# On the thermodynamics and phase structure of QCD

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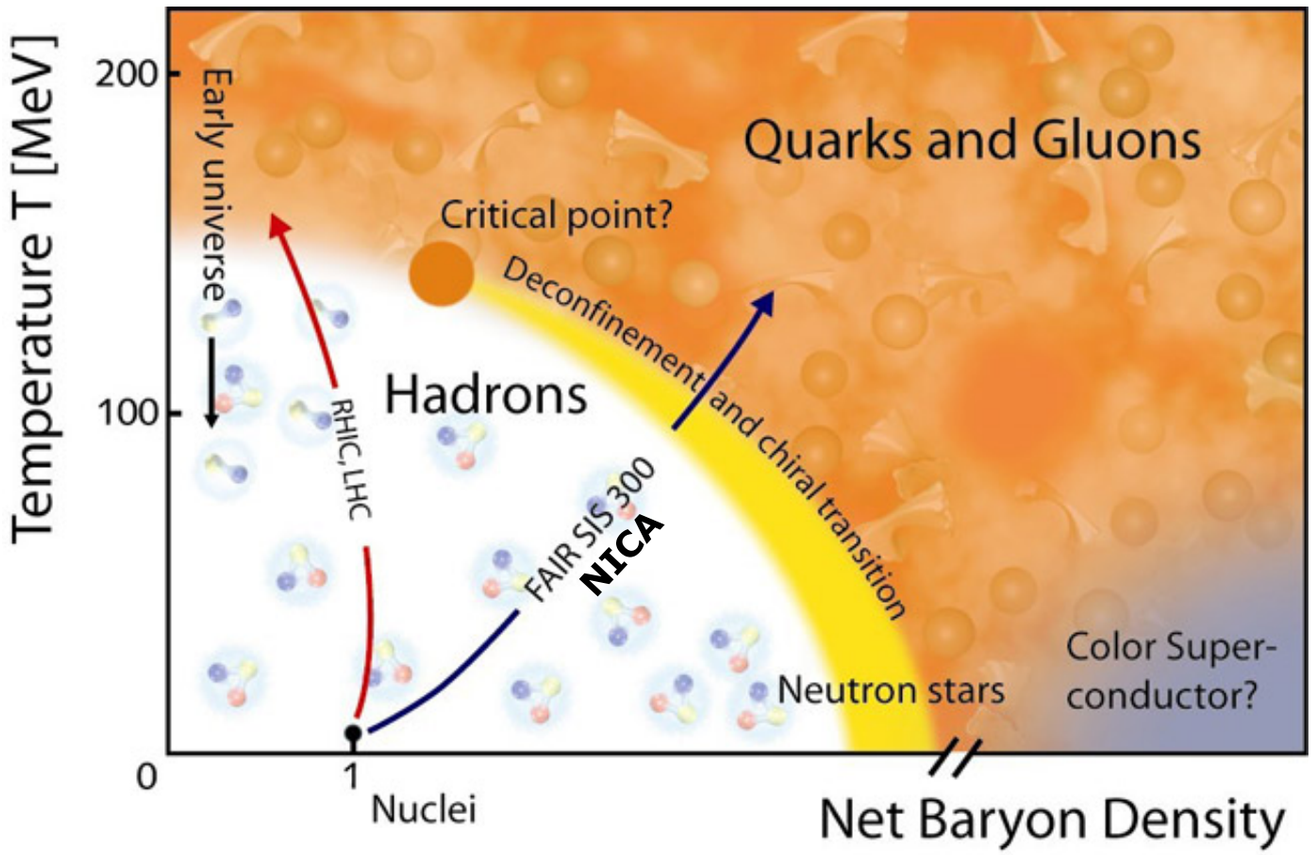
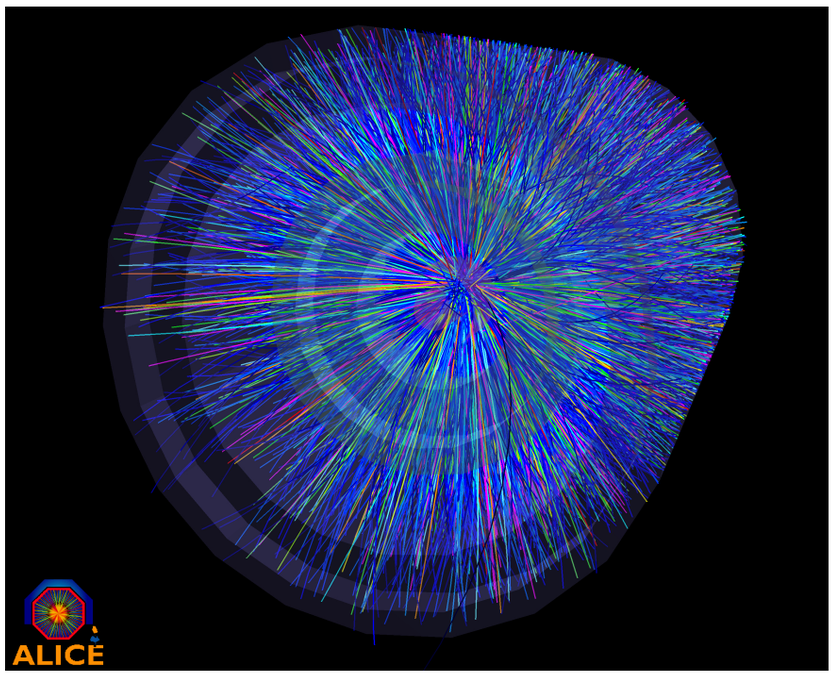
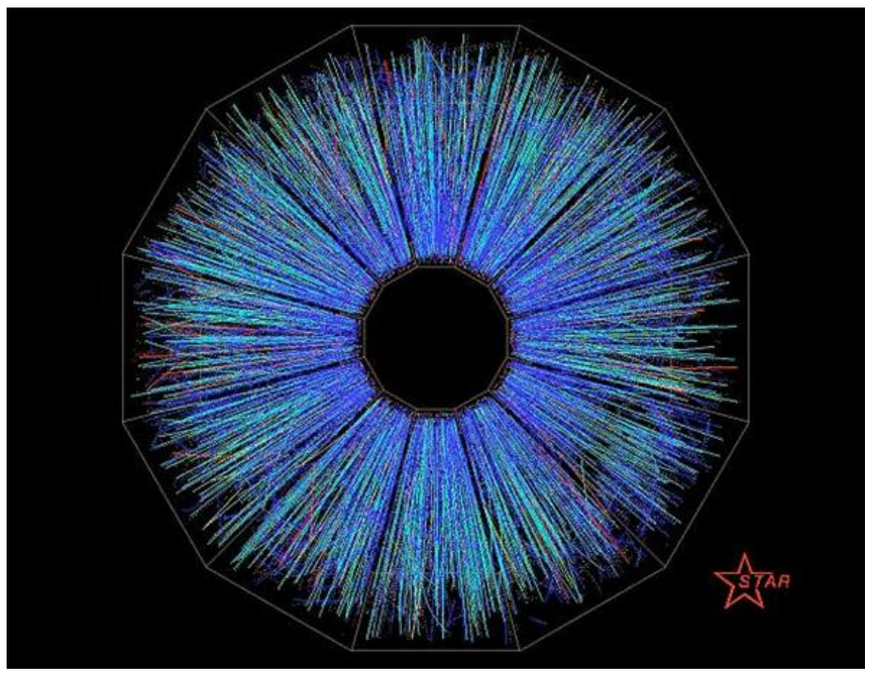
**Universität Heidelberg & ExtreMe Matter Institute**

**Dubna, February 29th 2012**

# Heavy ion collisions



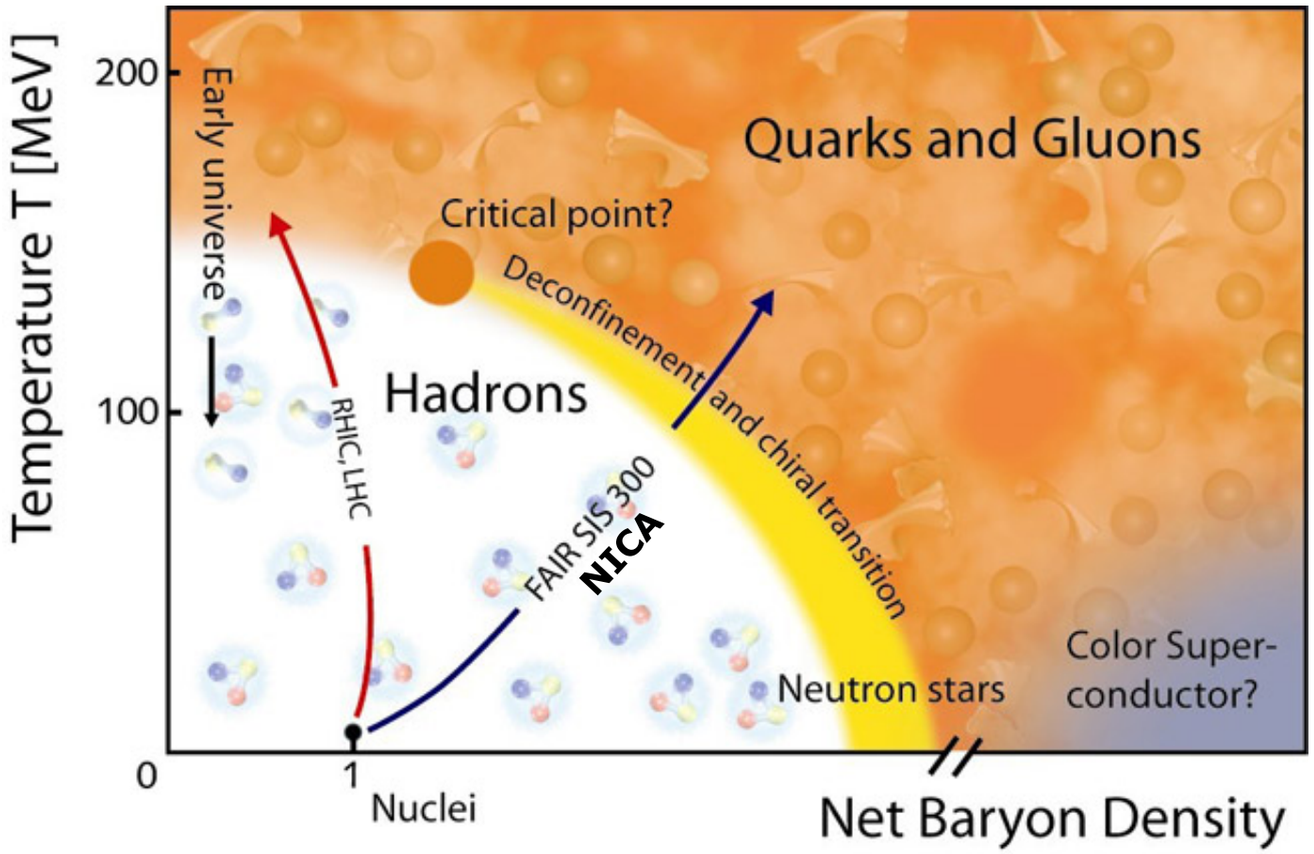

**ALICE, LHC**

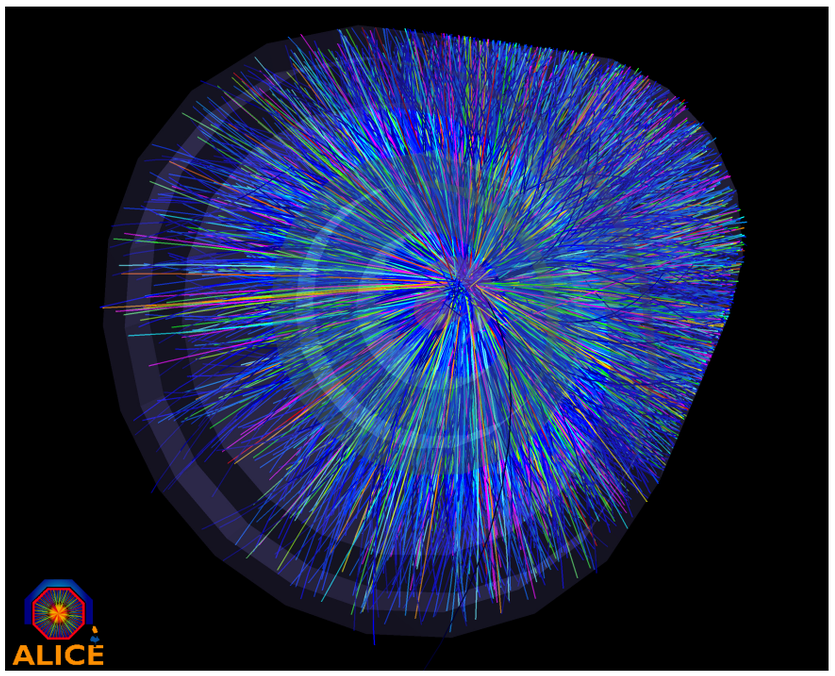
**UrQMD Frankfurt/M**

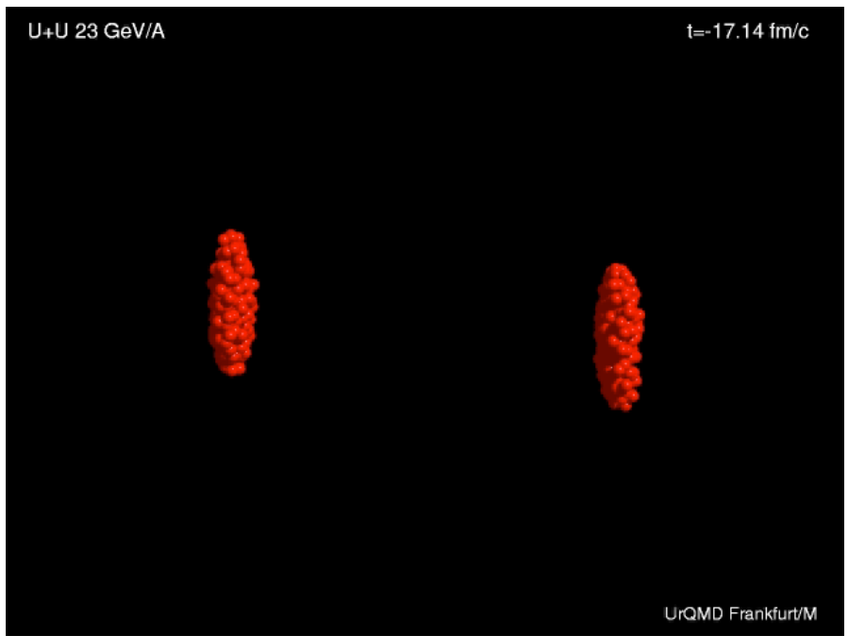
**Simulation of a heavy ion collision**

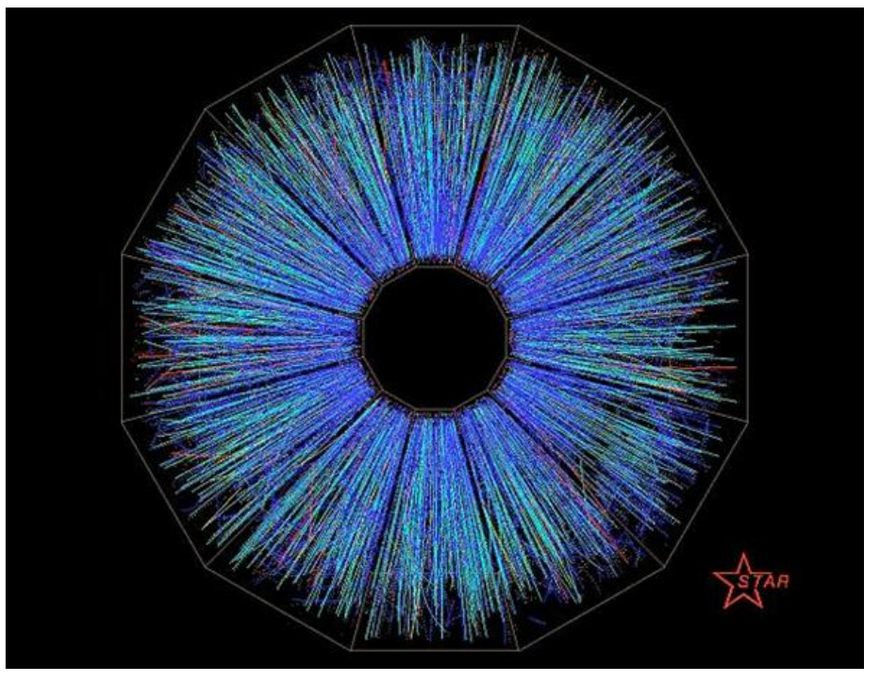
**STAR, RHIC**

# Heavy ion collisions




**ALICE, LHC**

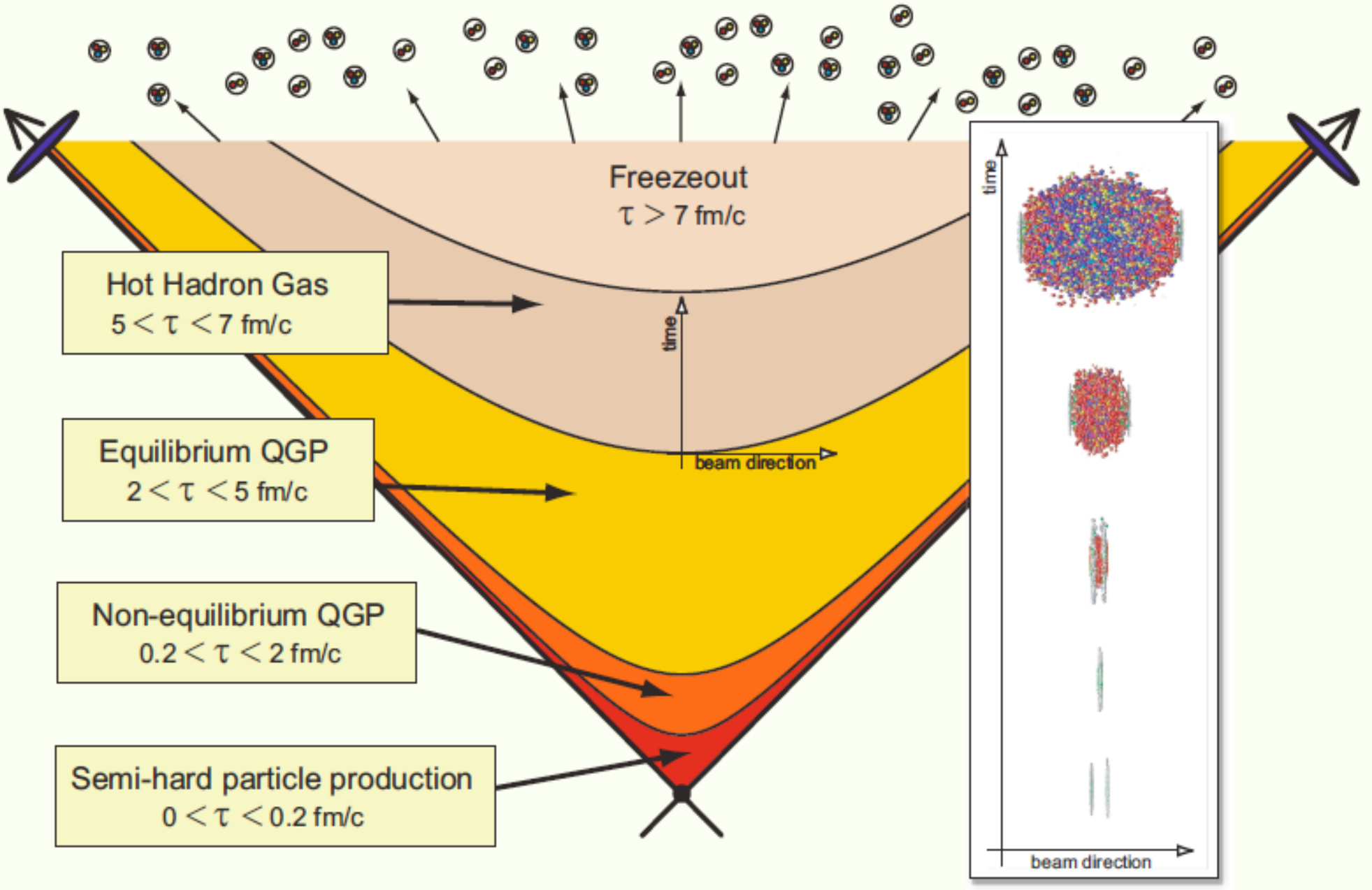
**UrQMD Frankfurt/M**

**Simulation of a heavy ion collision**

**STAR, RHIC**

# Heavy ion collisions

## Heavy-ion collision timescales and "epochs" @ RHIC




*1 fm/c $\simeq 3 \times 10^{-24}$ seconds

# Phase diagram of QCD


Temperature $T$

Quark-Gluon Plasma

sQGP

*Critical Point*

Inhomogeneous S$\chi$B

Quarkyonic Matter

Hadronic Phase

*Liquid-Gas*

?

2SC

uSC
dSC
CFL

Color Superconductors

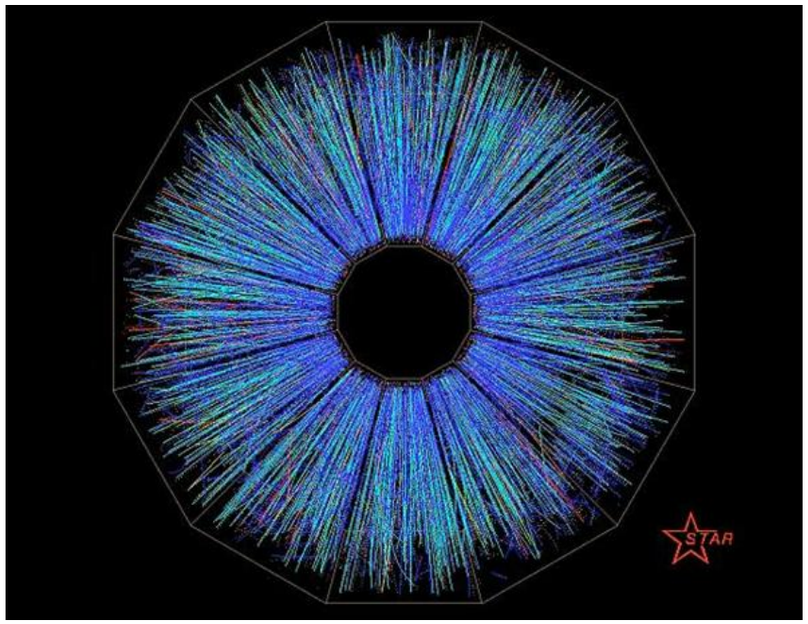
Nuclear Superfluid

CFL-$K^0$, Crystalline CSC
Meson supercurrent
Gluonic phase, Mixed phase

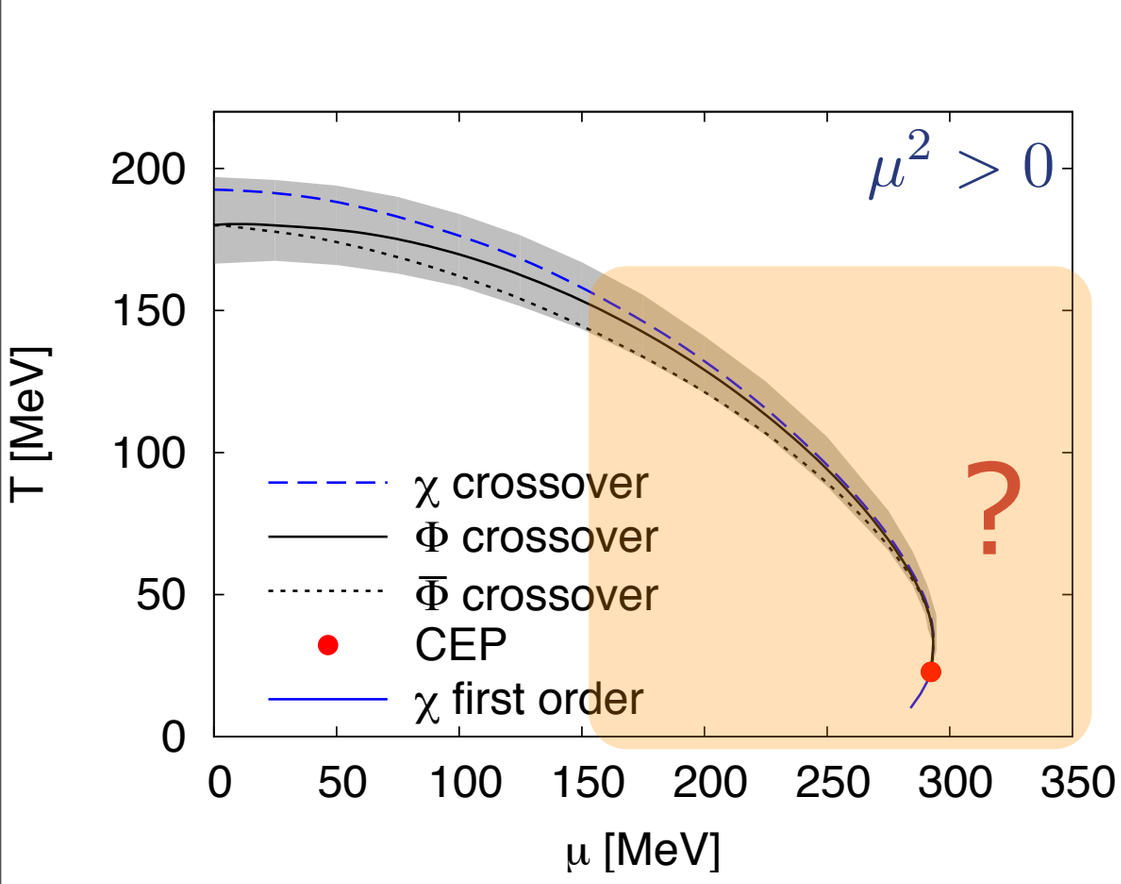
Baryon Chemical Potential $\mu_B$

baryonic density

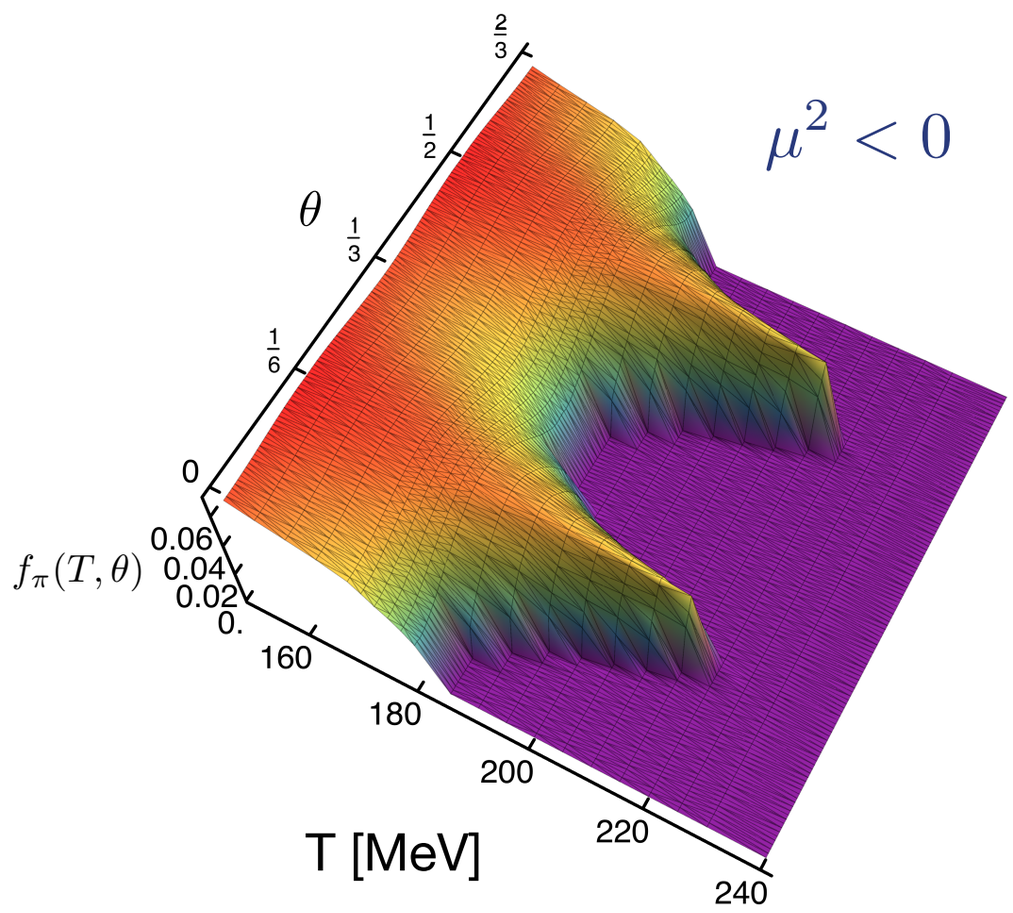
Fukushima




# From the quark-gluon plasma to the hadron gas




## Results

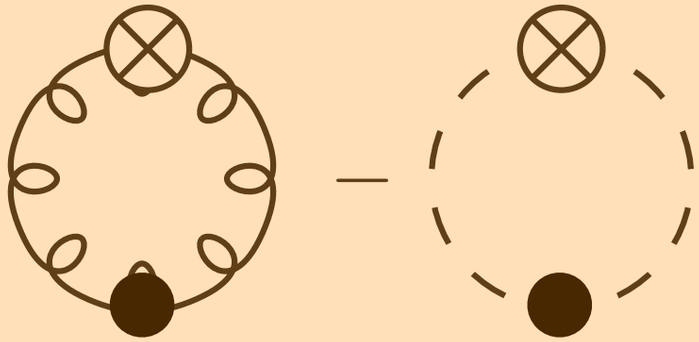


# Functional Methods for QCD

**for a short review see JMP, arXiv:1012.5075**

**glue quantum fluctuations**

**hadronic quantum fluctuations**

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}$$  $$+ \frac{1}{2}$$

**free energy**

**quark quantum fluctuations**

RG-scale k: $t = \ln k$

- **Gluons have cost us decades**
- **Fermions are straightforward** though 'physically' complicated
  - **no sign problem**
  - **chiral fermions**
- **bound states via dynamical hadronisation**

**Complementary to lattice!**

**e.g. finite volume scaling: Braun, Klein, Piasecki, Schaefer '10-11**

# Functional Methods for QCD

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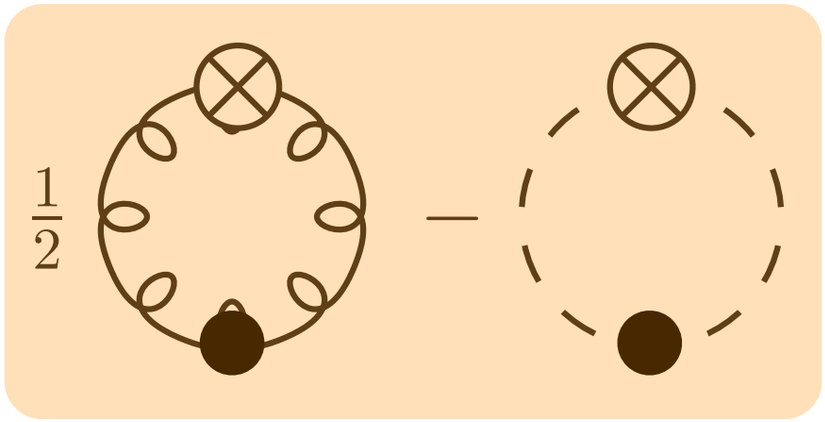
**glue**
**quantum fluctuations**

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}$$

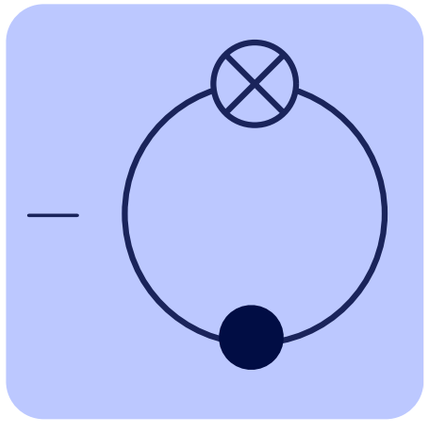


**free energy**

## Yang-Mills theory

# Functional Methods for QCD

$$\partial_t \Gamma_k[\phi] =$$



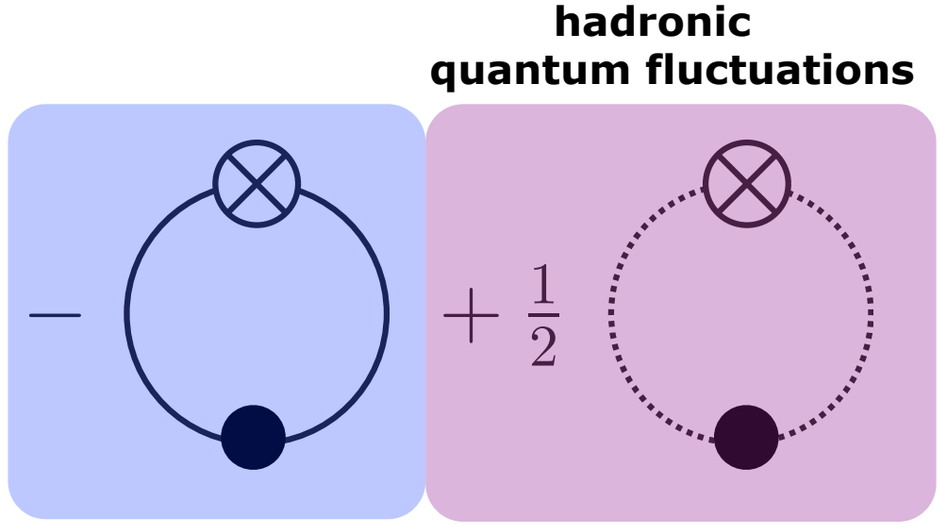
**free energy**

**quark quantum fluctuations**

## NJL/PNJL model

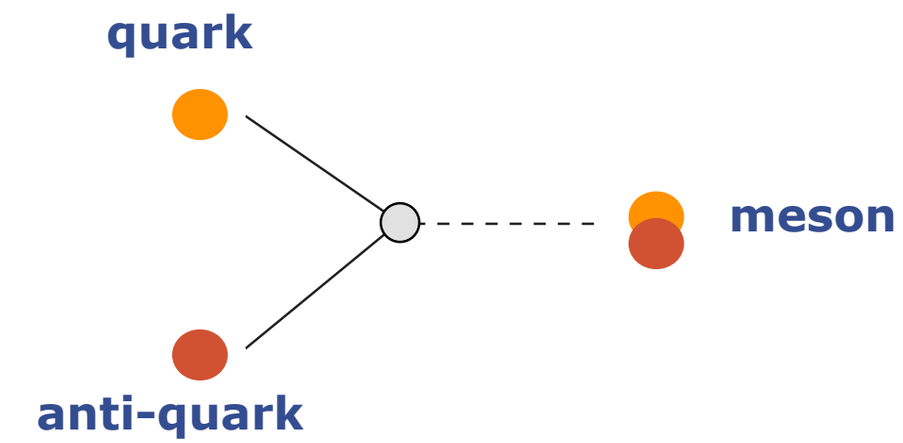
# Functional Methods for QCD

$$\partial_t \Gamma_k[\phi] =$$



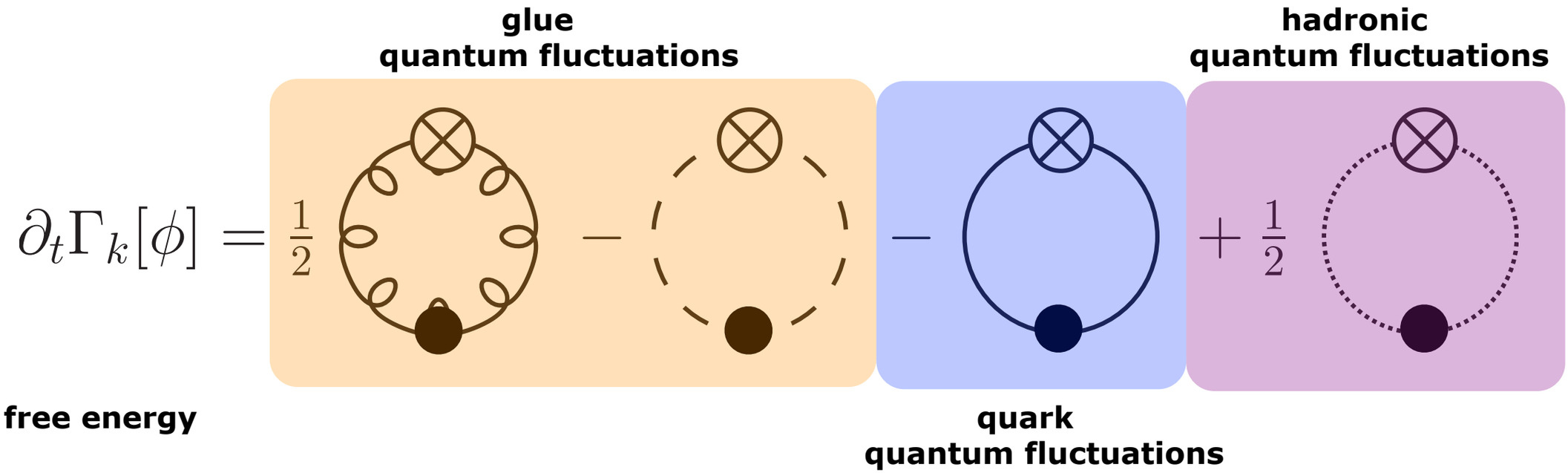

**free energy**

## Quark-hadron/PQH models



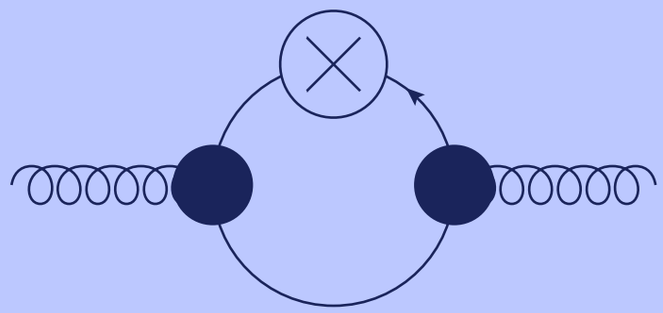

- **bound states via dynamical hadronisation**

# Functional Methods for QCD




**flow of gluon propagator**

**pure gauge theory flow** **+**

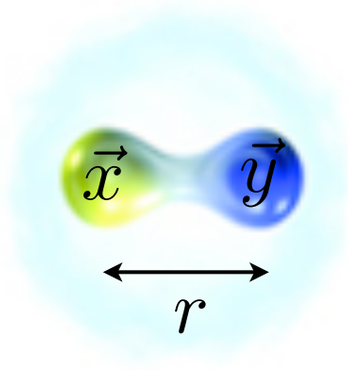


**+** **...**

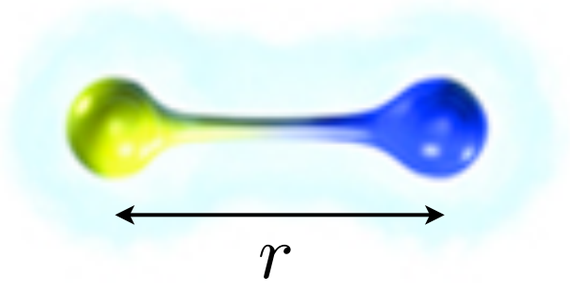
**Naturally** **encorporates PQM/PNJL models as** **specific low order trunations**

# Confinement

## Free energy $F_{q\bar{q}}$ of a quark - antiquark pair

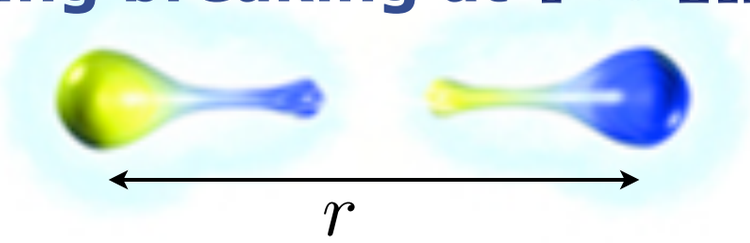


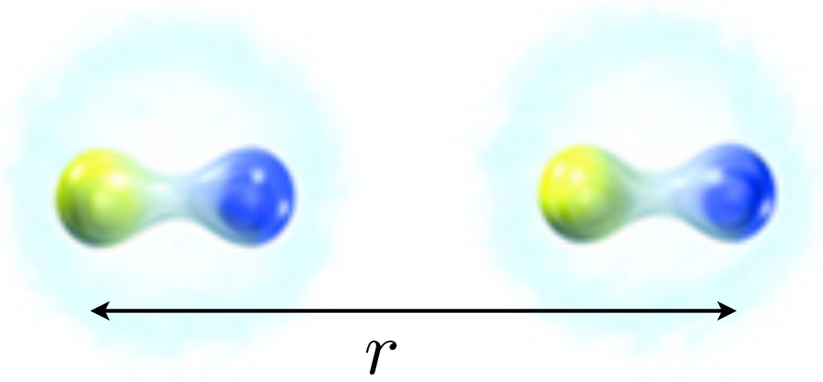

$$F_{q\bar{q}} \simeq -\frac{1}{r}$$




$$F_{q\bar{q}} \simeq \sigma r$$

**string breaking at** $\mathrm{r} \approx 1\mathrm{fm}$




$$F_{q\bar{q}} \simeq \text{const.}$$

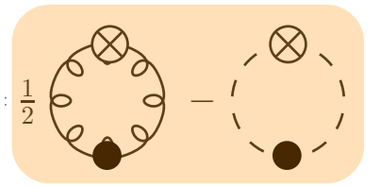
**Order parameter** $\sim \langle q \rangle$

$$\Phi = e^{-\frac{1}{2T} F_{q\bar{q}}(\infty)}$$

- **Confinement** $\Phi = 0$
- **Deconfinement** $\Phi \neq 0$

**Polyakov loop**

$$\Phi = \frac{1}{3} \langle \mathrm{Tr}\, \mathcal{P} \exp\{ig \int_0^{1/T} dx_0 A_0\} \rangle$$

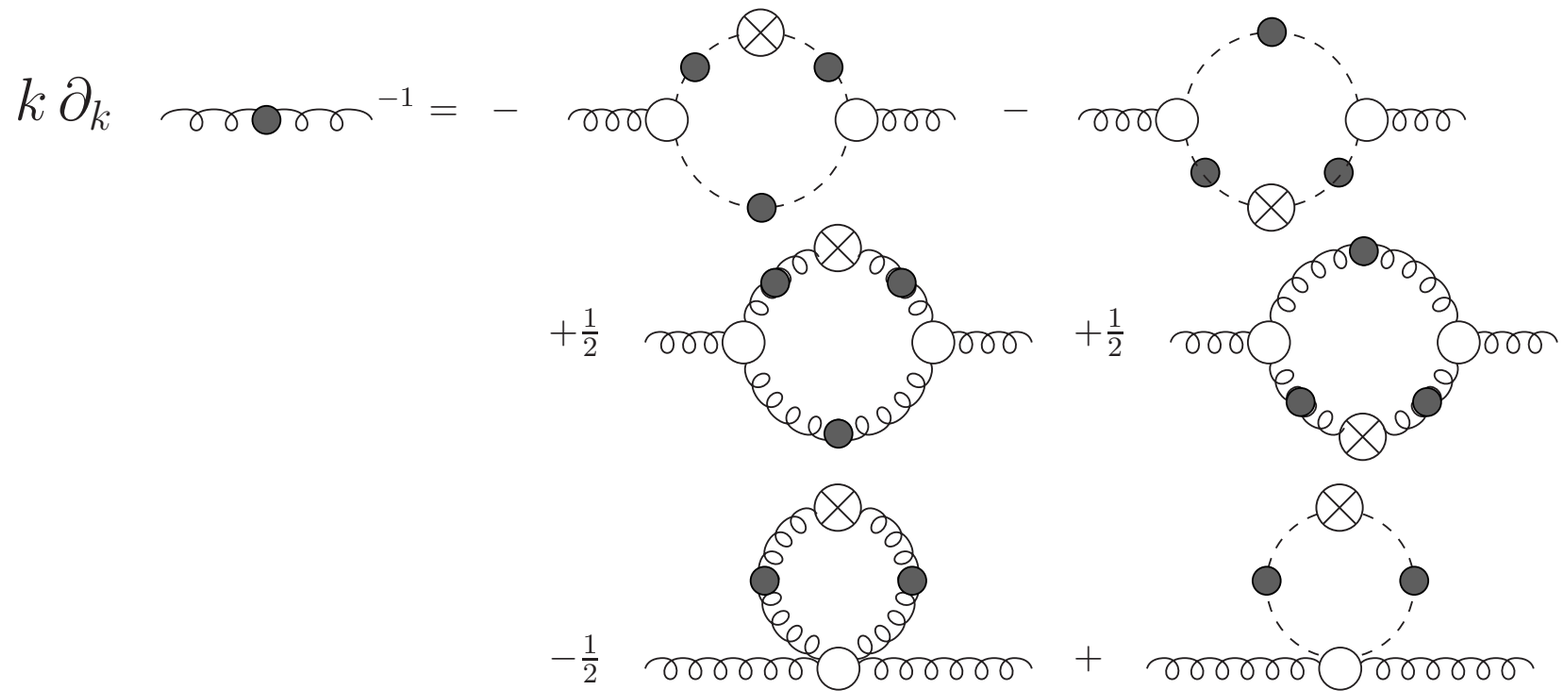


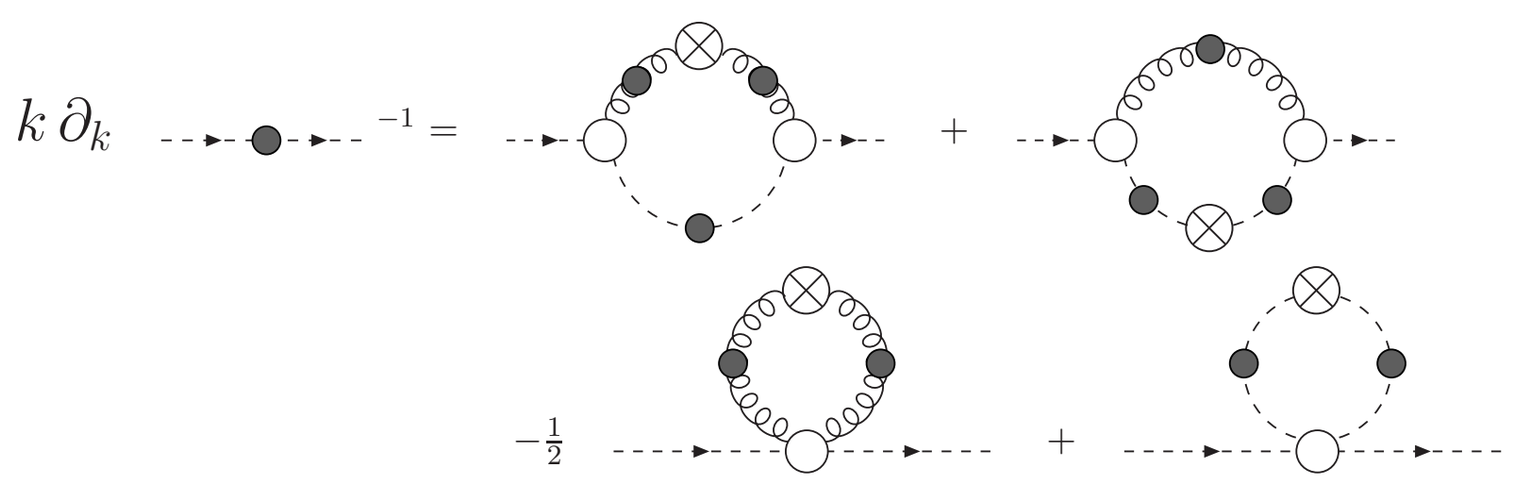
# Confinement

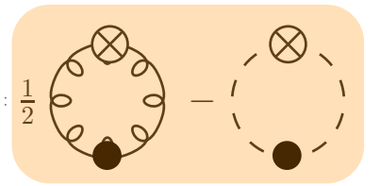
## effective potential

$$V[A_0] = -\frac{1}{2}\mathrm{Tr}\log\langle AA\rangle[A_0] + O(\partial_t\langle AA\rangle) + \mathrm{Tr}\log\langle C\bar{C}\rangle[A_0] + O(\partial_t\langle C\bar{C}\rangle)$$

**free energy**





# Confinement

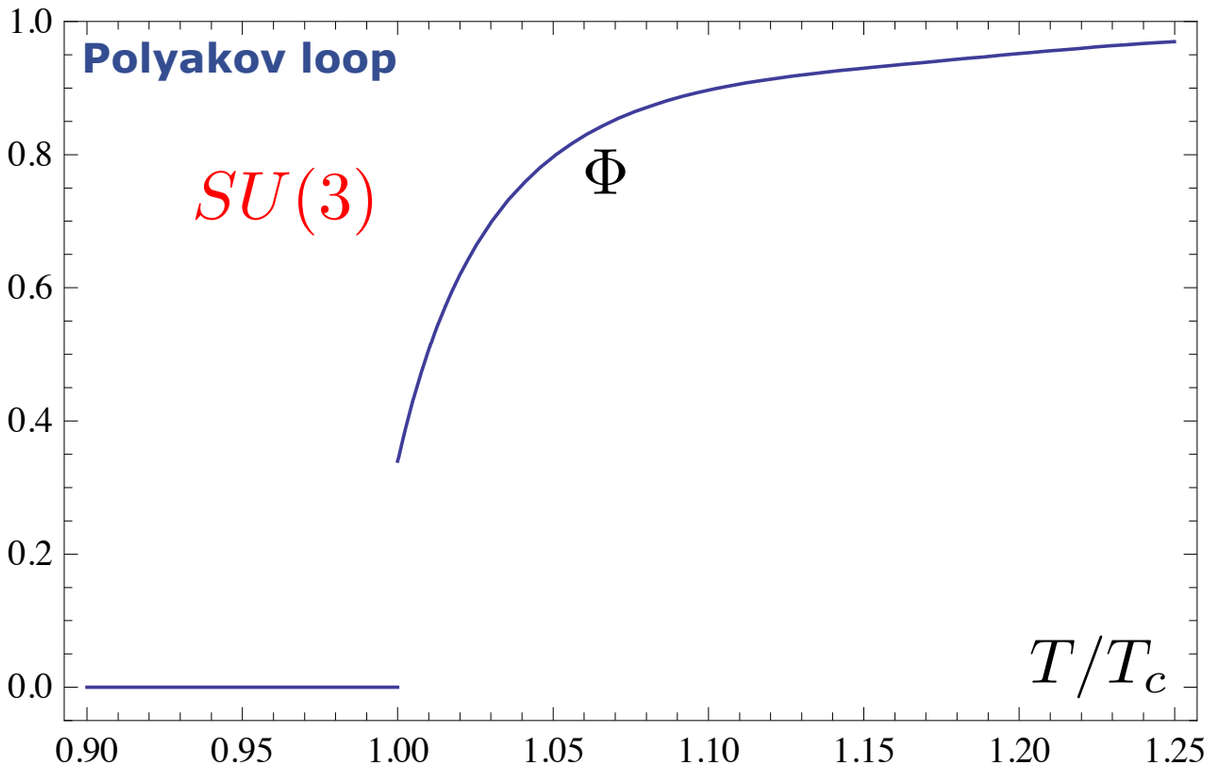
## Order parameter

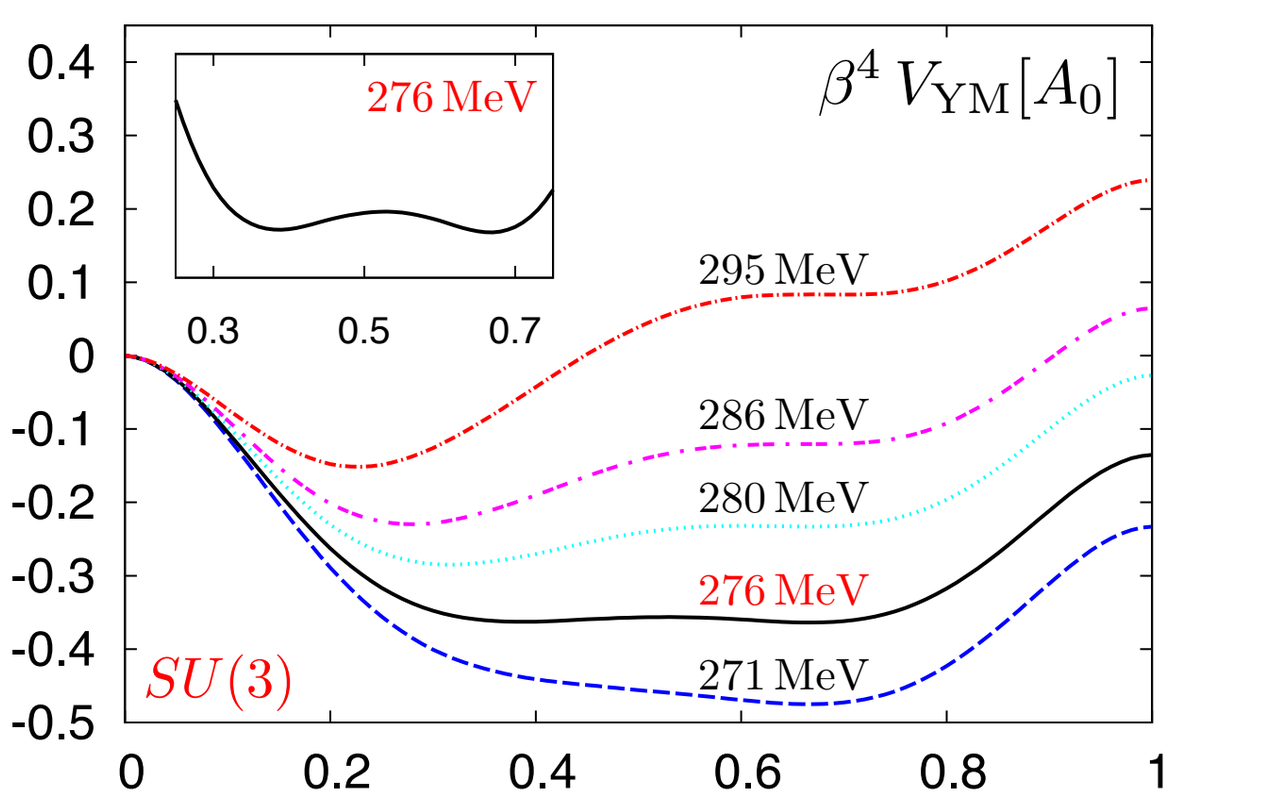
$$T_c = 276 \pm 10\,\mathrm{MeV}$$

$$T_c/\sqrt{\sigma} = 0.658 \pm 0.023$$

**Braun, Gies, JMP '07**

$$\text{lattice}: T_c/\sqrt{\sigma} = 0.646$$

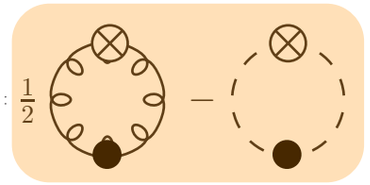



$$\Phi[A_0] = \frac{1}{3}(1 + 2\cos\frac{1}{2}\beta g A_0)$$

$$\Phi[\frac{4}{3}\pi\frac{1}{\beta g}] = 0$$

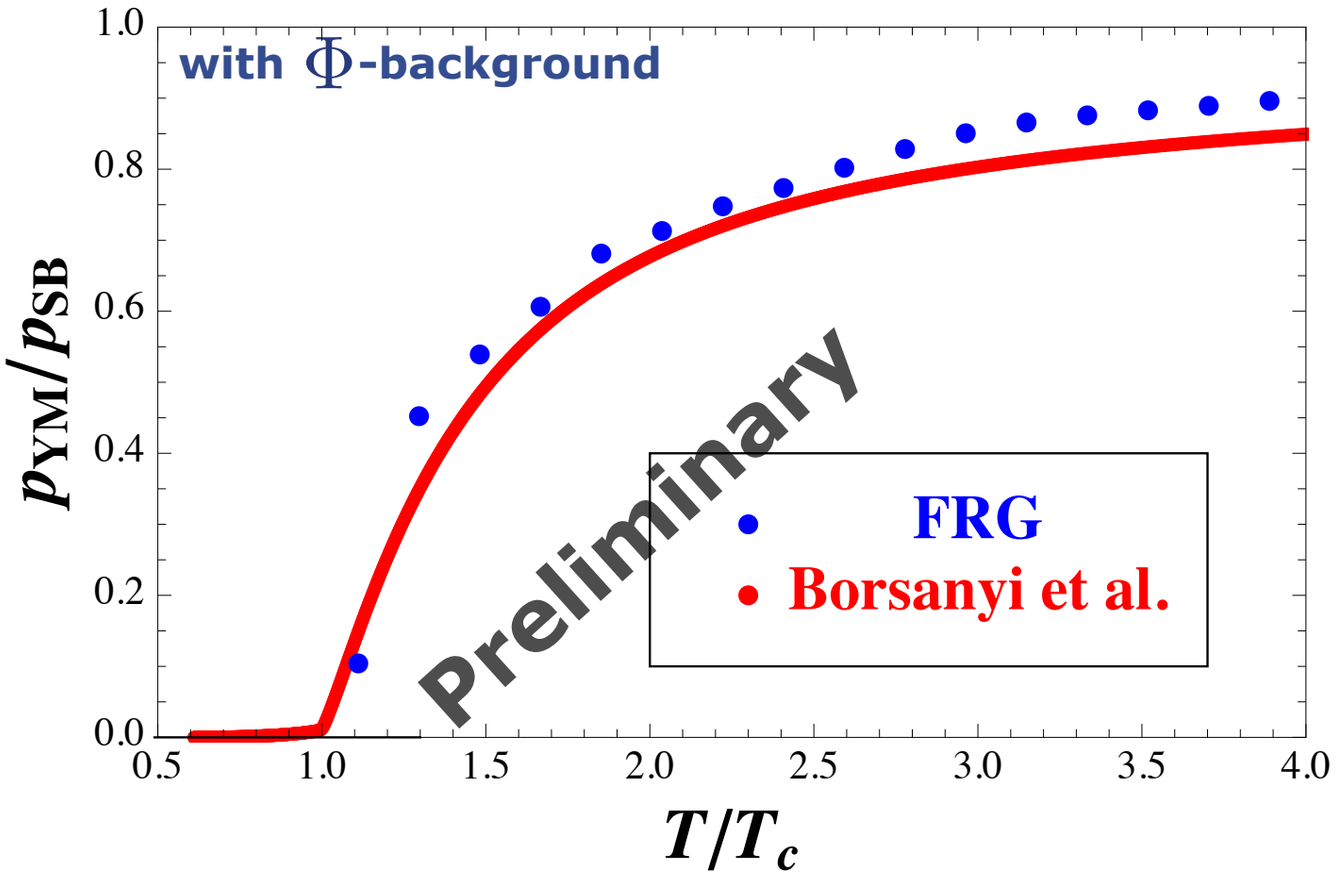
$$\frac{\beta g A_0}{2\pi}$$

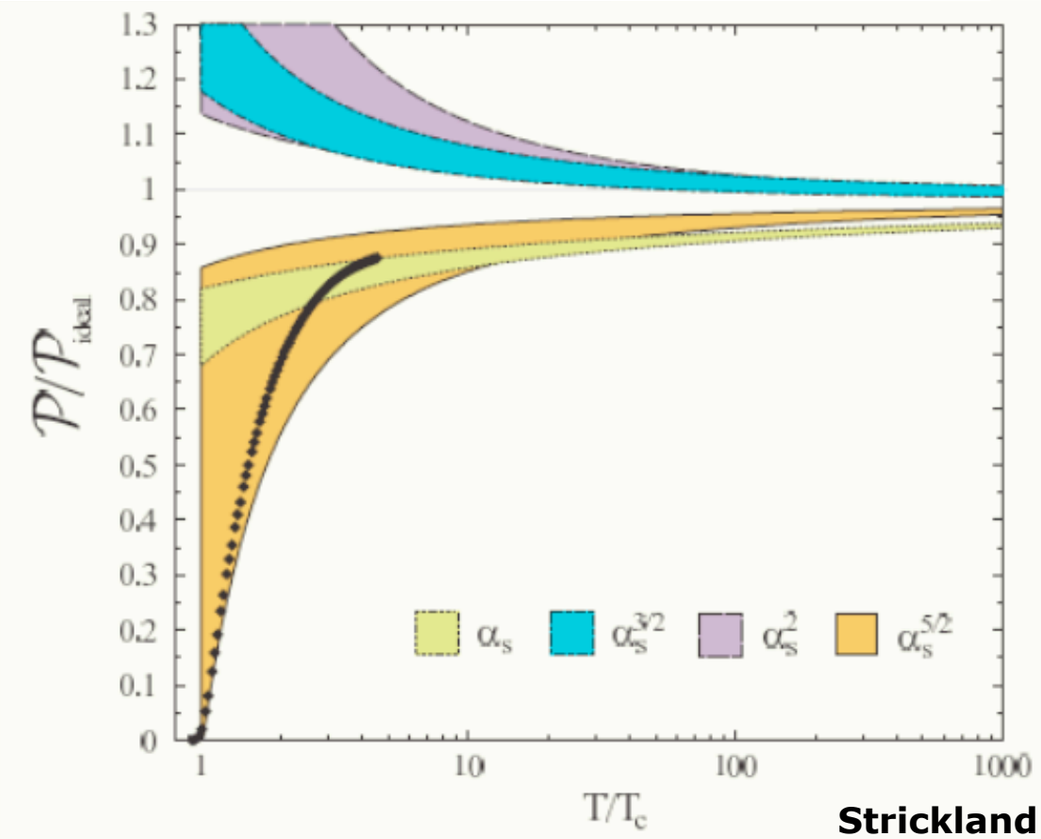


# thermodynamics

## Yang-Mills pressure

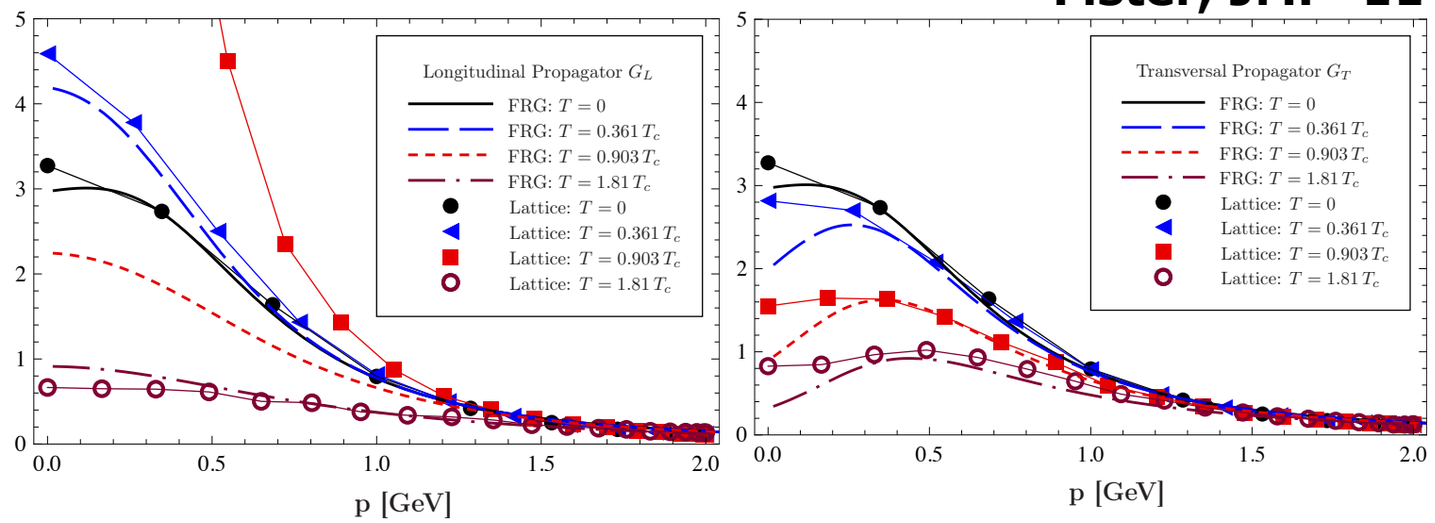
**Fister, JMP**

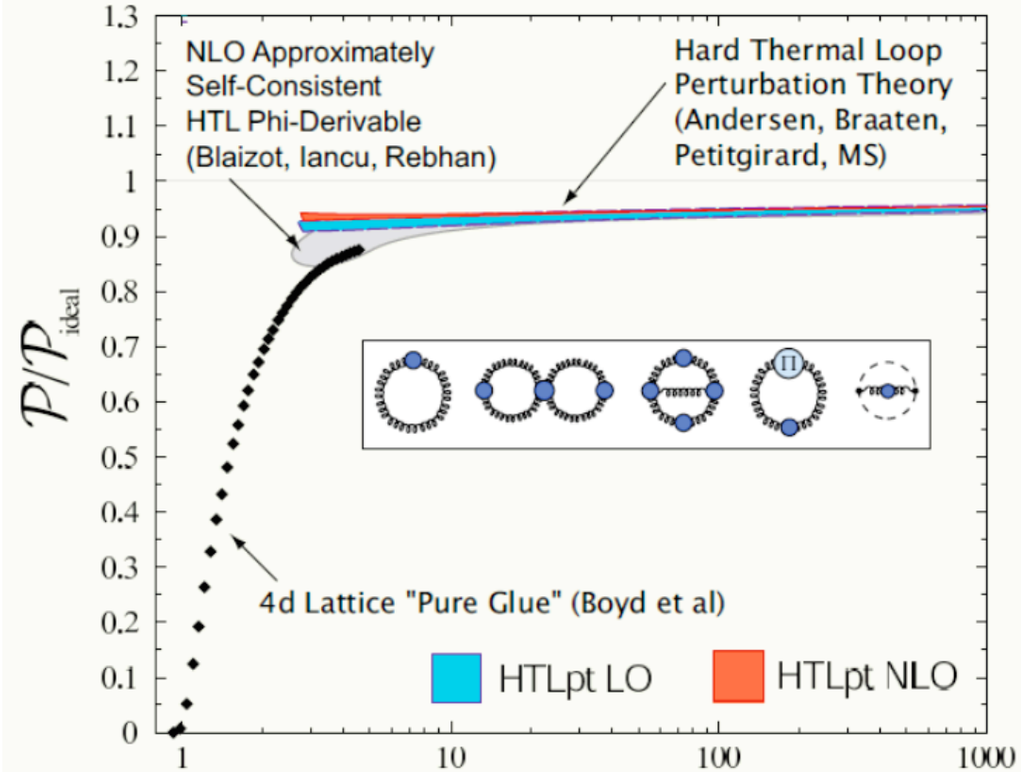



**Strickland**

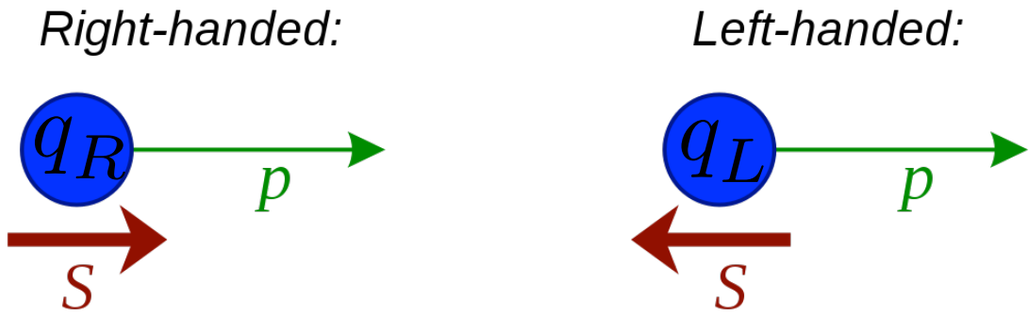
**Fister, JMP '11**



# Chiral symmetry breaking

- **Chirality for massless particles**

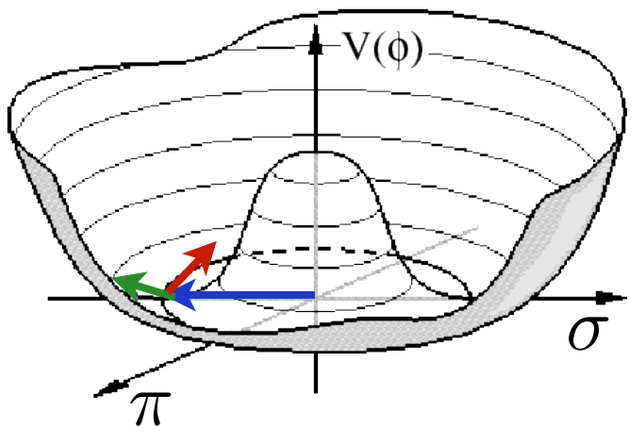



- **Order parameter**

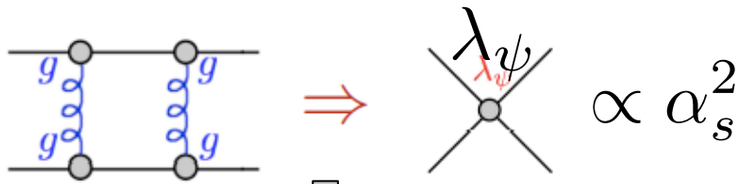
$$\sigma = \langle \bar{q}q \rangle$$ **chiral condensate**

- **chiral symmetry:** $\sigma = 0$
- **symmetry broken:** $\sigma \neq 0$

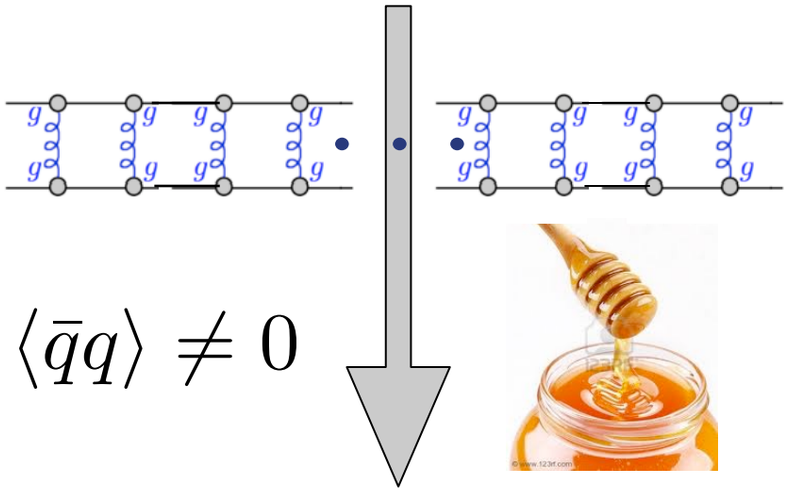
- **Meson potential**




## chiral symmetry



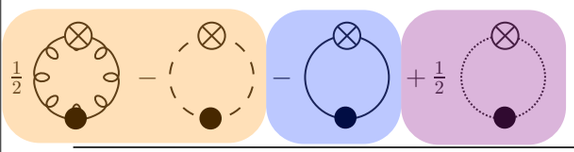

$$\int d^4x \, \lambda_\psi \left[ (\bar{q}q)^2 - (\bar{q}\gamma_5 q)^2 \right]$$




**mass term:** $\langle \bar{q}q \rangle \, \bar{q}q$

**chiral symmetry broken**

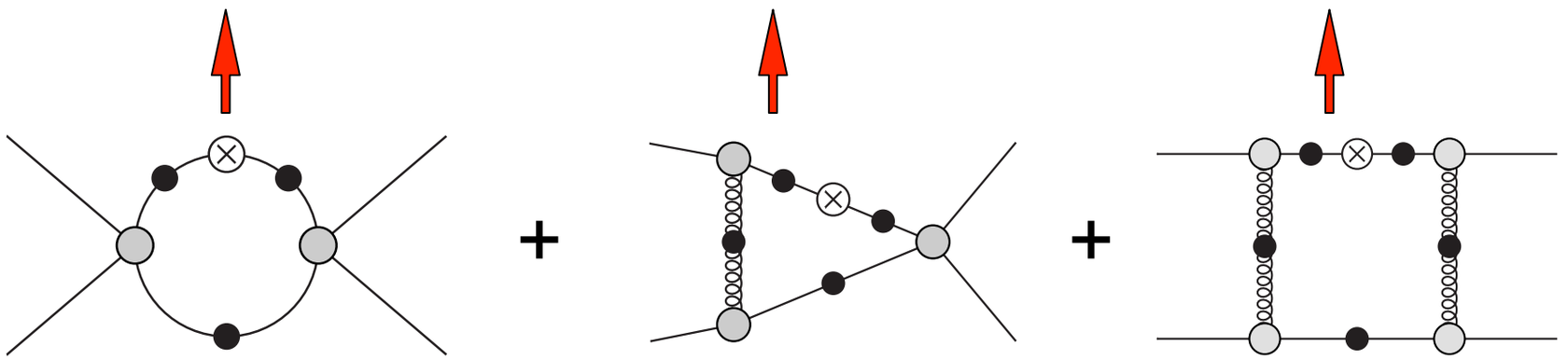
# Chiral symmetry breaking

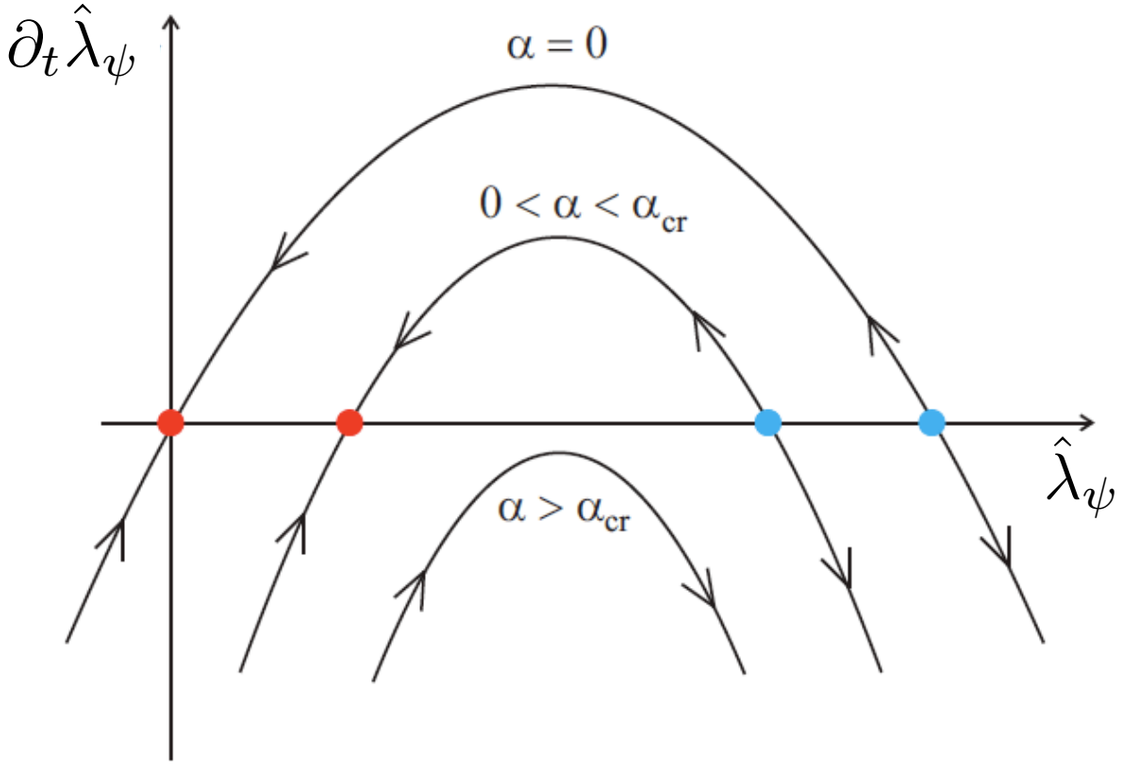


## RG picture

**Flow for four-fermion coupling $\hat{\lambda}_\psi = \lambda_\psi k^2$ with infrared scale $k$**

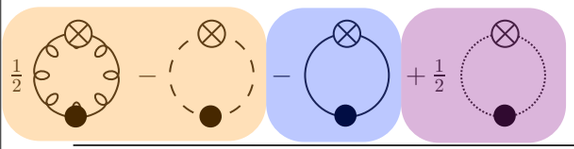
$$k\partial_k \hat{\lambda}_\psi \;=\; 2\hat{\lambda}_\psi \;-\; A\left(\frac{T}{k}\right)\,\hat{\lambda}_\psi^2 \;-\; B\left(\frac{T}{k}\right)\,\hat{\lambda}_\psi\,\alpha_s \;-\; C\left(\frac{T}{k}\right)\,\alpha_s^2 \;+\;\cdots$$

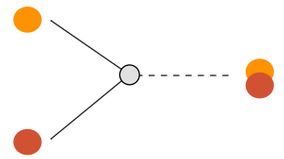




# Chiral symmetry breaking

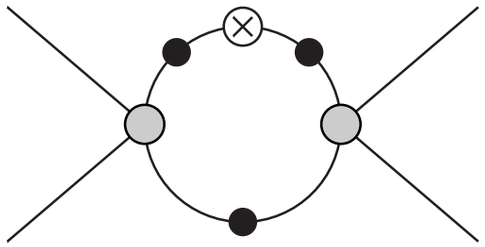
## dynamical hadronisation





**Flow for four-fermion coupling $\hat{\lambda}_\psi = \lambda_\psi k^2$ with infrared scale $k$**

$$k\partial_k \hat{\lambda}_\psi = 2\hat{\lambda}_\psi +$$



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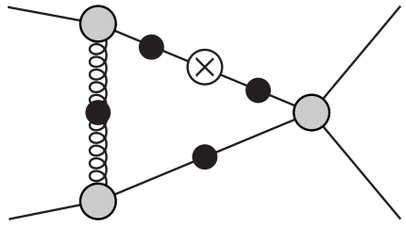

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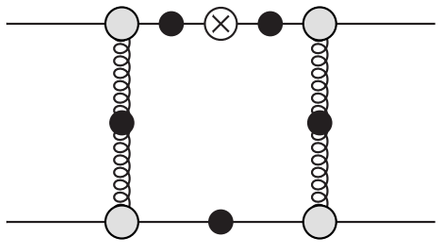

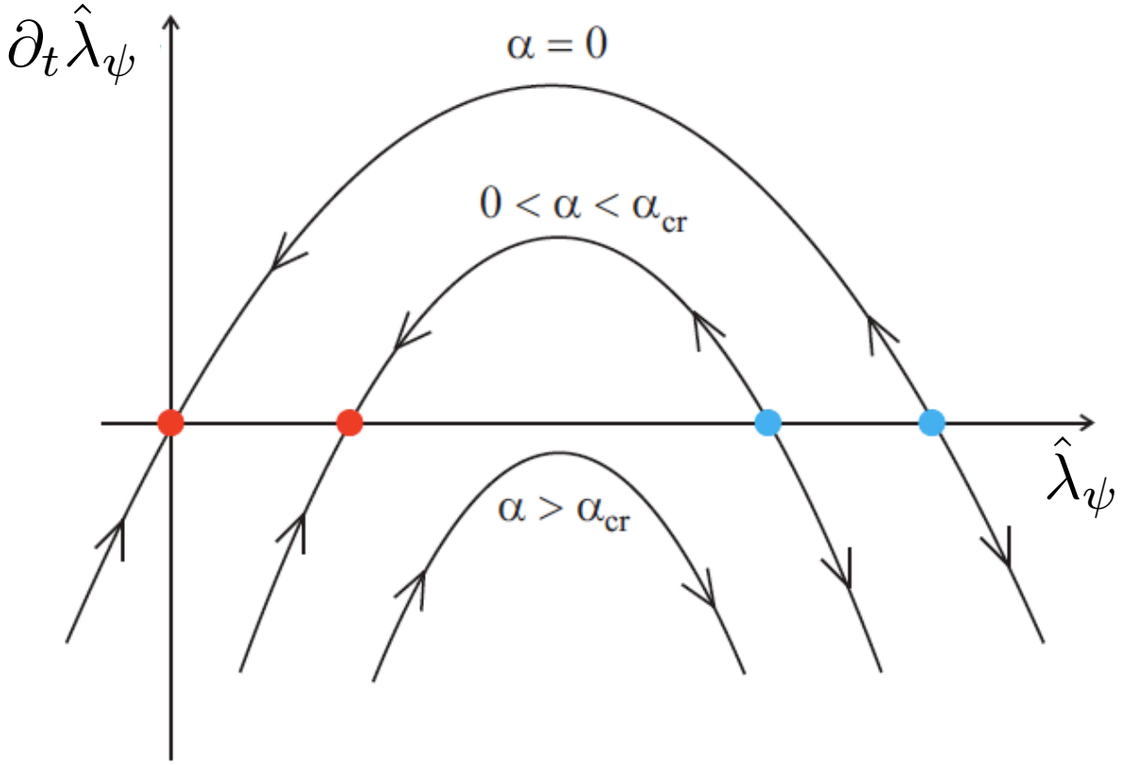


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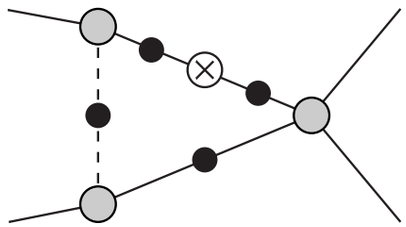

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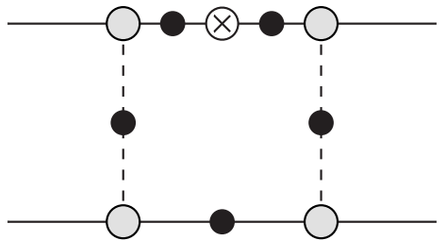
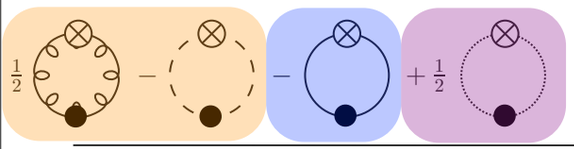


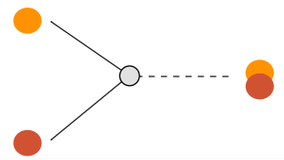
+ $\partial_t \dfrac{h^2}{m^2}$ **- terms**

+ …

# Chiral symmetry breaking

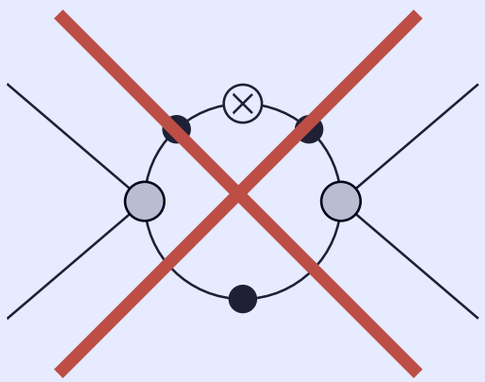
## dynamical hadronisation



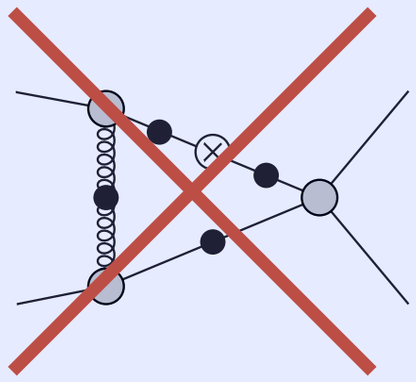


**Full bosonisation** $\hat{\lambda}_\psi = 0$

$k\partial_k \hat{\lambda}_\psi$ = $2\hat{\lambda}_\psi$ +



+



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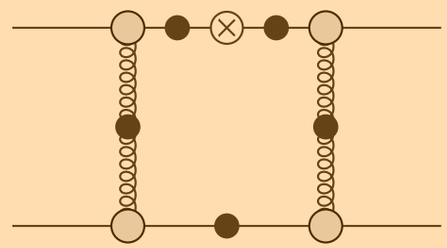

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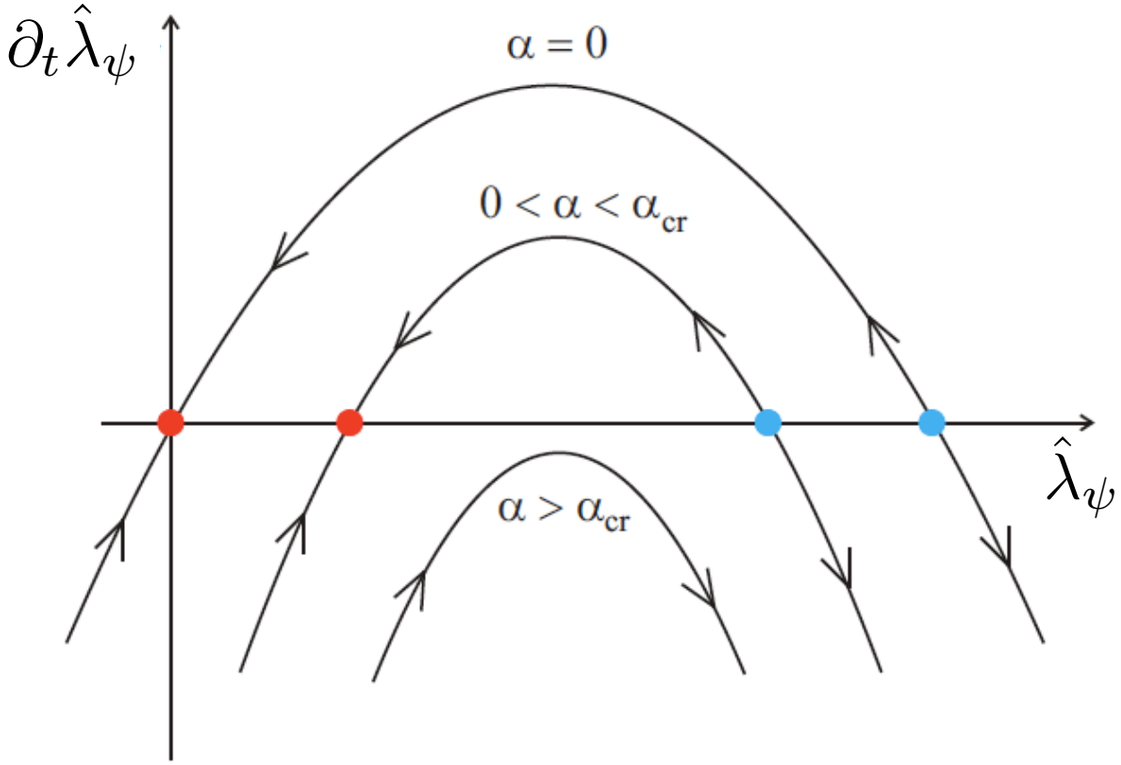


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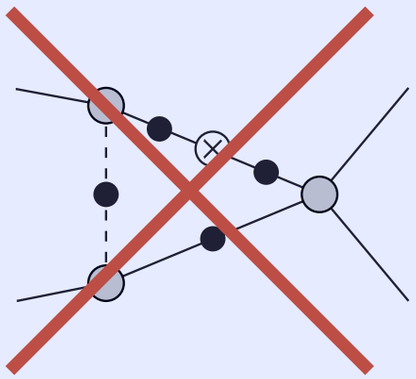
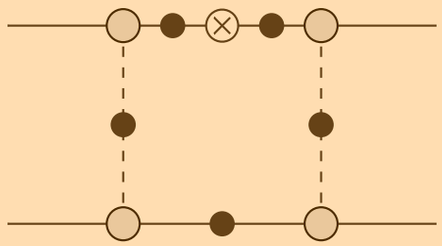


+



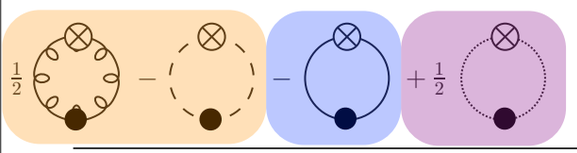
+ $\partial_t \dfrac{h^2}{m^2}$ **- terms**

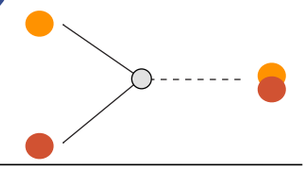
+ ...

$= 0$

# Chiral symmetry breaking

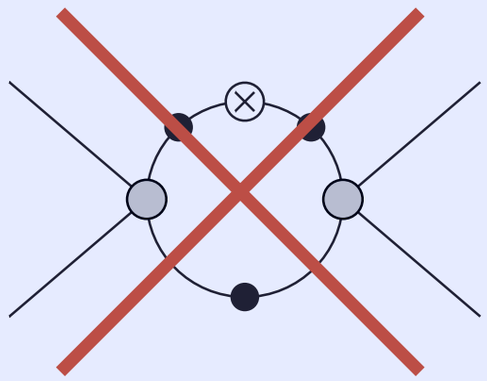
## dynamical hadronisation





**Full bosonisation** $\hat{\lambda}_\psi = 0$

$k\partial_k \hat{\lambda}_\psi$ = $2\hat{\lambda}_\psi$ +



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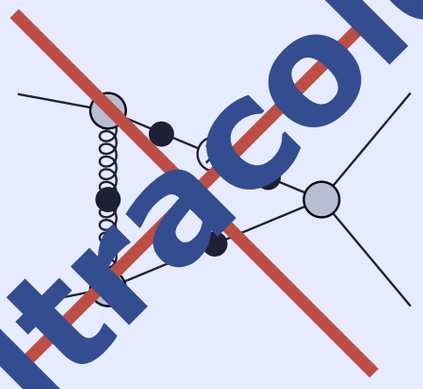

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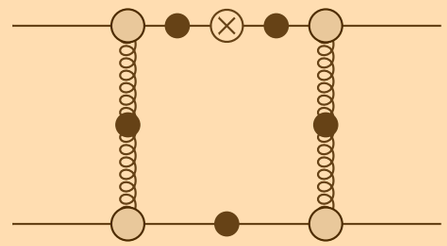

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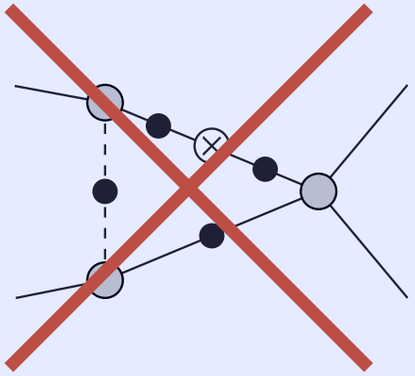

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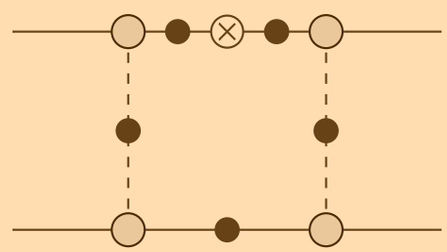
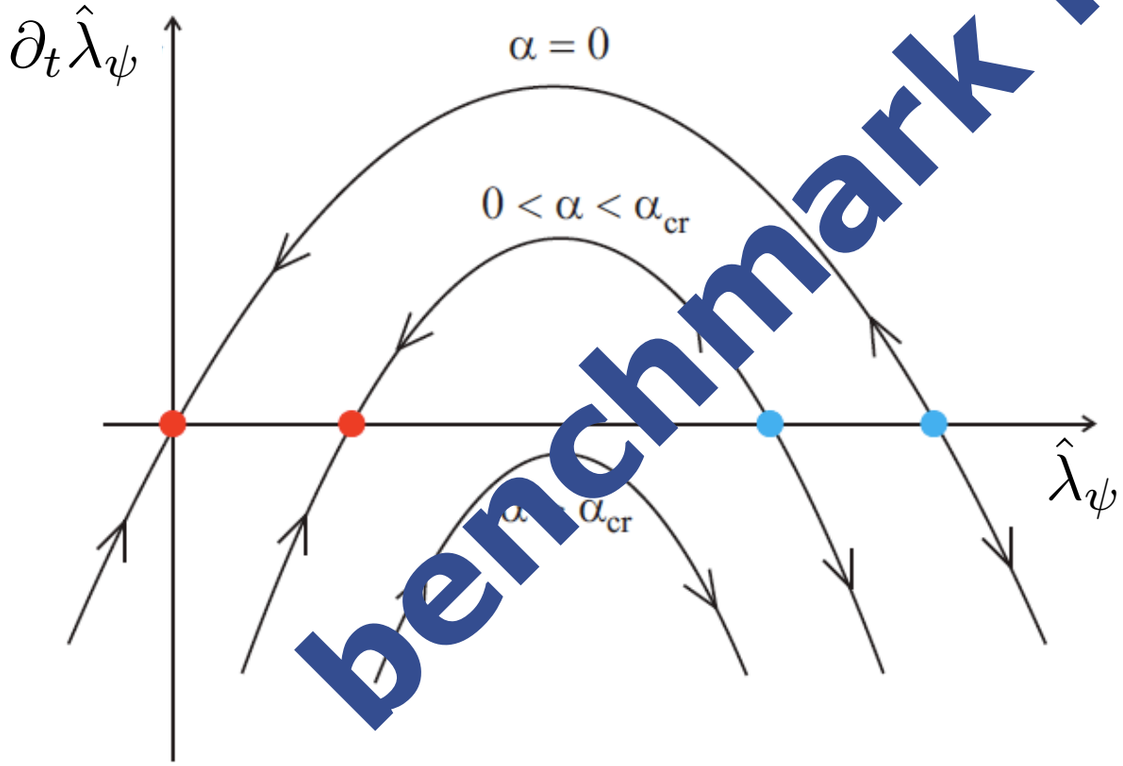


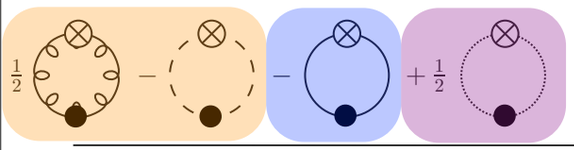
+ $\partial_t \dfrac{h^2}{m^2}$ **- terms**

+ ...

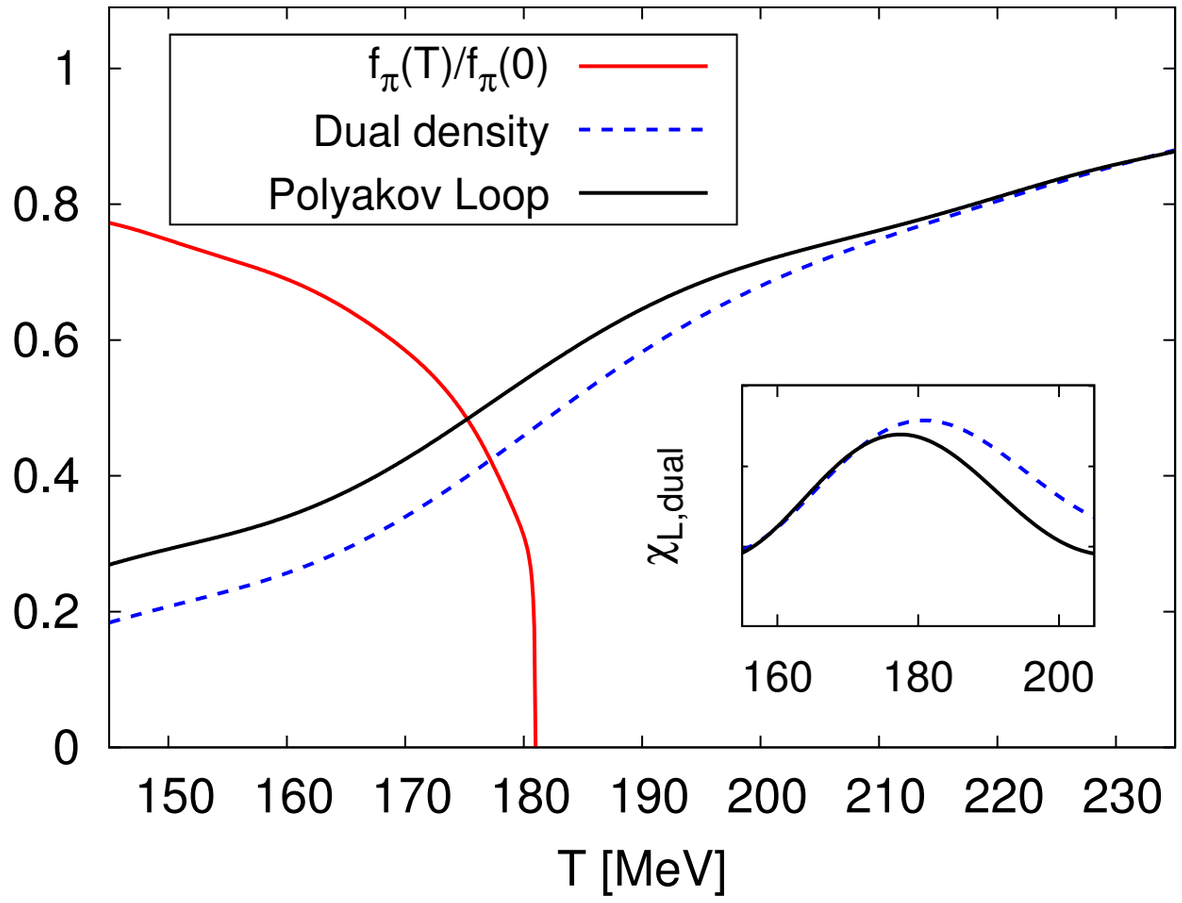
$= 0$



# Full dynamical QCD: $N_f = 2$ & chiral limit



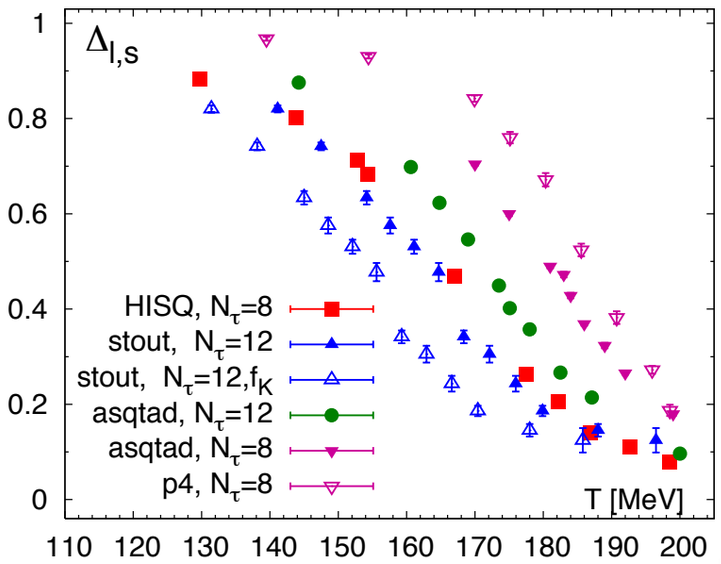
## Phase structure



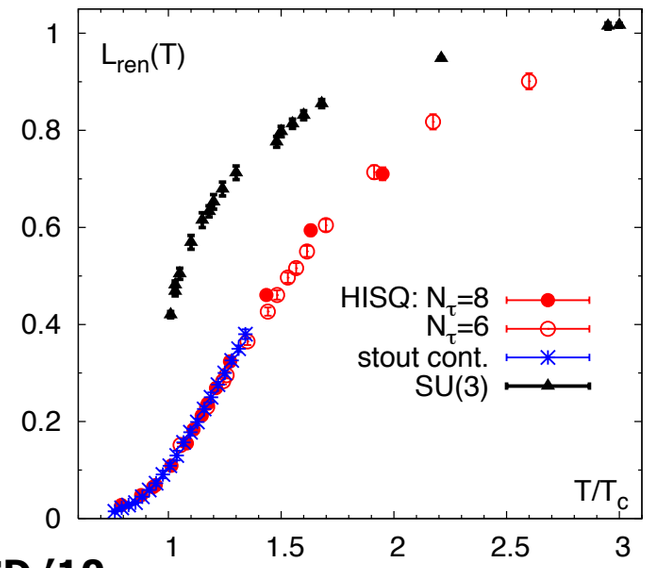

**Braun, Haas, Marhauser, JMP '09**

- $T_\chi \simeq T_{\rm conf} \simeq 180{\rm MeV}$
- **Width** $\Delta T_{\rm conf} \simeq \pm 20{\rm MeV}$
- $T_{\rm conf,FRG} \lesssim T_{\rm conf,lattice}$

### Chiral condensate

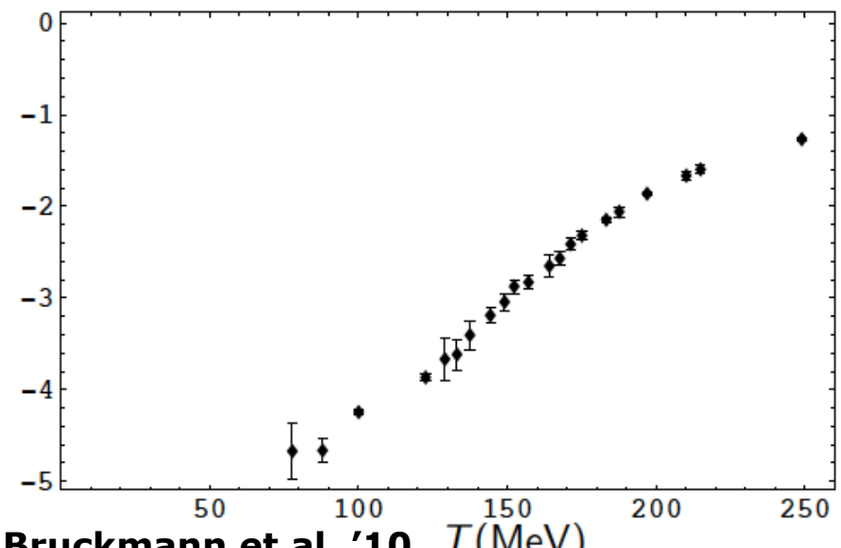



### Polyakov loop



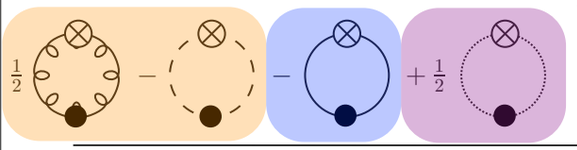

**hotQCD '10**

### Log of dual condensate, m=60 MeV

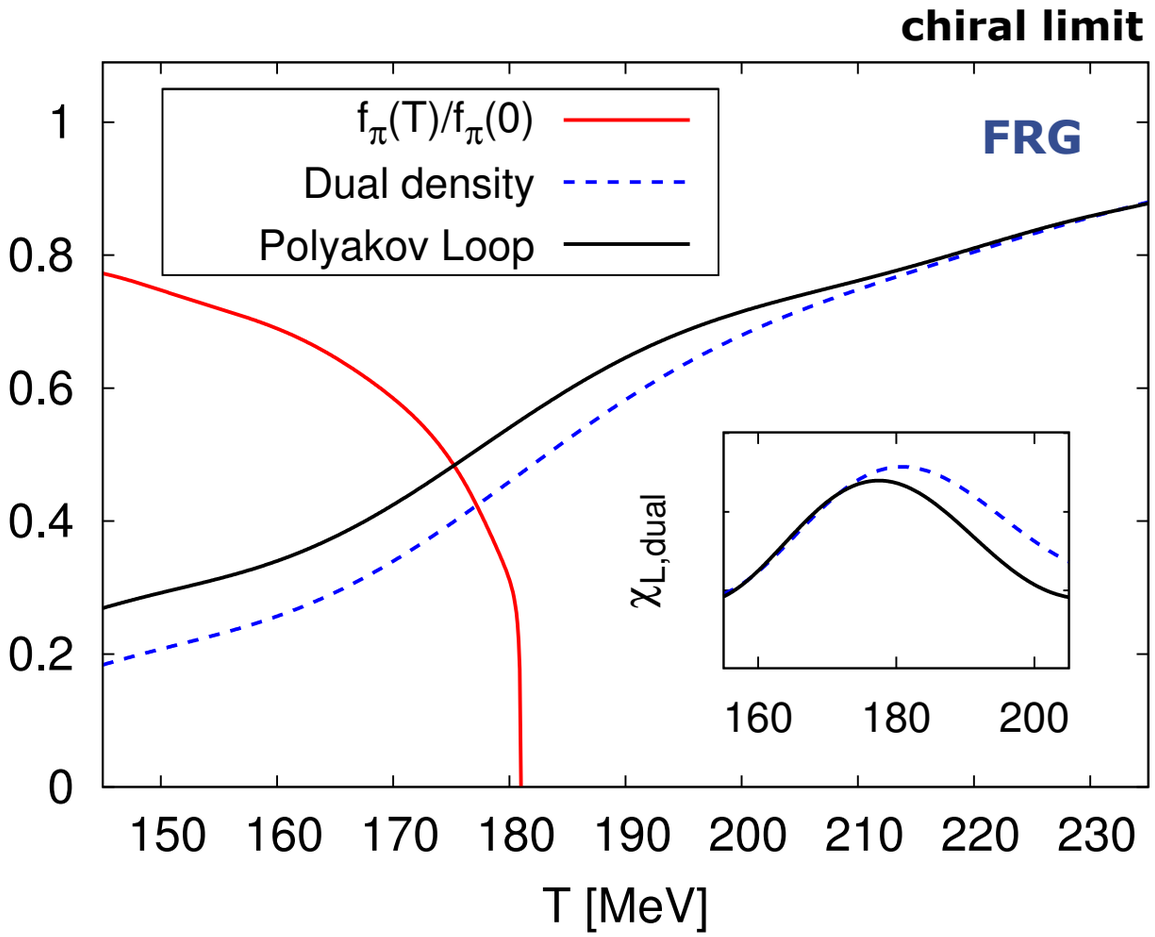



**Bruckmann et al '10**

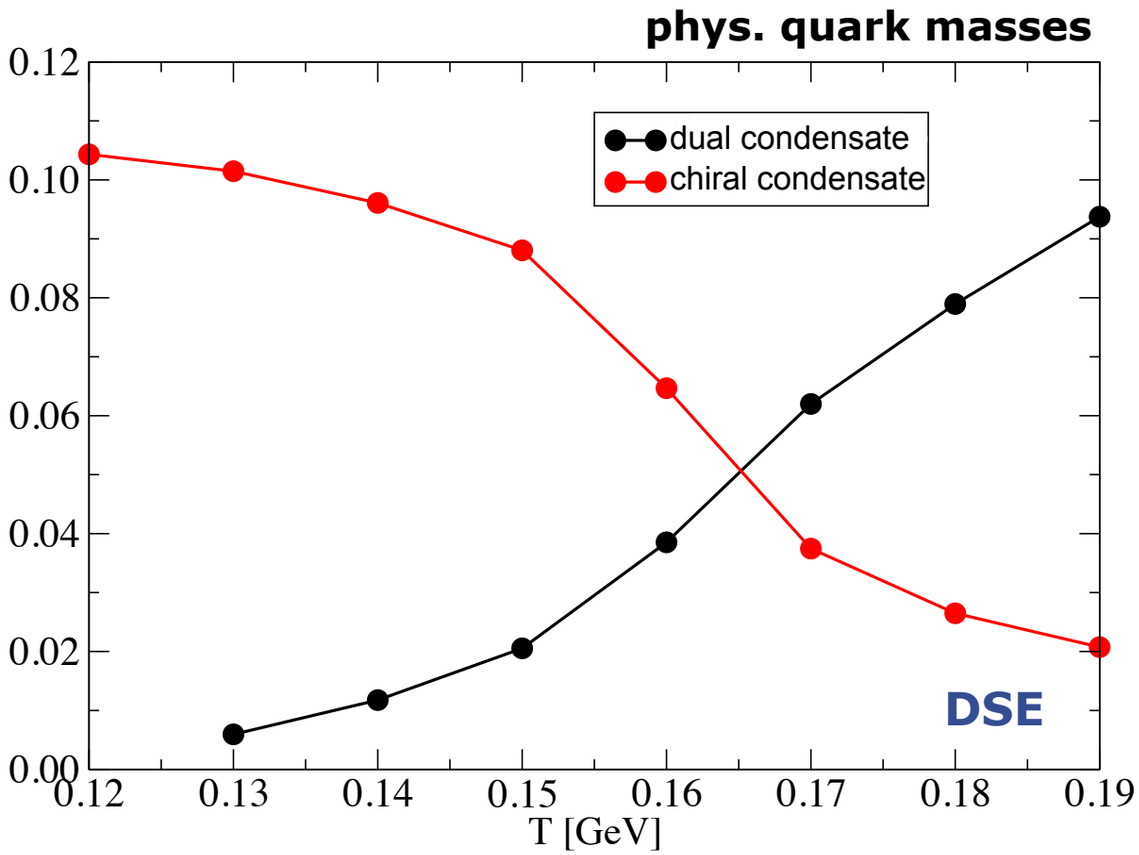
# Full dynamical QCD: $N_f = 2$ & chiral limit



## Phase structure



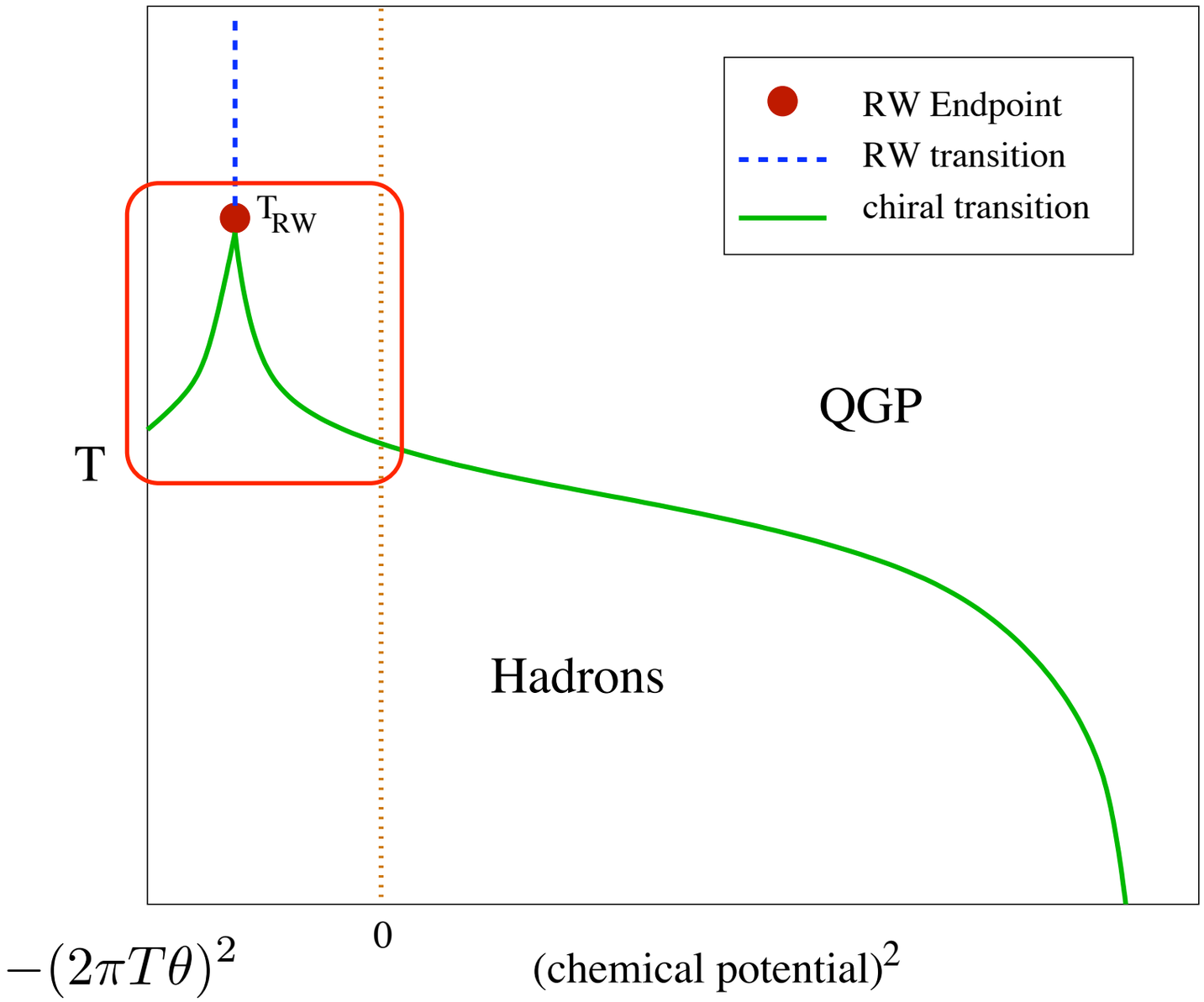

**Braun, Haas, Marhauser, JMP `09**




**Fischer, Lücker, Mueller `11**

# Imaginary chemical potential

$$\psi_\theta(t+\beta,\vec{x}) = -e^{2\pi i\theta}\psi_\theta(t,x) \qquad \textbf{with} \qquad \mu = \mathbf{2\pi i\, T\, \theta}$$



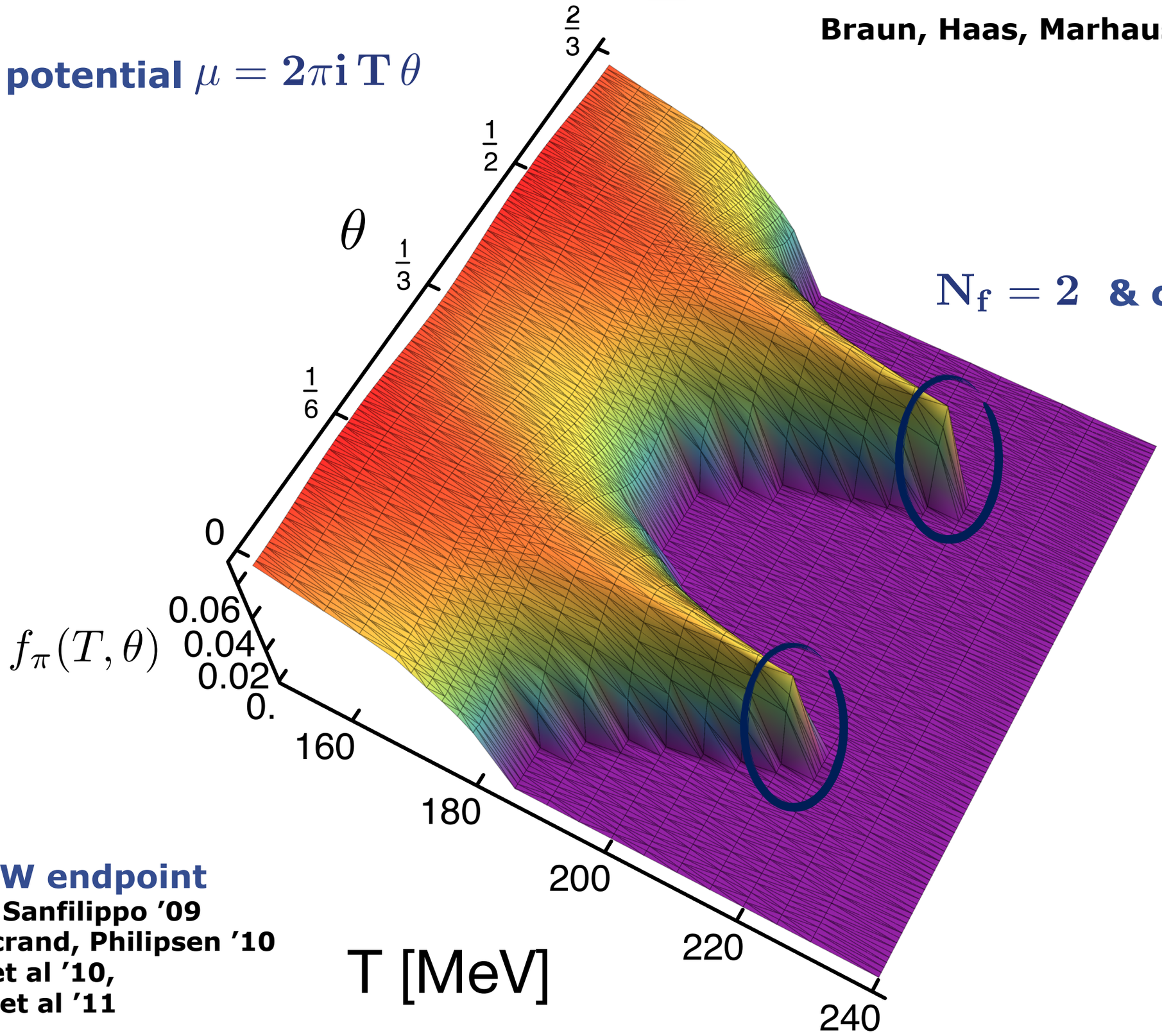

**Roberge-Weiss symmetry:** $\theta \to \theta + 1/3$

# Imaginary chemical potential

## Chiral phase structure

**chemical potential** $\mu = 2\pi \mathrm{i}\, \mathrm{T}\, \theta$

**Braun, Haas, Marhauser, JMP '09**




$\mathrm{N_f} = 2$ **& chiral limit**

**Nature of RW endpoint**

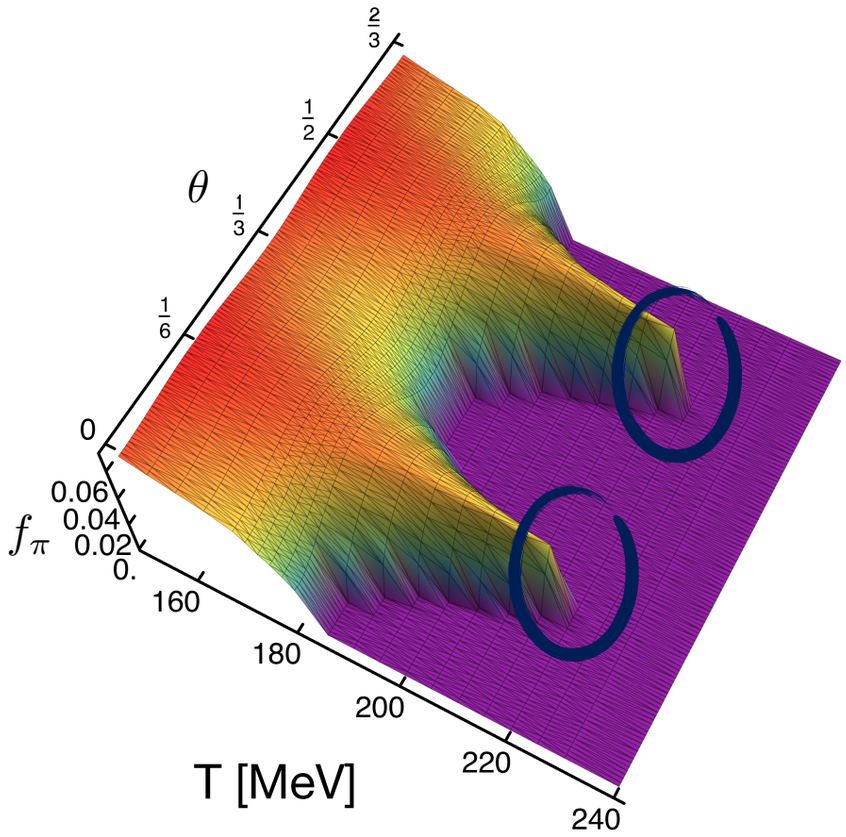
**lattice: D'Elia, Sanfilippo '09**
**de Forcrand, Philipsen '10**

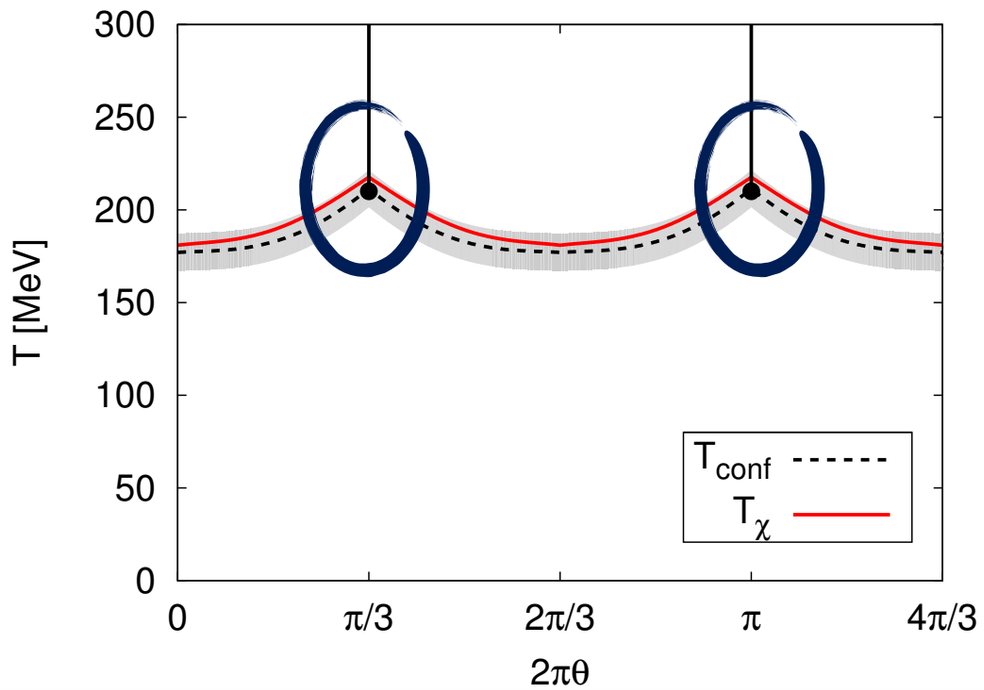
**PNJL: Sakai et al '10,**
**Morita et al '11**

# Imaginary chemical potential

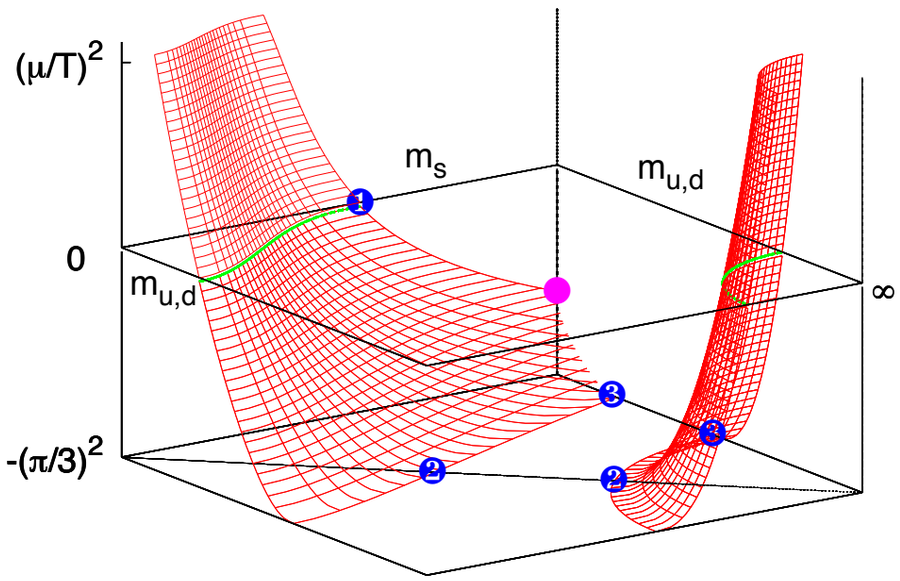
## Nature of the RW endpoint

**chemical potential** $\mu = 2\pi \mathrm{i}\, \mathrm{T}\, \theta$

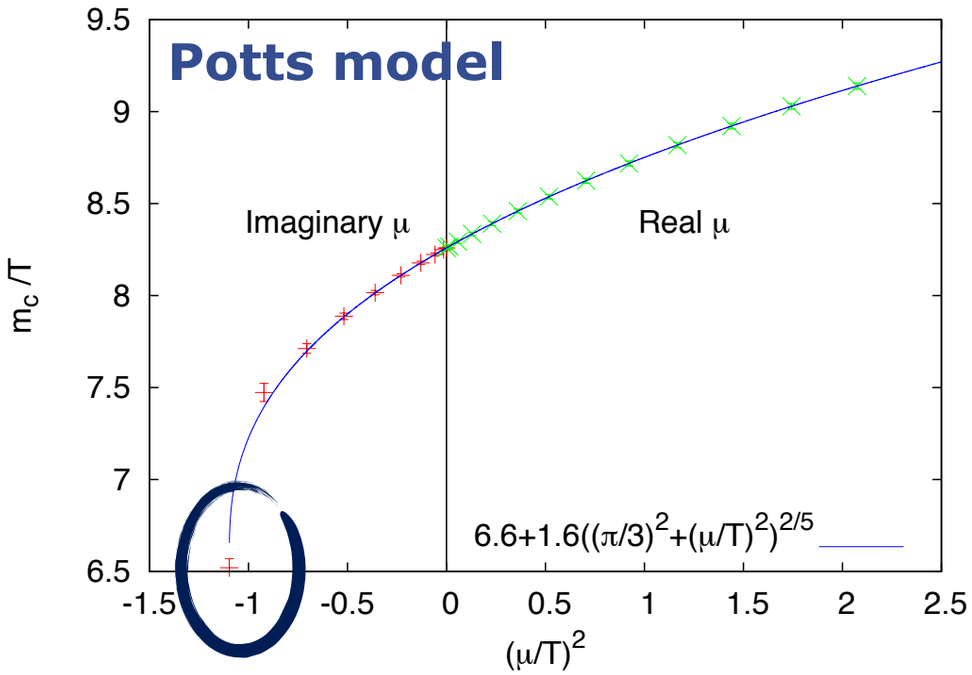



**RW endpoint**




**O. Philipsen '11**

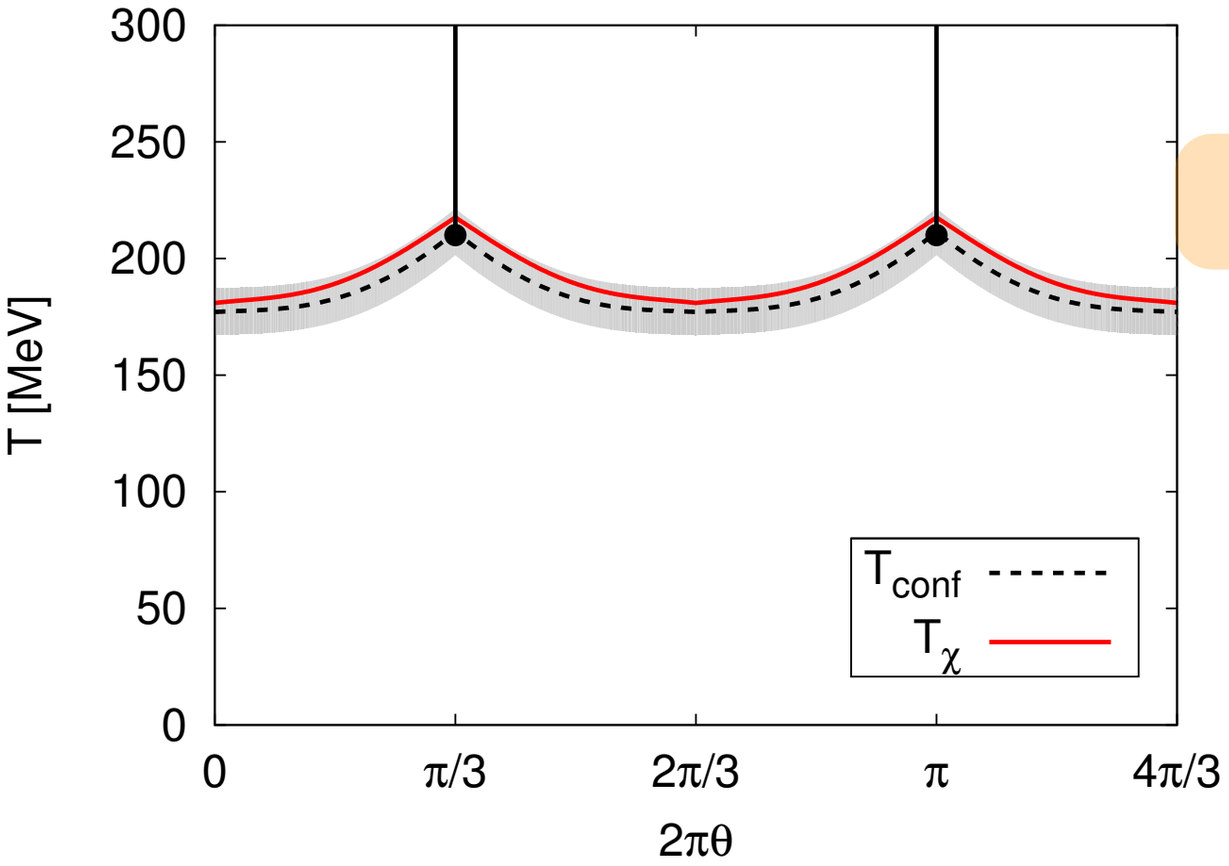


# Imaginary chemical potential

## Phase structure

**chemical potential** $\mu = 2\pi \mathrm{i}\, \mathrm{T}\, \theta$

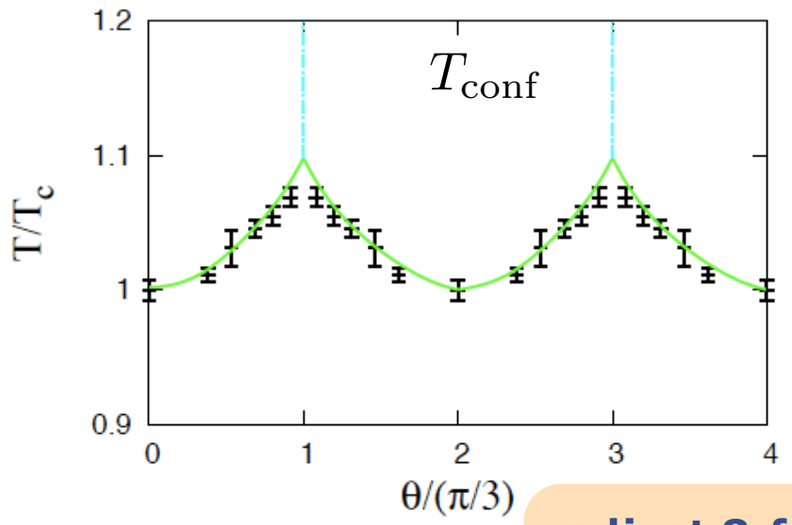
**Braun, Haas, Marhauser, JMP '09**

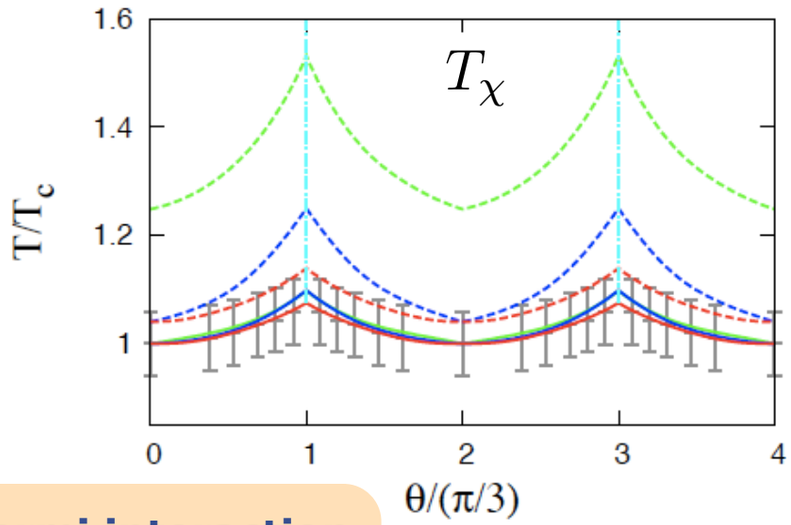



**compatibility**

**lattice results, e.g.**
**Kratochvila et al '06,**
**Wu et al '06,**
**D'Elia et al '07, ....**

**Polyakov-NJL model**
**Sakai et al '09, ...**

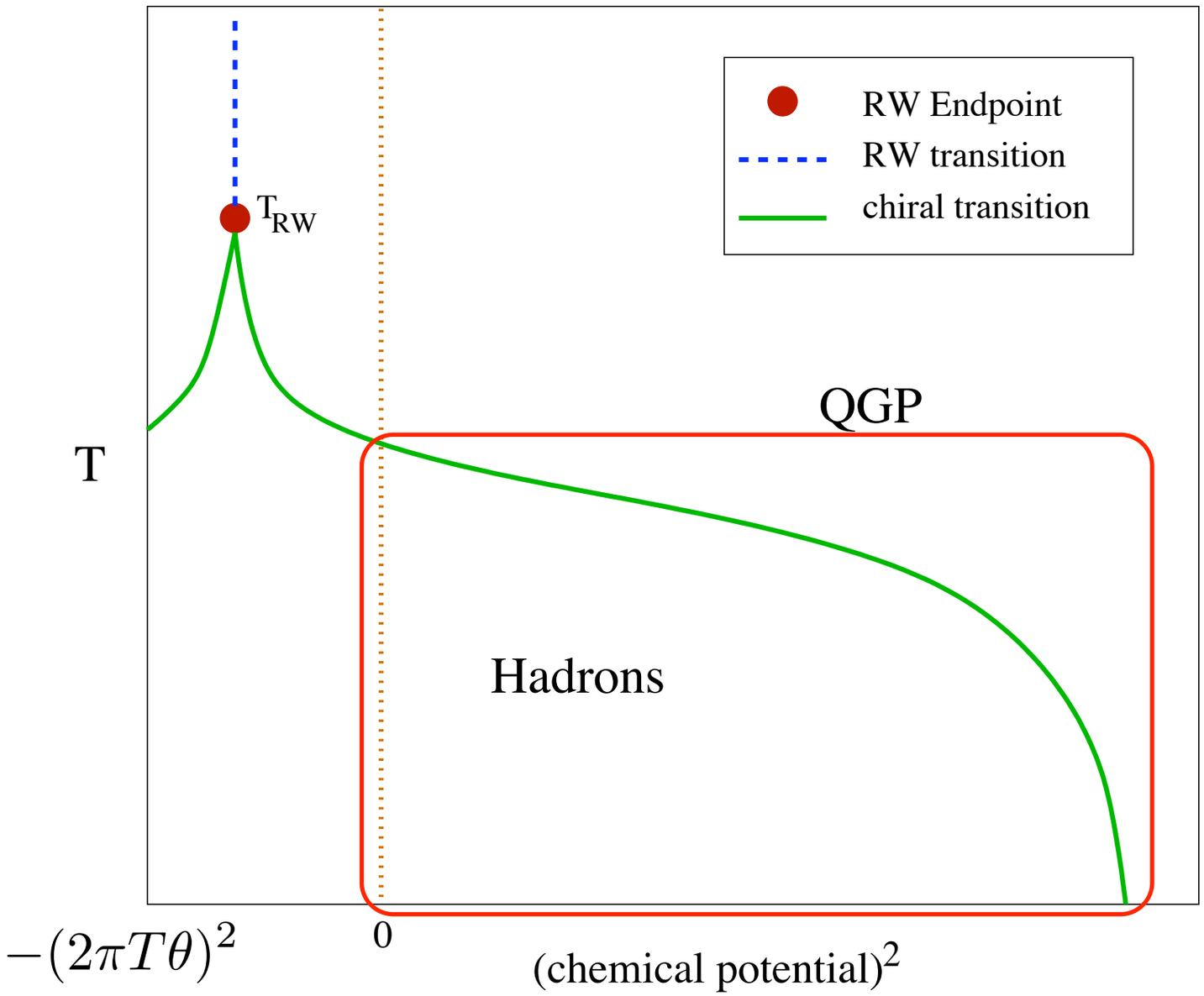



**adjust 8-fermi interaction**

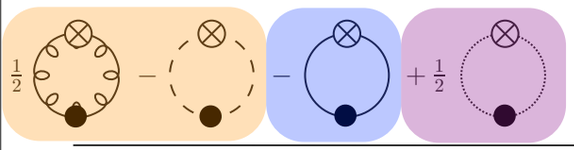
# Real chemical potential

$$\psi_\theta(t+\beta,\vec{x}) = -\psi(t,x)$$

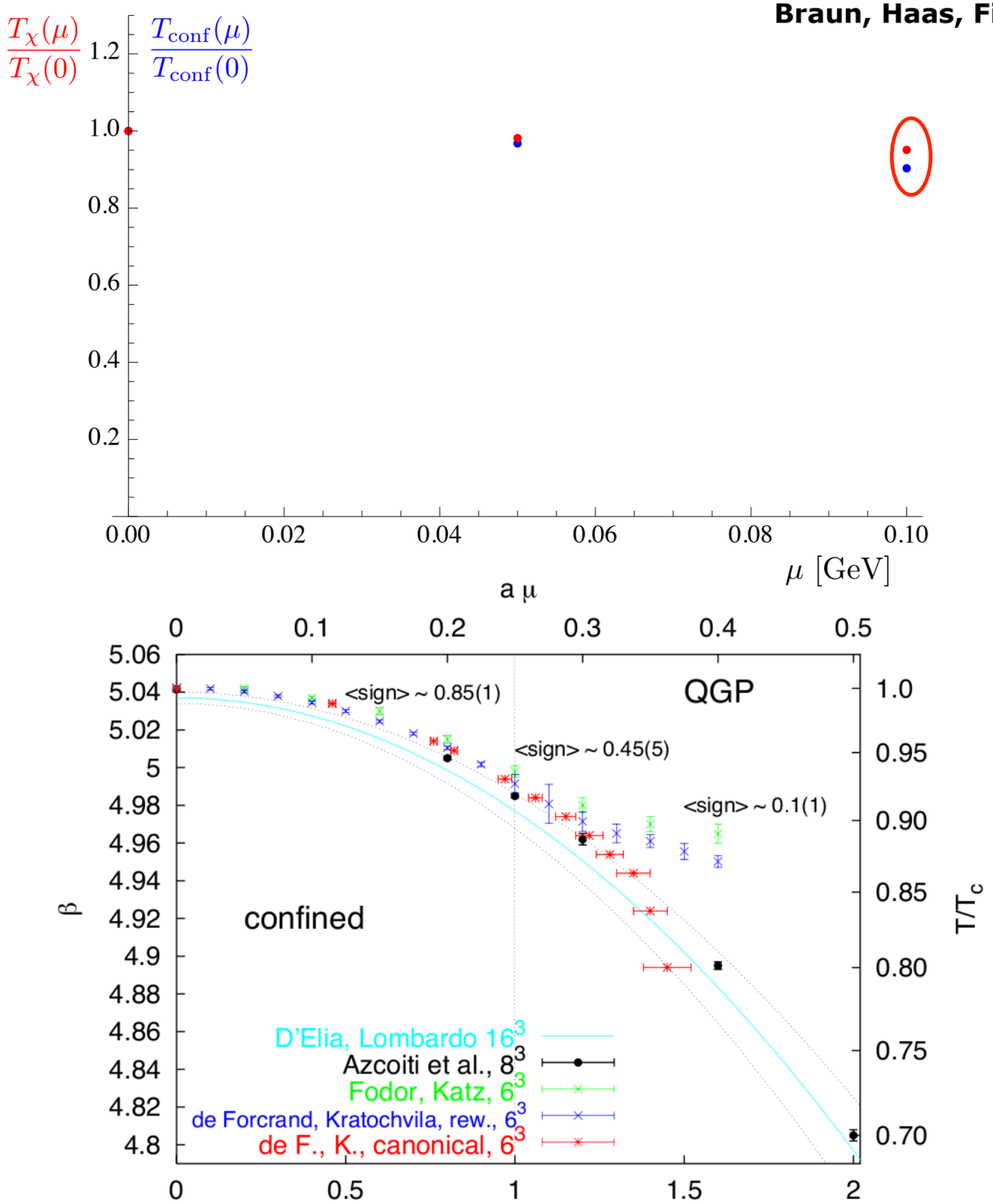


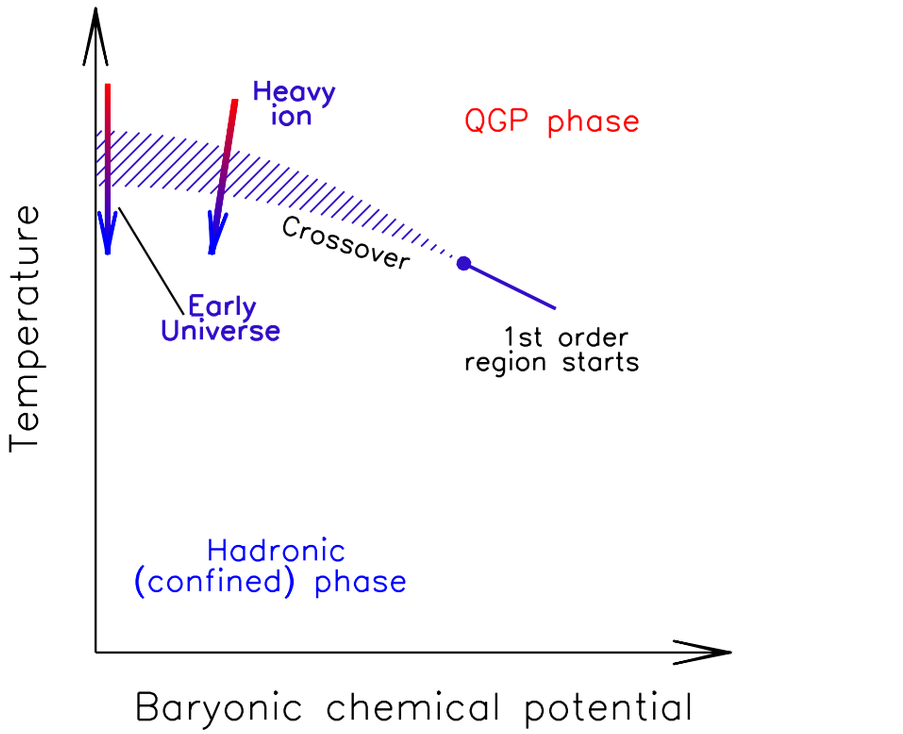
# Real chemical potential

## Full dynamical QCD

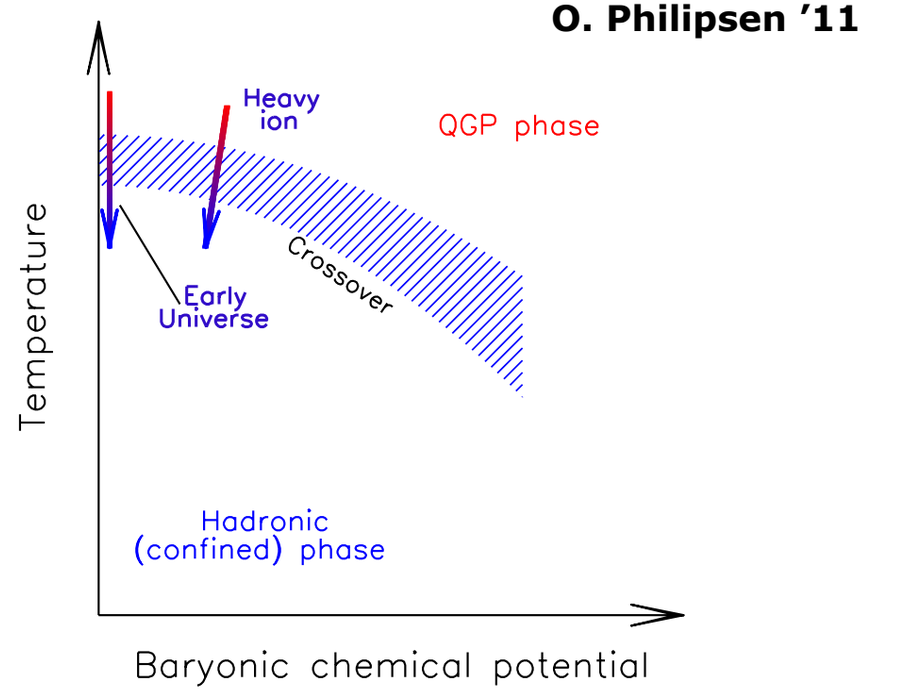



**Braun, Haas, Fister, JMP**



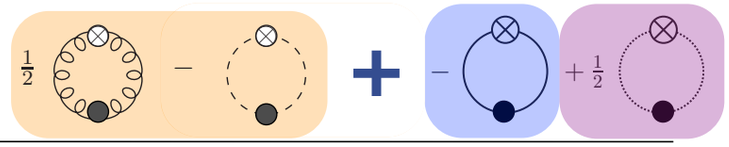




**O. Philipsen '11**

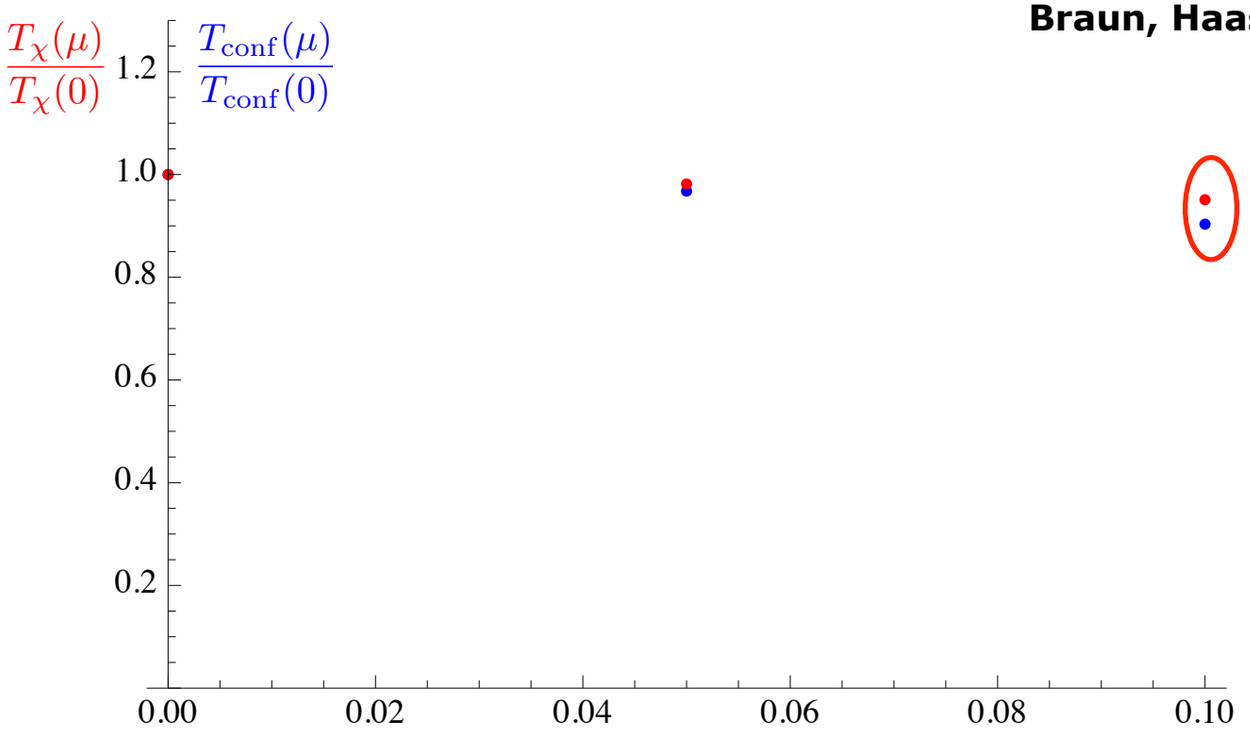


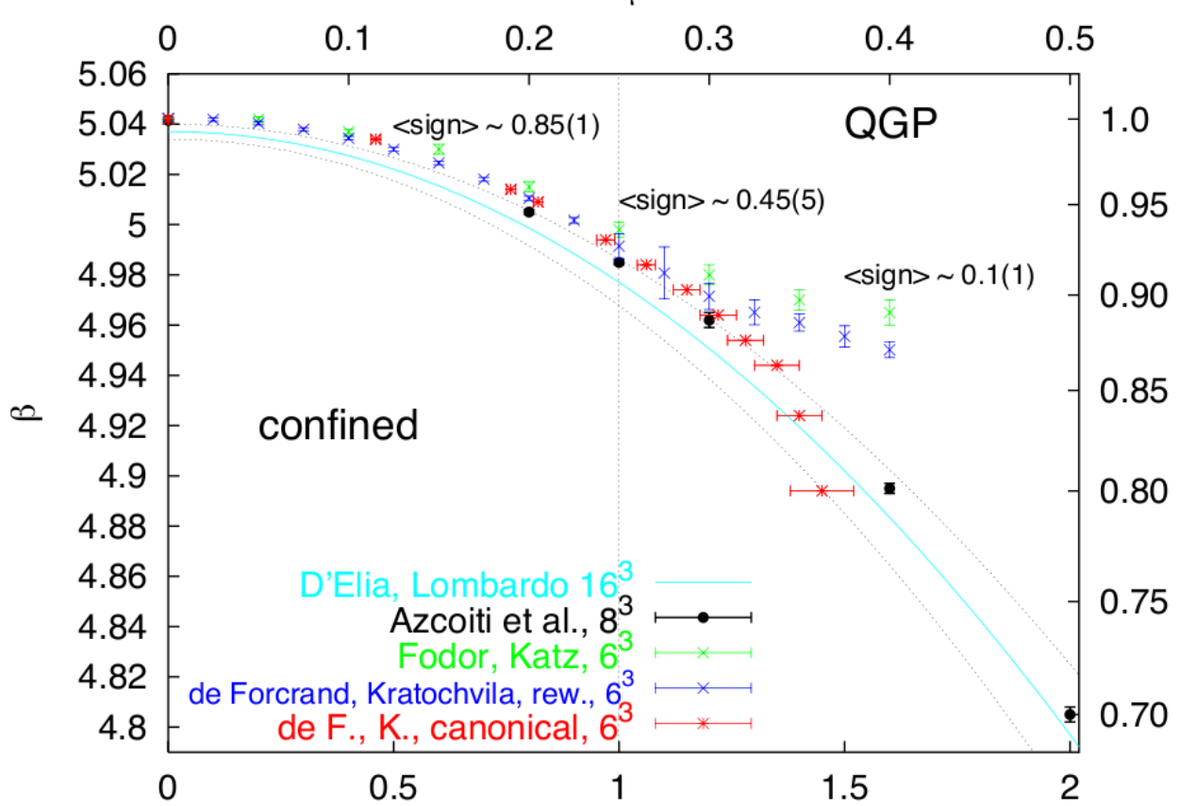
# Real chemical potential

## Full dynamical QCD

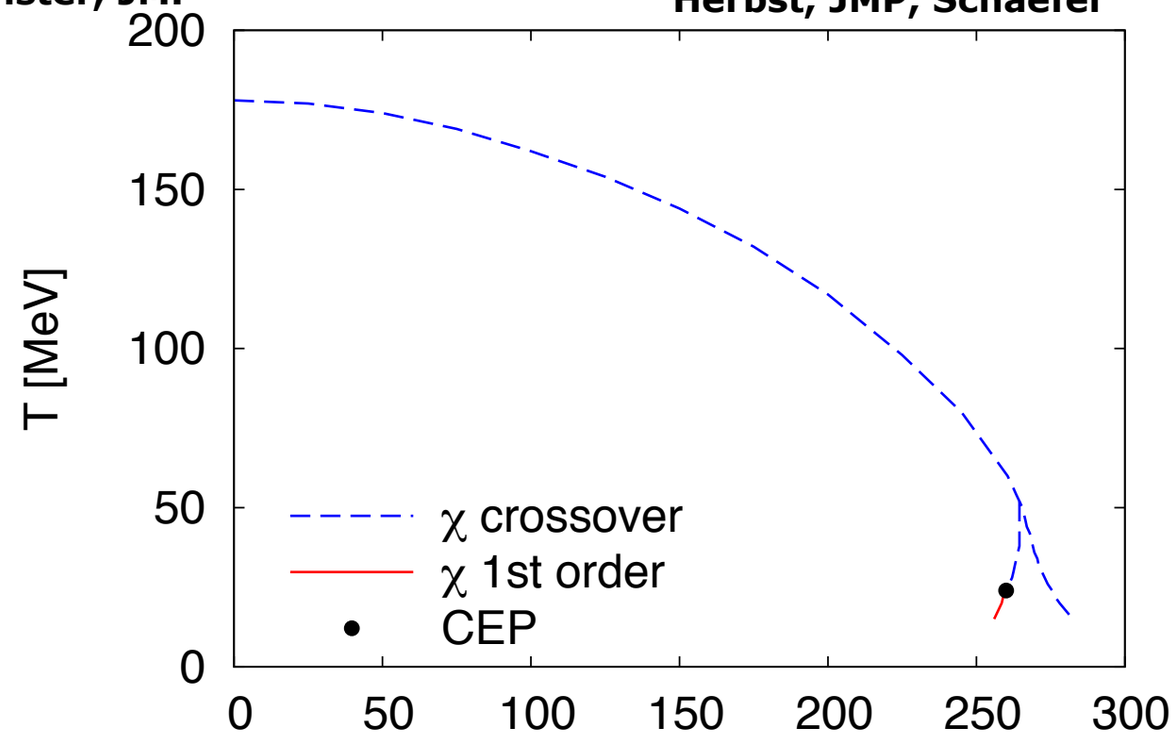


Braun, Haas, Fister, JMP

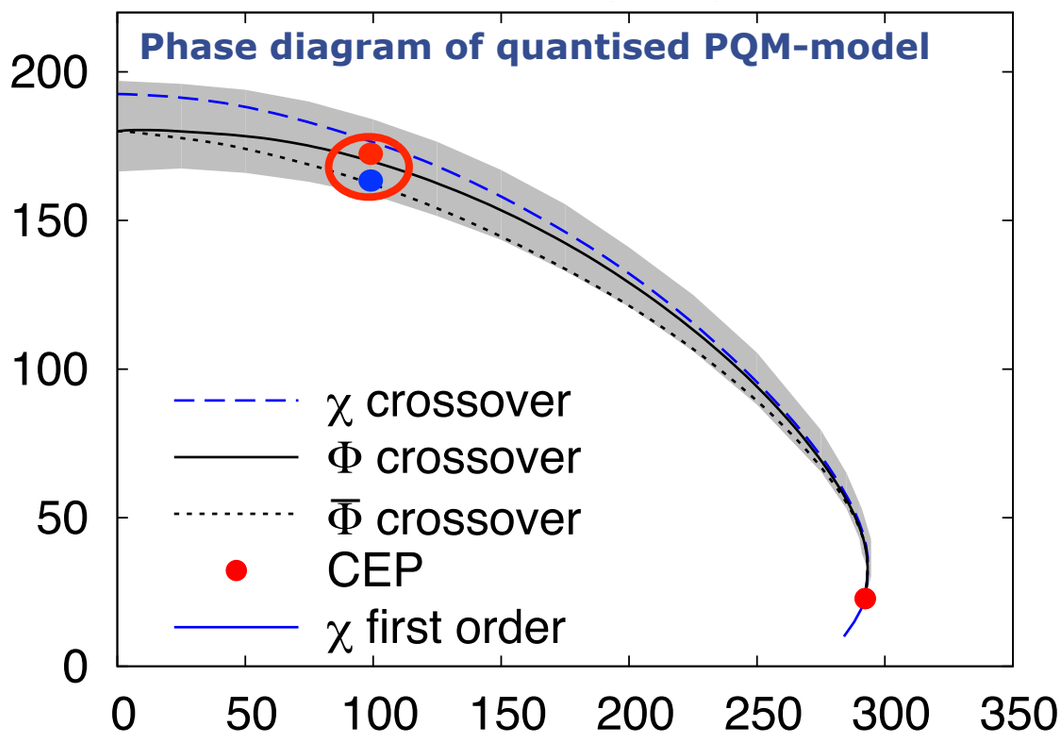



Herbst, JMP, Schaefer

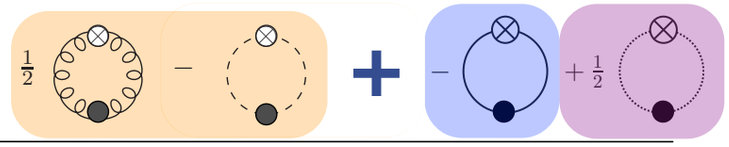



Herbst, JMP, Schaefer '10



# Real chemical potential

## Polyakov-extended models



## Potential

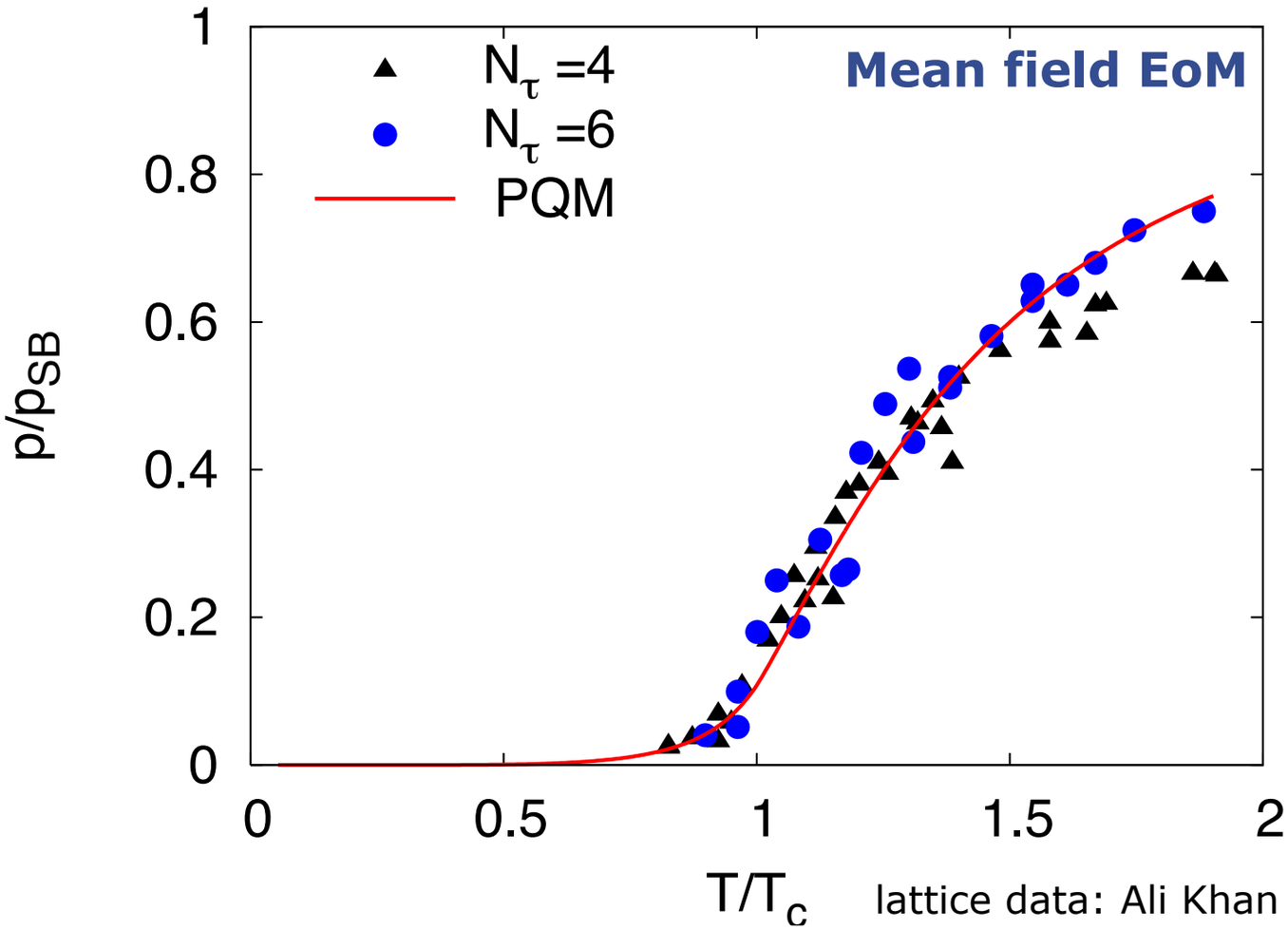
**Polyakov-loop Potential** $U[\Phi, \bar{\Phi}]$ — Fit to YM-thermodynamics

$+$

**Fermionic fluctuations** $\Omega[\Phi, \bar{\Phi}, \sigma, \vec{\pi}]$ — One loop computation

$+$

**Mesonic potential** $V[\sigma, \vec{\pi}]$ — Fit of meson phenomenology




lattice data: Ali Khan et al '01

Meisinger, Ogilvie '96

Pisarski '00

PNJL

Fukushima '04

Ratti, Thaler, Weise '06

Megias, Ruiz Arriola, Salcedo '06

Ghosh, Mukherjee, Mustafa, Ray '06

C. Sasaki, B. Friman and K. Redlich '06

PQM

Schaefer, JMP, Wambach '07

# Real chemical potential

## dynamical Polyakov-extended models

## Potential

Herbst, JMP, Schaefer ’10

**Polyakov-loop Potential** $U[\Phi,\bar{\Phi}]$ — Fit to YM-thermodynamics

$+$ **Fermionic fluctuations** $\Omega[\Phi,\bar{\Phi},\sigma,\vec{\pi}]$ — fermionic fluctuations

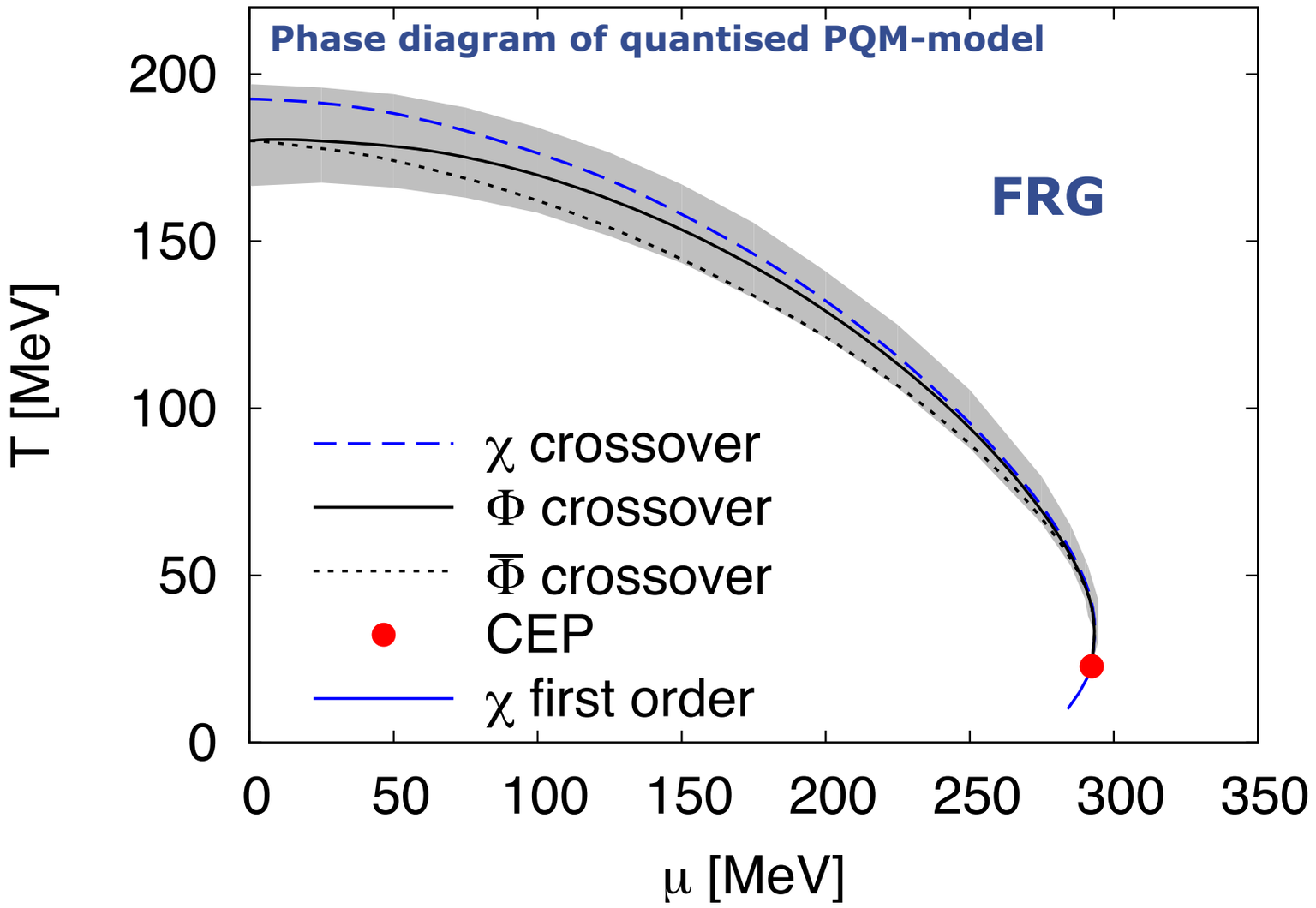
$+$ **Mesonic potential** $V[\sigma,\vec{\pi}]$ — mesonic fluctuations

**quark fluctuations change glue dynamics**

$$T_{0\mathrm{YM}} \to T_0(N_f,\mu;m_q)$$

**estimated via HTL/HDL computation**

**Schaefer, JMP, Wambach ’07**

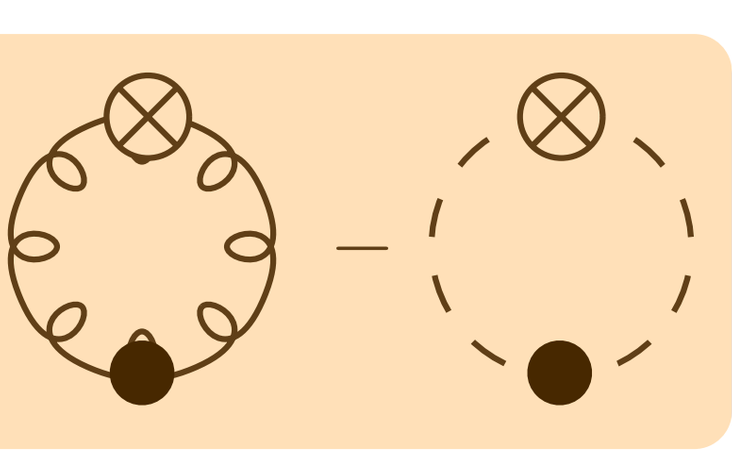
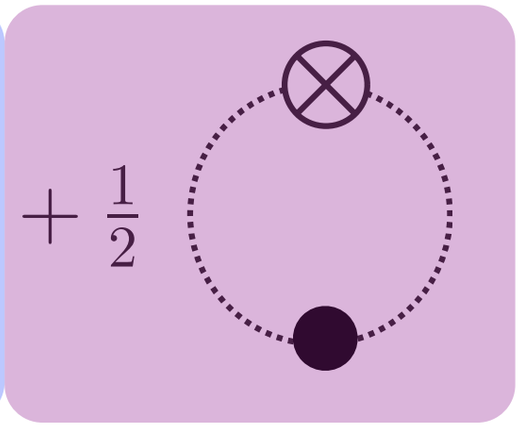


# Real chemical potential

## Polyakov-extended models as reduced QCD

## Potential

**Polyakov-loop Potential** $U[\Phi, \bar{\Phi}]$ + **Fermionic fluctuations** $\Omega[\Phi, \bar{\Phi}, \sigma, \vec{\pi}]$ + **Mesonic potential** $V[\sigma, \vec{\pi}]$

$$\partial_t \Gamma_k[\phi] = \frac{1}{2}$$  $$+ \frac{1}{2}$$ 

**Flow equation for QCD**

# Real chemical potential

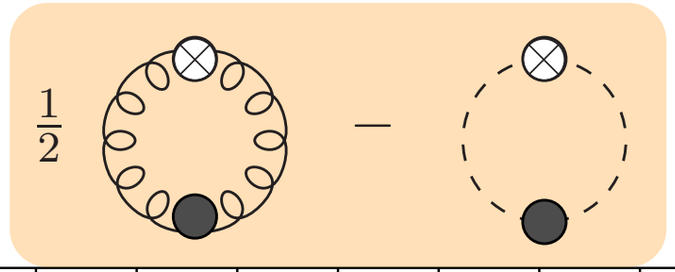
## Polyakov-extended models as reduced QCD

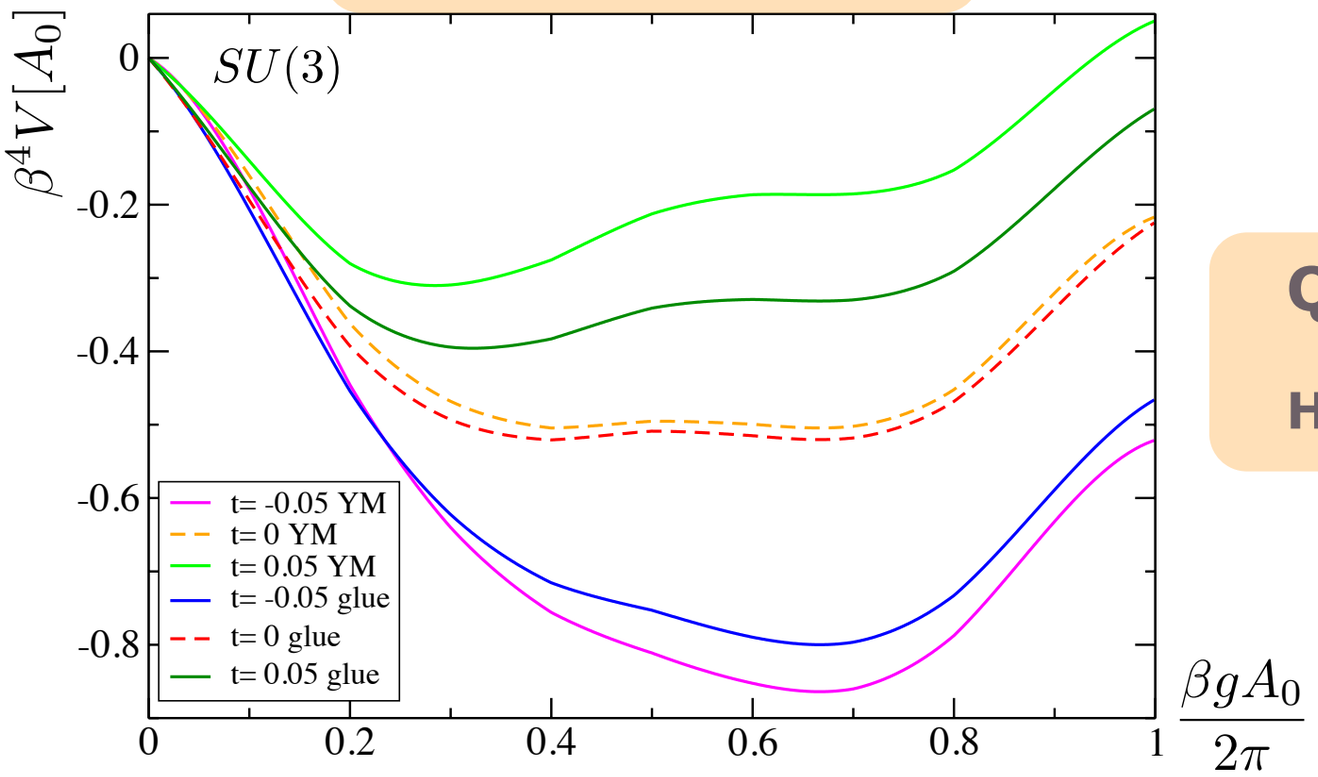
## Towards QCD

Braun, Fister, Haas, JMP

JMP '10

**Polyakov-loop Potential** $U[\Phi, \bar{\Phi}]$ + **Fermionic fluctuations** $\Omega[\Phi, \bar{\Phi}, \sigma, \vec{\pi}]$ + **Mesonic potential** $V[\sigma, \vec{\pi}]$





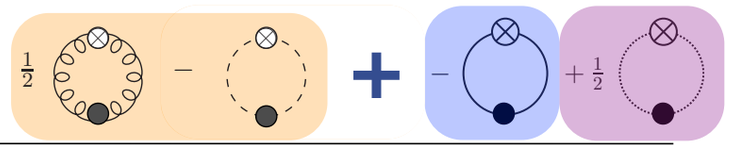

**QCD confirmation of**

**HTL/HDL quark estimate**

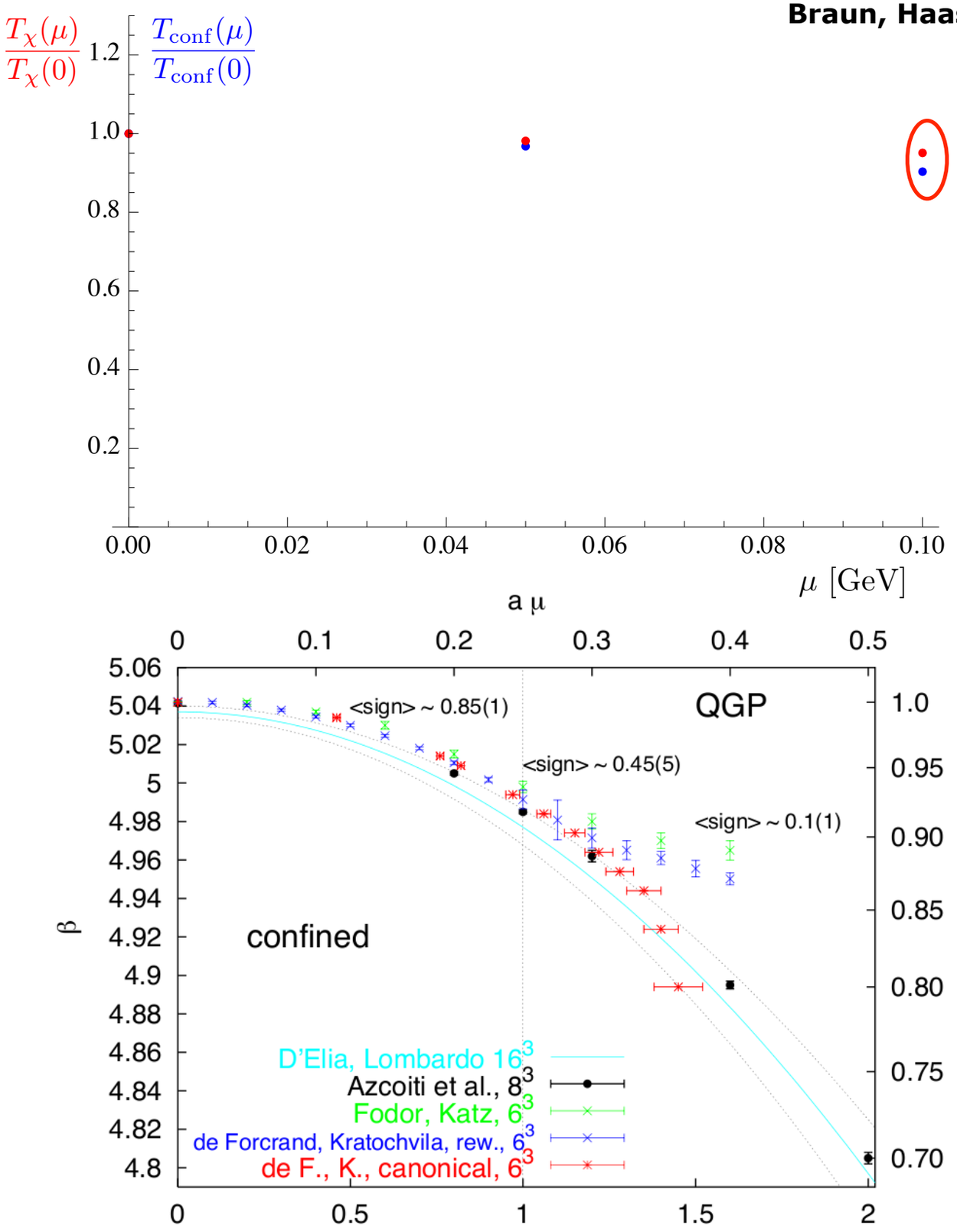
$$(\beta^4 V)_{\rm glue}[t\,,\,A_0] \simeq (\beta^4 V)_{\rm YM}[t_{\rm YM}(t)\,,\,A_0]$$

# Real chemical potential

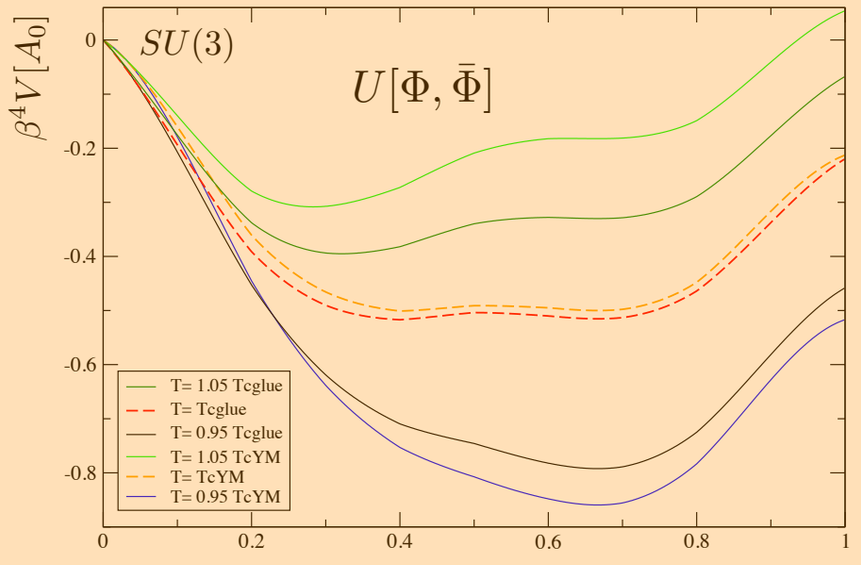
## Full dynamical QCD



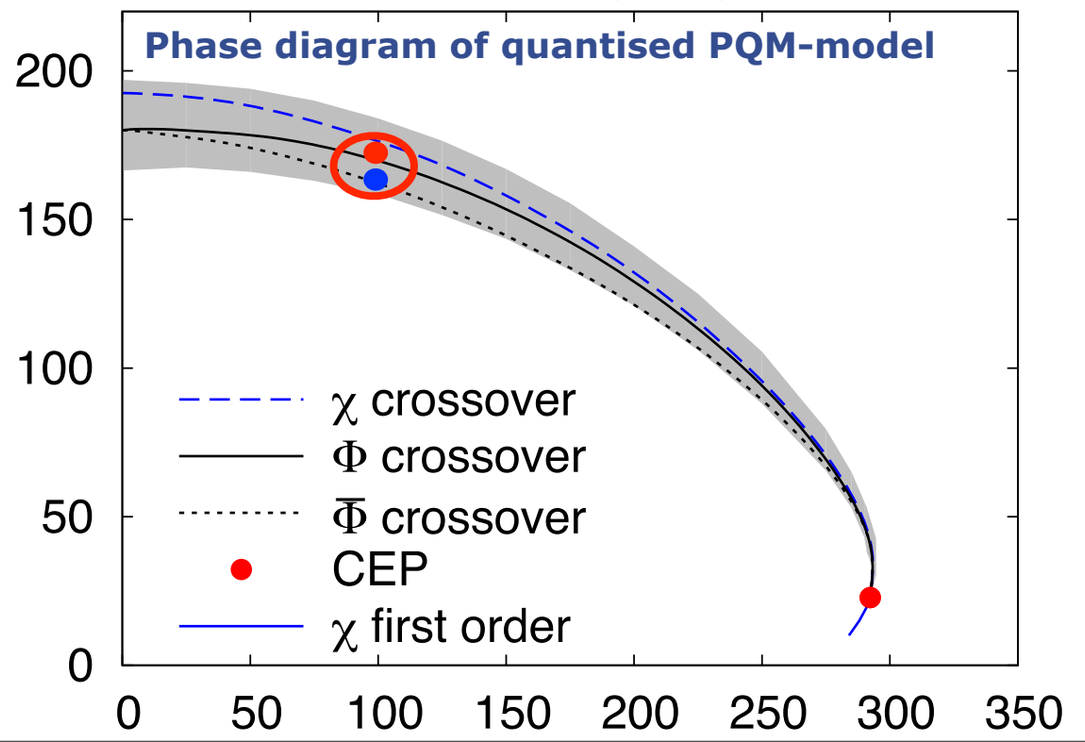
**Braun, Haas, Fister, JMP**




### Polyakov loop potential in full QCD

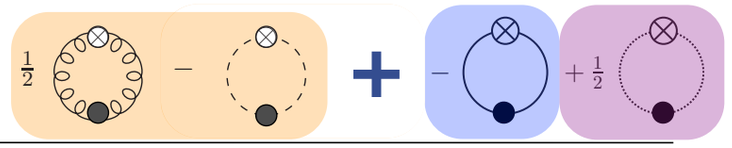



**Herbst, JMP, Schaefer '10**

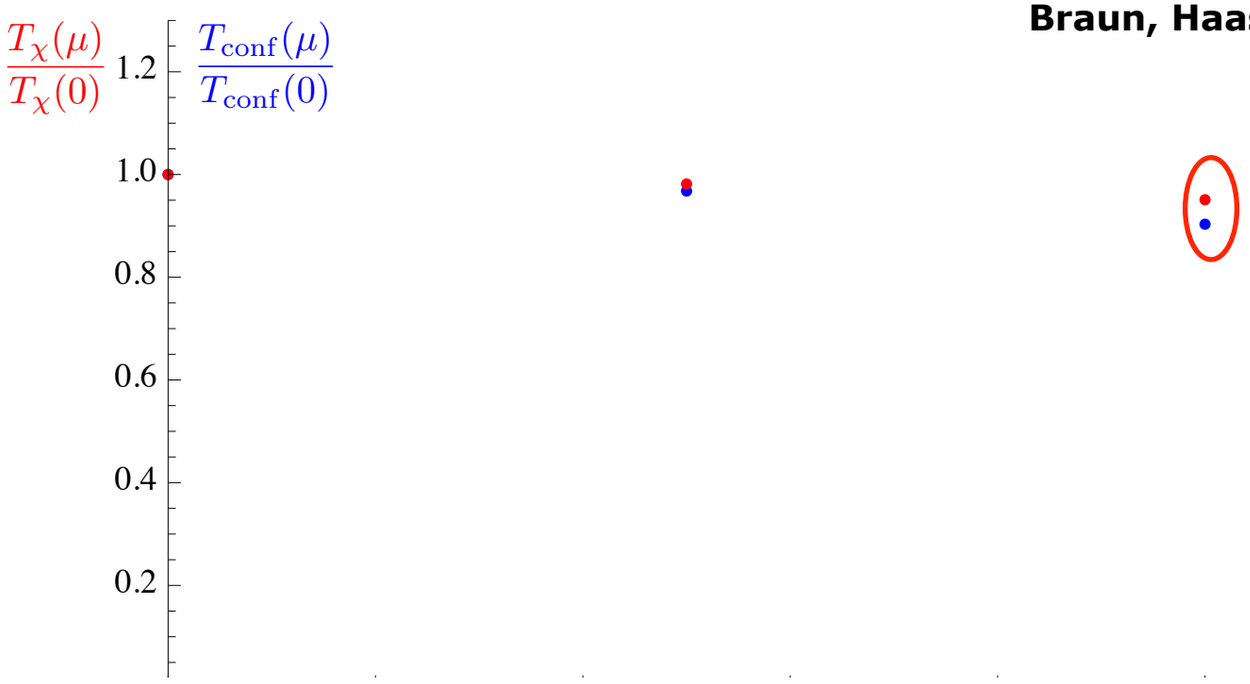


# Real chemical potential

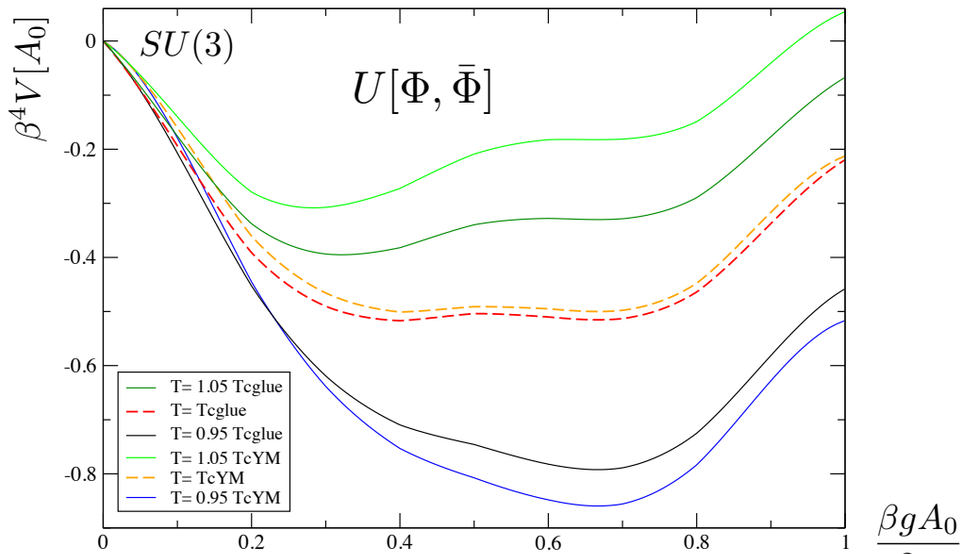
## Full dynamical QCD

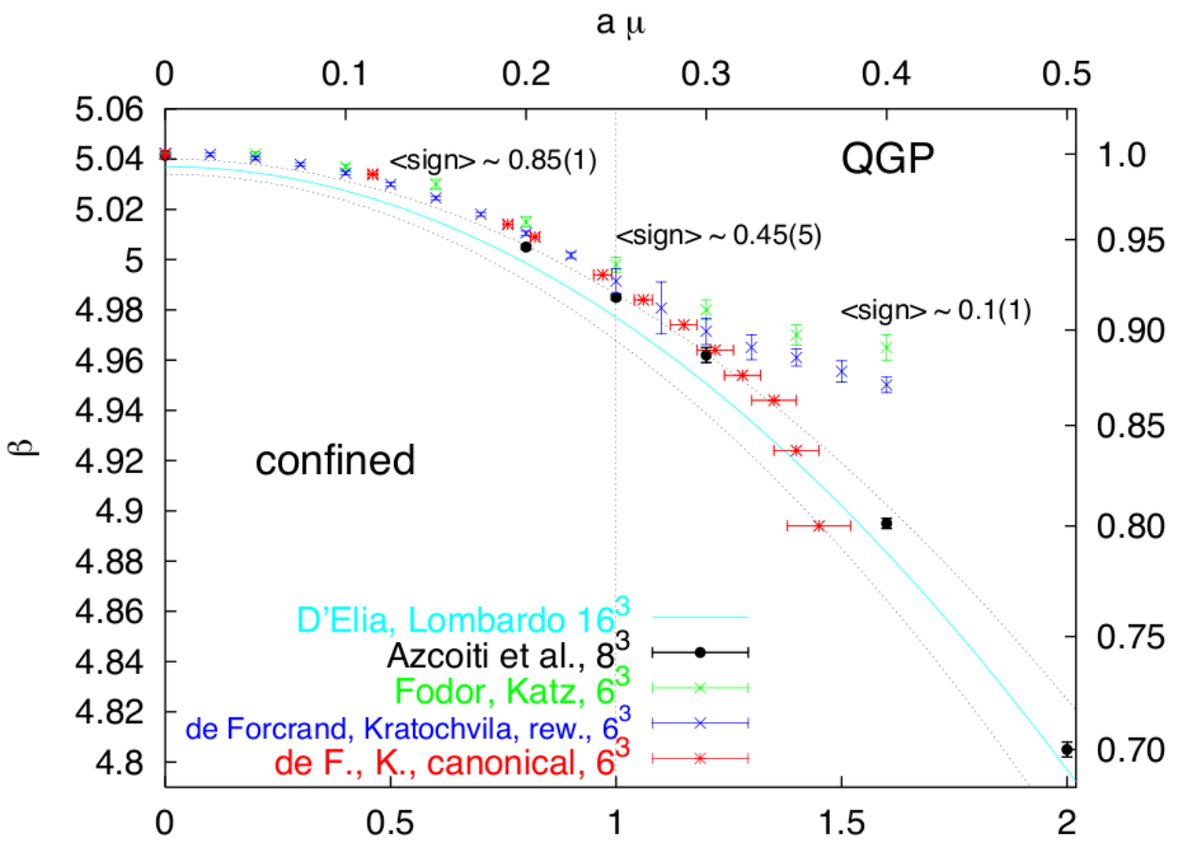


**Braun, Haas, Fister, JMP**



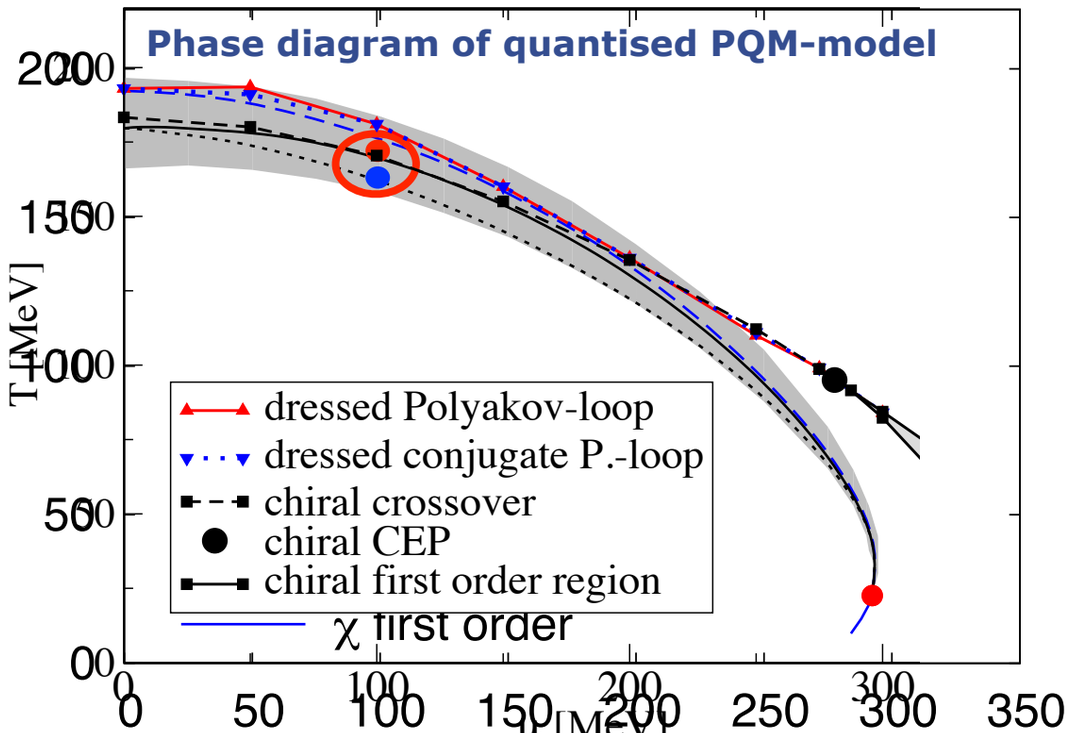

**Polyakov loop potential in full QCD**




**DSE: Fischer, Lücker, Mueller '11**

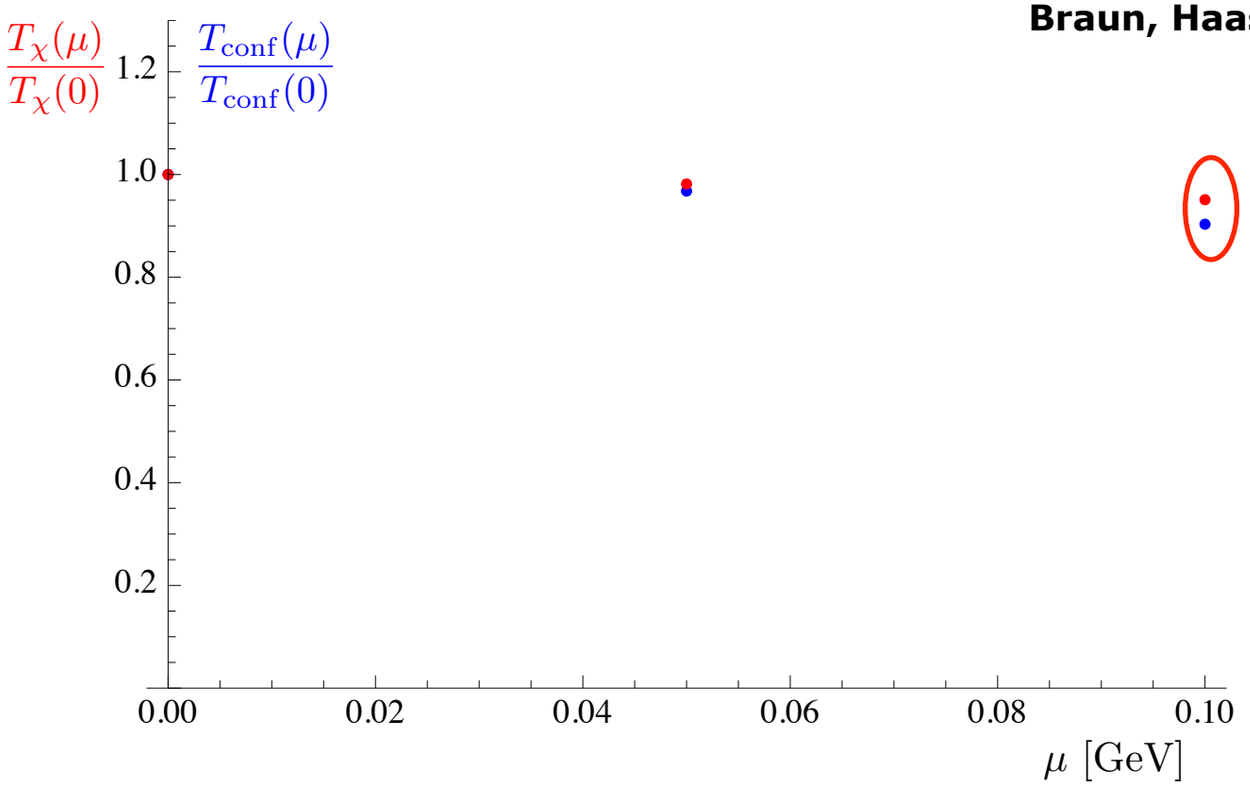
**Herbst, JMP, Schaefer '10**



# Real chemical potential

## Full dynamical QCD

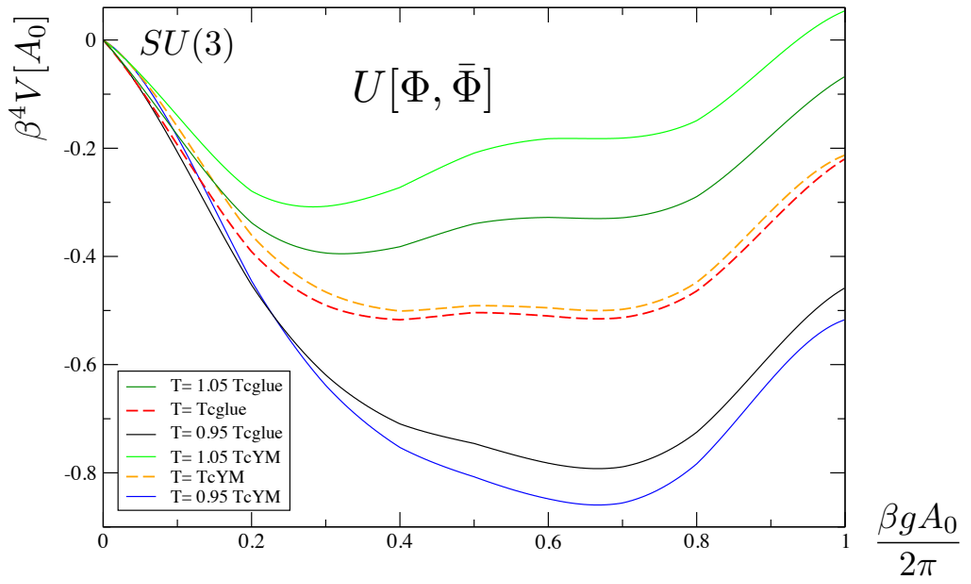
**Braun, Haas, Fister, JMP**




**Critical point unlikely for**

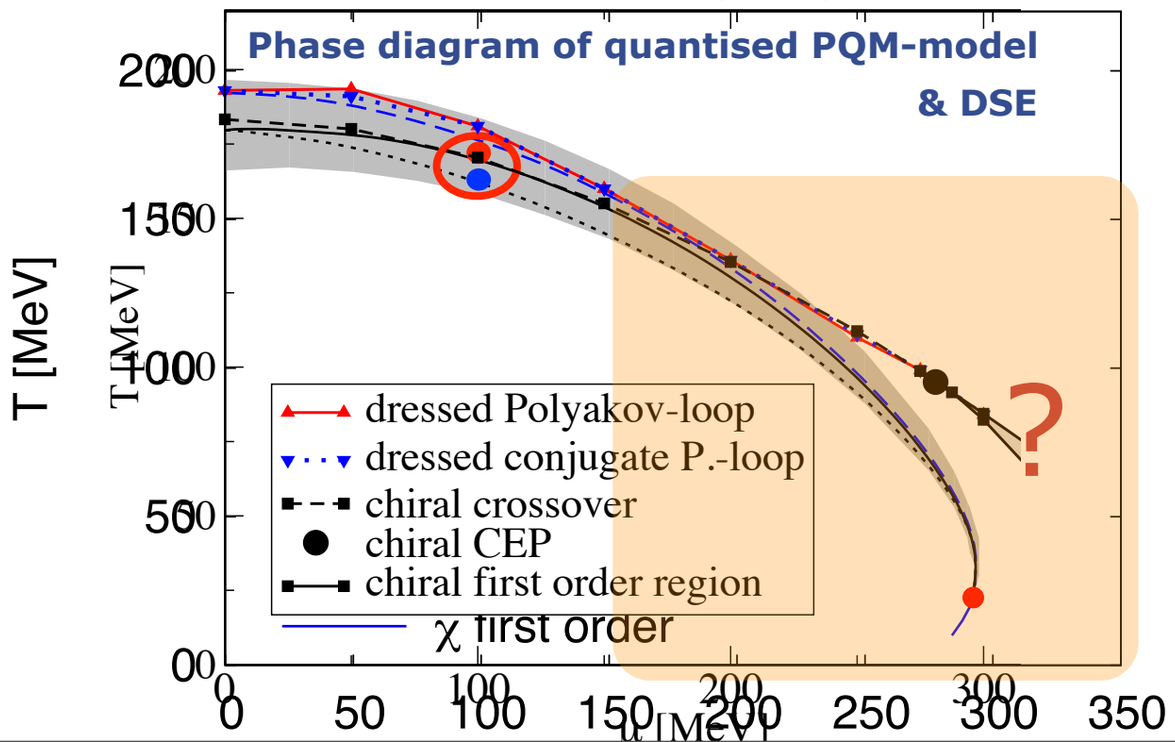
$$\frac{\mu_B}{T} < 2$$

**Polyakov loop potential in full QCD**




**DSE: Fischer, Lücker, Mueller '11**

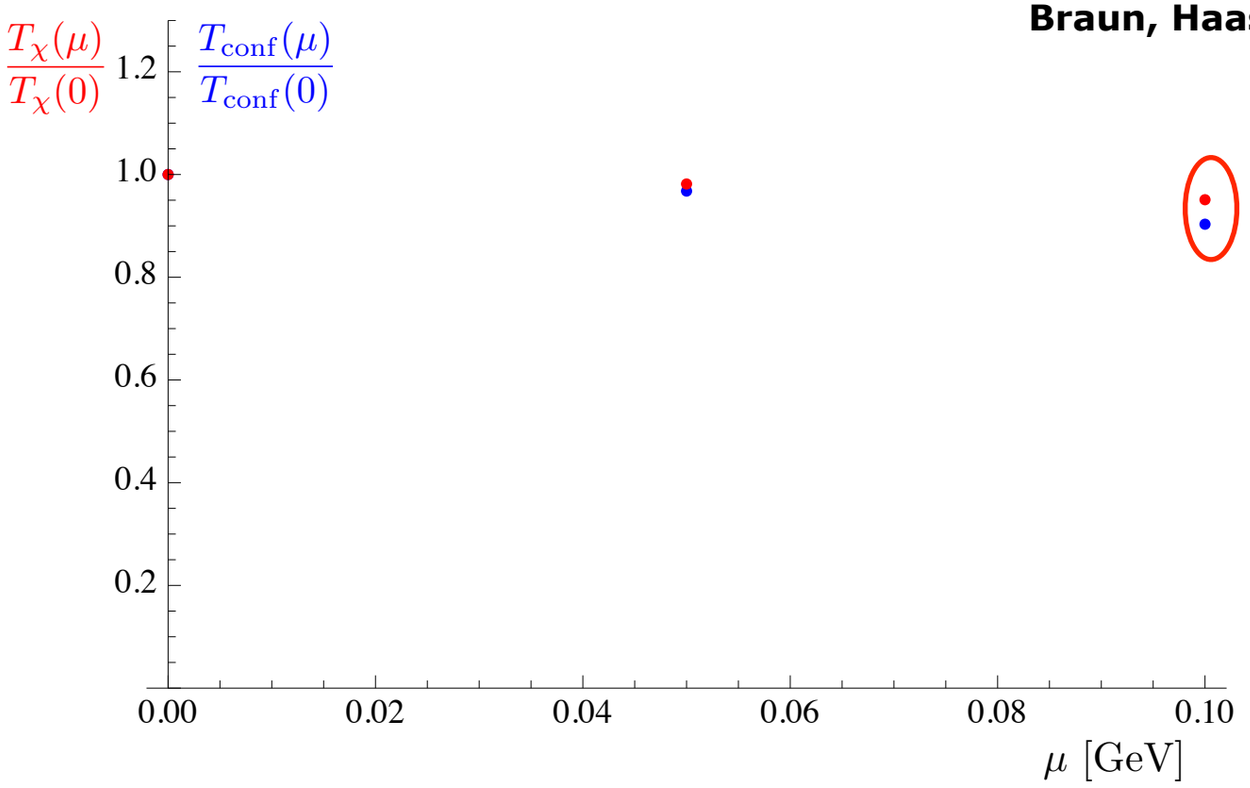
**Herbst, JMP, Schaefer '10**



# Real chemical potential

## Full dynamical QCD

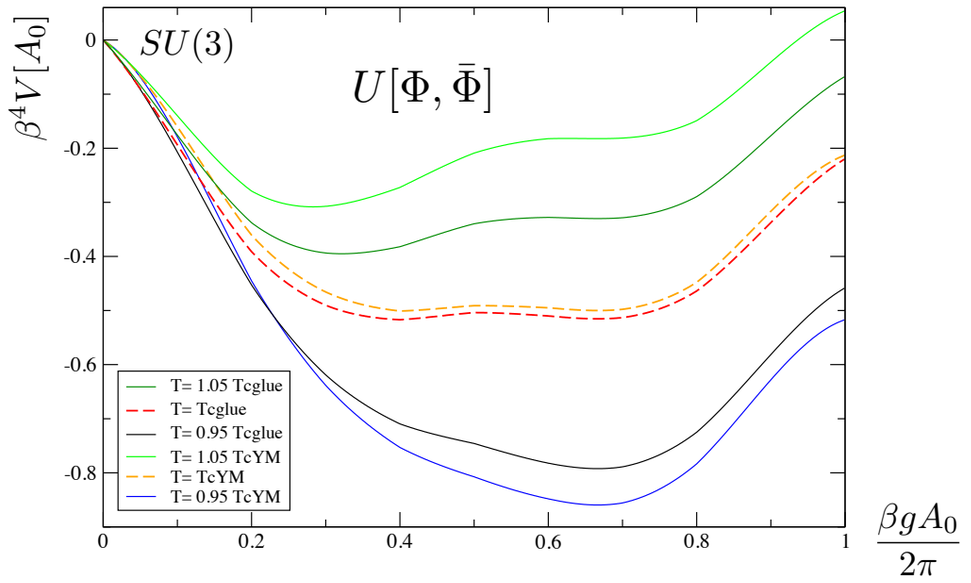
**Braun, Haas, Fister, JMP**




**Critical point unlikely for**

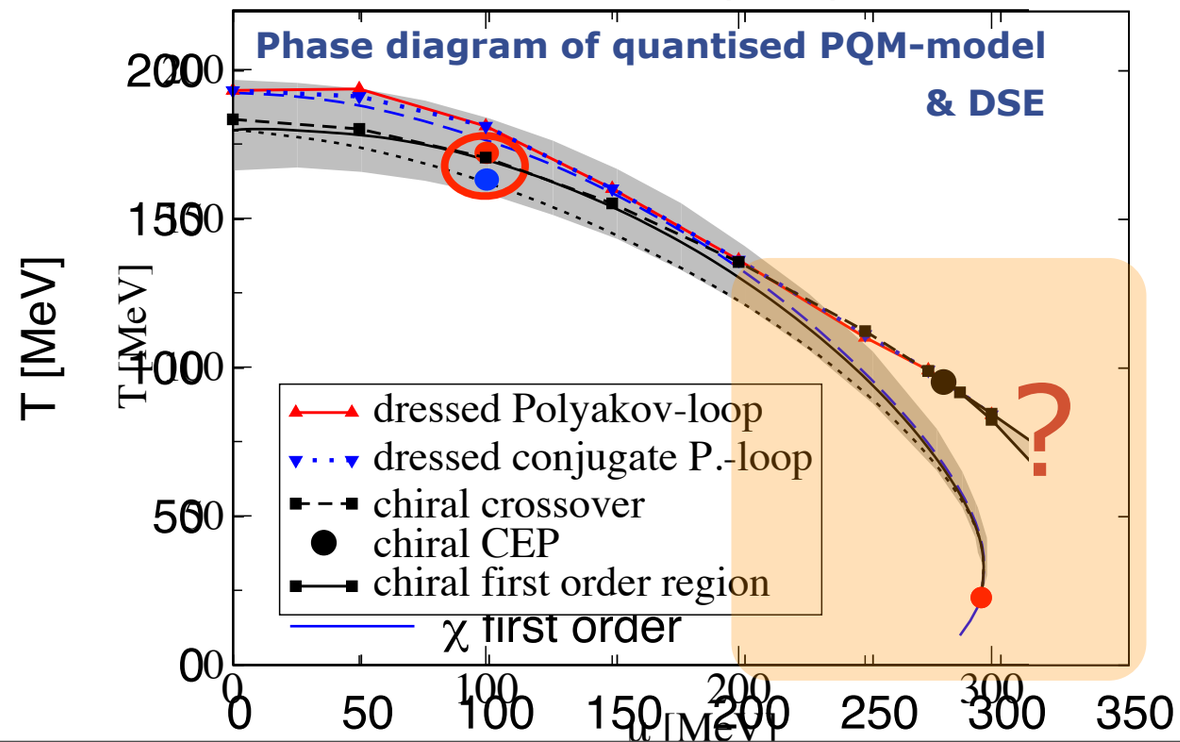
$$\frac{\mu_B}{T} < 4.5$$

**Polyakov loop potential in full QCD**



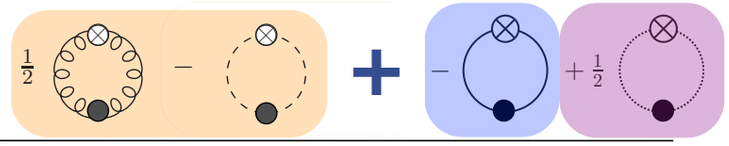

**DSE: Fischer, Lücker, Mueller '11**

**Herbst, JMP, Schaefer '10**

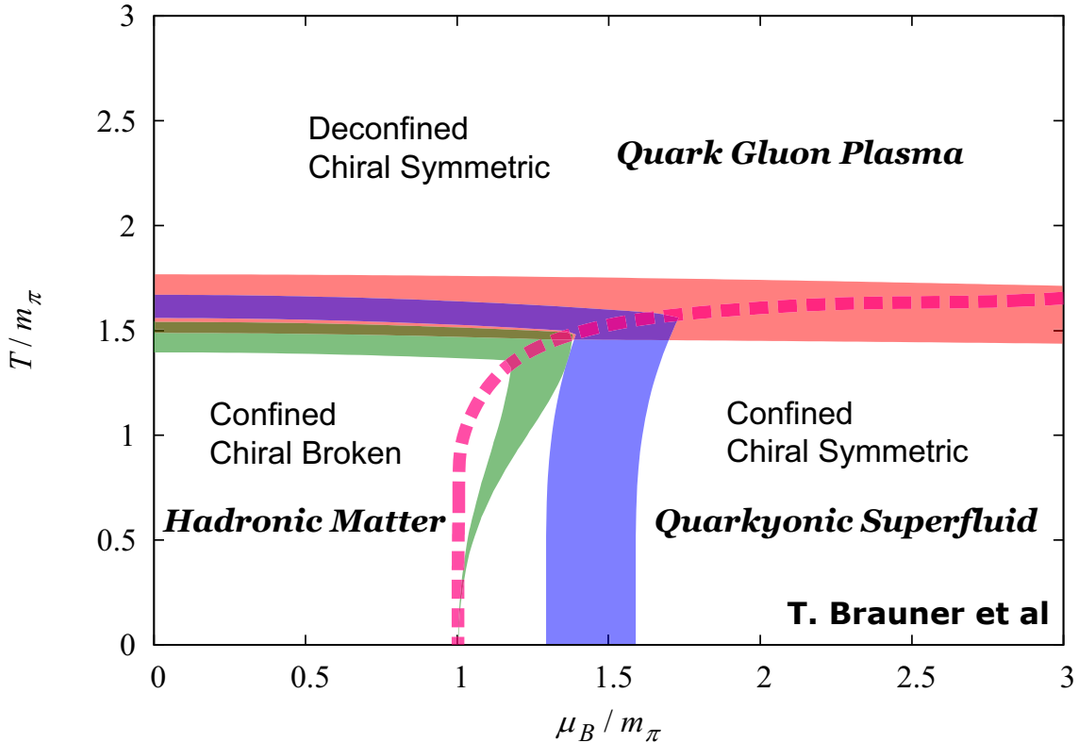


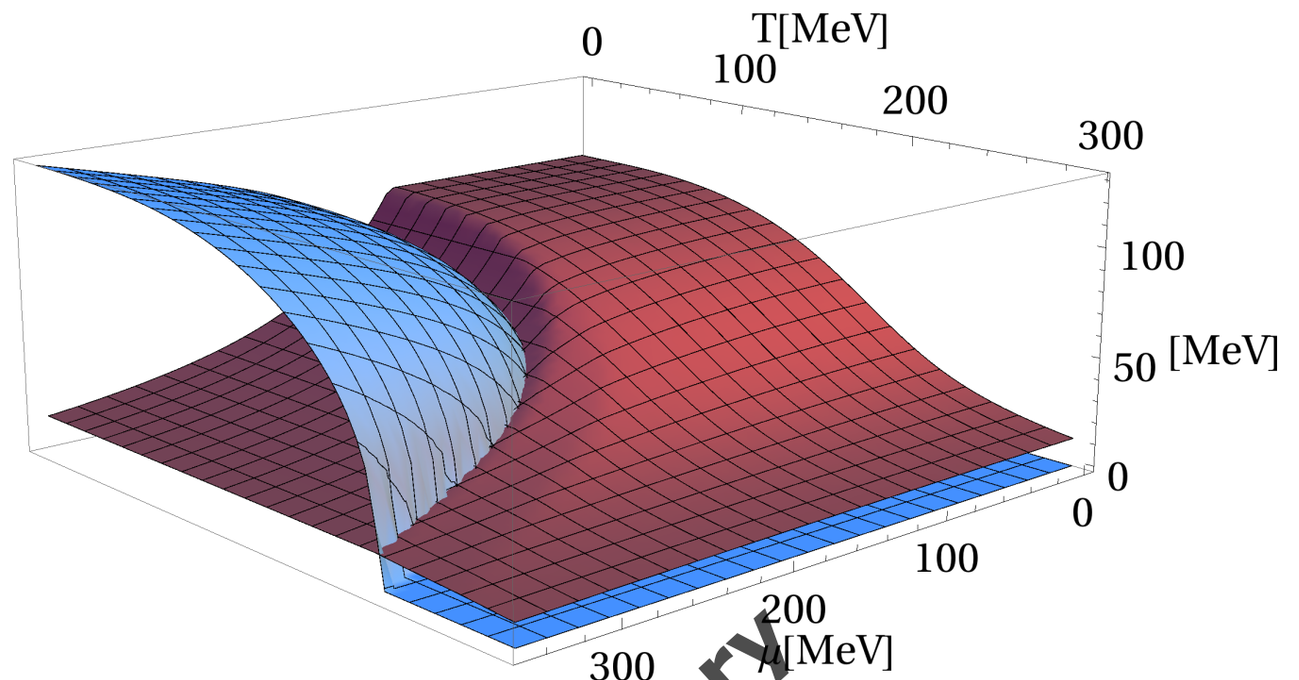
# Real chemical potential

## a glimpse at baryons



**Haas, Khan, JMP, Rennecke, Scherer**

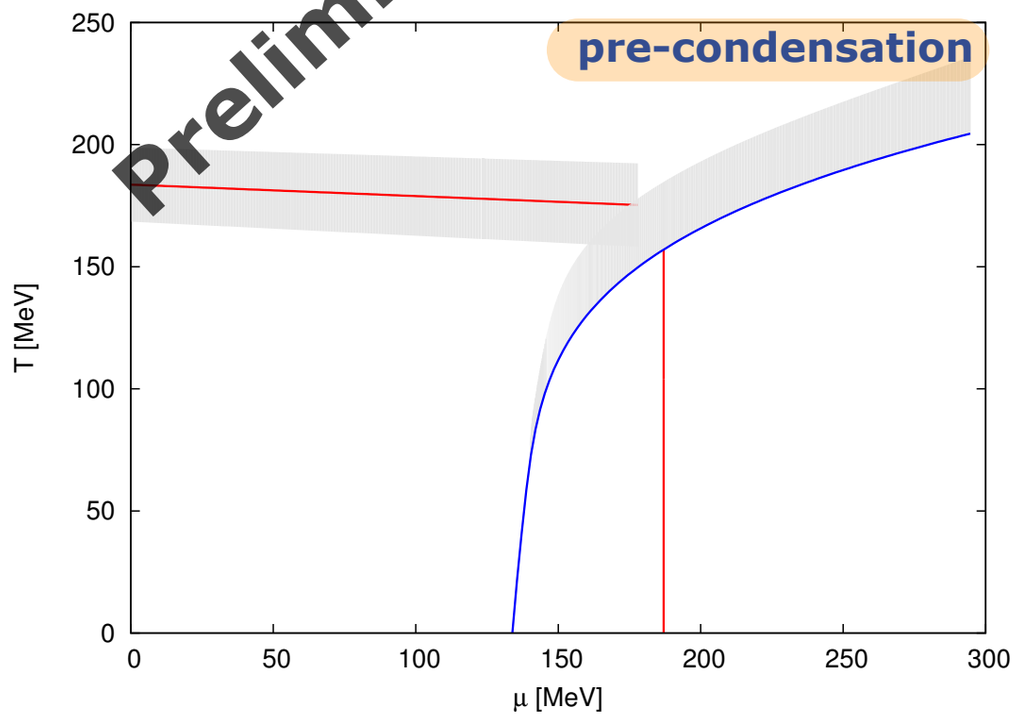



## two colour QCD

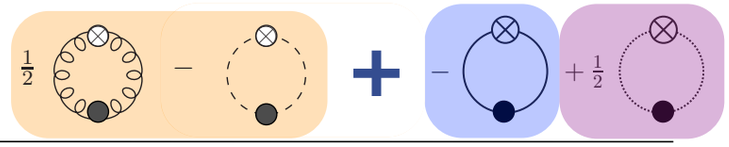
**…, Ratti et al ’04, …, Brauner et al ’08, …., Strodthoff et al ’11**

**FRG**



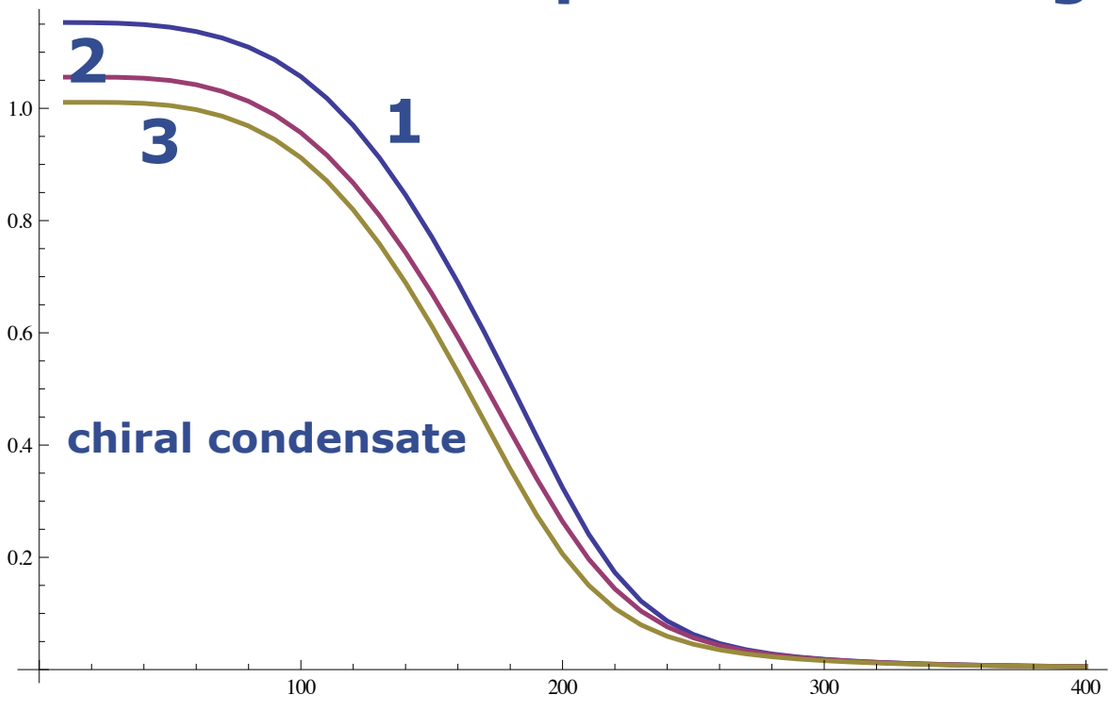
# Real chemical potential

## a glimpse at multi-scatterings



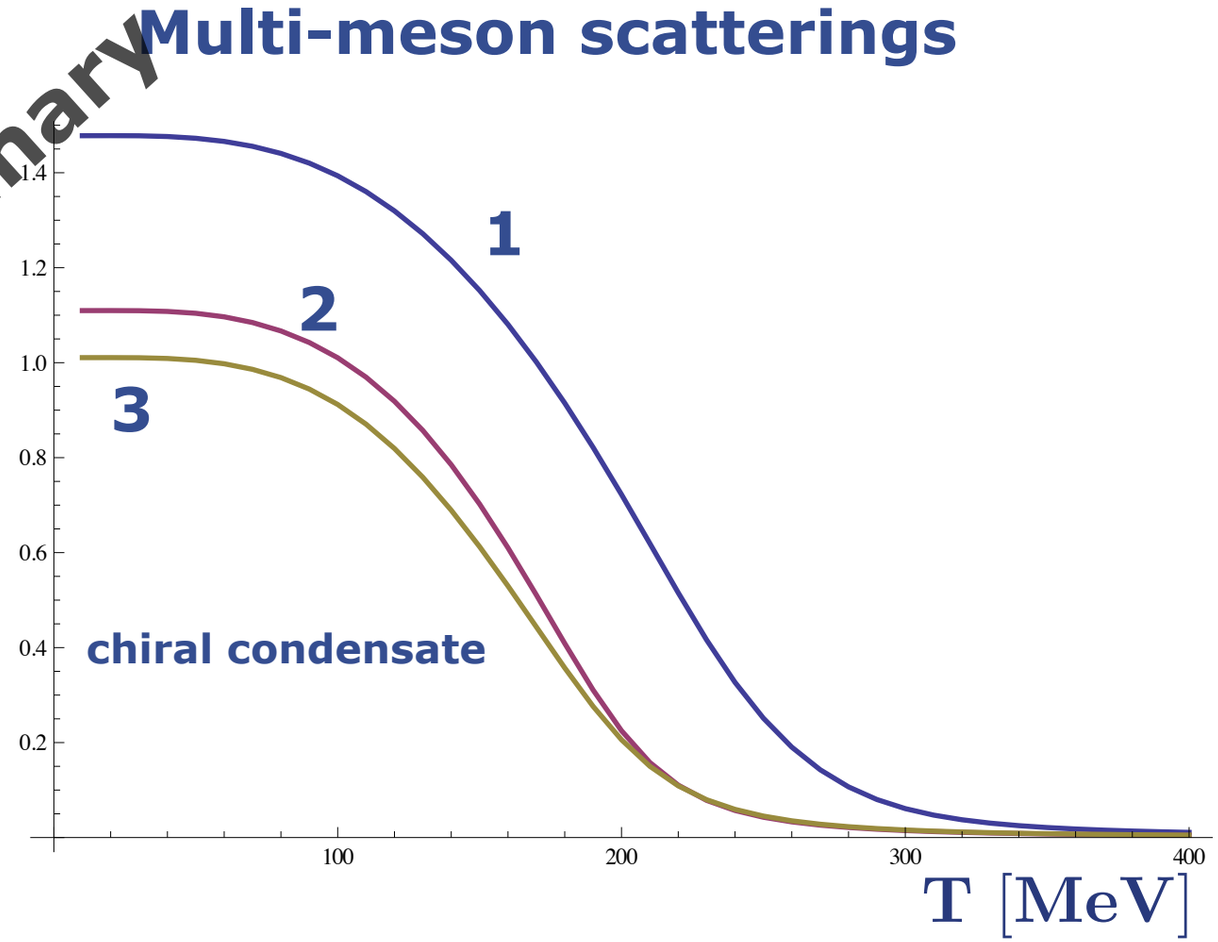
**JMP, Rennecke**

## Multi-meson-quark scatterings




Preliminary

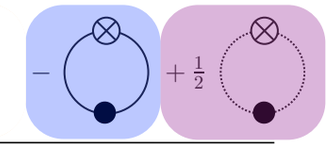
## Multi-meson scatterings



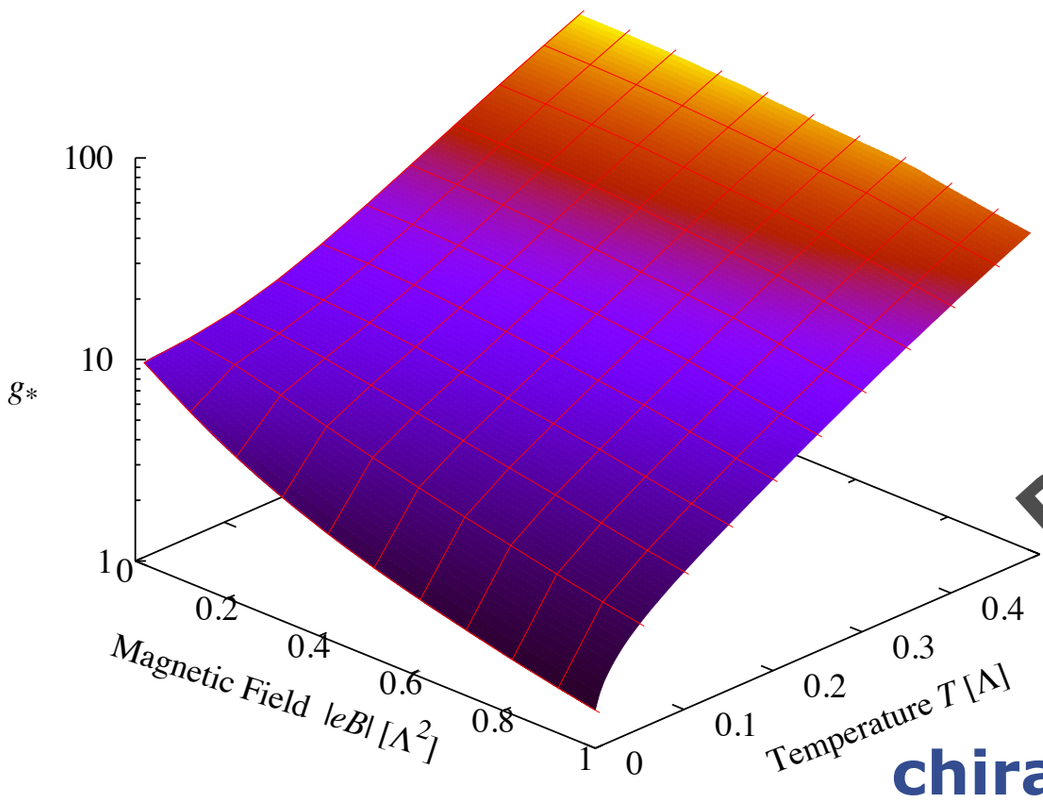

## PQM model

# Real chemical potential

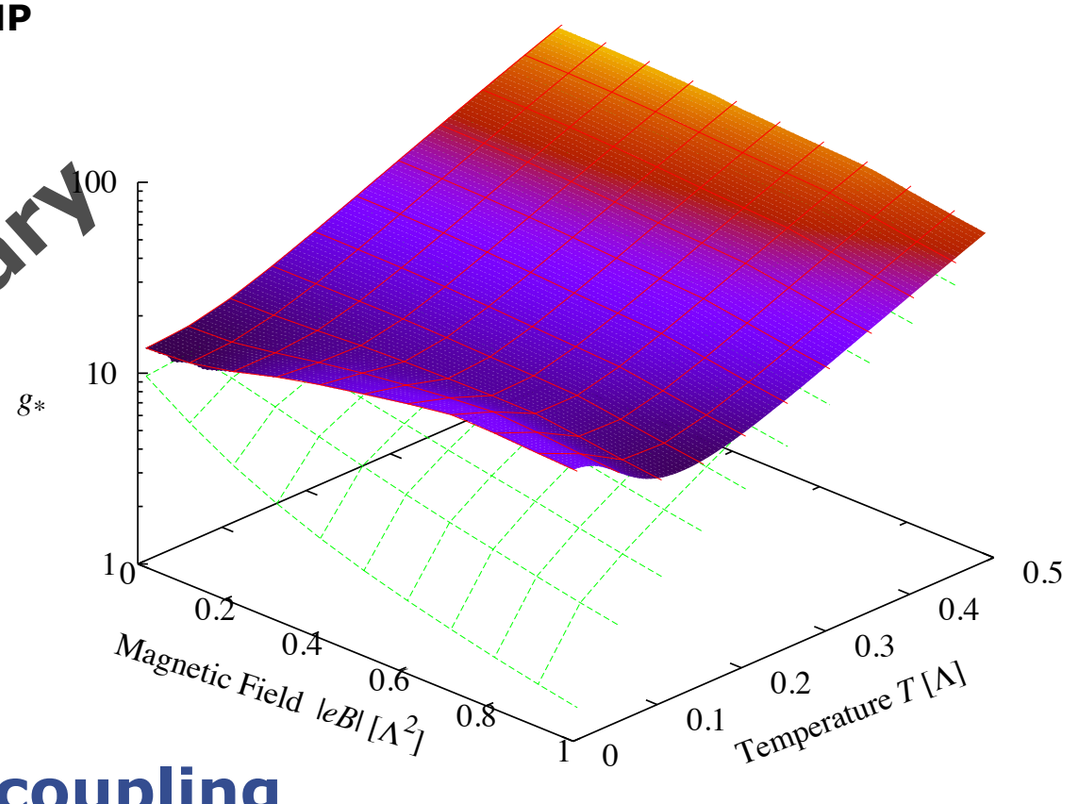
## a glimpse at strong magnetic fields



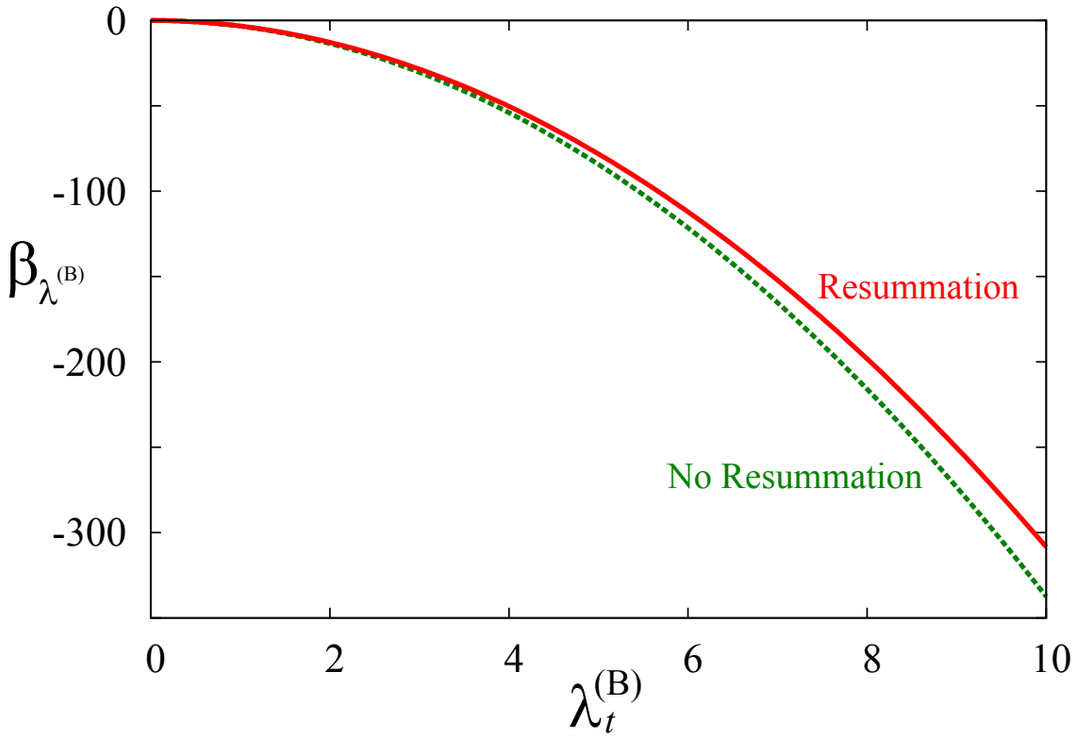

**Fukushima, JMP**

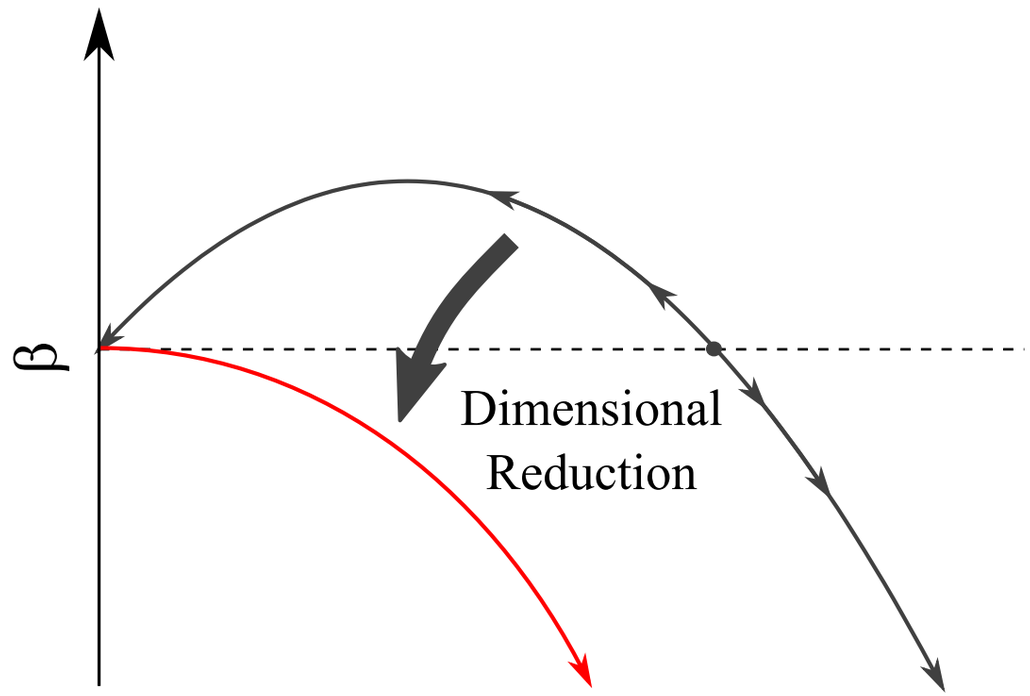



Preliminary




## chiral critical coupling

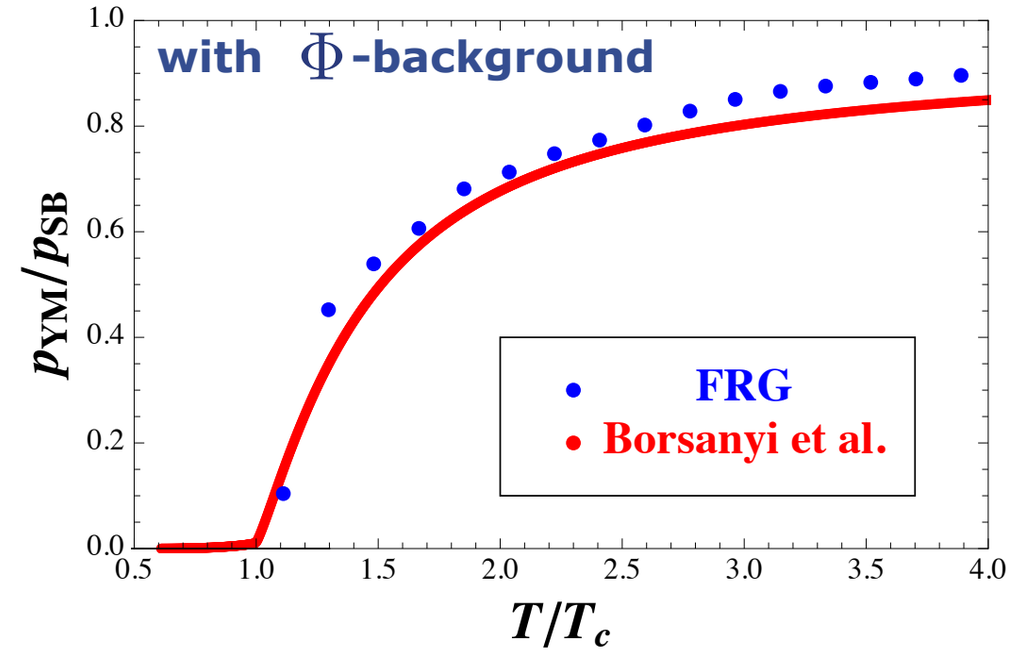


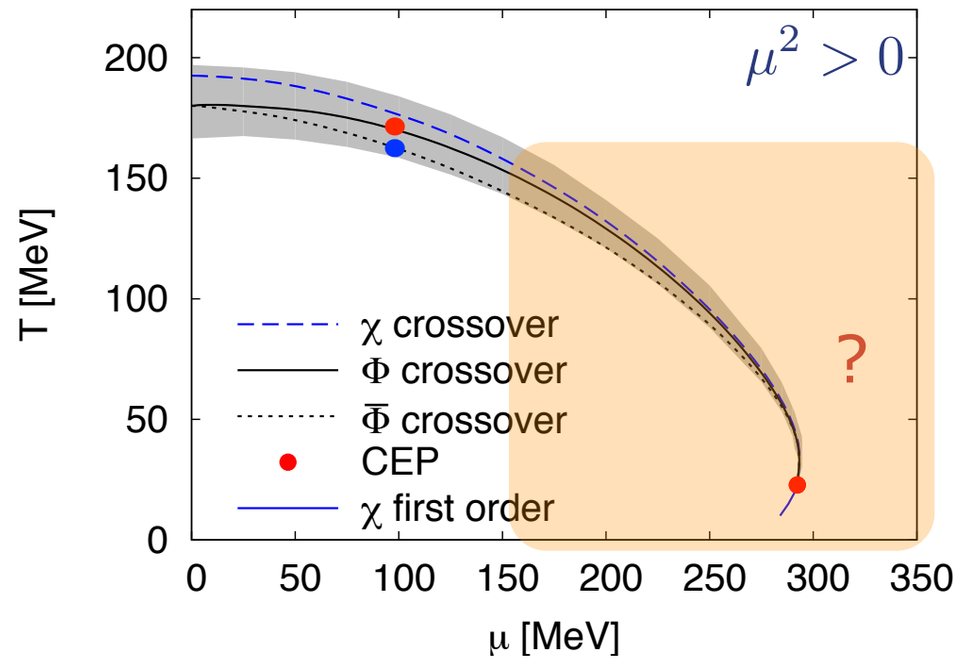



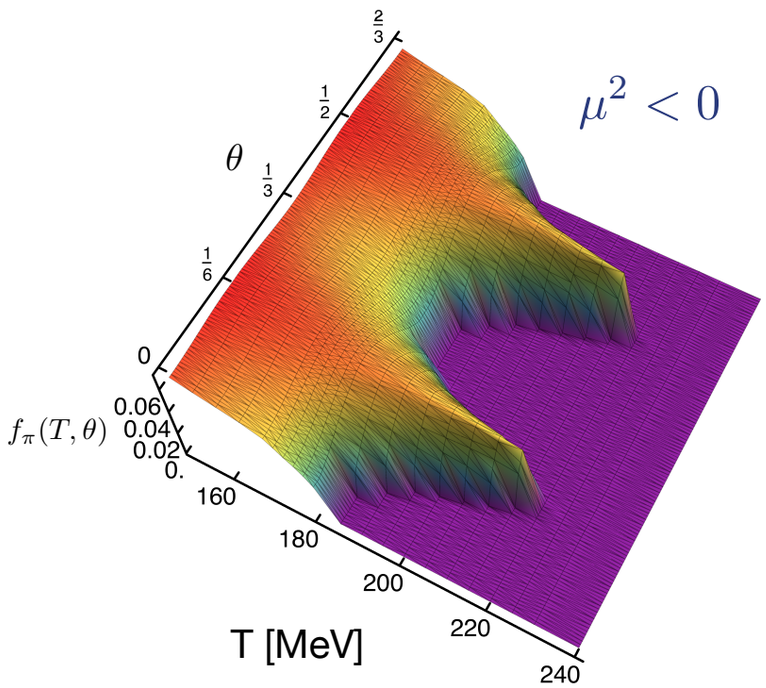
# Summary & outlook

- **Phase diagram of QCD**

  - **Phase structure and thermodynamics at finite $T$ & $\mu$**



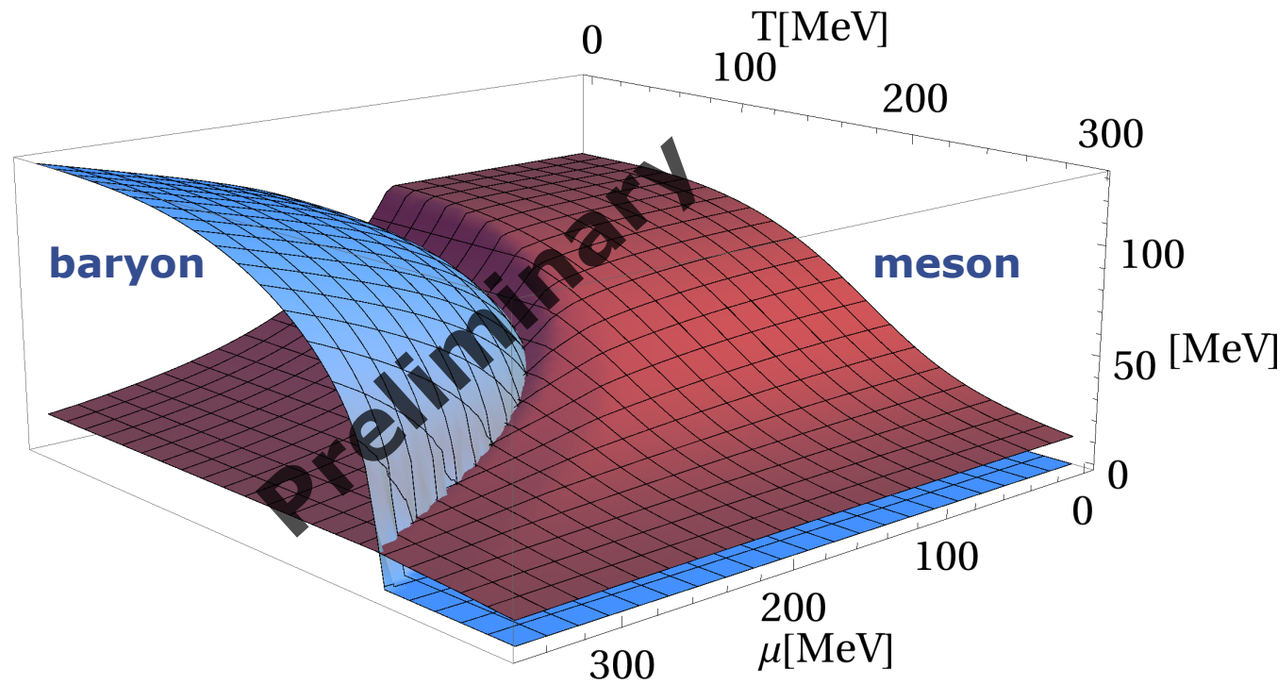

**Critical point unlikely for**

$$\frac{\mu_B}{T} < 2$$

# Summary & outlook

- **Phase diagram of QCD**
  - **Phase structure and thermodynamics at finite $T$ & $\mu$**
  - **2+1 flavours, baryons, phenomenology, dynamics**
  - **QCD meets cold quantum gases: two-colour QCD**

**Haas, Khan, JMP, Rennecke, Scherer**




**diquark condensate** **chiral condensate**

# Summary & outlook

- **Phase diagram of QCD**
  - **Phase structure and thermodynamics at finite $T$ & $\mu$**
  - **2+1 flavours, baryons, phenomenology, dynamics**
  - **QCD meets cold quantum gases: two-colour QCD**

**EpisodeIII: QGP meets ultracold atoms (Hirschegg August 25th -31st)**

- **Hadronic properties**
  - **dynamical hadronisation**
  - **dynamics**