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Solvable non-conformal holographic models for QCD

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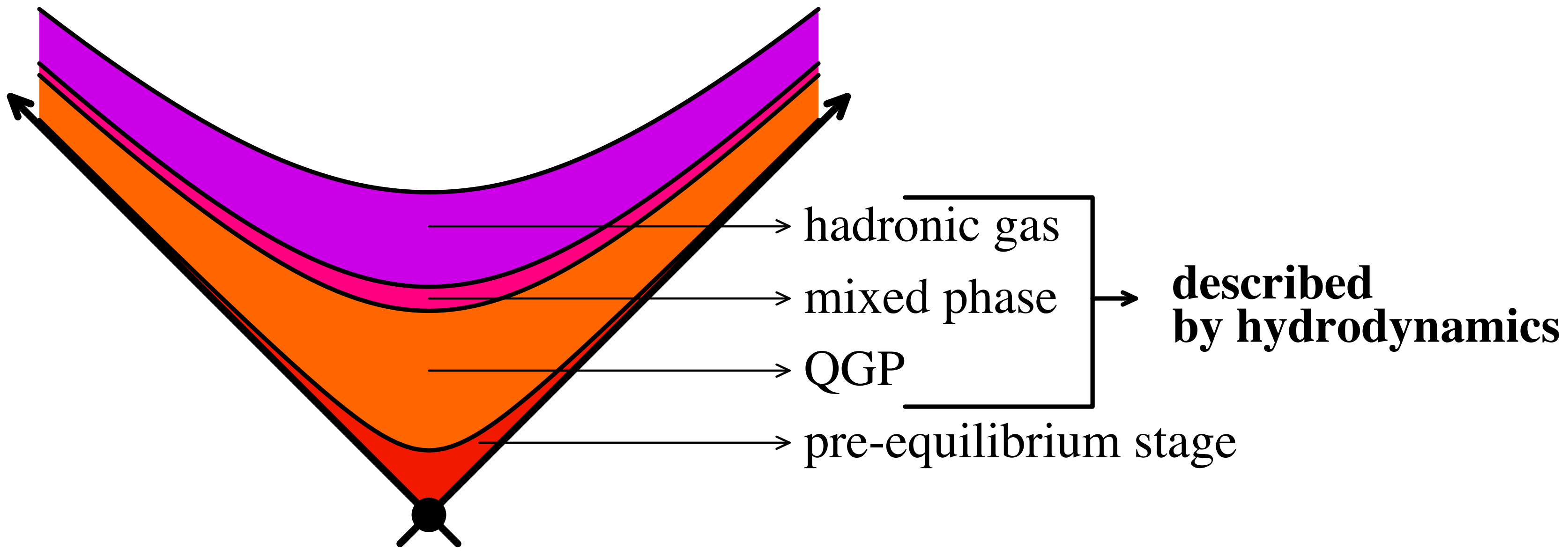
Outline

- Quark-gluon plasma and hydro
- Bjorken flow of CR solutions
- Quasi-normal modes of CR
- Solutions for two-exp potentials
- Conclusions

Conventional picture of QGP dynamics



[Heller, Janik, Peschanski]

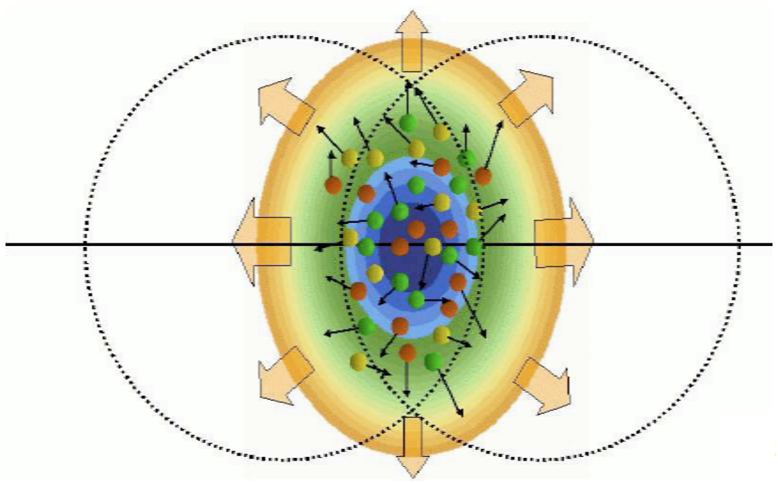
Early stages: Glauber, CGC, problem of inital conditions

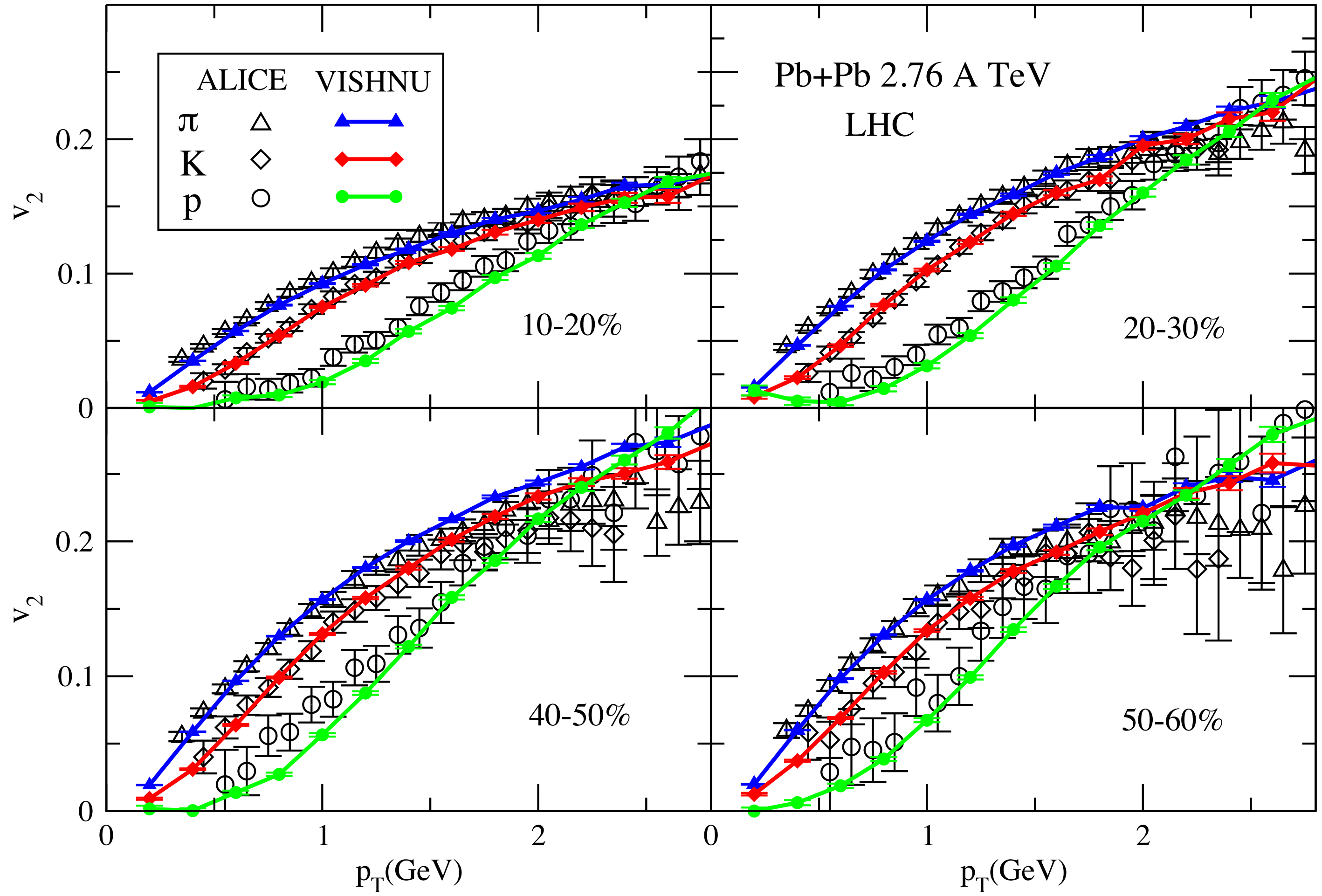
Middle: low-viscosity hydrodynamics

Late: hadronization

Evidence for QGP phase: elliptic flow

$$v_2 = \langle cos(2\phi) \rangle$$





$$\frac{\eta}{s} = \frac{2}{4\pi}$$

[arXiv:1311.0157]

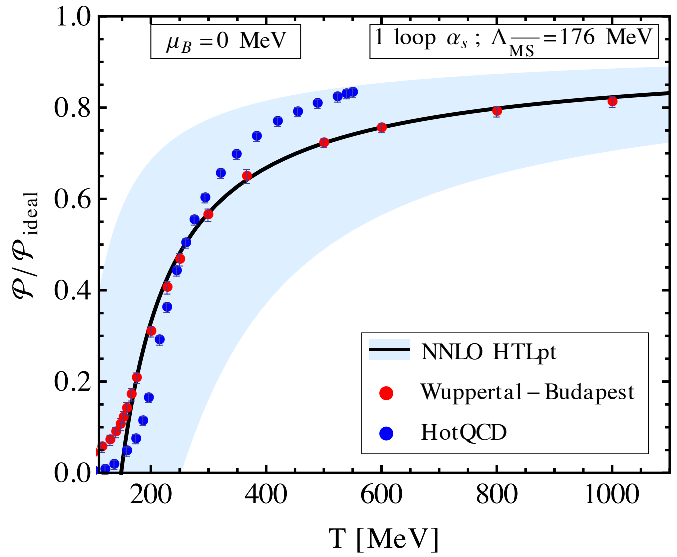
This scenario requires fast thermalization ~ 1 fm

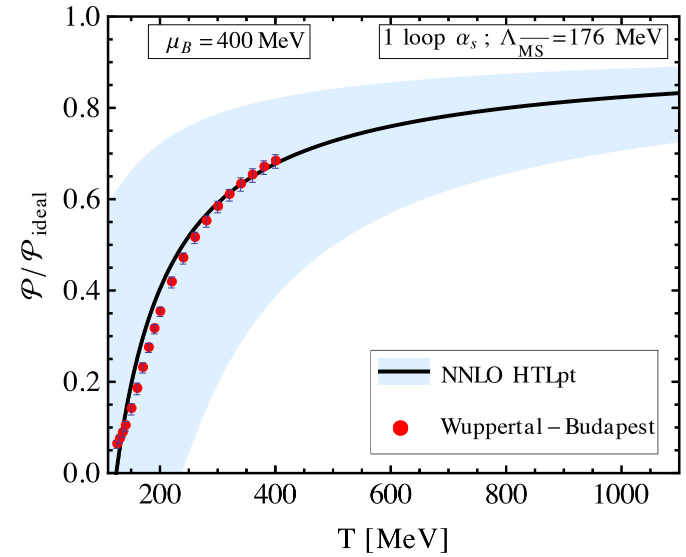
Recent simulations cast some doubt and allow for slower build-up of the flow

Deviation from conformality

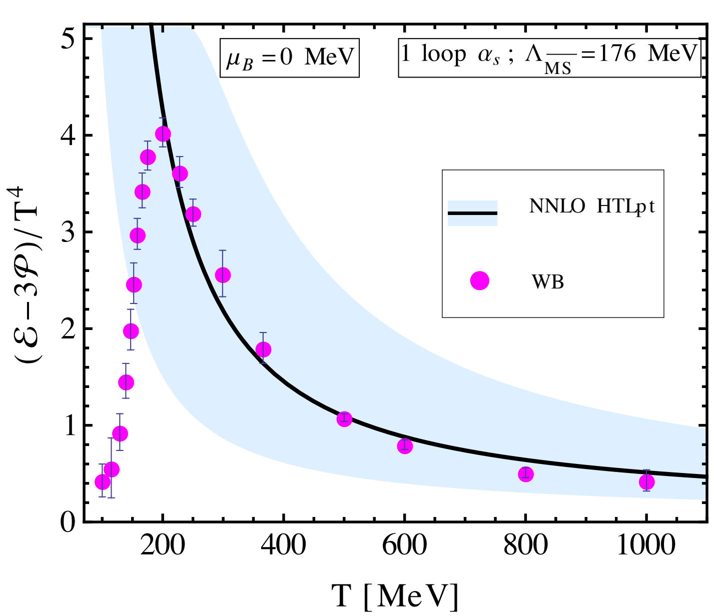
[arXiv:1402.6907]

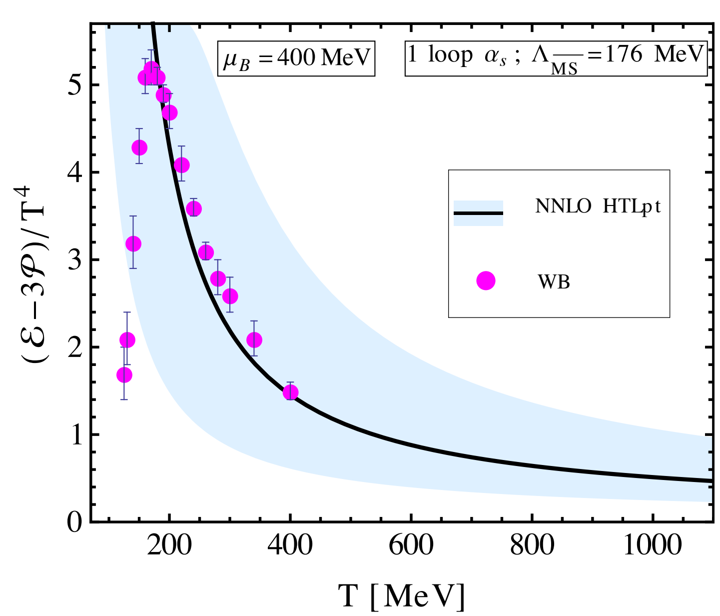
pressure





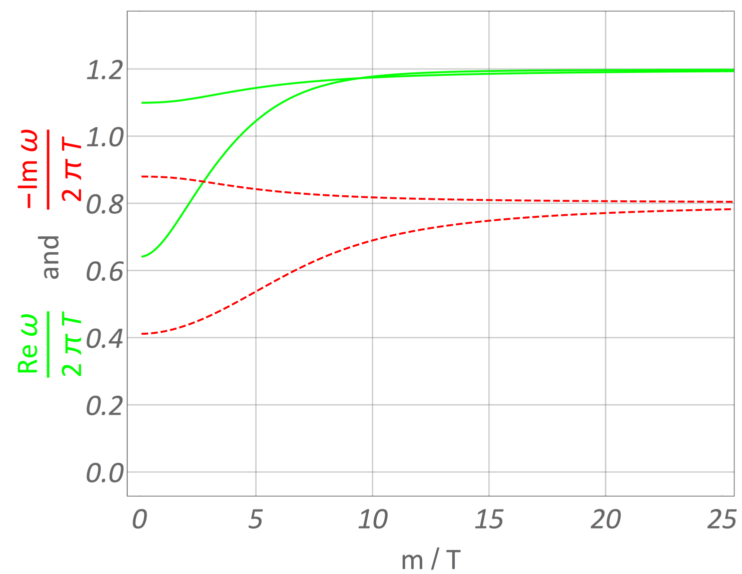
trace anomaly



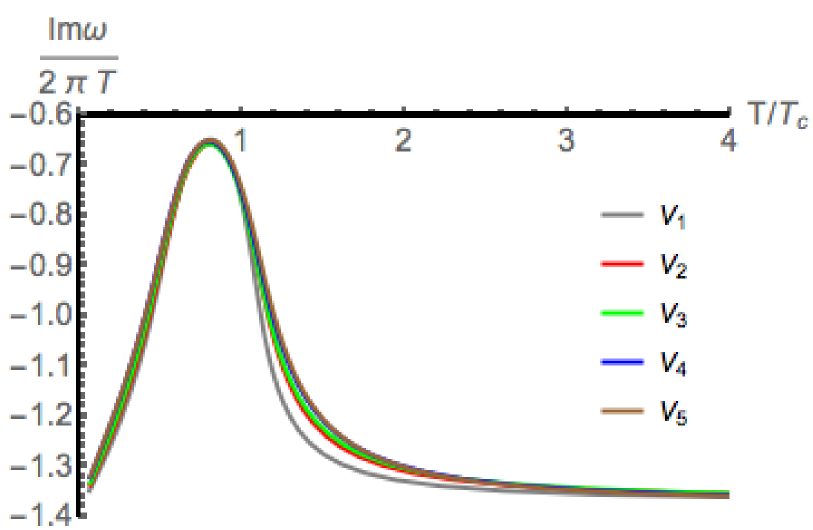


The study of deviations from the conformal behavior in the QGP dynamics has not been thoroughly investigated

[Buchel, Heller, Myers '15][Janik, Plewa, Soltanpanahi, Spalinski] consider the equilibration rate determined by lowest quasi-normal modes in non-conformal theories



N=2*



Einstein-scalar with

$$V(\phi) = cosh(\phi) + \phi^2 + \phi^4 + \phi^6$$

Variation of the imaginary part = attenuation rate by factor of ~ 2

Bottom-up non-conformal models [Gursoy, Kiritsis, Nitti et al]

Einstein-dilaton gravity

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left(R - \frac{4}{3}(\partial\phi)^2 - V(\phi)\right) - \frac{1}{\kappa^2}\int d^4x\sqrt{-h}\,K$$

The potential can be tuned to reproduce the beta-function

For asymptotically AdS UV $\quad V = V_0 + v_1\lambda + v_2\lambda^2 + \ldots$

$$\lambda = e^{\phi} = g_{YM}^2 N_c \qquad \beta = -b_1\lambda^2 - b_2\lambda^3 + \ldots$$

$$b_1 = v_1,\; b_2 = v_2 - v_1^2,\; \ldots$$

For confinement in the IR $\quad V \sim \lambda^Q(\log\lambda)^P$

$$Q > 4/3 \,\text{or}\, Q = 4/3\,,\, P \geq 0$$

Confinement $\Longleftrightarrow$ finite-T transition between thermal gas and BH

We consider a simple setup with an exponential potential

$$V = V_0(1 - X^2)e^{-\frac{8}{3}X\phi}. \qquad X < 0 \quad \text{(confining for} \quad X < -\frac{1}{2}\text{)}$$

For $X > -\frac{1}{2}$ analytic BH solution [Chamblin,Reall '99]

$$ds^2 = e^{2A(u)}\left(-f(u)dt^2 + \delta_{ij}dx^i dx^j\right) + \frac{du^2}{f(u)}$$

$$e^A = e^{A_0}\lambda^{\frac{1}{3X}} \qquad f = 1 - C_2\lambda^{-\frac{4(1-X^2)}{3X}}$$

$$\lambda \equiv e^{\phi} = \left(C_1 - 4X^2\frac{u}{\ell}\right)^{\frac{3}{4X}}$$

Thermodynamics $\quad \beta = \pi\ell\frac{e^{-A_0}C_2^{-\frac{\frac{1}{4}-X^2}{1-X^2}}}{1 - X^2}$

for $X < -\frac{1}{2}$
negative specific heat

$$-T^{\mu}_{\mu} = E + 3F = 3c_s\frac{X^2}{1 - X^2}\left(T\ell\right)^{\frac{4(1-X^2)}{1-4X^2}} \qquad p = \frac{1 - 4X^2}{3}\epsilon$$

Boost-invariant CR flow

Trace condition $$-T_{\tau\tau} + \frac{1}{\tau^2}T_{yy} + 2T_{xx} = -c\,T^{\xi} \qquad \xi = \frac{4(1-X^2)}{1-4X^2}$$

$$T_{\mu\nu} = \mathrm{d}iag\left(\epsilon(\tau),\ -\tau^3\partial_\tau\epsilon - \tau^2\epsilon,\ \epsilon + \frac{\tau}{2}\partial_\tau\epsilon - \frac{c}{2}T^{\xi}, \epsilon + \frac{\tau}{2}\partial_\tau\epsilon - \frac{c}{2}T^{\xi}\right)$$

Assuming $T = T_0\tau^{-\alpha}$ the energy is determined

$$\epsilon(\tau) = \epsilon_0\tau^{-\frac{4}{3}} + \frac{c\,T_0^{\xi}}{4-3\alpha\xi}\tau^{-\alpha\xi}$$

If $\alpha\xi < \dfrac{4}{3}$ the trace anomaly determines the late time behaviour

Ansatz for metric and dilaton

$$ds^2 = z^{-\frac{2}{1-4X^2}} \left(dz^2 - e^{a(v)}d\tau^2 + e^{b(v)}\tau^2 dy^2 + e^{c(v)}dx_\perp^2\right)$$

$$\lambda = z^{-\frac{3X}{1-4X^2}} e^{\lambda_1(v)} \qquad\qquad v = \frac{z}{\tau^{s/4}}$$

Complicated system of equations for late time…

Basis of solutions

$$\begin{aligned}
a(v) &= A(v) - 2\left(1-4X^2\right)m(v) + 2Xn(v)\\
b(v) &= A(v) + 2\left(s-1+4X^2\right)m(v) + 2Xn(v)\\
c(v) &= A(v) - \left(s-2+8X^2\right)m(v) - 2Xn(v)\\
\lambda_1(v) &= \frac{3}{2}XA(v) + X\left(1-4X^2\right)m(v) + \left(1-X^2\right)n(v)
\end{aligned}$$

The equations decouple

$$A(w) = \frac{2}{\chi}w - \frac{1}{2}\log m'(w) + \text{const.}\,, \qquad n(w) = \kappa\, m(w) + \text{const.}$$

$$w = \log v\,, \qquad \chi = \frac{1-4X^2}{1-X^2}$$

The dual stress-energy tensor can be obtained by holographic renormalization in 5d, or more easily lifting the solution by a generalized dimensional reduction

$$S = \frac{1}{16\pi \tilde{G}_N} \int d^{d+1}x \, d^{2\sigma - d}y \, \sqrt{-\tilde{g}} \left(\tilde{R} - 2\Lambda \right)$$

Reducing on $\mathbb{R}^{d+1} \times T^{2\sigma - d}$ $\qquad \widetilde{ds}^2 = e^{-\delta_1 \phi(x)} dx^2 + e^{\delta_2 \phi(x)} dy^2$

$$\delta_1 = \frac{4\sqrt{2\sigma - d}}{\sqrt{3(d-1)(2\sigma - 1)}}, \quad \delta_2 = \frac{4\sqrt{d-1}}{\sqrt{3(2\sigma - 1)(2\sigma - d)}} \qquad 2\sigma - d = \frac{4(d-1)^2 X^2}{3 - 4(d-1)X^2}$$

The uplifted metric is AAdS $\qquad \langle T^{\mu\nu} \rangle_{2\sigma} = \frac{2\sigma l^{2\sigma - 1}}{16\pi \tilde{G}_N} \tilde{g}^{\mu\nu}_{(2\sigma)}$

$T_{\mu\nu}$ consistent with perfect fluid $\qquad \boxed{\epsilon(\tau) \sim \tau^{-\frac{4}{3}(1 - 4X^2)}}$

leading w.r.t. the conformal form

Estimate of thermalization time

Viscous e.m. tensor

$$T_{\mu\nu} = \begin{pmatrix} \epsilon(\tau) & & & \\ & p(\tau) - \frac{4}{3}\frac{\eta}{\tau} & & \\ & & p(\tau) + \frac{2}{3}\frac{\eta}{\tau} & \\ & & & p(\tau) + \frac{2}{3}\frac{\eta}{\tau} \end{pmatrix}$$

CGC e.m. tensor

$$T^{CGC}_{\mu\nu} = \begin{pmatrix} \epsilon(\tau) & & & \\ & 0 & & \\ & & p(\tau) & \\ & & & p(\tau) \end{pmatrix} \qquad \epsilon^{CGC}(\tau) \sim \frac{A}{\tau}$$

matching at time τ_0

$$p(\tau_0) = \frac{4}{3}\frac{\eta}{\tau_0} \propto T(\tau_0)^{\xi}$$

using $\eta \sim T^{\xi-1}$ $\qquad T \sim \tau^{-\frac{1}{3}(1-4X^2)}$

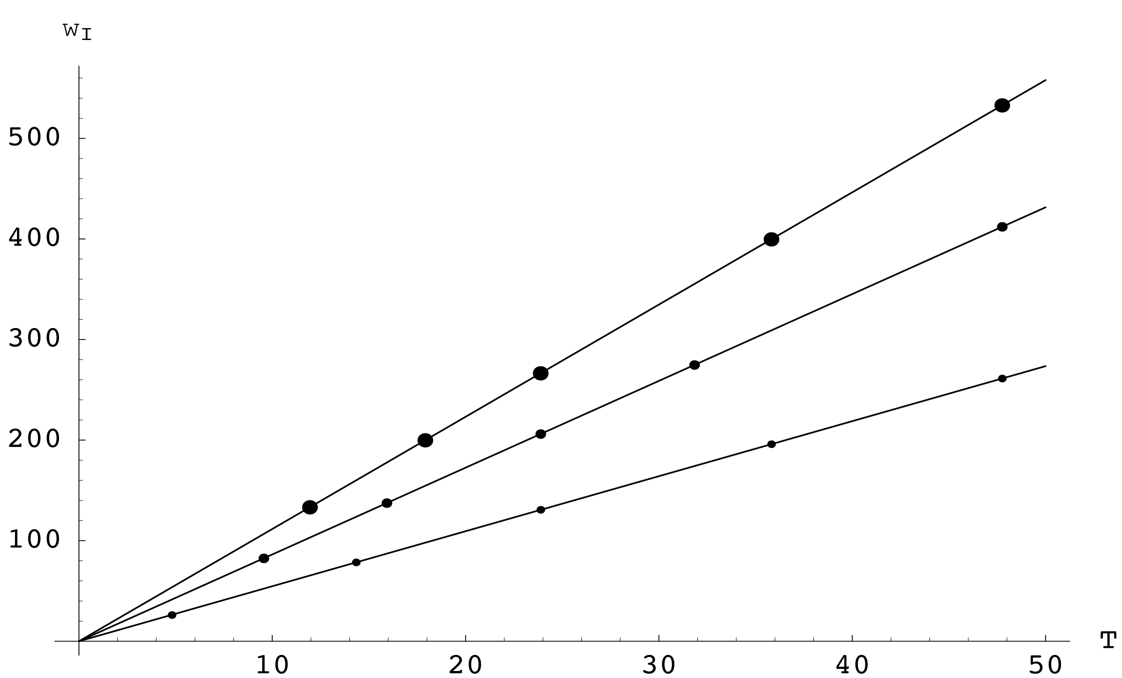
$$\tau_0 \sim \left(\frac{\eta}{T^{\xi-1}}\right)^{\frac{2}{3(1-2X^2)}}$$

General form of finite-T correlators

$$G^R(k,\omega) = \frac{A_n(k)}{\omega - \omega_n(k) + i\Gamma_n(k)}$$

The lowest-lying modes encode the thermalization rate

In gravity they correspond to Quasi-Normal Modes (black hole ringdown)



[Horowitz, Hubeny 99]

$\omega_I = 11.16\ T \qquad$ for $d = 4$

$\omega_I = 8.63\ T \qquad$ for $d = 5$

$\omega_I = 5.47\ T \qquad$ for $d = 7$

$$t_{th} \sim 0.5\mathrm{fm}$$

Fluctuations around the CR solution

$$\delta g = e^{2A_0}\hat{r}^{-\frac{2}{1-4X^2}}\left[-H_{vv}dv^2 + 2H_{vi}dv dx^i + H_{ij}dx^i dx^j\right]$$
$$\delta\lambda = \hat{r}^{-\frac{3X}{1-4X^2}}\psi \ .$$

Spin-2 modes $\quad H_{23}\,, \quad \dfrac{H_{22}-H_{33}}{2}$

Spin-0 mode $\quad \dfrac{H_{22}+H_{33}}{2} - \dfrac{2}{3X}\psi \qquad$ are decoupled and degenerate

$$rf(r)\zeta''(r) + (2ir\omega + f(r) - \xi)\zeta'(r) - (k^2 r + (\xi - 1)i\omega)\zeta(r) = 0$$

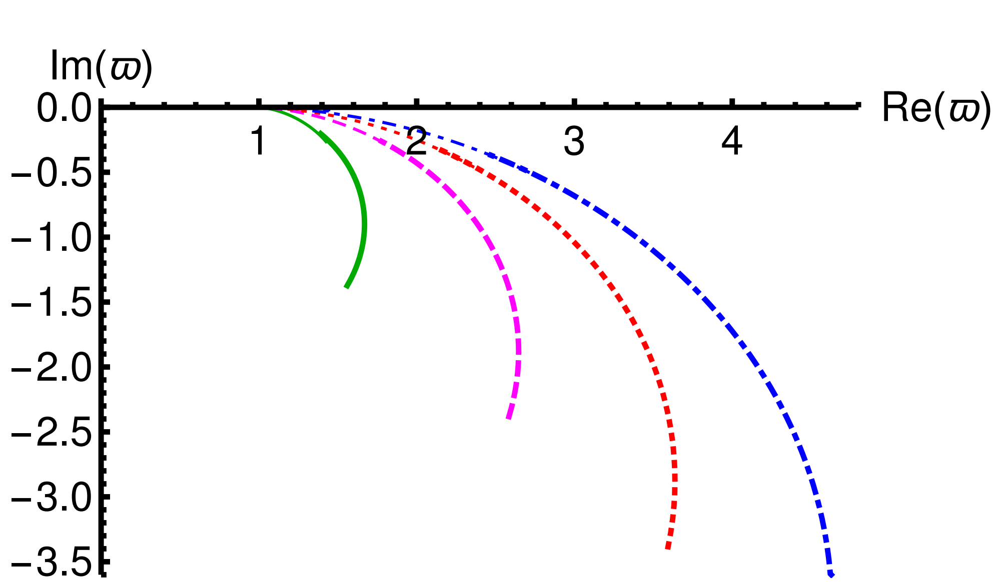
It can be solved analytically at large ξ or in the UV expansion

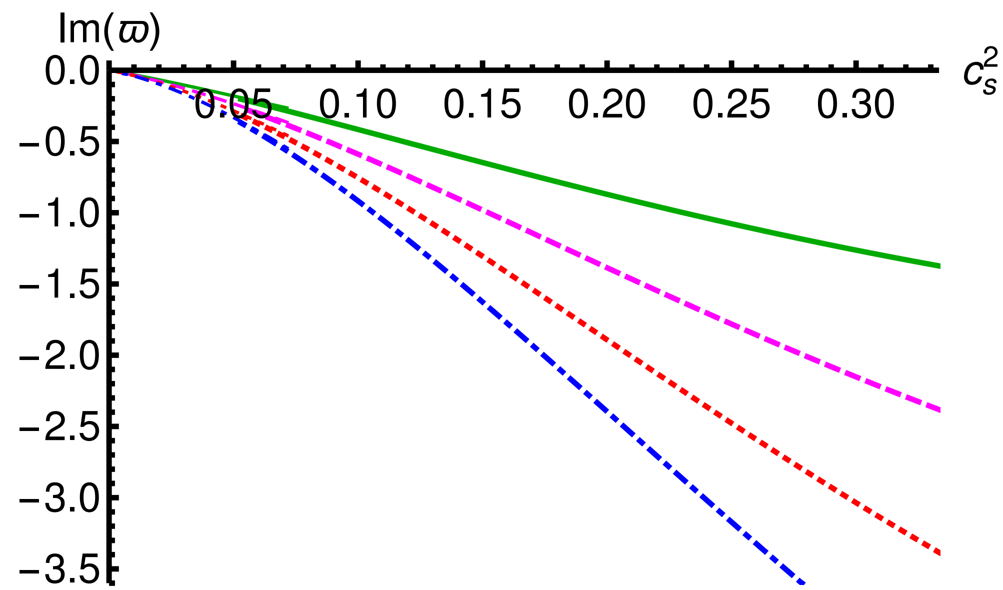
Matching the solutions gives an analytic form of the correlator

$$
\begin{aligned}
G(k,\omega) &= \frac{2\pi\,\xi^{\xi}\hat{r}_h^{-\xi}}{\Gamma\left(\frac{\xi}{2}\right)\Gamma\left(1+\frac{\xi}{2}\right)}\left(\frac{(\varpi^2-q^2)}{16}\right)^{\frac{\xi}{2}} \\
&\times\left[i-\left(\frac{1+i\widetilde{S}}{1-i\widetilde{S}}\right)^{\frac{\xi}{2}} e^{-i\xi\widetilde{S}}\;\frac{\Gamma\left(1-i\widetilde{S}\right)}{\Gamma\left(1+i\widetilde{S}\right)}\frac{\Gamma\left(\frac{1}{2}\left(1-i\varpi+i\widetilde{S}\right)\right)^2}{\Gamma\left(\frac{1}{2}\left(1-i\varpi-i\widetilde{S}\right)\right)^2}\right]^{-1}
\end{aligned}
$$

$$
q=\frac{k}{2\pi T} \qquad \varpi=\frac{\omega}{2\pi T} \qquad \widetilde{S}=\sqrt{\varpi^2-q^2-1}
$$

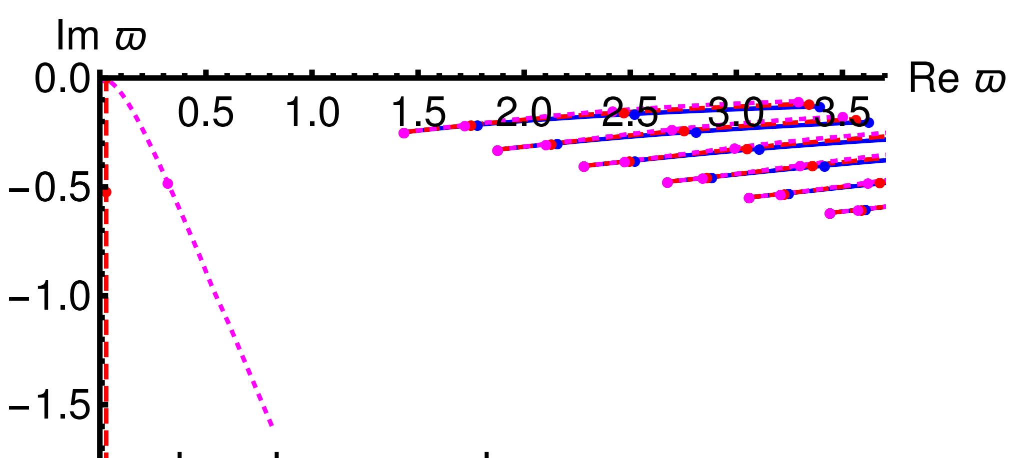
Dependence of QNM on X at q=0 $\quad -0.46 < X < 0$

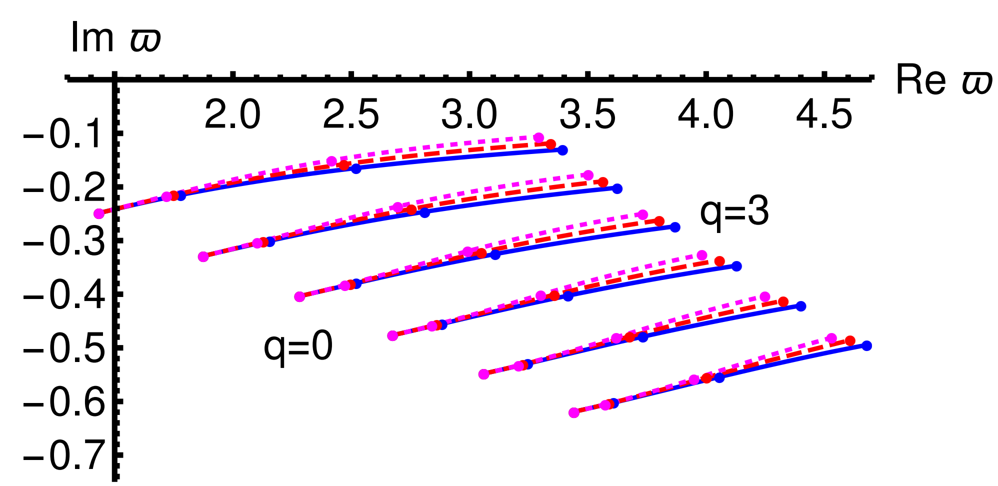




$$\mathrm{Im}\,\omega \sim \xi^{-1}$$

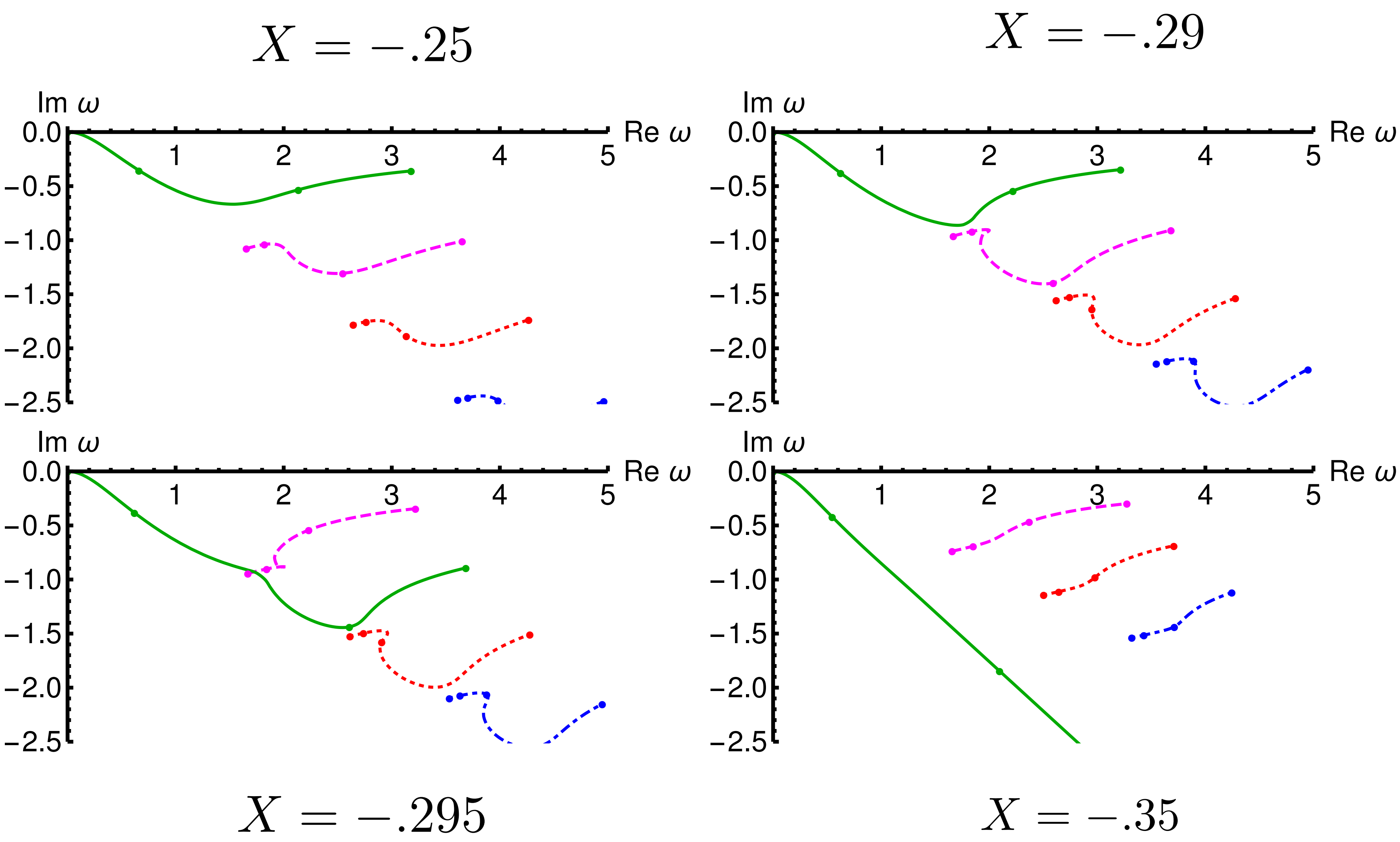
Dependence of QNM on q at X=-0.45





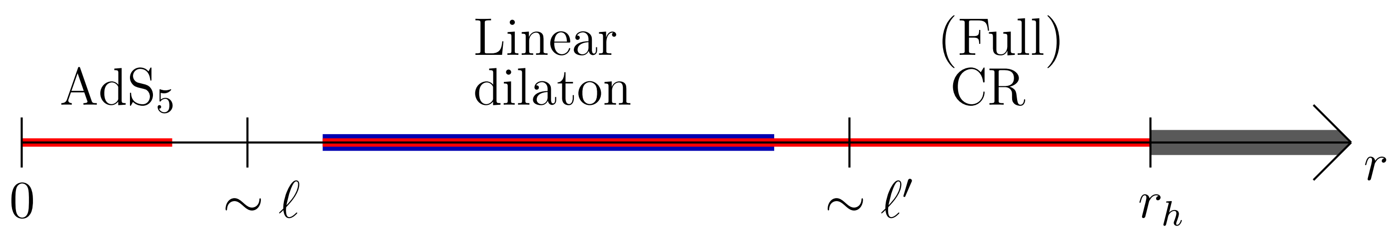
Crossover of hydro and non-hydro modes at $\quad q^* \sim \xi^{-1/2}$

Dependence of QNM on q in the sound channel



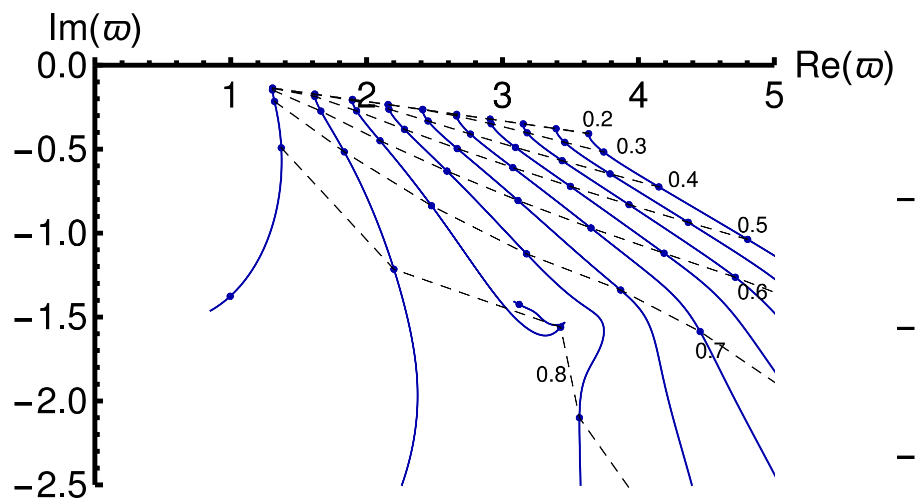
The Chamblin-Reall solution has bad UV behavior, not AAdS (it is hyperscaling-violating)

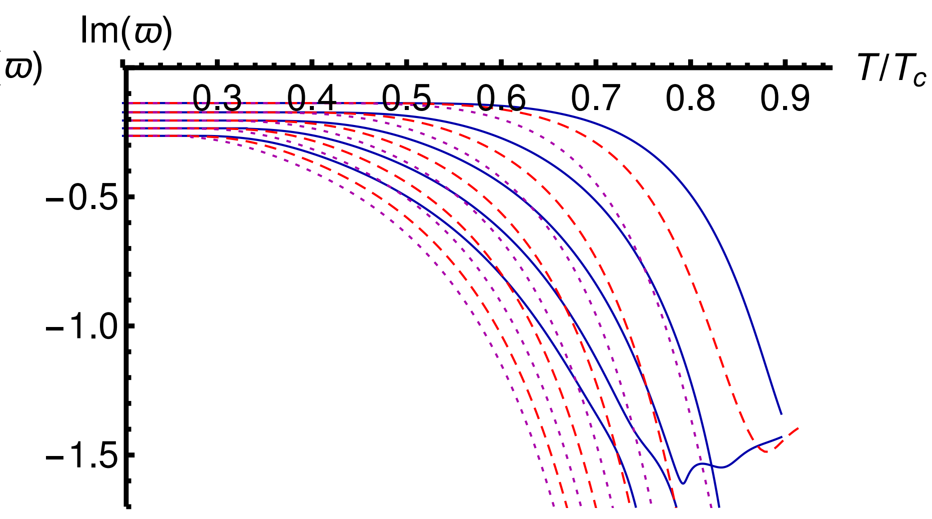
A simple regularization: attach a slice of AdS in the UV



$$\ell' = \xi \ell$$

The QNM depend non-trivially on T





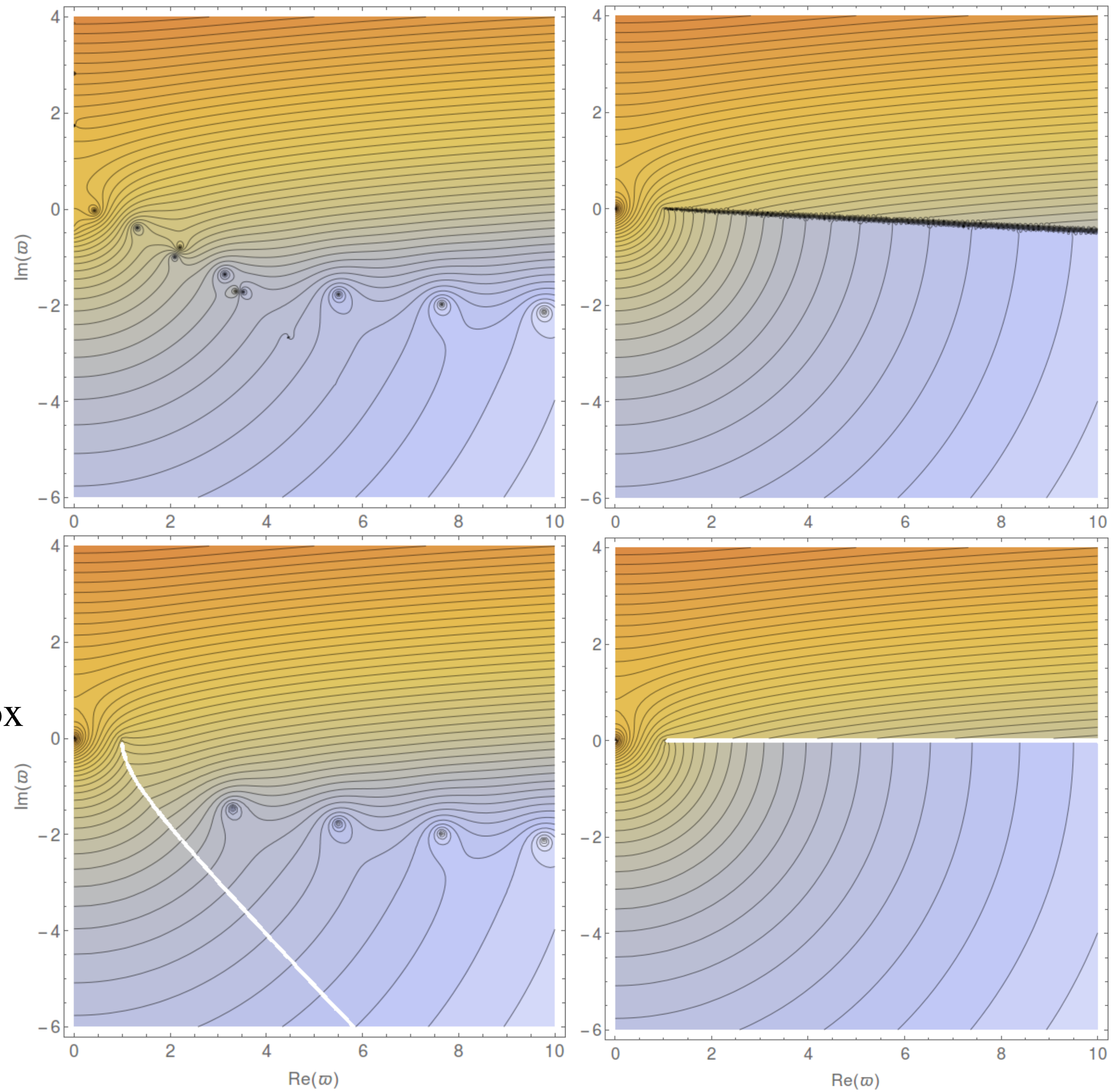
QNM of the X=-1/2 UV-completed CR geometry

$\frac{r_h}{r_c} = 2$

high-T

$\frac{r_h}{r_c} = 20$

low-T



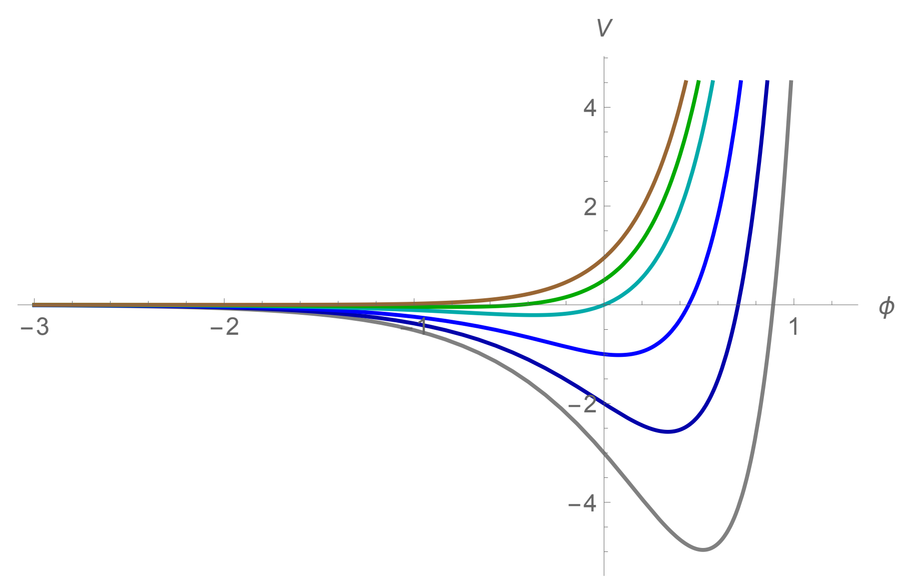
large ω approx

A better model: interpolate between CR and AdS with a smooth potential

A simple Ansatz $$V = C_1\, e^{2k_1\phi} + C_2\, e^{2k_2\phi}$$

gives a model that 1) allows for non-trivial flows between fixed points

2) is solvable



A change of variables turns the Einstein equations into an integrable (Toda lattice) system if $$k_2 = \frac{16}{9k_1}$$

$$
\begin{aligned}
ds^2 &= F_1^{\frac{8}{9k^2-16}} F_2^{\frac{9k^2}{2(16-9k^2)}} \left(-e^{2\alpha^1 u} dt^2 + e^{-\frac{2}{3}\alpha^1 u} d\vec{y}^{\,2}\right) + F_1^{\frac{32}{9k^2-16}} F_2^{\frac{18k^2}{16-9k^2}} du^2 \\
\phi &= -\frac{9k}{9k^2-16}\ln F_1 + \frac{9k}{9k^2-16}\ln F_2
\end{aligned}
$$

$$
F_s(u-u_{0s}) = \begin{cases}
\sqrt{\frac{|C_s|}{2|E_s|}}\sinh\left[\mu_s(u-u_{0s})\right], & \text{if} \quad \eta_{ss}C_s>0, \eta_{ss}E_s>0, \\
\sqrt{\frac{|C_s|}{2|E_s|}}\sin\left[\mu_s(u-u_{0s})\right], & \text{if} \quad \eta_{ss}C_s>0, \eta_{ss}E_s<0, \\
\sqrt{\frac{C_s}{2}}|\mu_s(u-u_{0s})|, & \text{if} \quad \eta_{ss}C_s>0, E_s=0, \\
\sqrt{\frac{|C_s|}{2|E_s|}}\cosh\left[\mu_s(u-u_{0s})\right], & \text{if} \quad \eta_{ss}C_s<0, \eta_{ss}E_s>0,
\end{cases}
$$

$$
s=1,2, \quad \mu_1 = \sqrt{|\frac{3E_1}{2}(k^2-\frac{16}{9})|}, \quad \mu_2 = \sqrt{\left|\frac{3E_2}{2}\left(\left(\frac{16}{9}\right)^2\frac{1}{k^2}-\frac{16}{9}\right)\right|}.
$$

$$
E_1 + E_2 + \frac{2}{3}\alpha_1^2 = 0 \qquad\qquad u_{01}, u_{02}
$$

$\alpha_1 = 0$ Poincaré invariant vacuum solutions

$\alpha_1 \neq 0$ finite-temperature solutions

Regularity of the horizon fixes $E_1, E_2(\alpha_1)$

Vacuum flows

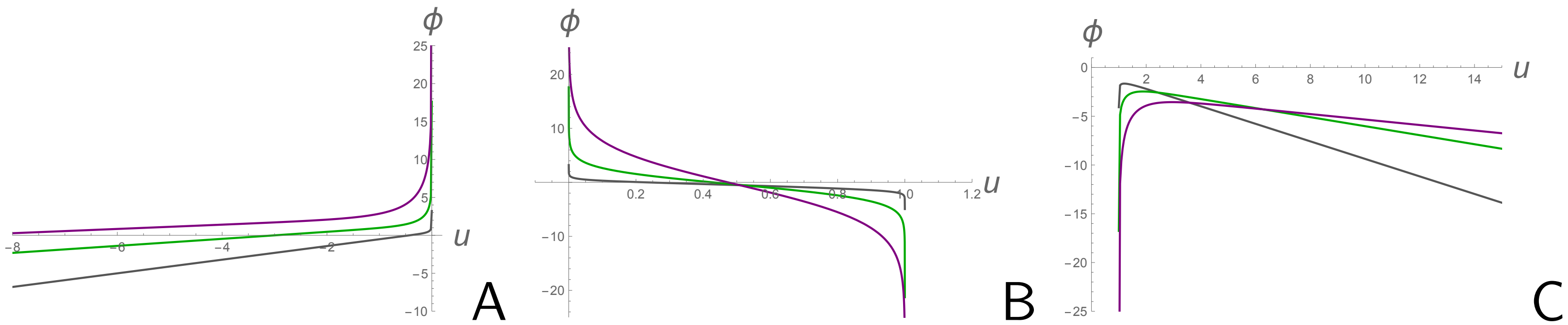
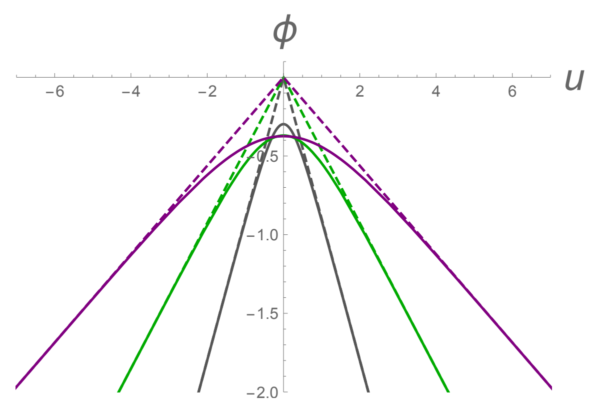


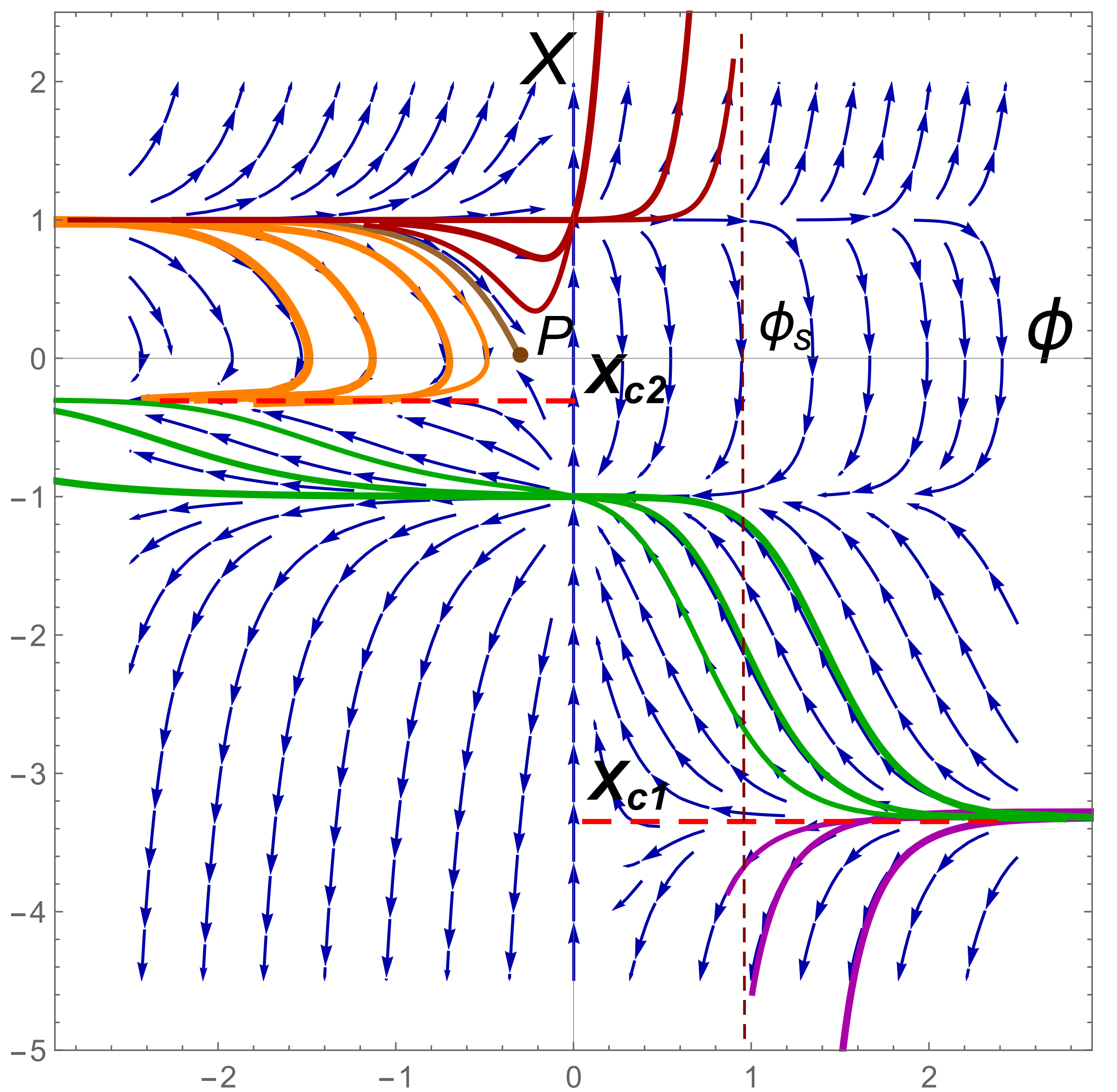
Figure: Dilaton as a function of u: A) $u < u_{02}$, B) $u_{02} < u < u_{01}$, C) the dilaton for $u > u_{01}$, $u_{01} = 1$. For all $u_{01} = 1$, $u_{02} = 0$,$E_1 = -E_2 = -1$, $C_1 = -C_2 = -1$, $k = 0.4, 1, 1.2$.



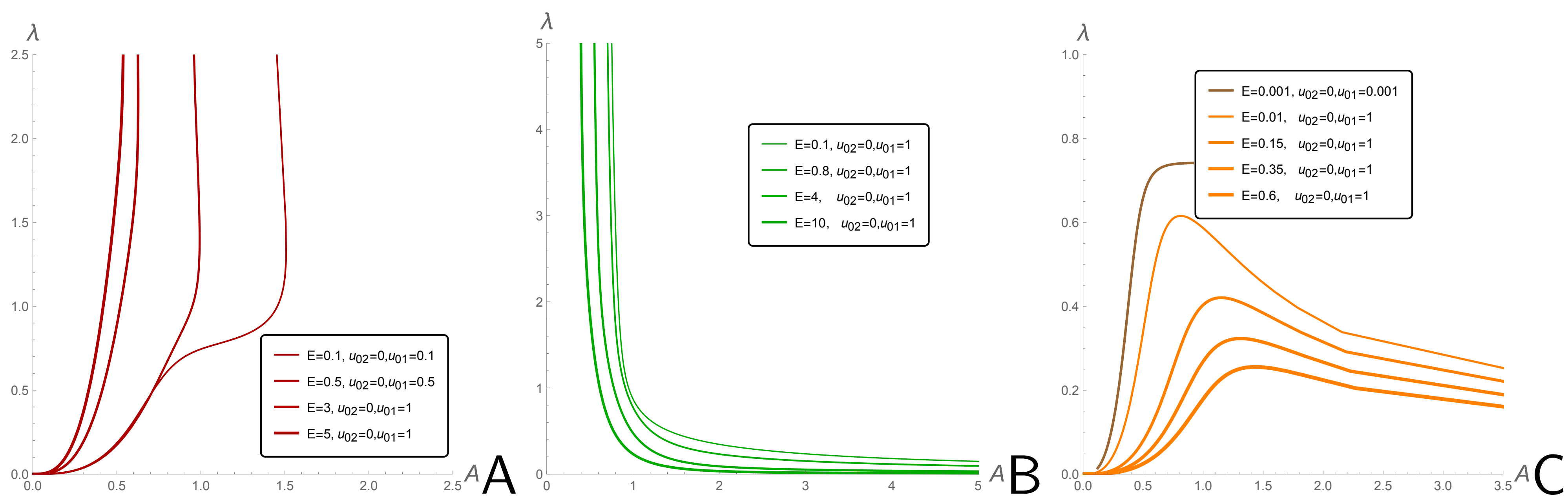
flow to AdS

Figure: The behaviour of the dilaton (solid lines) and its asymptotics at infinity (dashed lines) for $u_{01} = u_{02} = 0$, $C_1 = -C_2 = -1$, $E_1 = -E_2 = -1$ and different values of k. From bottom to top $k = 0.4, 1, 1.2$.

$$X = \frac{\beta(\lambda)}{3\lambda} = \frac{\phi'}{A'}$$



Running of the coupling



Only the type C solutions are non-singular (in the Gubser's sense) and can be promoted to regular BH solutions at finite T

There are BH solutions that interpolate from CR(UV) to AdS(IR)

Summary

Bottom-up holographic models can be used to approach a more realistic description of the QGP phase

Simple models can be useful to gain insight into general aspects

CR solutions: deviation from conformality results in longer thermalization and even breakdown of hydro at the critical point

More refined potentials can be used to embed CR into a realistic model (work in progress)

Outlook

Match the collective modes of CR to some hydro model, understand the appearance of branch cut

Relevant for RHIC / LHC ??

Explore the thermodynamics and fluctuations of the flows

Charged BH solutions

Other solvable potentials (three exponentials…)