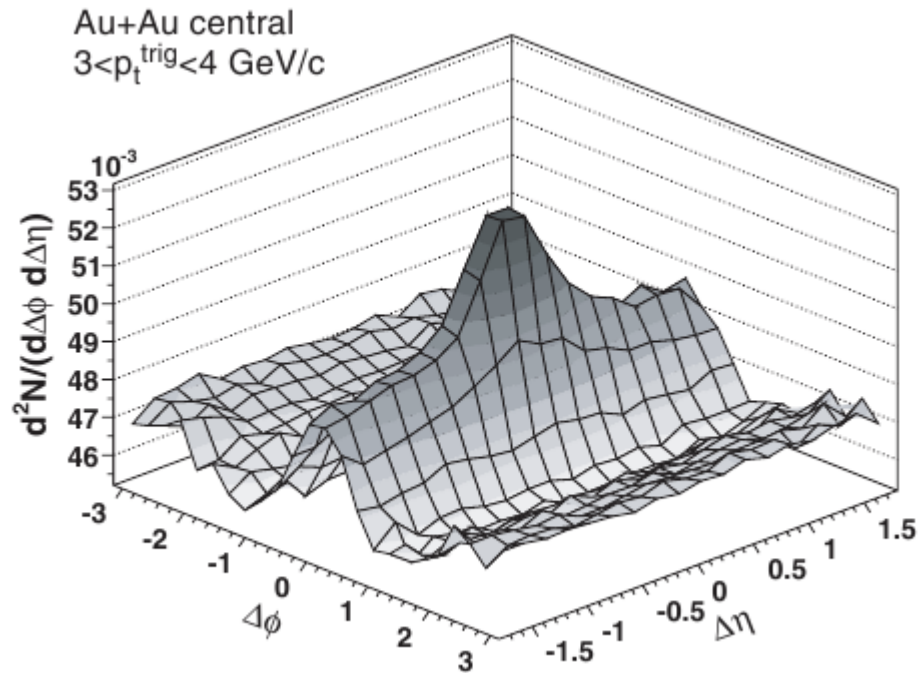
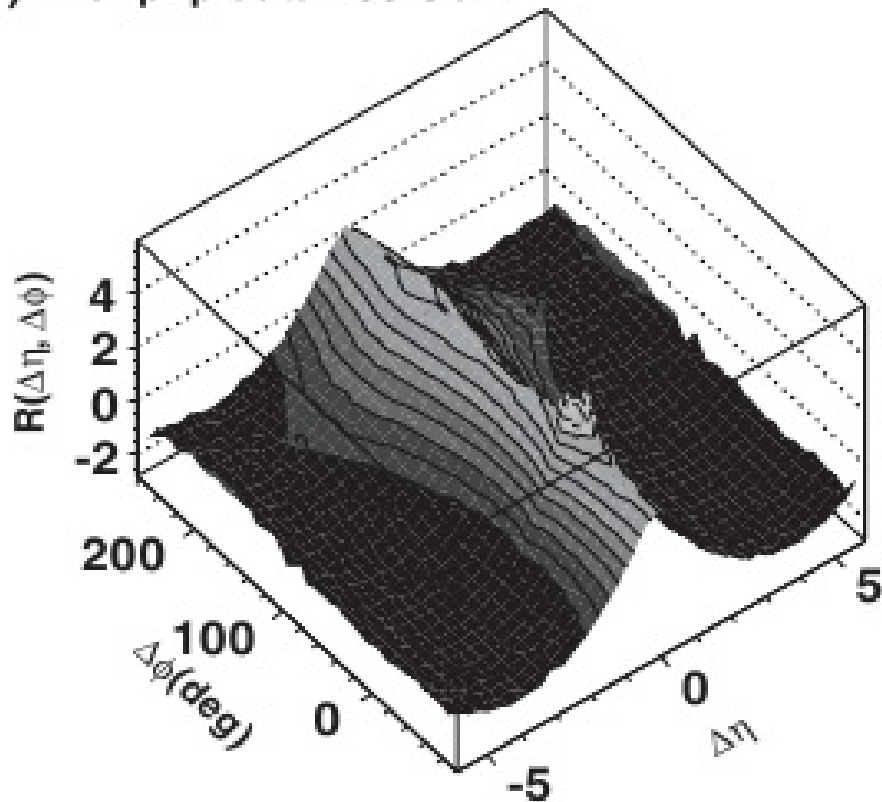


# Ridge @ RHIC

PHYSICAL REVIEW C 75, 054913 (2007)

PHYSICAL REVIEW C 80, 064912 (2009)

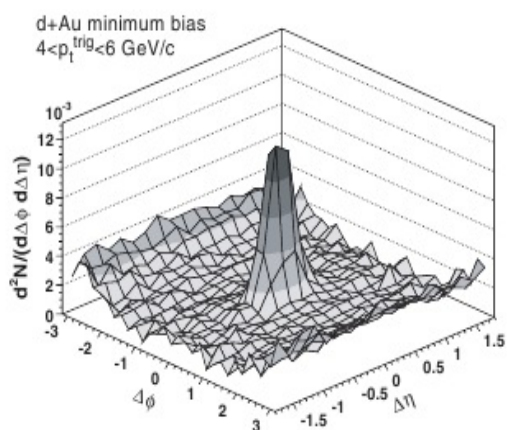
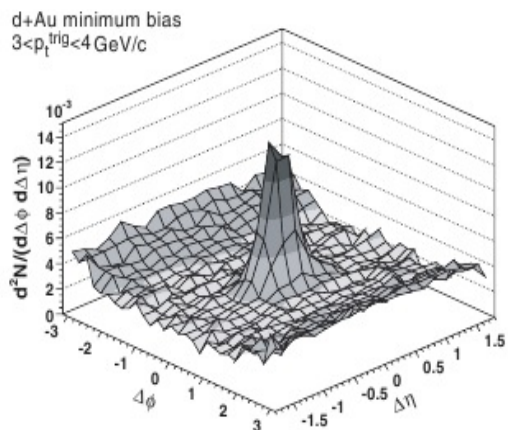
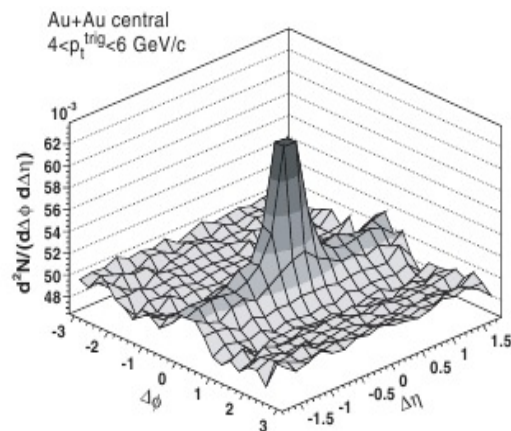
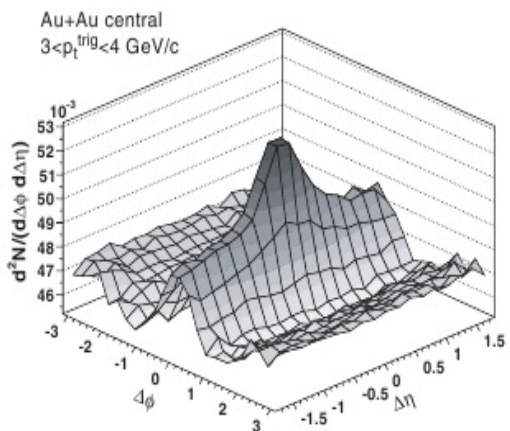
(a) final p+p data 200 GeV



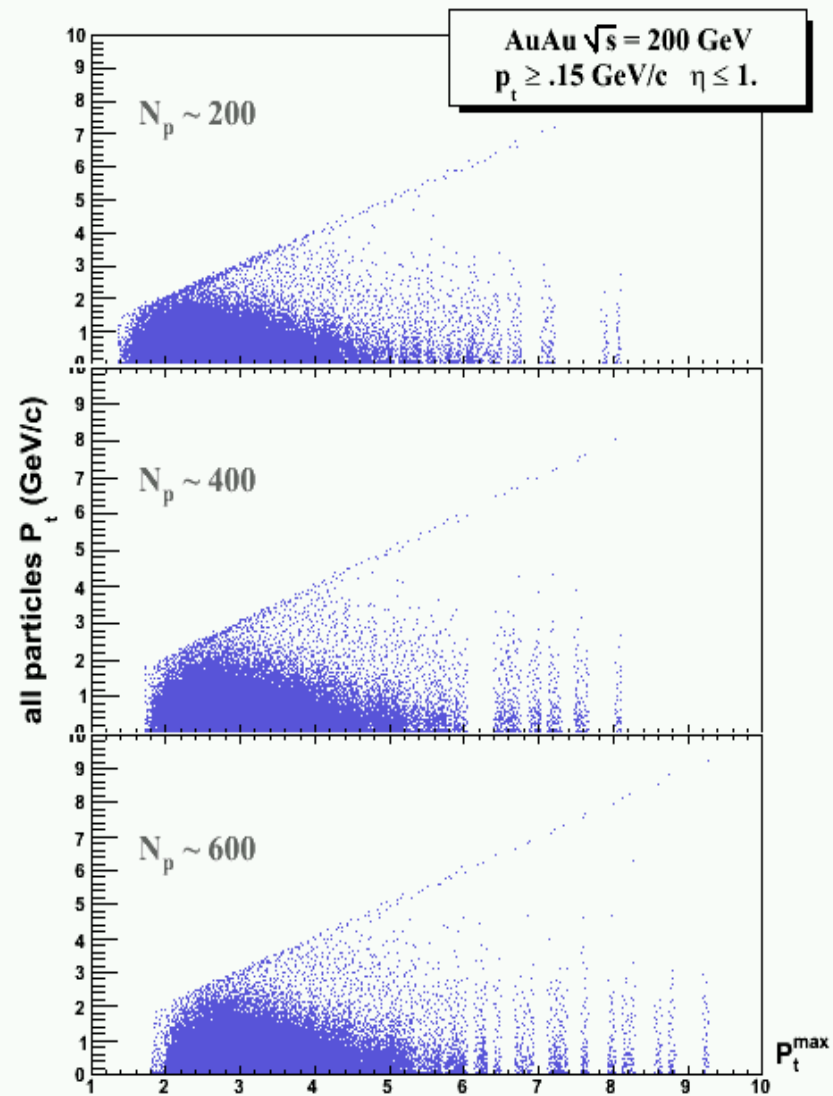
$$2 \text{ GeV}/c < p_T^{\text{assoc}} < p_T^{\text{trig}}$$

They cover an acceptance of  $3 < |\eta| < 4.5$  and  $-180^\circ < \phi < 180^\circ$ . About  $5 \times 10^5$  200-GeV and  $8 \times 10^5$  410-GeV p+p events were selected for further analysis by requiring that the main collision vertex fell within  $|z_{\text{vtx}}| < 10$  cm along the beam axis.

# Ridge dAu vs AuAu



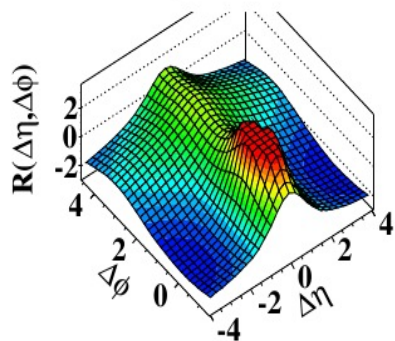
$$2 \text{ GeV/c} < p_T^{\text{assoc}} < p_T^{\text{trig}}$$



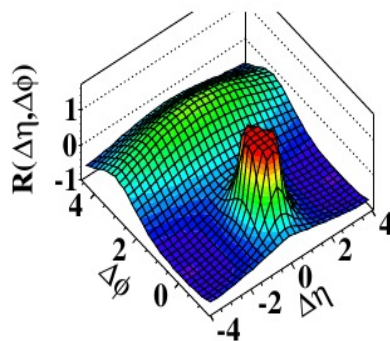
# Ridge @ LHC

## pp @ 7 TeV

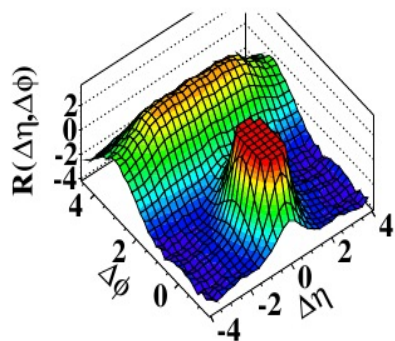
(a) CMS MinBias,  $p_T > 0.1 \text{ GeV}/c$



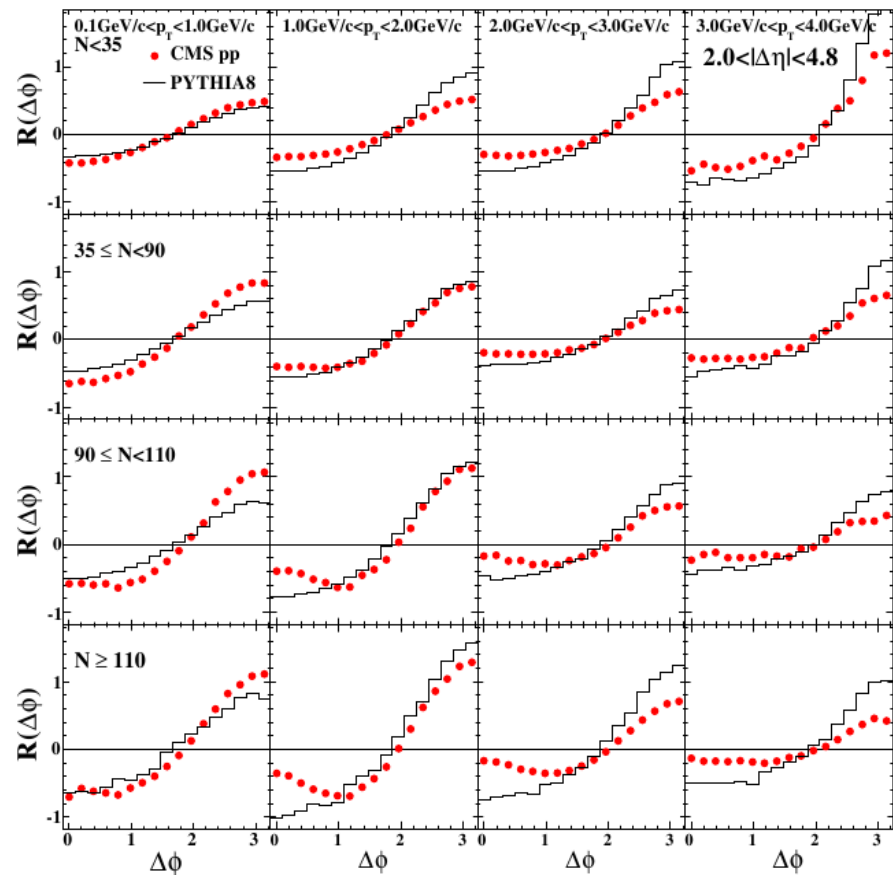
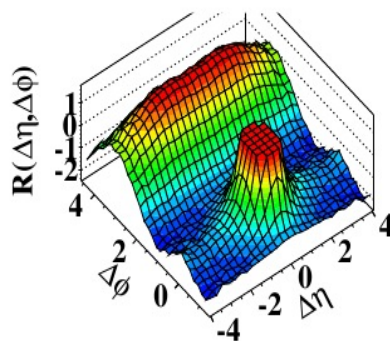
(b) CMS MinBias,  $1.0 \text{ GeV}/c < p_T < 3.0 \text{ GeV}/c$



(c) CMS  $N \geq 110$ ,  $p_T > 0.1 \text{ GeV}/c$



(d) CMS  $N \geq 110$ ,  $1.0 \text{ GeV}/c < p_T < 3.0 \text{ GeV}/c$

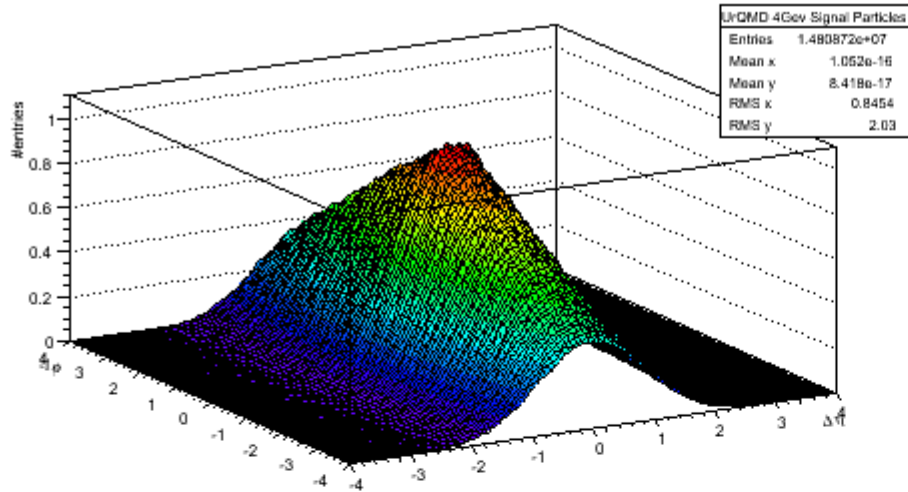


# Summary

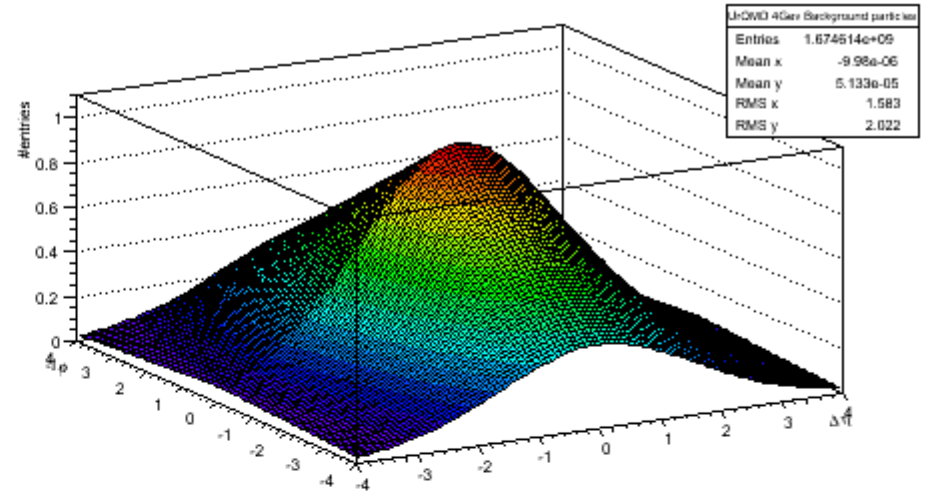
*Some features in multiparticle collisions could be explained by its event structure.*

# UrQMD 4 GeV AuAu

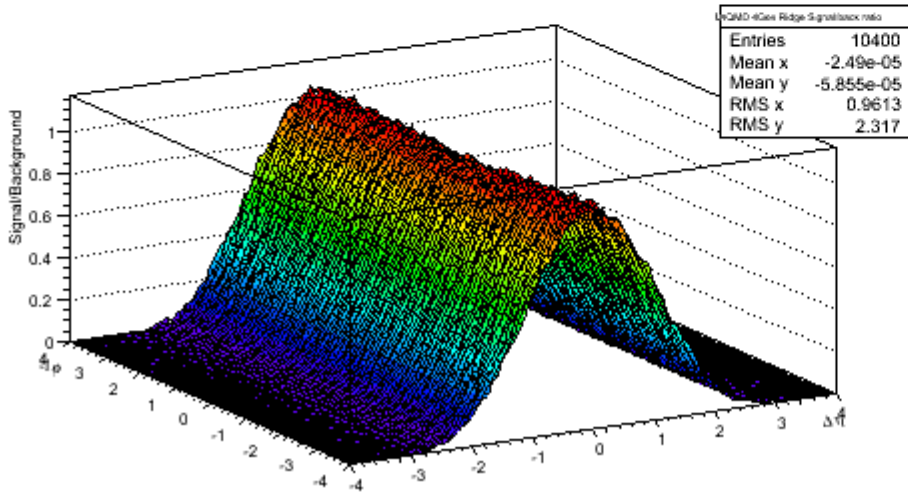
4 GeV 2500 event  $3 > Pt > 1$  Signal particles (CASCADE hydro disabled)



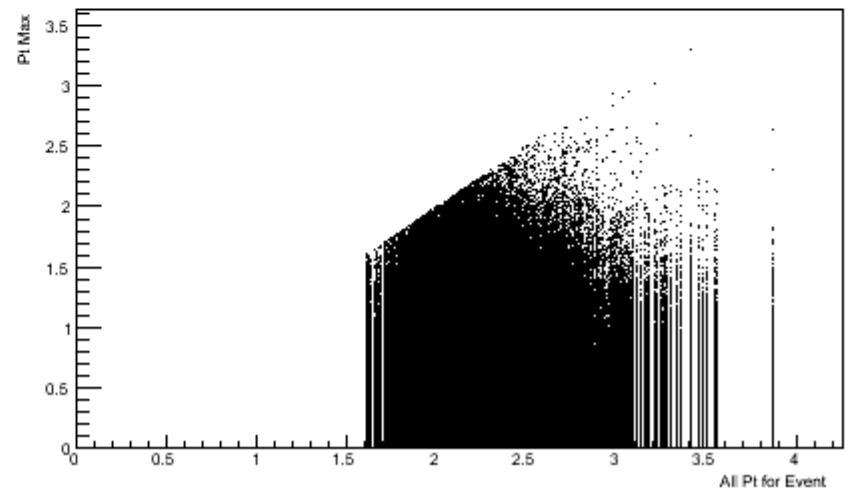
4 GeV 2500 event All Pt Background particles (CASCADE mode hydro disabled)



4 GeV 2500 event  $3 > Pt > 1$  signal/background ratio (CASCADE mode hydro disabled)



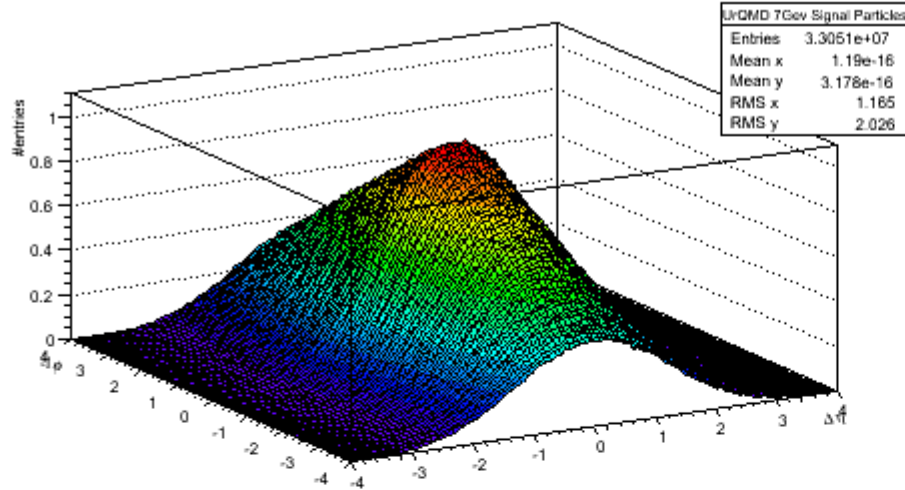
All Pt 4GeV



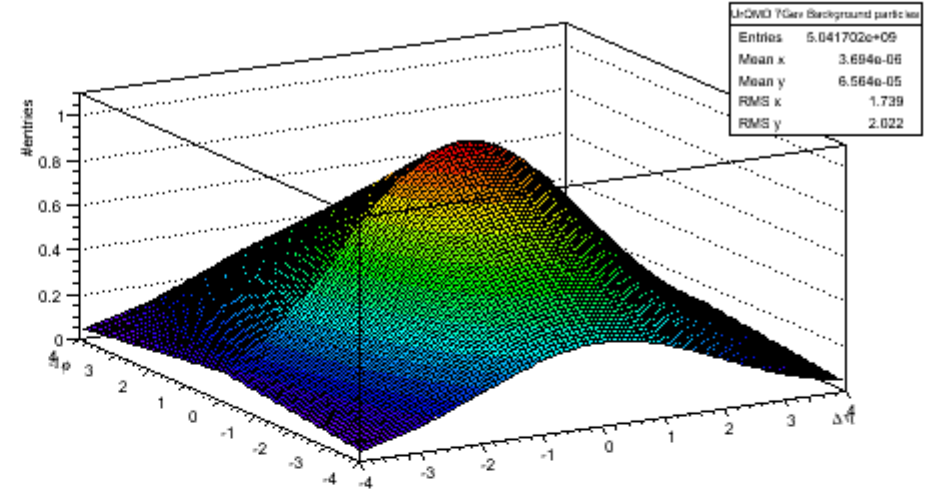


# UrQMD 7 GeV AuAu

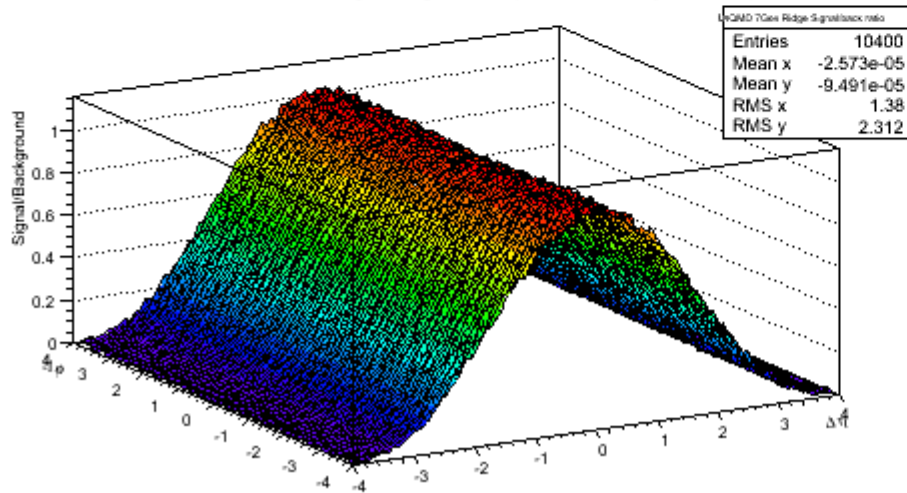
7 GeV 2500 event  $3 > p_T > 1$  Signal particles (CASCADE hydro disabled)



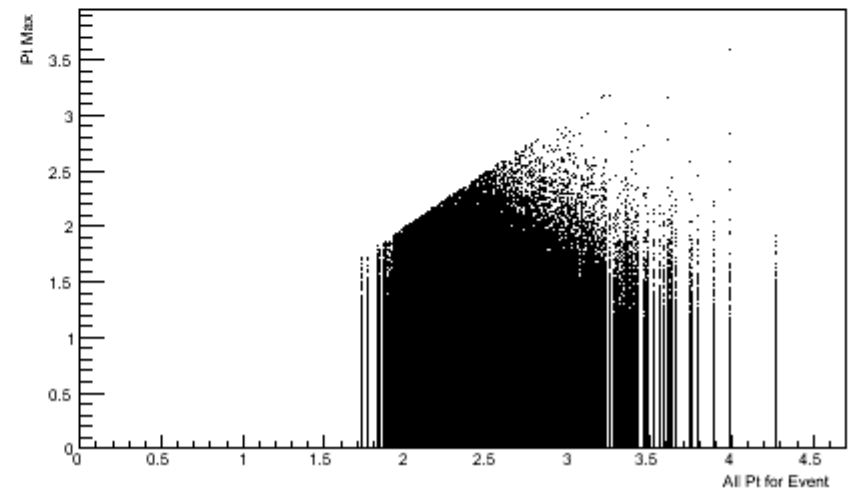
7 GeV 2500 event All  $p_T$  Background particles (CASCADE mode hydro disabled)



7 GeV 2500 event  $3 > p_T > 1$  signal/background ratio (CASCADE mode hydro disabled)

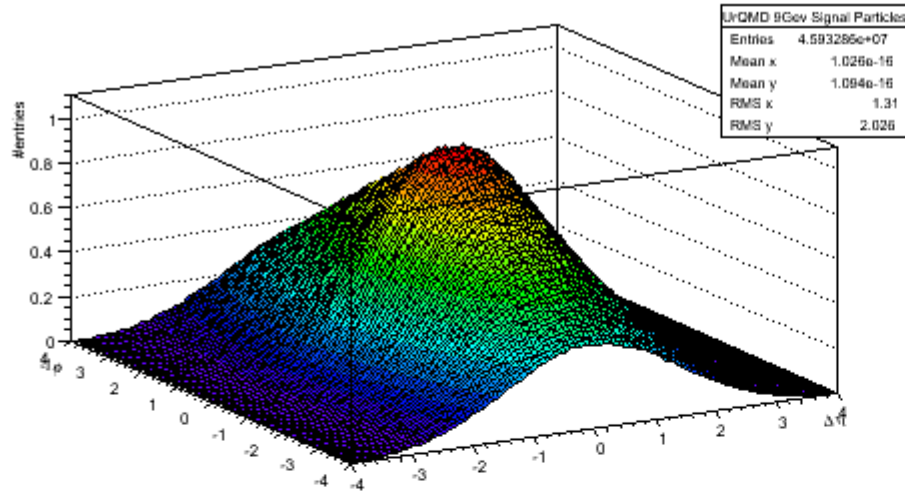


All Pt 7GeV

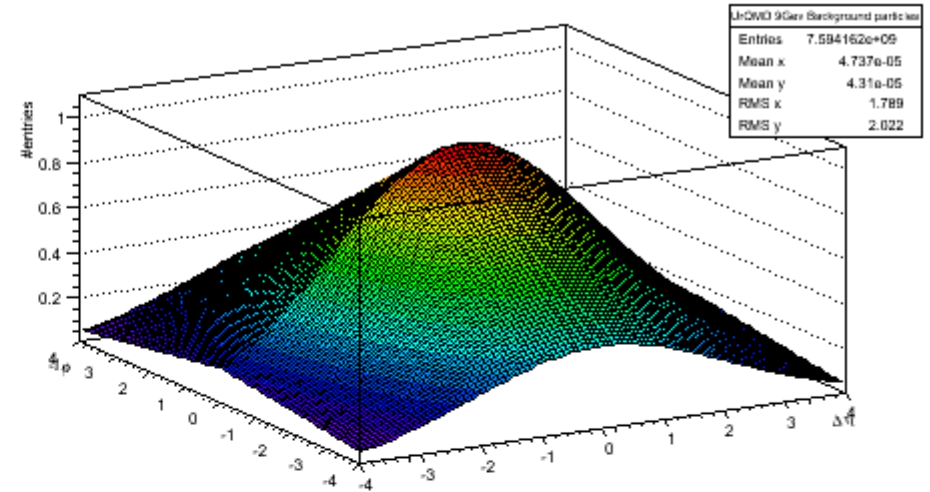


# UrQMD 9 GeV AuAu

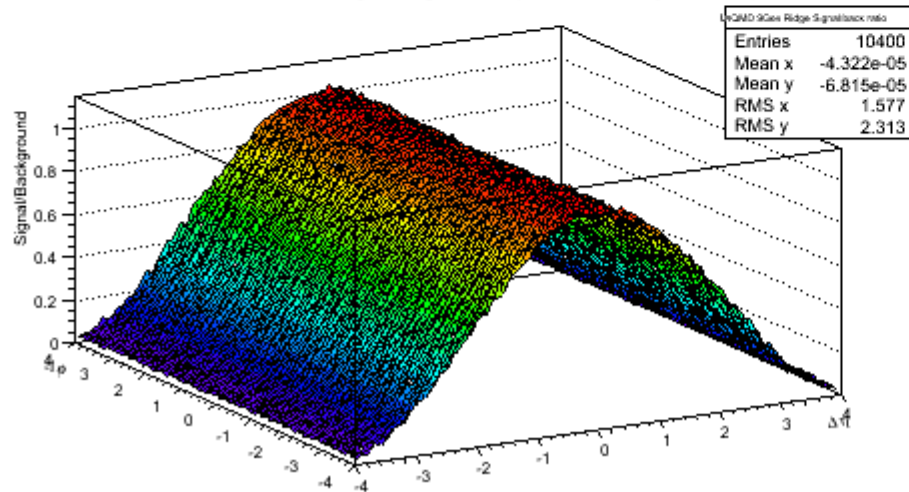
9 GeV 2500 event  $3 > p_T > 1$  Signal particles (CASCADE hydro disabled)



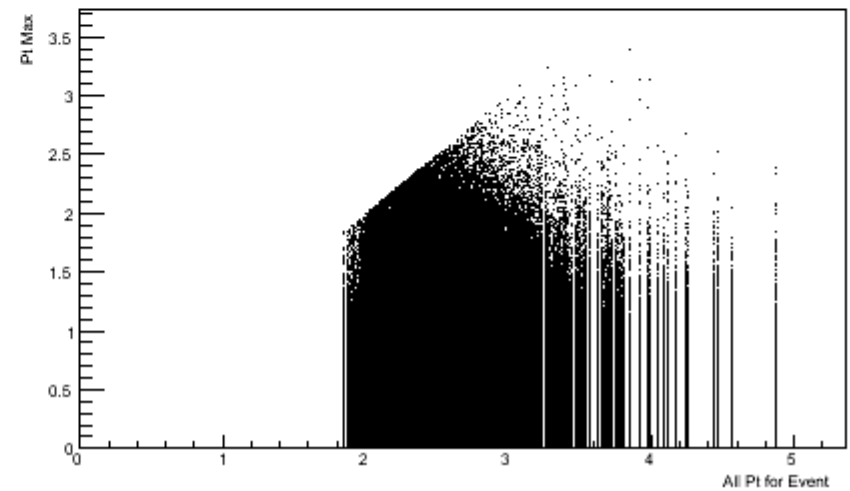
9 GeV 2500 event All  $p_T$  Background particles (CASCADE mode hydro disabled)



9 GeV 2500 event  $3 > p_T > 1$  signal/background ratio (CASCADE mode hydro disabled)

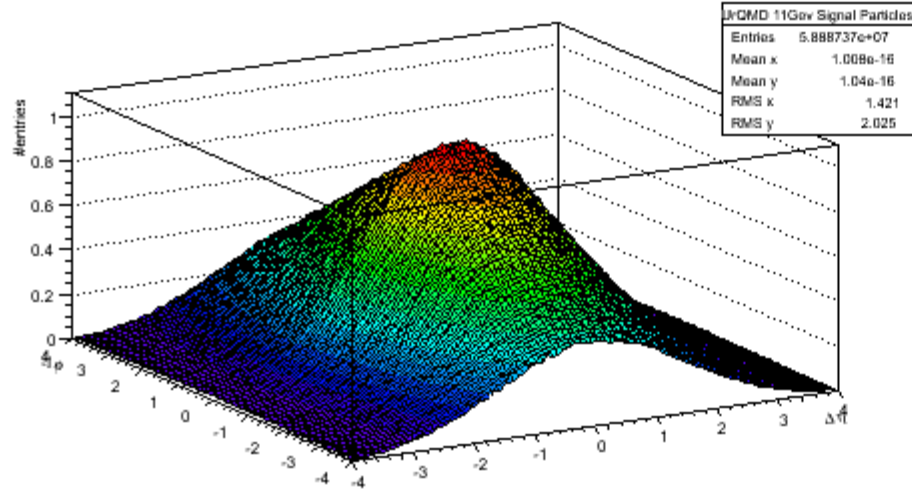


All Pt 9GeV

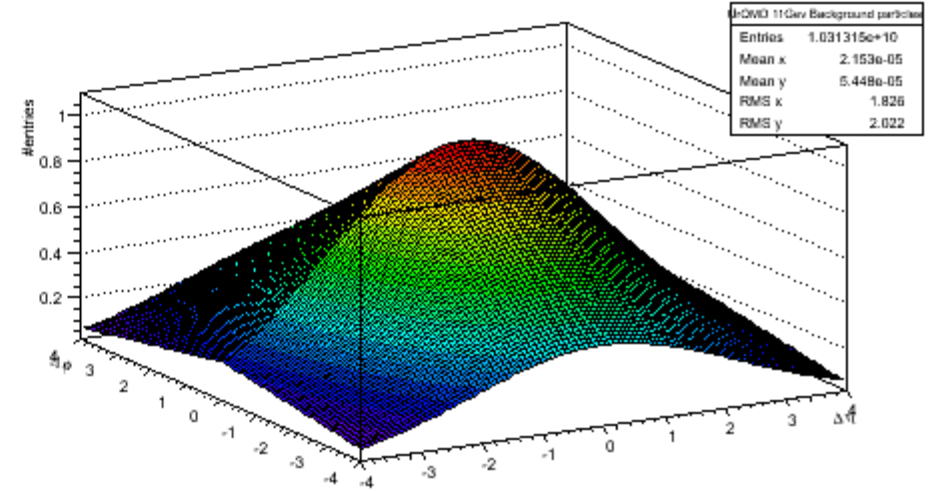


# UrQMD 11 GeV AuAu

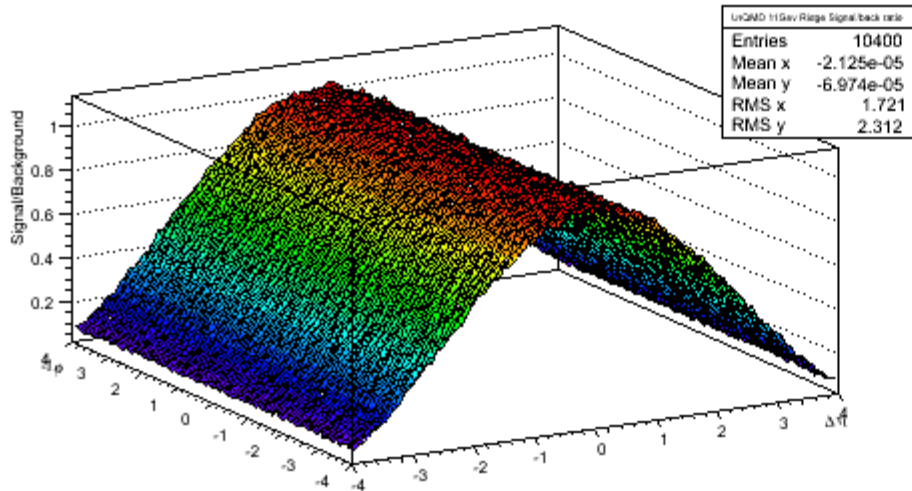
11 GeV 2500 event  $3 > Pt > 1$  Signal particles (CASCADE hydro disabled)



11 GeV 2500 event All Pt Background particles (CASCADE mode hydro disabled)



11 GeV 2500 event  $3 > Pt > 1$  signal/background ratio (CASCADE mode hydro disabled)



All Pt 11GeV

