Low energy QCD in terms of gauge invariant dynamical variables[1]

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- H.-P. P., *$SU(2)$ Yang-Mills quantum mechanics of spatially constant fields*, Phys. Lett. B **648** (2007) 97-106.
- H.-P. P., *Expansion of the Yang-Mills Hamiltonian in spatial derivatives and glueball spectrum*, Phys. Lett. B **685** (2010) 353-364.
- H.-P. P., *SU(2) Dirac-Yang-Mills quantum mechanics of spatially constant quark and gluon fields*, Phys. Lett. B **700** (2011) 265-276.
- H.-P. P., *Unconstrained Hamiltonian formulation of low energy SU(3) Yang-Mills quantum theory* , arXiv: 1205.2237v1 [hep-th] (2012).
- H.-P. P., *QCD in terms of gauge invariant dynamical variables*, PoS (Confinement X) (2013) 071, arXiv: 1303.3763 v1 [hep-th] (2013).
- H.-P. P., *Unconstrained Hamiltonian formulation of low energy QCD*, EPJ Web of Conferences **71**, 00104 (2014).

Outline of the talk

- Gribov ambiguity (1978) $\rightarrow$ Attempt of an exact resolution of the non-Abelian Gauss-laws to have an QCD Hamiltonian at low energy (a.o. Jackiw+Goldstone, Faddeev, T.D.Lee, t'Hooft)
- Unconstrained Hamiltonian of 2-color QCD
- Derivative expansion
- Spectrum: fermions, Lorentz-inv and renormalisation in the IR
- Extension to SU(3) $\rightarrow$ Gribov-copies and horizons

Aim: Alternative nonperturbative formulation of QCD

The QCD action

$$\mathcal{S}[A,\psi,\overline{\psi}] := \int d^4x \left[-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \overline{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi\right]$$

$$F^a_{\mu\nu} := \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu \ , \quad a = 1,..,8$$

$$D_\mu := \partial_\mu - igA^a_\mu \tau_a/2$$

is invariant under the $SU(3)$ gauge transformations $U[\omega(x)] \equiv \exp(i\omega_a\tau_a/2)$

$$\psi^\omega(x) = U[\omega(x)]\ \psi(x)$$

$$A^\omega_{a\mu}(x)\tau_a/2 = U[\omega(x)]\left(A_{a\mu}(x)\tau_a/2 + \frac{i}{g}\partial_\mu\right)U^{-1}[\omega(x)]$$

chromoelectric : $E^a_i \equiv F^a_{i0}$ and chromomagnetic $B^a_i \equiv \frac{1}{2}\epsilon_{ijk}F^a_{jk}$

$\Pi_{ai} = -E_{ai}$ momenta can. conj. to the spatial A_{ai} → canonical Hamiltonian

$$H_C = \int d^3x \Big[\frac{1}{2}E^2_{ai} + \frac{1}{2}B^2_{ai}(A) - gA_{ai}\, j_{ia}(\psi) + \overline{\psi}\left(\gamma_i\partial_i + m\right)\psi$$
$$- gA_{a0}\left(D_i(A)_{ab}E_{bi} - \rho_a(\psi)\right)\Big]$$

with the covariant derivative $D_i(A)_{ab} \equiv \delta_{ab}\partial_i - g f_{abc}A_{ci}$

Constrained Quantisation

Exploit the time dependence of the gauge transformations to put

$$A_{a0} = 0 \ , \qquad a = 1, .., 8 \qquad \text{(Weyl gauge)}$$

The dynamical vaiables A_{ai}, $-E_{ai}$, $\psi_{\alpha r}$ and $\psi^*_{\alpha r}$ are quantized in the Schrödinger functional approach imposing the equal-time (anti-)CR, e.g. $-E_{ai} = -i\partial/\partial A_{ai}$. The physical states Φ satisfy

$$\begin{aligned} H\Phi &= \int d^3x \left[\frac{1}{2}E^2_{ai} + \frac{1}{2}B^2_{ai}[A] - A_{ai}\, j_{ia}(\psi) + \overline{\psi}\,(\gamma_i\partial_i + m)\,\psi\right]\Phi = E\Phi \ , \\ G_a(x)\Phi &= [D_i(A)_{ab}E_{bi} - \rho_a(\psi)]\,\Phi = 0 \ , \quad a = 1, .., 8 \ . \end{aligned}$$

The Gauss law operators G_a are the generators of the residual time independent gauge transformations, satisfying $[G_a, H] = 0$ and $[G_a, G_b] = if_{abc}G_c$.

Angular momentum operators $\quad [J_i, H] = 0$

$$J_i = \int d^3x \left[-\epsilon_{ijk}A_{aj}E_{ak} + \Sigma_i(\psi) + \text{orbital parts}\right] \ , \quad i = 1, 2, 3 \ ,$$

The matrix element of an operator O is given in the Cartesian form

$$\langle\Phi'|O|\Phi\rangle \ \propto \int dA\; d\overline{\psi}\; d\psi\;\; \Phi'^*(A, \overline{\psi}, \psi)\, O\, \Phi(A, \overline{\psi}, \psi) \ .$$

For $SU(3)$ Yang-Mills QM of spat.const.gluon fields: P. Weisz and V. Ziemann (1986)

Point trafo to new set of adapted coordinates,
$A_{ai}, \psi_\alpha \to$ 3 gauge angles q_j, the pos. definite symmetric 3×3 matrix S, and new ψ'_β

$$A_{ai}\left(q,S\right) = O_{ak}\left(q\right) S_{ki} - \frac{1}{2g}\epsilon_{abc}\left(O\left(q\right)\partial_i O^T\left(q\right)\right)_{bc}, \quad \psi_\alpha\left(q,\psi'\right) = U_{\alpha\beta}\left(q\right)\psi'_\beta$$

orthog. $O(q)$ and unitary $U\left(q\right)$ related via $\quad O_{ab}(q) = \frac{1}{2}\mathrm{Tr}\left(U^{-1}(q)\tau_a U(q)\tau_b\right)$.

Generalisation of the polar decomposition of A and corresponds to (A. Khvedelidze and H.-P. P. 1999)

$$\chi_i(A) = \epsilon_{ijk}A_{jk} = 0 \quad (\text{"symmetric gauge"}).$$

Preserving the CCR $\to$ old canonical momenta in terms of the new variables

$$-E_{ai}(q,S,p,P) = O_{ak}\left(q\right)\left[P_{ki} + \epsilon_{kil}{}^*D^{-1}_{ls}(S)\left(\Omega^{-1}_{sj}(q)p_j + \rho_s(\psi') + D_n(S)_{sm}P_{mn}\right)\right]$$

$$\Rightarrow \qquad G_a\Phi \equiv O_{ak}(q)\Omega^{-1}_{ki}(q)p_i\Phi = 0 \quad \Leftrightarrow \quad \frac{\delta}{\delta q_i}\Phi = 0 \quad (\text{Abelianisation})$$

$$\text{Ang. mom. op.} \quad J_i = \int d^3x\left[-2\epsilon_{ijk}S_{mj}P_{mk} + \Sigma_i(\psi') + \rho_i(\psi') + \text{orbital parts}\right]$$

$\to$ S colorless spin 0,2 glueball field, ψ' colorless reduced quark fields of spin 0,1
Reduction: Color $\to$ Spin (unusual spin-statistics relation specific to SU(2) !)

The correctly ordered physical quantum Hamiltonian (Christ and Lee 1980) in terms of the physical variables $S_{ik}(\mathbf{x})$ and the can. conj. $P_{ik}(\mathbf{x}) \equiv -i\delta/\delta S_{ik}(\mathbf{x})$ reads

$$H(S,P) = \frac{1}{2}\mathcal{J}^{-1}\int d^3\mathbf{x}\, P_{ai}\, \mathcal{J} P_{ai} + \frac{1}{2}\int d^3\mathbf{x}\left[B^2_{ai}(S) - S_{ai}\, j_{ia}(\psi') + \overline{\psi}'\,(\gamma_i\partial_i + m)\,\psi'\right]$$
$$-\mathcal{J}^{-1}\int d^3\mathbf{x}\int d^3\mathbf{y}\Big\{\Big(D_i(S)_{ma}P_{im} + \rho_a(\psi')\Big)(\mathbf{x})\mathcal{J}$$
$$\langle \mathbf{x}\ a|{}^*D^{-2}(S)|\mathbf{y}\ b\rangle\ \ \Big(D_j(S)_{bn}P_{nj} + \rho_b(\psi')\Big)(\mathbf{y})\Big\}$$

with the FP operator

$${}^*D_{kl}(S) \equiv \epsilon_{kmi}D_i(S)_{ml} = \epsilon_{kli}\partial_i - g(S_{kl} - \delta_{kl}\mathrm{tr}S)$$

and the Jacobian $\mathcal{J} \equiv \det|{}^*D|$

The matrix element of a physical operator O is given by

$$\langle\Psi'|O|\Psi\rangle \propto \int_{\mathrm{S\ pos.def.}}\int_{\overline{\psi}',\psi'}\prod_{\mathbf{x}}\left[dS(\mathbf{x})d\overline{\psi}'(\mathbf{x})d\psi'(\mathbf{x})\right]\mathcal{J}\,\Psi'^*[S,\overline{\psi}',\psi']O\Psi[S,\overline{\psi}',\psi']$$

The inverse of the FP operator and hence the physical Hamiltonian can be expanded in the number of spatial derivatives $\equiv$ expansion in $\lambda = g^{-2/3}$

Introduce UV cutoff a: infinite spatial lattice of granulas $G(\mathbf{n}, a)$ at $\mathbf{x} = a\mathbf{n}$ $(\mathbf{n} \in Z^3)$ and averaged variables

$$S(\mathbf{n}) := \frac{1}{a^3} \int_{G(\mathbf{n},a)} d\mathbf{x}\ S(\mathbf{x})$$

and the discretised spatial derivatives.
Expansion of the Hamiltonian in $\lambda = g^{-2/3}$

$$H = \frac{g^{2/3}}{a} \left[\mathcal{H}_0 + \lambda \sum_\alpha \mathcal{V}_\alpha^{(\partial)} + \lambda^2 \left(\sum_\beta \mathcal{V}_\beta^{(\Delta)} + \sum_\gamma \mathcal{V}_\gamma^{(\partial\partial \neq \Delta)} \right) + \mathcal{O}(\lambda^3) \right]$$

The "free" Hamiltonian

$$\mathcal{H}_0 \quad = \quad \sum_{\mathbf{n}} \mathcal{H}_0^{QM}(\mathbf{n})$$

is the sum of the Hamiltonians of $SU(2)$-Yang-Mills quantum mechanics of constant fields in each box (which include the local gluon-gluon interactions !).
The interaction terms

$$\mathcal{V}^{(\partial)}, \mathcal{V}^{(\Delta)}, ..$$

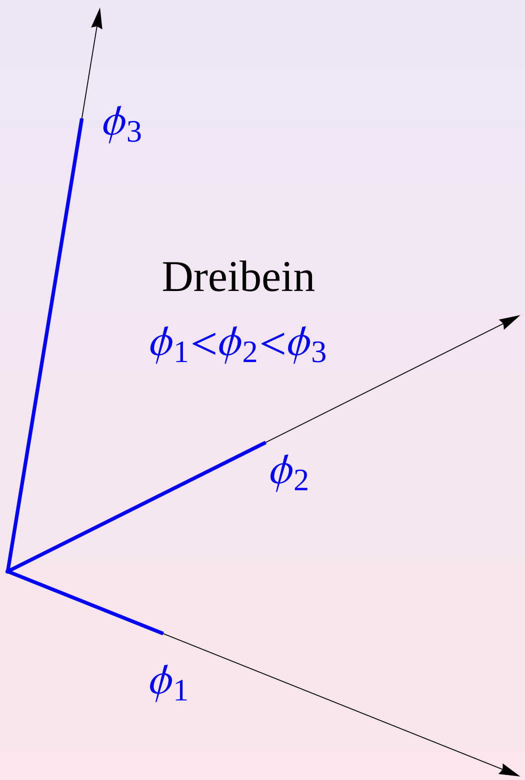
lead to interactions between the granulas (non-local terms).

H.-P. P., Phys. Lett. B **685** (2010) 353-364.

Intrinsic System of symmetric tensor S

Intrinsic system: $S = R^T(\alpha,\beta,\gamma)\begin{pmatrix}\phi_1 & 0 & 0\\ 0 & \phi_2 & 0\\ 0 & 0 & \phi_3\end{pmatrix}R(\alpha,\beta,\gamma)$

Jacobian: $\sin(\beta)\,(\phi_3-\phi_1)(\phi_3-\phi_2)(\phi_2-\phi_1)$



$\rightarrow$ local part of magn.pot. $B^2 = \phi_2^2\phi_3^2 + \phi_3^2\phi_1^2 + \phi_1^2\phi_2^2$

$B^2 = 0$: "$\phi_1 = \phi_2 = 0$, ϕ_3 arbitrary" (0-valleys)

Zeroth order Hamiltonian

$$H = \frac{g^{2/3}}{V^{1/3}}\left[\mathcal{H}^G + \mathcal{H}^D + \mathcal{H}^C\right] + \frac{1}{2}m\left[\left(\widetilde{u}_L^{(0)\dagger}\widetilde{v}_R^{(0)} + \sum_{i=1}^{3}\widetilde{u}_L^{(i)\dagger}\widetilde{v}_R^{(i)}\right) + h.c.\right]$$

$$\mathcal{H}^G := \frac{1}{2}\sum_{ijk}^{\text{cyclic}}\left(-\frac{\partial^2}{\partial\phi_i^2} - \frac{2}{\phi_i^2-\phi_j^2}\left(\phi_i\frac{\partial}{\partial\phi_i} - \phi_j\frac{\partial}{\partial\phi_j}\right) + (\xi_i - \widetilde{J}_i^Q)^2\frac{\phi_j^2+\phi_k^2}{(\phi_j^2-\phi_k^2)^2} + \phi_j^2\phi_k^2\right),$$

$$\mathcal{H}^D := \frac{1}{2}(\phi_1+\phi_2+\phi_3)\left(\widetilde{N}_L^{(0)} - \widetilde{N}_R^{(0)}\right) + \frac{1}{2}\sum_{ijk}^{\text{cyclic}}(\phi_i - (\phi_j+\phi_k))\left(\widetilde{N}_L^{(i)} - \widetilde{N}_R^{(i)}\right),$$

$$\mathcal{H}^C := \sum_{ijk}^{\text{cyclic}}\frac{\widetilde{\rho}_i(\xi_i - \widetilde{J}_i^Q + \widetilde{\rho}_i)}{(\phi_j+\phi_k)^2},$$

and the total spin $J_i = R_{ij}(\chi)\,\xi_j$, $[J_i, H] = 0$

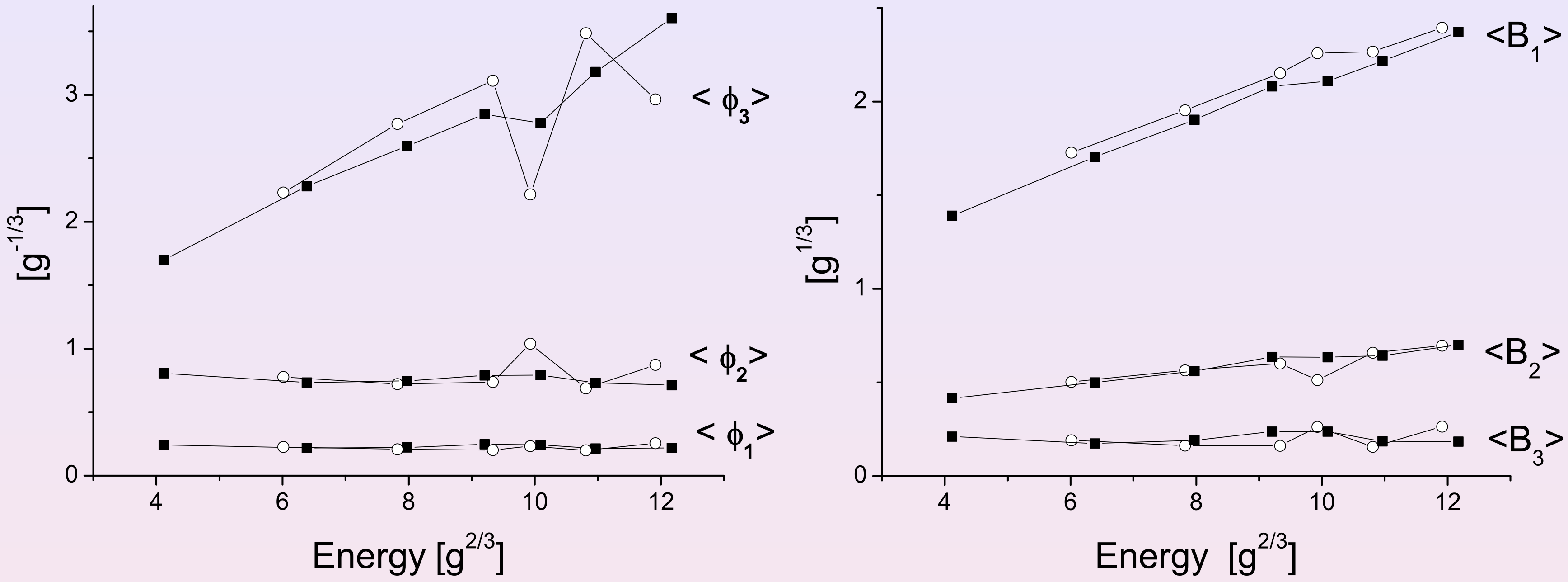
in terms of the intrinsic spin ξ_i satisfying $[J_i, \xi_j] = 0$ and $[\xi_i, \xi_j] = -i\epsilon_{ijk}\xi_k$

The matrix elements become

$$\langle\Phi_1|\mathcal{O}|\Phi_2\rangle = \int d\alpha\sin\beta d\beta d\gamma\int_{0<\phi_1<\phi_2<\phi_3} d\phi_1 d\phi_2 d\phi_3\,(\phi_1^2-\phi_2^2)(\phi_2^2-\phi_3^2)(\phi_3^2-\phi_1^2)\int d\overline{\psi}'d\psi'\,\Phi_1^*\mathcal{O}\Phi_2\,.$$

H.-P. P., Phys. Lett. B **700** (2011) 265-276.

$\langle \phi_i \rangle$ and $\langle B_i \rangle \equiv \langle g\phi_j\phi_k \rangle$ expectation values for $0^+$and 2^+



$\langle \phi_3 \rangle$ is raising with increasing excitation, whereas $\langle \phi_1 \rangle$ and $\langle \phi_2 \rangle$ are practically constant. $\langle B_3 \rangle$ is practically constant with increasing excitation, whereas $\langle B_1 \rangle$ and $\langle B_2 \rangle$ are raising.

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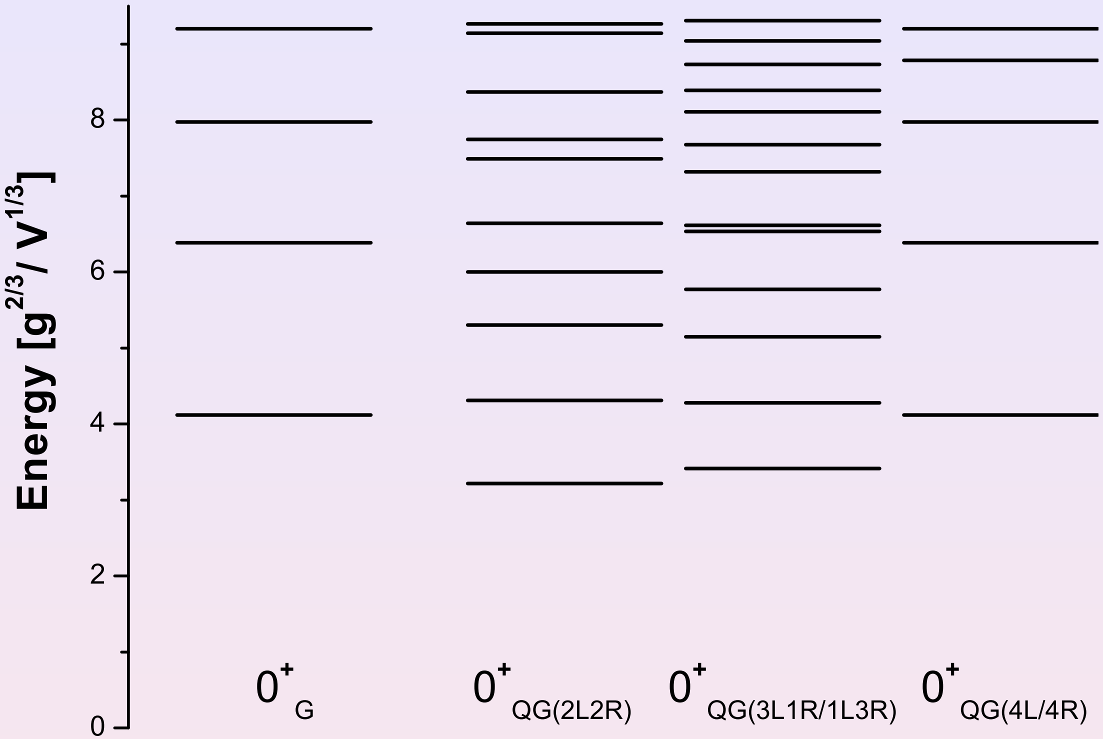
0^+ energy spectrum for the pure-gluon and the quark-gluon cases



The energy of the quark-gluon groundstate 3.22 $(5.63, -2.43, 0.02)$ is lower than that of the pure-glueball groundstate 4.11

The energy for the sigma-antisigma excitation is lower than that for the lowest glueball excitation (about one fifth)

H.-P. P., Phys. Lett. B **700** (2011) 265-276.

1st and 2nd order pert. theory in $\lambda = g^{-2/3}$ give the result (for the $(+)$ b.c.)

$$E^{+}_{\rm vac} = \mathcal{N}\left[4.1167 + 29.894\lambda^2 + \mathcal{O}(\lambda^3)\right]\frac{g^{2/3}}{a}$$

for the energy of the interacting glueball vacuum, and

$$E_1^{(0)+}(k) - E^{+}_{\rm vac} = [\ 2.270 + 13.511\lambda^2 + \mathcal{O}(\lambda^3)]\ \frac{g^{2/3}}{a} + 0.488\frac{a}{g^{2/3}}k^2 + \mathcal{O}((a^2k^2)^2)$$

for the energy spectrum of the interacting spin-0 glueball.

Lorentz invariance : $E = \sqrt{M^2 + k^2} \simeq M + \frac{1}{2M}\ k^2$

$\longrightarrow$ Consider $J = L + S$ states:

H.-P. P., Phys. Lett. B **685** (2010) 353-364.

Glueball mass and coupling constant renormalisation in the IR

Consider the physical mass

$$M = \frac{g_0^{2/3}}{a}\left[\mu + c\, g_0^{-4/3}\right]$$

Demanding its independence of box size a, one obtains

$$\gamma(g_0) \equiv a\frac{d}{da}g_0(a) = \frac{3}{2}g_0\frac{\mu + c\, g_0^{-4/3}}{\mu - c\, g_0^{-4/3}}$$

vanishes for $g_0 = 0$ (pert. fixed point) or $g_0^{4/3} = -c/\mu$ (IR fixed point, if $c < 0$)

My (incomplete) result $c_1^{(0)}/\mu_1^{(0)} = 5.95(1.34)$ suggests, that no IR fixed points exist.

$$\text{for } c > 0: \quad g_0^{2/3}(Ma) = \frac{Ma}{2\mu} + \sqrt{\left(\frac{Ma}{2\mu}\right)^2 - \frac{c}{\mu}}, \qquad a > a_c := 2\sqrt{c\mu}/M$$

critical coupling $g_0^2|_c = 14.52\ (1.55)$ and

$$\text{for } M \sim 1.6 \text{ GeV}: \quad a_c \sim 1.4 \text{ fm } (0.9 \text{ fm}) .$$

Use idea of *minimal embedding* of $su(2)$ in $su(3)$ by Kihlberg + Marnelius (1982)

$$\tau_1 := \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \tau_2 := -\lambda_5 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix} \quad \tau_3 := \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\tau_4 := \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \tau_5 := \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \tau_6 := \lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\tau_7 := \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \tau_8 := \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

The corresponding non-trivial non-vanishing structure constants $[\frac{\tau_a}{2}, \frac{\tau_b}{2}] = i c_{abc} \frac{\tau_c}{2}$, have at least one index $\in \{1, 2, 3\}$

"symmetric gauge" for SU(3) : $\quad \chi_a(A) = \sum_{b=1}^{8} \sum_{i=1}^{3} c_{abi} A_{bi} = 0 \ , \quad a = 1, ..., 8$

H.-P. P., arXiv: 1205.2237 [hep-th] (2012);
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H.-P. P., EPJ Web of Conferences **71**, 00104 (2014).

Symmetric gauge for SU(3): Unconstrained representation

Carrying out the coordinate transformation (generalized polar decomposition)

$$A_{ak}\left(q_1,..,q_8,\widehat{S}\right) = O_{a\hat{a}}(q)\,\widehat{S}_{\hat{a}k} - \frac{1}{2g}c_{abc}\left(O(q)\,\partial_k O^T(q)\right)_{bc},$$

$$\psi_\alpha\left(q_1,..,q_8,\psi^{RS}\right) = U_{\alpha\hat{\beta}}(q)\,\psi^{RS}_{\hat{\beta}}$$

$$\widehat{S}_{\hat{a}k} \equiv \binom{S_{ik}}{S_{Ak}} = \begin{pmatrix} & S_{ik}\ \text{pos. def.} & \\ \hline W_0 & X_3 - W_3 & X_2 + W_2 \\ X_3 + W_3 & W_0 & X_1 - W_1 \\ X_2 - W_2 & X_1 + W_1 & W_0 \\ -\frac{\sqrt{3}}{2}Y_1 - \frac{1}{2}W_1 & \frac{\sqrt{3}}{2}Y_2 - \frac{1}{2}W_2 & W_3 \\ -\frac{\sqrt{3}}{2}W_1 - \frac{1}{2}Y_1 & \frac{\sqrt{3}}{2}W_2 - \frac{1}{2}Y_2 & Y_3 \end{pmatrix}, \quad c_{\hat{a}\hat{b}k}\widehat{S}_{\hat{b}k} = 0$$

exists and unique : $\widehat{S}_{\hat{a}i}\widehat{S}_{\hat{a}j} = A_{ai}A_{aj}$ (6) $\quad d_{\hat{a}\hat{b}\hat{c}}\widehat{S}_{\hat{a}i}\widehat{S}_{\hat{b}j}\widehat{S}_{\hat{c}k} = d_{abc}A_{ai}A_{bj}A_{ck}$ (10)

reduced gluons (glueballs): Spin 0,1,2,3 reduced quarks: Spin $3/2$ Rarita-Schwinger

Reduction: Color $\rightarrow$ Spin, consequ.for Spin-Physics? $\Delta^{++}(3/2) : (3/2, +1/2, -1/2)$?

Symmetric gauge for SU(3): Intrinsic system

Rotate into the intrinsic frame of submatrix S representing the embedded $su(2)$

$$\widehat{S} \quad = \left(\begin{array}{c} S \\ \hline \overline{S} \end{array}\right) = \left(\begin{array}{c|c} R(\alpha,\beta,\gamma) & 0 \\ \hline 0 & D^{(2)}(\alpha,\beta,\gamma) \end{array}\right) \cdot \left(\begin{array}{c} \text{diag}(\phi_1,\phi_2,\phi_3) \\ \hline \begin{array}{c} \overline{S}(X_i \to x_i \\ Y_i \to y_i \\ W_i \to w_i) \end{array} \end{array}\right) \cdot R^T(\alpha,\beta,\gamma)$$

The magnetic potential V_{magn} has the zero-energy valleys ("constant Abelian fields")

$$B^2 = 0 \quad : \quad \phi_3 \text{ and } y_3 \text{ arbitrary} \quad \wedge \quad \text{all others zero}$$

At the bottom of the valleys the string-interaction becomes diagonal

$$\mathcal{H}^D_{\text{diag}} = \frac{1}{2}\widetilde{\psi}_L^{(1,\frac{1}{2})\dagger}\left[(\phi_3\lambda_3 + y_3\lambda_8)\otimes\sigma_3\right]\widetilde{\psi}_L^{(1,\frac{1}{2})} - \frac{1}{2}\widetilde{\psi}_R^{(\frac{1}{2},1)\dagger}\left[\sigma_3\otimes(\phi_3\lambda_3 + y_3\lambda_8)\right]\widetilde{\psi}_R^{(\frac{1}{2},1)}$$

Symmetric gauge for SU(3): Faddeev-Popov operator

Faddeev-Popov operator for ymmetric gauge for SU(3)

$$\gamma_{\hat{a}\hat{b}} \quad = \quad c_{\hat{a}\hat{c}i} D_i(S)_{\hat{c}\hat{b}} = c_{\hat{a}\hat{c}i}\left(\delta_{\hat{b}\hat{c}}\partial_i - c_{\hat{b}\hat{c}\hat{d}}\widehat{S}_{\hat{d}i}\right) = -c_{\hat{a}\hat{c}i}c_{\hat{b}\hat{c}\hat{d}}\widehat{S}_{\hat{d}i} + c_{\hat{a}\hat{b}i}\partial_i$$

Explicit form of the intrinsic $\widetilde{\gamma}$,

$$\left(\begin{array}{ccc|ccc|cc}
\phi_2+\phi_3 & 0 & 0 & \multicolumn{5}{c}{} \\
0 & \phi_3+\phi_1 & 0 & \multicolumn{5}{c}{-2\overline{S}^T(-\frac{3}{2}v,w)} \\
0 & 0 & \phi_1+\phi_2 & \multicolumn{5}{c}{} \\
\hline
 & & & 4\phi_1+\phi_2+\phi_3 & 0 & 0 & 0 & 0 \\
 & & & 0 & \phi_1+4\phi_2+\phi_3 & 0 & 0 & 0 \\
\multicolumn{3}{c|}{-2\overline{S}\left(-\frac{3}{2}v,w\right)} & 0 & 0 & \phi_1+\phi_2+4\phi_3 & 0 & 0 \\
\cline{4-8}
 & & & 0 & 0 & 0 & \phi_1+\phi_2+4\phi_3 & -\sqrt{3}(\phi_1-\phi_2) \\
 & & & 0 & 0 & 0 & -\sqrt{3}(\phi_1-\phi_2) & 3(\phi_1+\phi_2)
\end{array}\right)$$

In contrast to the $SU(2)$ case, transition to the intrinsic system does not completely diagonalize γ.

Symmetric gauge for SU(3): 1 spatial dimension

In one spatial dimension the symmetric gauge for SU(3) reduces to ($\sim$ t'Hooft gauge)

$$A^{(1d)} = \begin{pmatrix} 0 & 0 & A_{13} \\ 0 & 0 & A_{23} \\ 0 & 0 & A_{33} \\ 0 & 0 & A_{43} \\ 0 & 0 & A_{53} \\ 0 & 0 & A_{63} \\ 0 & 0 & A_{73} \\ 0 & 0 & A_{83} \end{pmatrix} \rightarrow S^{(1d)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \phi_3 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_3 \end{pmatrix} \quad (\sim A_3^a \lambda_a = U(\phi_3 \lambda_3 + y_3 \lambda_8) U^+)$$

which consistently reduces the above equs. for S for given A_3 to

$$\phi_3^2 + y_3^2 = A_{a3} A_{a3} \quad \wedge \quad \phi_3^2\, y_3 - 3\, y_3^3 = d_{abc} A_{a3} A_{b3} A_{c3}$$

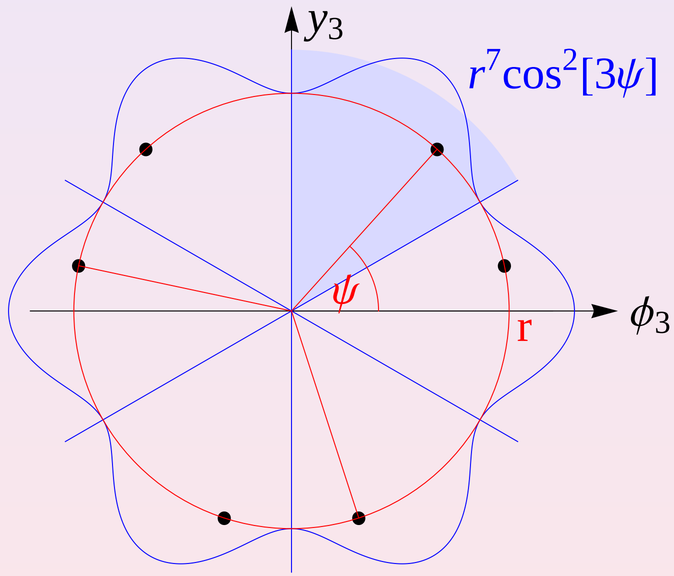
ϕ_3=r cos[ψ] $\quad$ y_3=r sin[ψ]

$$r^2 = A_3^a A_3^a$$

$$r^3 \sin[3\psi] = \sqrt{3}\, d_{abc} A_3^a A_3^b A_3^c$$

FP–det $\rightarrow$ $r^7 \cos^2[3\psi]$

$\rightarrow$ 6 Weyl–chambers



with 6 sol. separated by 0-lines of the FP-det $\phi_3^2\,(\phi_3^2 - 3y_3^2)^2$ ("Gribov-horizons").
Exactly 1 sol. exists in the "fundamental domain" (blue area) and we can replace

$$\int_{-\infty}^{+\infty} \prod_{a=1}^{8} dA_{a3} \quad \rightarrow \quad \int_0^\infty d\phi_3 \int_{\phi_3/\sqrt{3}}^\infty dy_3\; \phi_3^2\,(\phi_3^2 - 3y_3^2)^2 \propto \int_0^\infty r^7 dr \int_{\pi/6}^{\pi/2} d\psi \cos^2(3\psi)$$

Symmetric gauge for SU(3): 2 spatial dimensions

For two spatial dimensions, one can show that (putting $W_1 \equiv X_1, W_2 \equiv -X_2$)

$$A^{(2d)} = \begin{pmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ A_{31} & A_{32} & 0 \\ A_{41} & A_{42} & 0 \\ A_{51} & A_{52} & 0 \\ A_{61} & A_{62} & 0 \\ A_{71} & A_{72} & 0 \\ A_{81} & A_{82} & 0 \end{pmatrix} \to \widehat{S}^{(2d)}_{\rm intr} = \left(\begin{array}{ccc} \phi_1 & 0 & 0 \\ 0 & \phi_2 & 0 \\ 0 & 0 & 0 \\ \hline 0 & x_3 & 0 \\ x_3 & 0 & 0 \\ 2x_2 & 2x_1 & 0 \\ -\frac{\sqrt{3}}{2}y_1 - \frac{1}{2}x_1 & \frac{\sqrt{3}}{2}y_2 + \frac{1}{2}x_2 & 0 \\ -\frac{\sqrt{3}}{2}y_1 + \frac{1}{2}x_1 & -\frac{\sqrt{3}}{2}y_2 + \frac{1}{2}x_2 & 0 \end{array}\right)$$

consistently reduces the above equs. for S to a system of 7 equs. $(i, j, k = 1, 2)$

$$\widehat{S}_{\hat{a}i}\widehat{S}_{\hat{a}j} = A_{ai}A_{aj} \ (3) \qquad d_{\hat{a}\hat{b}\hat{c}}\widehat{S}_{\hat{a}i}\widehat{S}_{\hat{b}j}\widehat{S}_{\hat{c}k} = d_{abc}A_{ai}A_{bj}A_{ck} \ (4)$$

for 8 physical fields (incl. rot.-angle γ), which, adding as an 8th equ. $(d_{\hat{a}\hat{b}\hat{c}}\widehat{S}_{\hat{b}1}\widehat{S}_{\hat{c}2})^2 = (d_{abc}A_{b1}A_{c2})^2$, can be solved numerically for randomly gen. $A^{(2d)}$, again yielding solutions separated by horizons. Restricting to a fundamental domain

$$\int_{-\infty}^{+\infty} \prod_{a,b=1}^{8} dA_{a1}dA_{b2} \quad \to \quad \int_0^{2\pi} d\gamma \int_0^{\infty} r^{15} dr \int_{0<\hat{\phi}_1<\hat{\phi}_2<1} d\hat{\phi}_1 d\hat{\phi}_2 (\hat{\phi}_2 - \hat{\phi}_1) \int_{\rm fund.\ domain} d\alpha_1\, d\alpha_2\, d\alpha_3\, d\alpha_4\, \mathcal{J}$$

in terms of the compact variables $\hat{\phi}_i \equiv \phi_i / r$ (i=1,2) and α_k (k=1,..4). Due to the difficulty of the FP-determinant $\mathcal{J}$, I have, however, not yet found a satisfactory description of the fundamental domain.

Symmetric gauge for SU(3): 2 spatial dimensions: example

A_{ai}	$a=1$	$a=2$	$a=3$	$a=4$	$a=5$	$a=6$	$a=7$	$a=8$
$i=1$	0.00145	-0.05643	0.38156	0.2994	0.35597	-0.15409	-0.24694	0.26102
$i=2$	0.03221	0.14621	-0.15238	-0.30259	-0.06546	0.37797	-0.33890	-0.28775

S	S_{11}	S_{22}	S_3	X_1	X_2	X_3	Y_1	Y_2
1	0.44613	0.59928	-0.39002	-0.04911	-0.11429	0.23885	0.43251	-0.08321
2	0.65224	0.58752	-0.27742	0.07114	0.04073	0.30854	-0.02577	-0.19945
3	0.52058	0.40845	0.04142	0.34919	-0.26980	0.16909	-0.15906	0.14053
4	0.40472	0.37685	-0.07270	-0.19796	0.34328	0.42889	-0.06569	-0.32873
5	0.45038	0.36692	-0.46652	-0.12047	-0.02525	0.57146	-0.18583	0.21427
6	0.21869	0.36758	-0.22454	-0.35781	0.12015	0.34167	0.54196	0.00449

	$\phi^{(0)}$	$\phi^{(2)}$	x_3	$v^{(1)}$	$w^{(3)}$	$\sqrt{\phi^{(0)2}+\phi^{(2)2}}$	$\sqrt{x_3^2+v^{(1)2}+w^{(3)2}}$	$\mathcal{J}\propto$
2	0.88	0.28	0.31	0.22	0.10	0.92	0.39	1.70
5	0.58	0.47	0.57	0.20	0.28	0.75	0.67	0.14
4	0.55	0.08	0.43	0.61	0.37	0.56	0.83	0.20
1	0.74	0.40	0.24	0.39	0.28	0.84	0.54	- 0.61
3	0.66	0.09	0.17	0.59	0.42	0.66	0.75	- 0.17
6	0.41	0.25	0.34	0.78	0.19	0.48	0.88	- 0.33

Symmetric gauge for SU(3): 3 spatial dimensions

For 3 dimensions, I have found several solutions of the 16 S-equations $(i, j, k = 1, 2, 3)$

$$\widehat{S}_{\hat{a}i}\widehat{S}_{\hat{a}j} = A_{ai}A_{aj} \ (6) \qquad d_{\hat{a}\hat{b}\hat{c}}\widehat{S}_{\hat{a}i}\widehat{S}_{\hat{b}j}\widehat{S}_{\hat{c}k} = d_{abc}A_{ai}A_{bj}A_{ck} \ (10)$$

for the 16 physical fields numerically for a randomly generated A (see Table).

But to write the corresponding unconstrained integral over a fundamental domain

$$\int_{-\infty}^{+\infty} \prod_{a,b,c=1}^{8} dA_{a1}dA_{b2}dA_{c3} \quad \rightarrow \quad \int d\alpha \sin\beta \, d\beta \, d\gamma \int_0^\infty r^{23} dr$$

$$\int_{0<\hat{\phi}_1<\hat{\phi}_2<\hat{\phi}_3<1} d\hat{\phi}_1 d\hat{\phi}_2 d\hat{\phi}_3 \prod_{i<j}(\hat{\phi}_i - \hat{\phi}_j)\int_{\text{fund. domain}} d\alpha_1 ... d\alpha_9 \ \mathcal{J}$$

in terms of the compact variables $\hat{\phi}_i \equiv \phi_i/r$ and α_k is a difficult, but I think solvable, future task.

Symmetric gauge for SU(3): 3 spatial dimensions: example

$\phi^{(0)}$	$\phi^{(2)}$	$v^{(1)}$	$w^{(3)}$	$\sqrt{\phi^{(0)2}+\phi^{(2)2}}$	$\sqrt{v^{(1)2}+w^{(3)2}}$	$\prod_{i<j}(\phi_i-\phi_j)$	$\mathcal{J} \propto$
0.88	0.29	0.12	0.35	0.928	0.372	0.016	2.800
0.83	0.37	0.16	0.39	0.906	0.423	0.034	1.035
0.79	0.40	0.16	0.43	0.890	0.456	0.046	0.543
0.77	0.35	0.11	0.51	0.851	0.526	0.030	0.476
0.77	0.38	0.14	0.50	0.857	0.515	0.040	0.285
0.74	0.37	0.10	0.55	0.826	0.564	0.035	0.223
0.75	0.31	0.24	0.54	0.808	0.590	0.011	0.078
0.69	0.23	0.18	0.67	0.724	0.689	0.004	0.036
0.73	0.45	0.20	0.47	0.857	0.516	0.062	-0.062
0.69	0.48	0.29	0.47	0.836	0.548	0.074	-0.254
0.70	0.41	0.20	0.55	0.813	0.582	0.046	-0.178
0.63	0.49	0.20	0.57	0.797	0.604	0.083	-0.094
0.67	0.40	0.11	0.61	0.787	0.618	0.047	-0.034
0.68	0.39	0.36	0.50	0.785	0.619	0.039	-0.286
0.59	0.47	0.33	0.57	0.754	0.657	0.060	-0.312
0.65	0.34	0.10	0.67	0.736	0.677	0.023	-0.035
0.83	0.25	0.17	0.47	0.864	0.503	-0.008	1.315
0.81	0.34	0.26	0.40	0.877	0.480	-0.020	0.739
0.80	0.27	0.25	0.48	0.841	0.540	-0.010	0.621
0.76	0.31	0.22	0.53	0.820	0.573	-0.013	0.328
0.74	0.20	0.23	0.60	0.767	0.642	-0.005	0.256
0.70	0.33	0.06	0.62	0.780	0.626	-0.026	0.119
0.58	0.45	0.34	0.58	0.739	0.674	-0.065	-0.168

Conclusions

- Using a canonical transformation of the dynamical variables, which Abelianises the non-Abelian Gauss-law constraints to be implemented, a reformulation of QCD in terms of gauge invariant dynamical variables can be achieved.
- Using minimal embedding, the symmetric gauge $\epsilon_{ijk}A_{jk} = 0$ for $SU(2)$ can be generalized to the corresponding $SU(3)$ symmetric gauge $c_{abi}A_{bi} = 0$.
- The exact implementation of the Gauss laws reduces the colored spin-1 gluons and spin-1/2 quarks to unconstrained colorless spin-0, spin-1, spin-2 and spin-3 glueball fields and colorless Rarita-Schwinger fields respectively.
- The obtained physical Hamiltonian admits a systematic strong-coupling expansion in powers of $\lambda = g^{-2/3}$, equiv to an expansion in spatial derivatives.
- leading-order term $\longrightarrow$ non-interacting hybrid-glueballs,
  Dirac-Yang-Mills QM of spatially constant fields $\longrightarrow$ low-lying mass spectrum
- higher-order terms in λ $\longrightarrow$ interactions between the hybrid-glueballs
  perturbation theory in λ $\longrightarrow$ e.g. energy-momentum relation of glueballs
  $\longrightarrow$ study of Lorentz invariance and renormalisation in the IR.
- conversion of color to spin $\longrightarrow$ new insights into low energy Spin-Physics
- Gauge reduced approach is difficult (due to the complicated Jacobian), but possible and direct. Some properties of the Gribov-copies and -horizons have been identified. The approach should be a useful alternative to lattice calculations.
- The investigation can be extended to flux-tubes (string-tension).