SCALED VARIABLES AND THE QUARK-HADRON DUALITY

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QCD PHASE DIAGRAM

- QCD is the accepted theory for strong interactions
- Lattice QCD is a Monte-Carlo statistical model for QCD theory
- The QCD phase diagram *(T,μ)* describes the hadron-quark-gluon phase transition in the grand canonical ensemble
- We have only one reliable point on the whole phase diagram obtained from the fundamental theory of QCD
- At present, LQCD provides trustworthy results only at $\mu=0$ because of unsolved sign problem
- The LQCD results suggest the crossover phase transition instead of the 1-st order one
- Let us see this

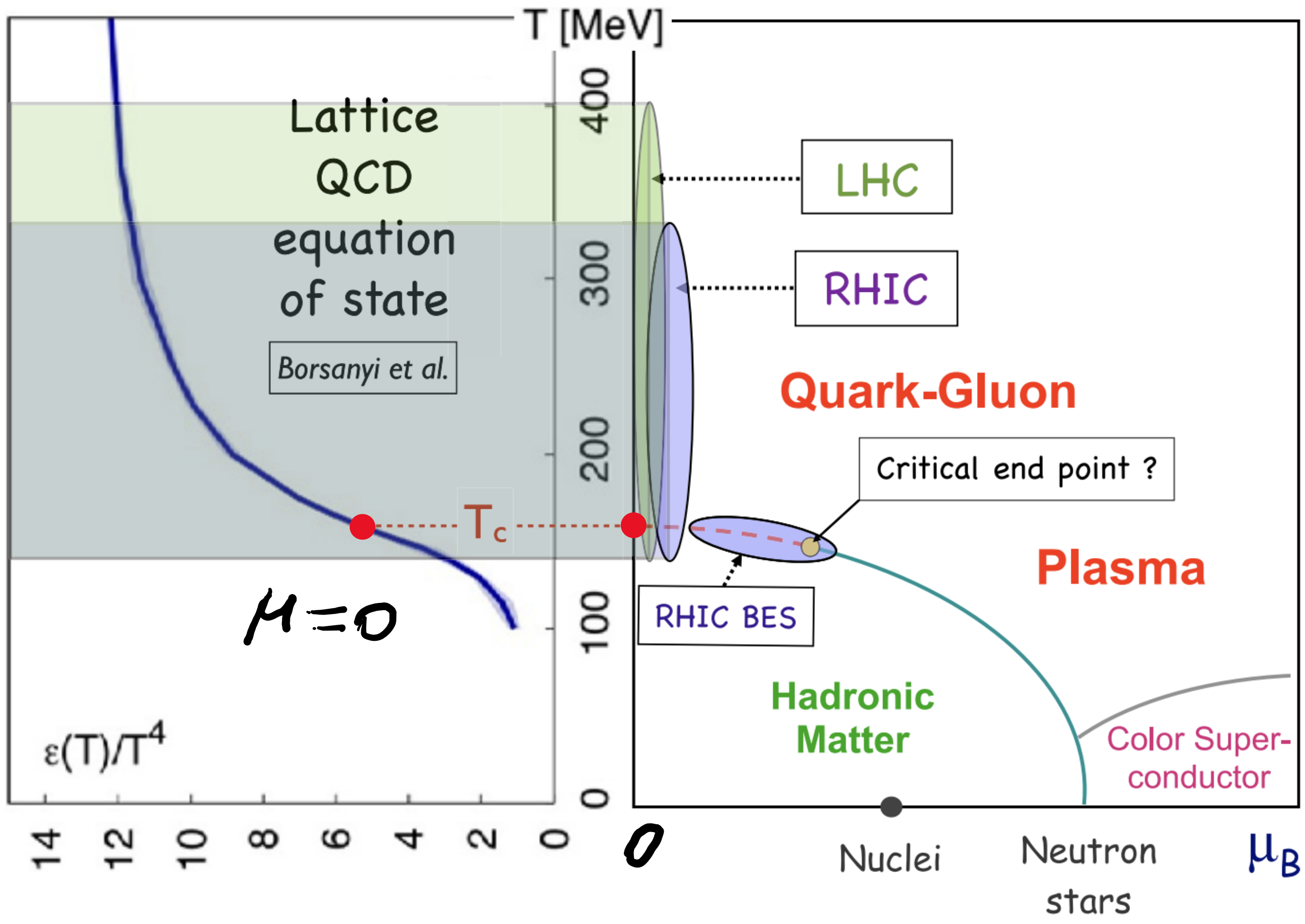


Figure is taken from Berndt Müller, Phys. Scr. T158 (2013) 014004

FIRST ORDER PHASE TRANSITION IN THE GRAND CANONICAL ENSEMBLE AND PHASE TRANSITION OF LQCD

- LQCD is a statistical model for quark-gluon fields in the grand canonical ensemble (T,V,μ)
- 1-st order phase transition in the grand canonical ensemble is characterized by the sharp jump into the energy density (latent heat) at fixed values of (T,μ)
- The energy density of LQCD does not have a jump and it has only a rapid smooth increase characteristic for crossover phase transition

ENERGY DENSITY OF LQCD MODEL IN THE GRAND CANONICAL ENSEMBLE

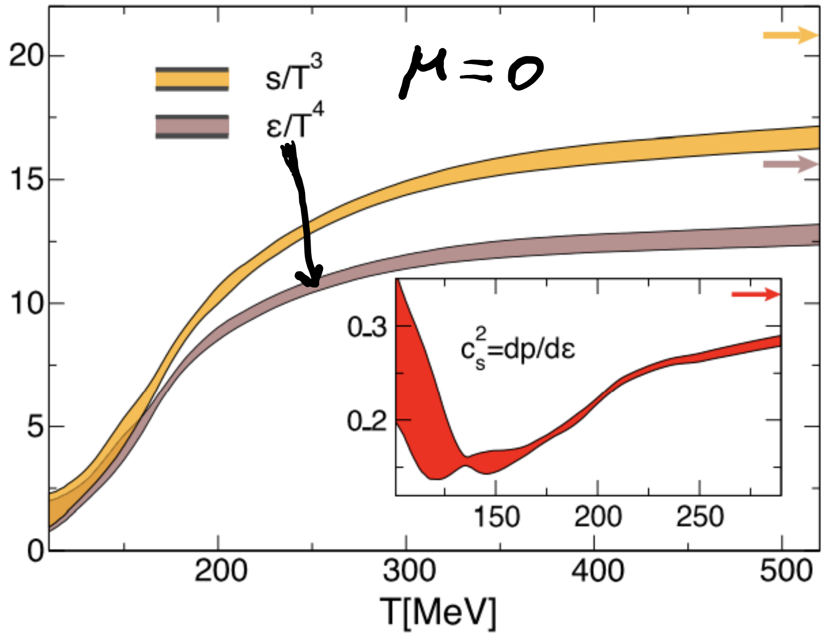
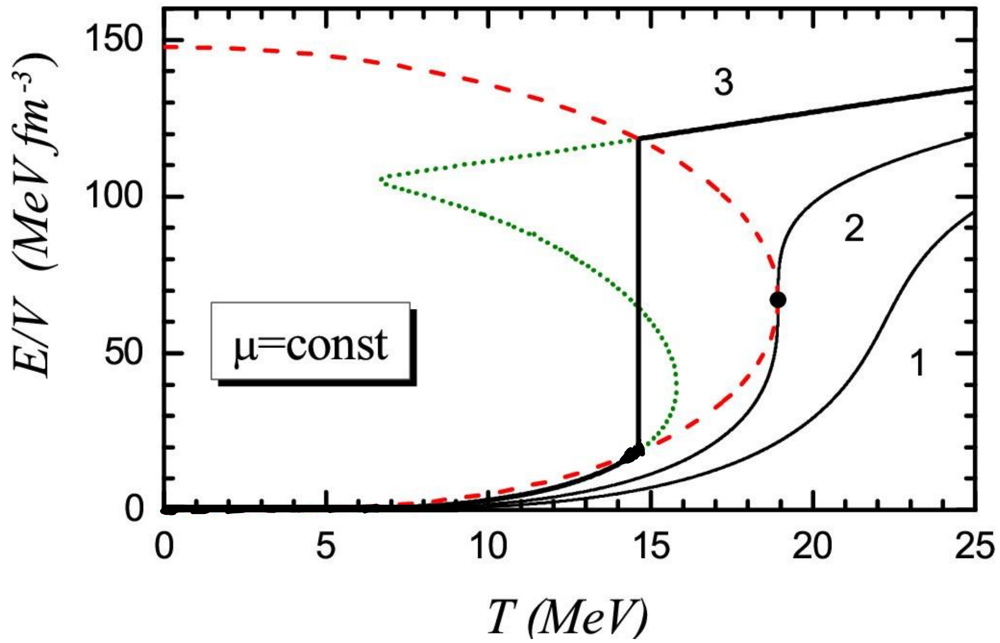


Figure is taken from S. Borsányi et al., Phys. Lett. B 730 (2014) 99

ENERGY DENSITY OF RELATIVISTIC MEAN-FIELD (RMF) MODEL IN THE GRAND CANONICAL ENSEMBLE



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FIRST ORDER PHASE TRANSITION IN THE GRAND CANONICAL ENSEMBLE AND PHASE TRANSITION OF LQCD

- In the grand canonical ensemble, the pressure is the specific thermodynamic potential
- In the point of the 1-st order phase transition pressure is continuous as function of T *at* μ=*const*, but it has a sharp corner (cusp).
- The pressure of LQCD does not have a cusp. It is a smooth function as for the crossover phase transition

PRESSURE OF LQCD MODEL IN THE GRAND CANONICAL ENSEMBLE

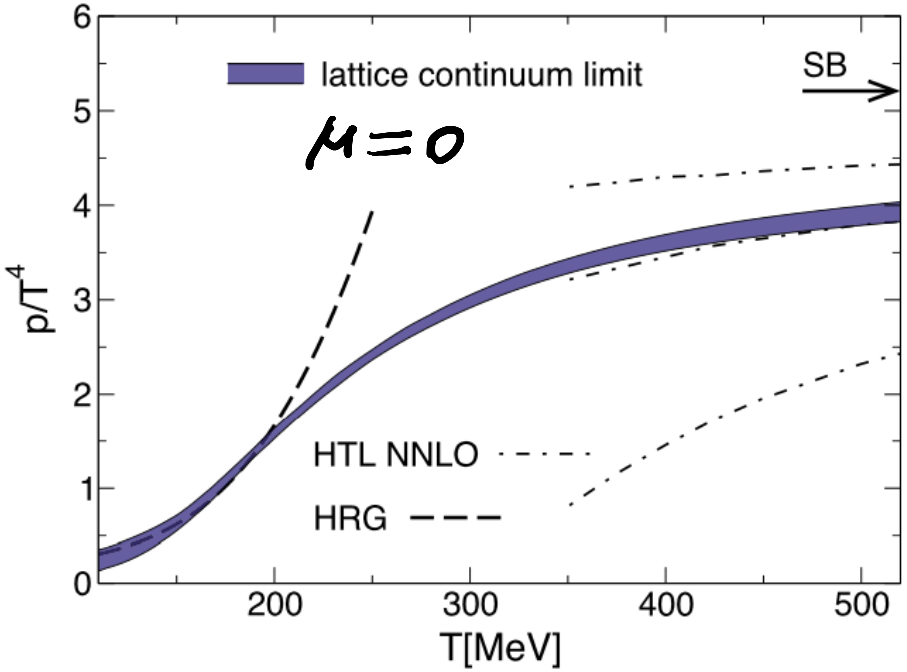
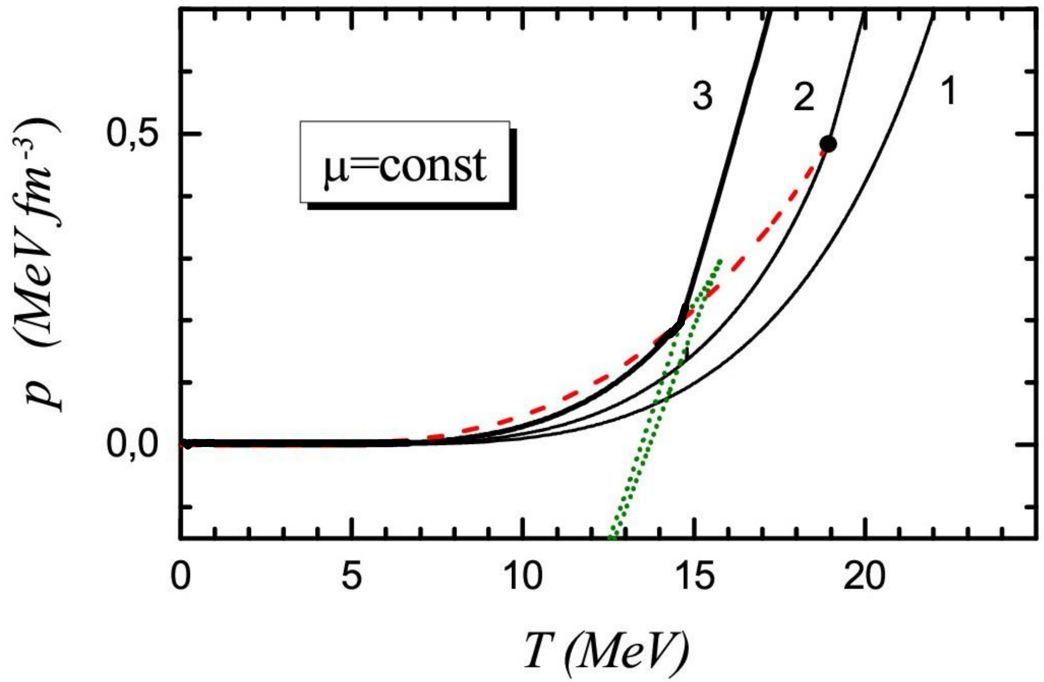


Figure is taken from S. Borsányi et al., Phys. Lett. B 730 (2014) 99

PRESSURE OF RMF MODEL IN THE GRAND CANONICAL ENSEMBLE



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FIRST ORDER PHASE TRANSITION IN THE GRAND CANONICAL ENSEMBLE AND PHASE TRANSITION OF LQCD

- 1-st order phase transition in the grand canonical ensemble is characterized by the sharp jump into the entropy density at fixed values of *(T,μ)*
- The entropy density of LQCD does not have a jump and it has only a rapid smooth increase characteristic for crossover phase transition

ENTROPY DENSITY OF LQCD MODEL IN THE GRAND CANONICAL ENSEMBLE

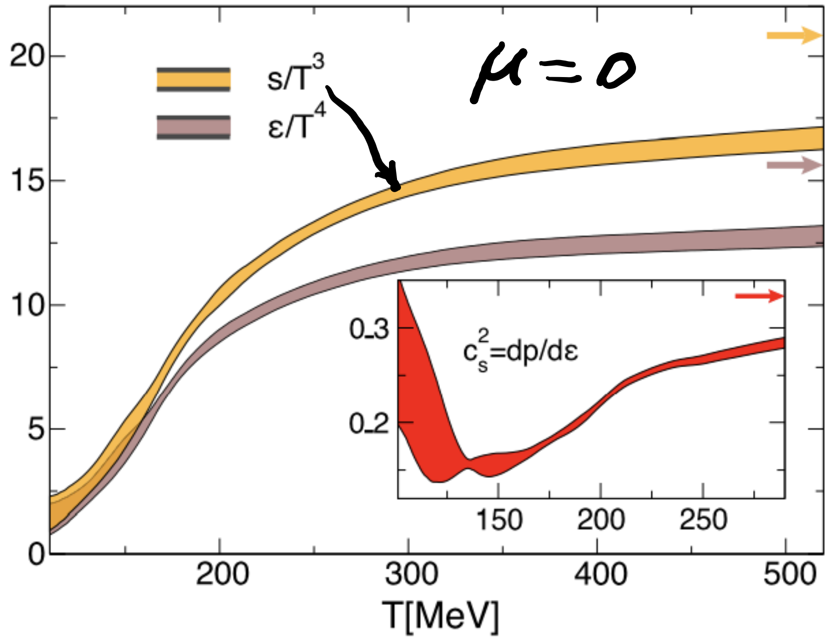
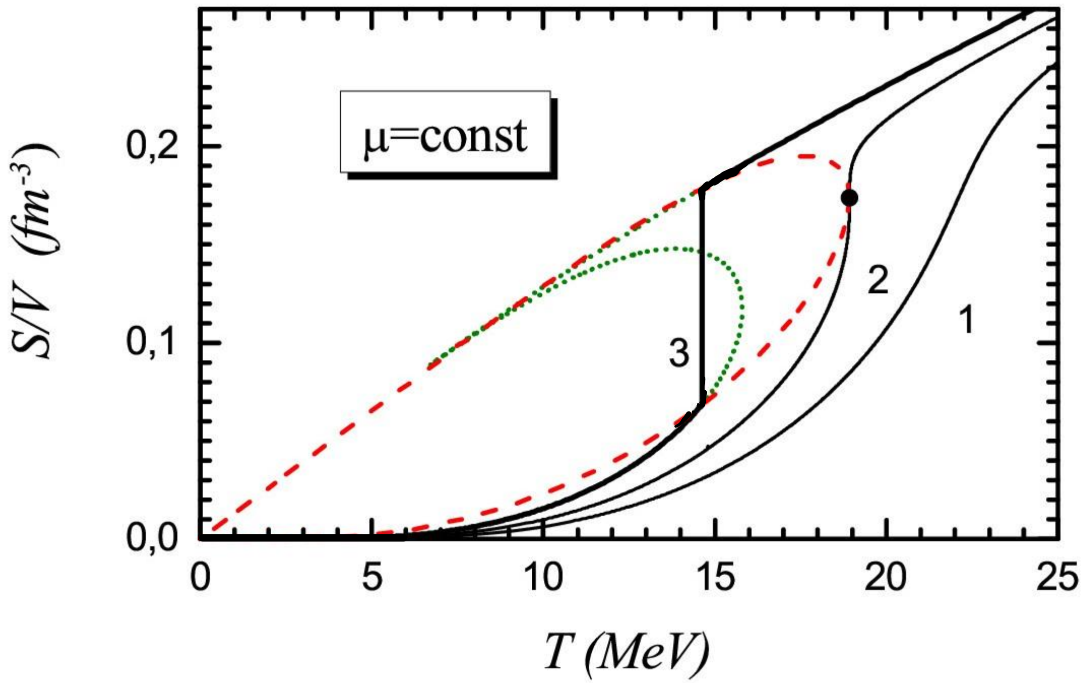


Figure is taken from S. Borsányi et al., Phys. Lett. B 730 (2014) 99

ENTROPY DENSITY OF RMF MODEL IN THE GRAND CANONICAL ENSEMBLE



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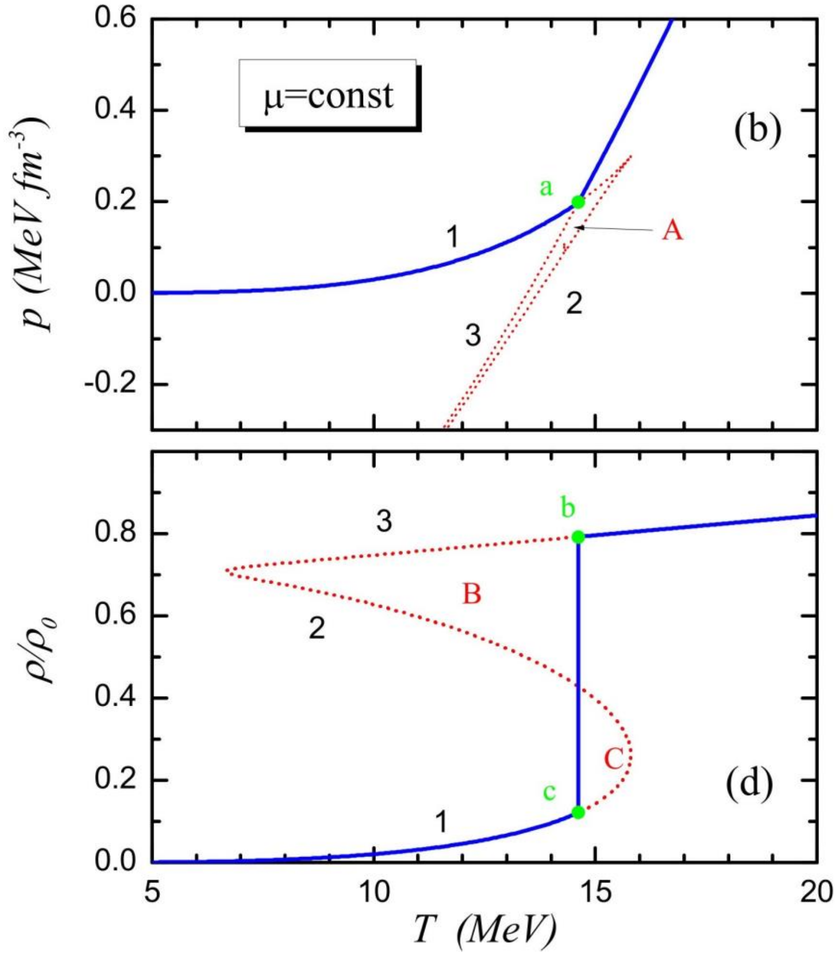
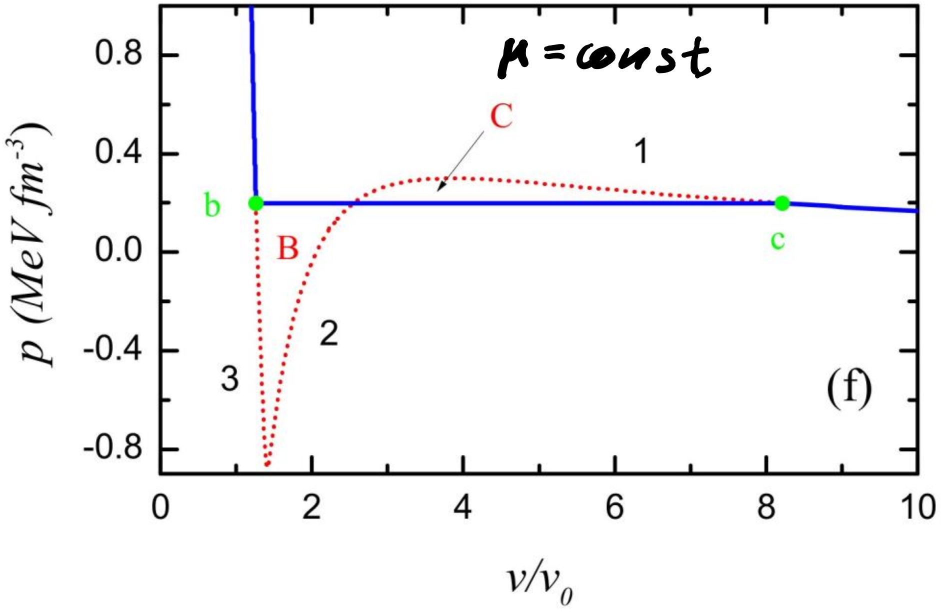
FIRST ORDER PHASE TRANSITION IN THE GRAND CANONICAL ENSEMBLE

The 1-st order phase transition in the grand canonical ensemble (T,V,μ) at fixed values of μ is characterized by the cusp in the pressure and jump in the density as functions of temperature, and the jump in the specific volume as function of pressure



RMF model



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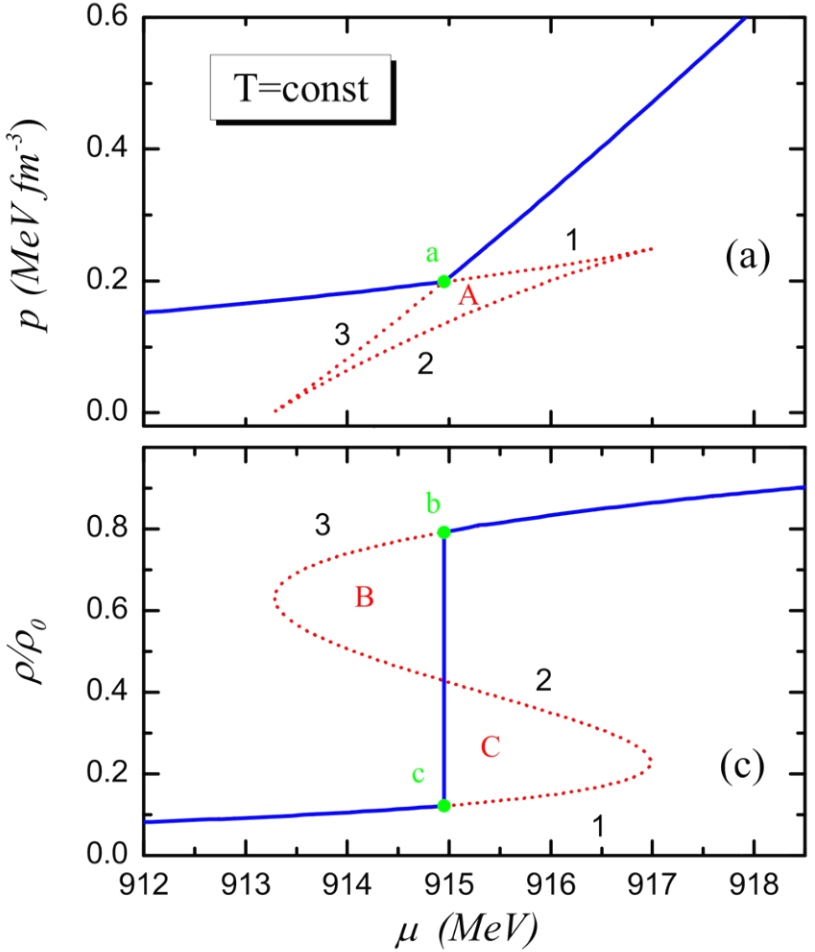
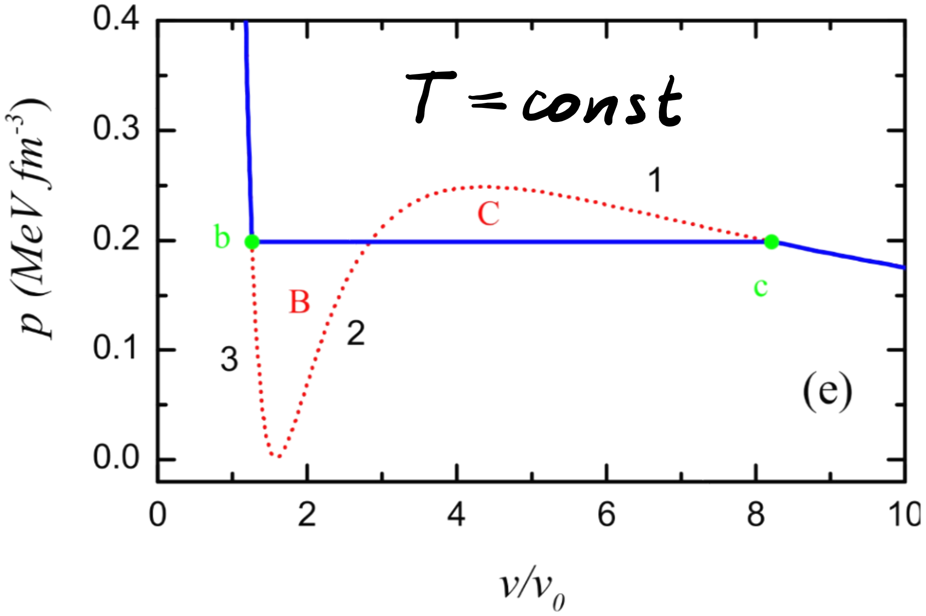
FIRST ORDER PHASE TRANSITION IN THE GRAND CANONICAL ENSEMBLE

The 1-st order phase transition in the grand canonical ensemble *(T,V,μ)* at fixed values of *T* is characterized by the cusp in the pressure and jump in the density as functions of chemical potential, and the jump in the specific volume as function of pressure



RMF model

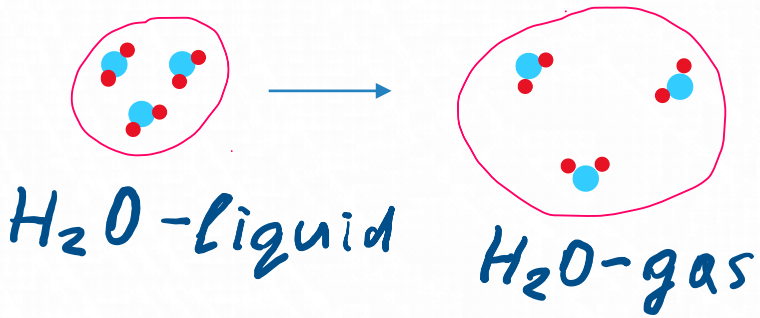


A.S.P., Nucl. Phys. A 887 (2012) 1-21

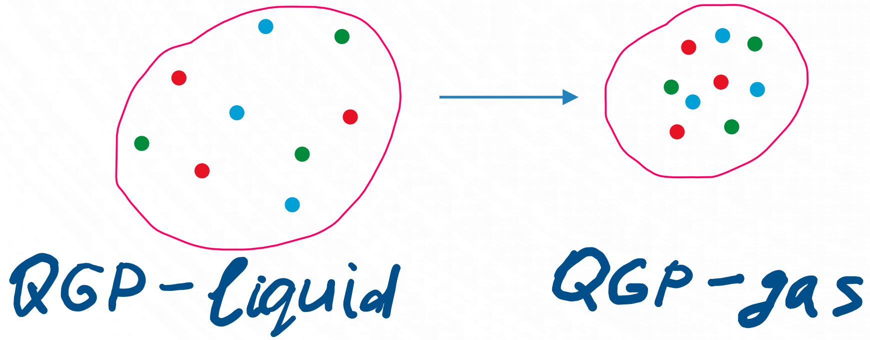
WHICH TYPE OF PHASE TRANSITION DOES THE LATTICE QCD DESCRIBE?

LIQUID-GAS PHASE TRANSITION (WEAKENING OF INTERACTION WITHOUT CHANGING COMPONENTS)

- MOLECULE-MOLECULE TRANSITION:

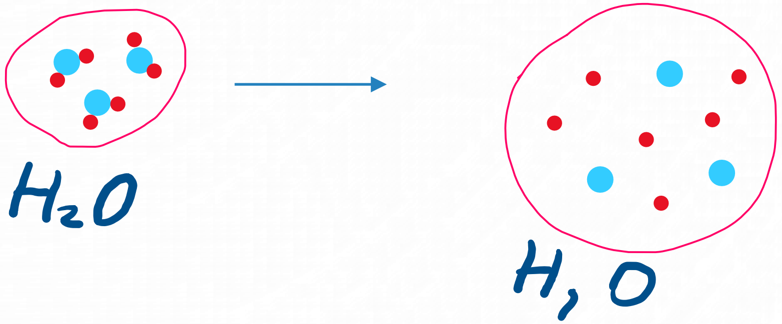


- QUARK-QUARK TRANSITION:

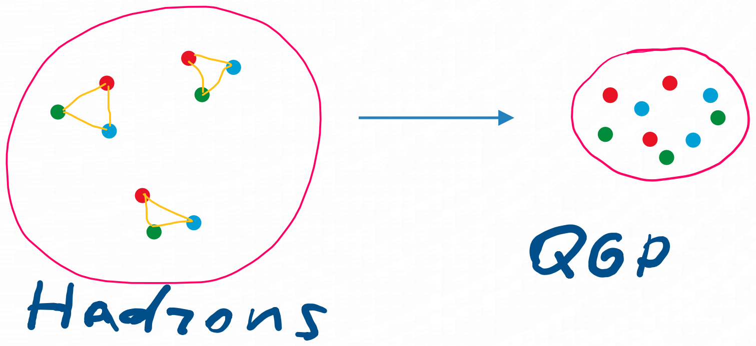


BREAKUP PHASE TRANSITION (DECONFINEMENT, DECAY INTO CONSTITUENT ELEMENTS)

- MOLECULE-ATOM TRANSITION:



- HADRON-QUARK TRANSITION:



QUARK-HADRON DUALITY

CONNECTION OF HADRONS WITH LQCD

- LQCD contains only quark-gluon degrees of freedom and it does not have any direct connection with hadrons
- The hadronic degrees of freedom are not contained in LQCD and therefore from the intrinsic calculations of LQCD it is not possible to prove the existence of the hadron phase
- LQCD cannot determine what is created at low temperatures: hadrons or bubble of strong interacting nonperturbative QGP (liquid)
- The only proof of connection of hadrons and LQCD is the concept of the so-called quark-hadron duality, which indicates the coincidence of the bulk thermodynamic quantities of LQCD and the Hadron Resonance Gas (HRG) model at low temperatures [A. Andronic et al., Nature 561, 321 (2018)]

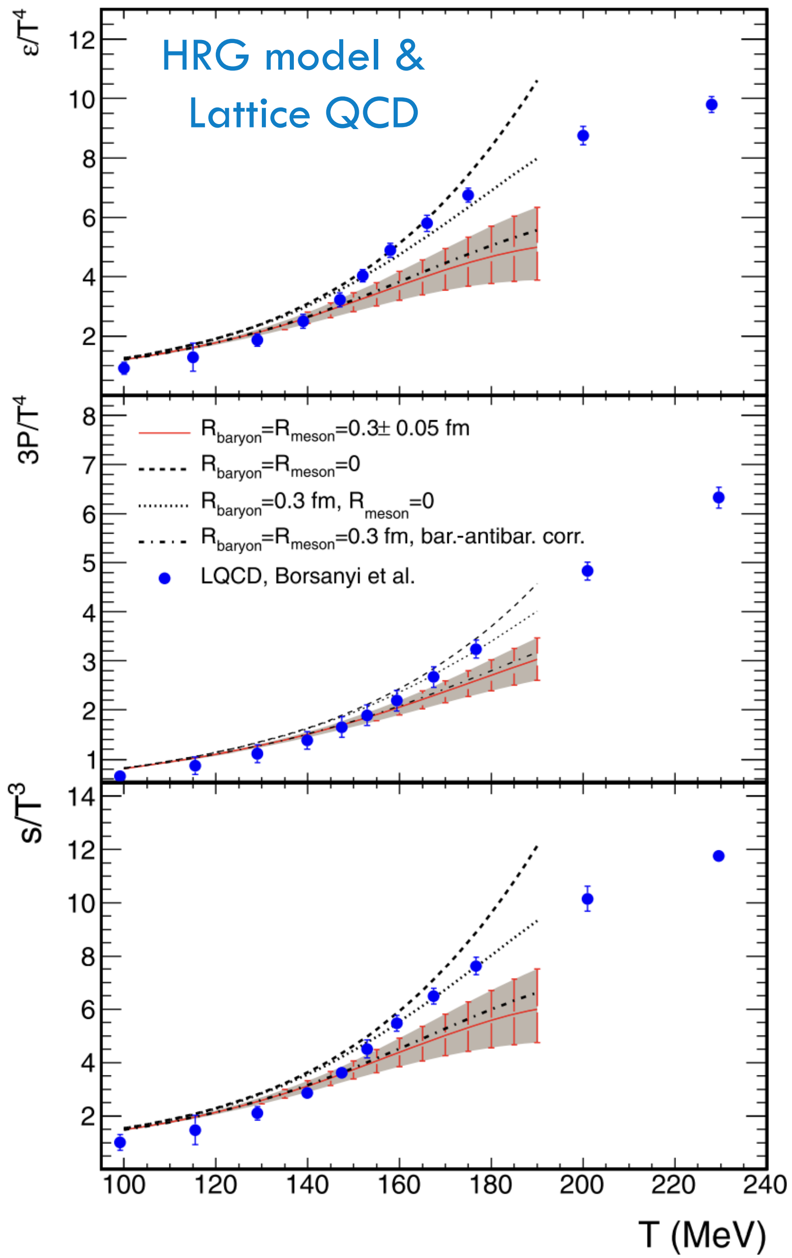


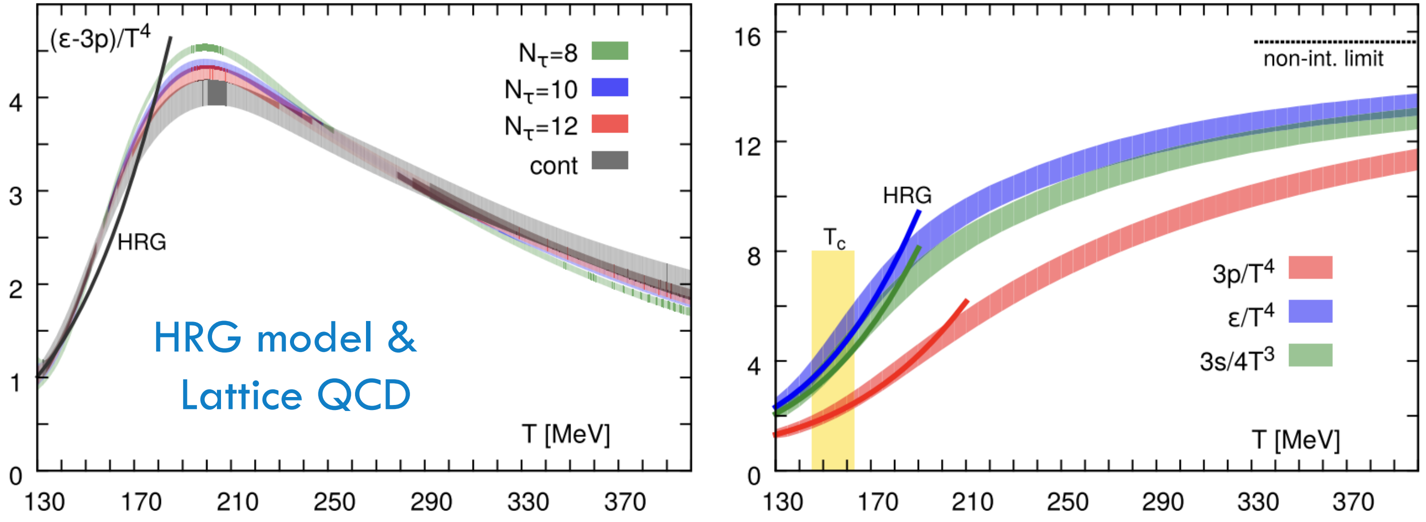
Fig. 4. Temperature dependence of thermodynamical quantities. The calculations with the hadron gas model are shown without (dashed line) and with (band for $R = 0.3 \pm 0.05$ fm) the excluded volume correction. The case of $R_{meson} = 0$ is shown with the dotted line, while the dot-dashed line denotes the effect of the baryon–antibaryon annihilation correction. They are compared to LQCD results of Borsányi et al. [37].

Figure and caption are taken from A. Andronic et al., Phys. Lett. B 718 (2012) 80–85

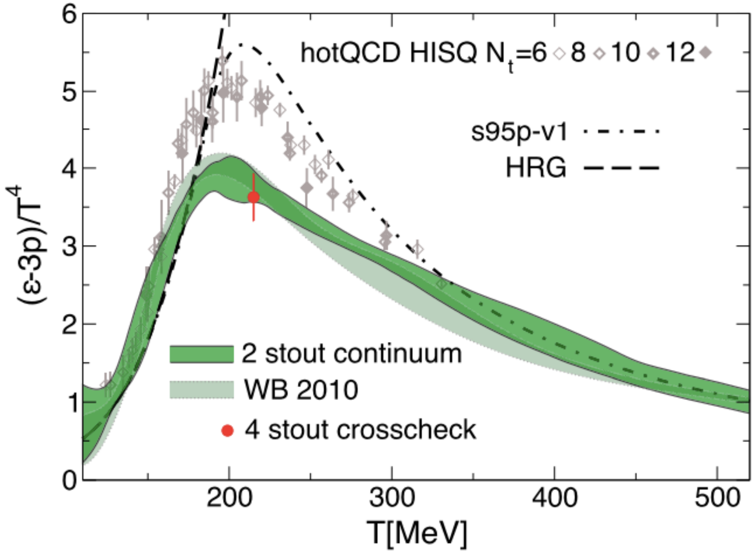
Figure is taken from A. Bazavov et al. (HotQCD Collaboration), Phys. Rev. D 90, 094503 (2014)

QUARK-HADRON DUALITY

CONNECTION OF HADRONS WITH LQCD



- This coincidence of the bulk thermodynamic quantities of LQCD and HRG model is the only proof that the lattice QCD may predict the creation of hadrons at low temperatures
- However, the LQCD lacks quantitative methods to identify specific hadrons that may be formed at low temperatures



Trace anomaly

Figure is taken from S. Borsányi et al., Phys. Lett. B 730 (2014) 99

IDEAL GAS OF HADRONS

IDEAL GAS OF BOSONS AND FERMIONS IN THE GRAND CANONICAL ENSEMBLE IN A FINITE VOLUME *V* AT TEMPERATURE *T* AND CHEMICAL POTENTIAL μ

- DENSITY OF THERMODYNAMIC POTENTIAL

$$\omega = -\frac{1}{\beta V \eta} \sum_{\vec{p},\sigma} \ln\left[1 + \eta e^{-\beta(\varepsilon_{\vec{p}} - \mu)}\right]$$

- PARTICLE SPECTRA

$$\varepsilon_{\vec{p}} = \sqrt{\vec{p}^{\,2} + m^2}$$

$$p_\alpha = \frac{2\pi}{L} k_\alpha, \quad k_\alpha = 0, \pm 1, \ldots, \quad \alpha = 1, 2, 3$$

- NOTATIONS

$$\beta = \frac{1}{T}, \qquad L = V^{1/3}, \qquad \mu = b\mu_B + q\mu_Q + s\mu_S$$

$\eta = 1$ -Fermi-Dirac statistics of particles

$\eta = -1$ -Bose-Einstein statistics

IDEAL GAS OF HADRONS

OCCUPATION NUMBERS

$$\langle n_{\vec{p}\sigma} \rangle = \frac{1}{e^{\beta(\varepsilon_{\vec{p}} - \mu)} + \eta}$$

ENTROPY DENSITY

$$s = -\beta(\omega - \varepsilon + \mu\varrho)$$

DENSITY OF HEAT CAPACITY

$$c_{V\mu} = \frac{\beta^2}{V} \sum_{\vec{p},\sigma} (\varepsilon_{\vec{p}} - \mu)^2 \langle n_{\vec{p}\sigma} \rangle (1 - \eta \langle n_{\vec{p}\sigma} \rangle)$$

- ENERGY DENSITY

$$\varepsilon = \frac{1}{V} \sum_{\vec{p},\sigma} \varepsilon_{\vec{p}} \langle n_{\vec{p}\sigma} \rangle$$

- PRESSURE

$$p = \frac{1}{3V} \sum_{\vec{p},\sigma} \frac{\vec{p}^{\,2}}{\varepsilon_{\vec{p}}} \langle n_{\vec{p}\sigma} \rangle,$$

- DENSITY OF PARTICLES

$$\varrho = \frac{1}{V} \sum_{\vec{p},\sigma} \langle n_{\vec{p}\sigma} \rangle$$

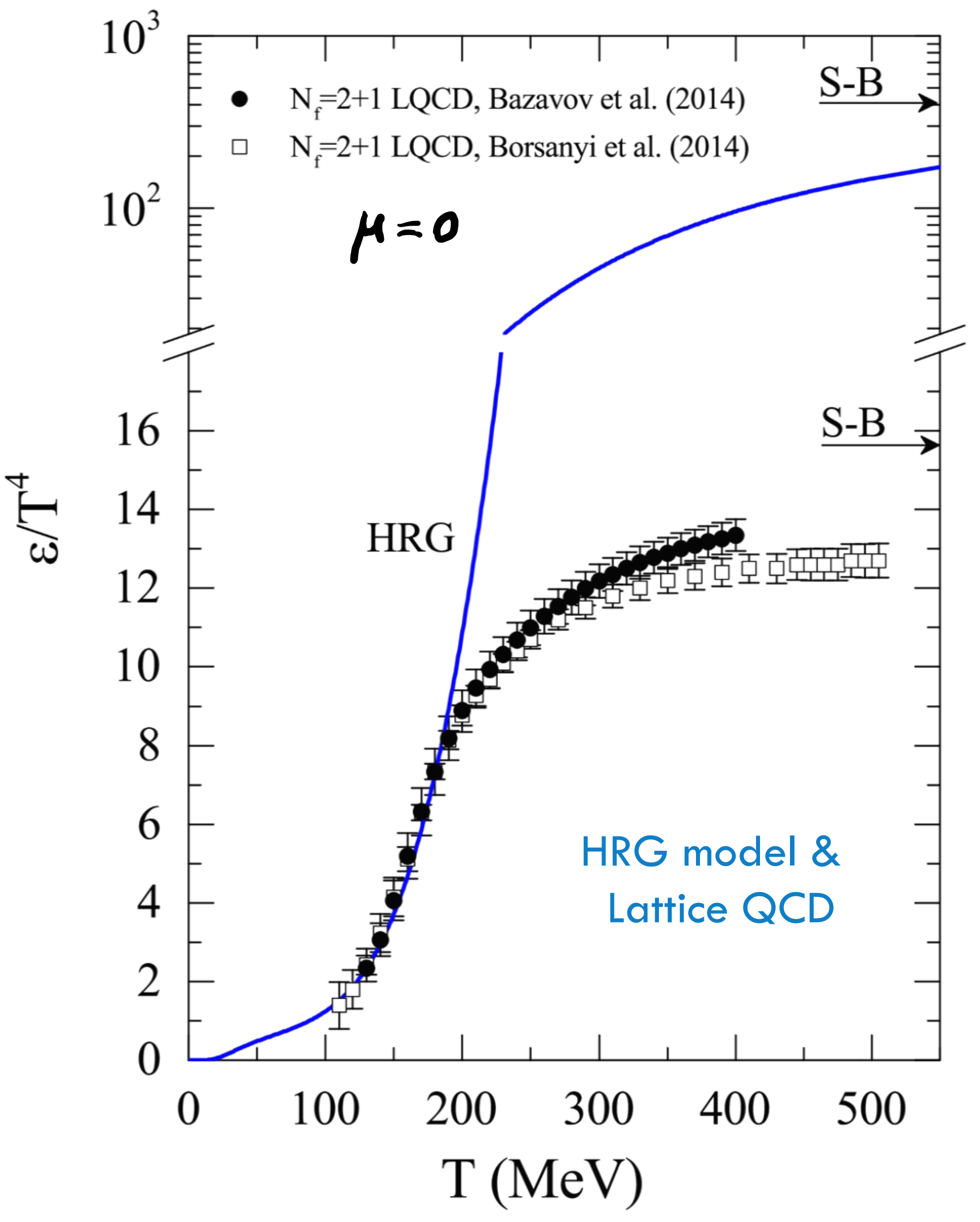
- TRACE ANOMALY

$$\beta^4 (\varepsilon - 3p) = \frac{\beta^4}{V} \sum_{\vec{p},\sigma} \frac{m^2}{\varepsilon_{\vec{p}}} \langle n_{\vec{p}\sigma} \rangle$$

COMPARISON OF HRG MODEL AND LQCD

QUARK-HADRON DUALITY AND THE STEFAN-BOLTZMANN LIMIT

- The results of HRG model and (2+1)-flavor LQCD were compared in the whole temperature region
- The quark-hadron duality at low temperatures was confirmed
- At high temperatures the curves for the energy density of the HRG model and LQCD have the same behavior but on the different scales which are determined by their own Stefan-Boltzmann limit
- Thus, these quantities are not commensurable since they correspond to different degrees of freedom
- Therefore, in order to compare them properly, we should bring them to the same scale reducing the degree of freedom by dividing the energy density to the effective degeneracy factor of the corresponding system.



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EFFECTIVE DEGENERACY FACTOR

LATTICE QCD

- EFFECTIVE DEGENERACY FACTOR OF THE QUARK AND GLUON FIELDS

$$g_{QCD} = g_g + \frac{7}{8} g_q,$$

$$g_g = 2_{spin} \times (N_c^2 - 1),$$

$$g_q = 2_{spin} \times 2_{q\bar{q}} \times N_c \times N_f$$

IDEAL HADRON GAS

- EFFECTIVE DEGENERACY FACTOR OF THE HADRON GAS

$$g_H = \sum_M g_M + \frac{7}{8} \sum_B g_B$$

$g_i = (2J_i + 1)(2I_i + 1)$ is the spin-isospin degeneracy factor of the (anti)mesons (i = M) and (anti) baryons (i = B)

The factor 7/8 appears from the difference of the Bose–Einstein and Fermi–Dirac statistics

Kohsuke Yagi et al., Quark-Gluon plasma: from big bang to little bang (Cambridge University Press, 2008);
G.E. Brown et al., Phys. Lett. B 263, 337 (1991);
L. Turko, D. Blaschke et al., EPJ Web Conf. 71, 00134 (2014)

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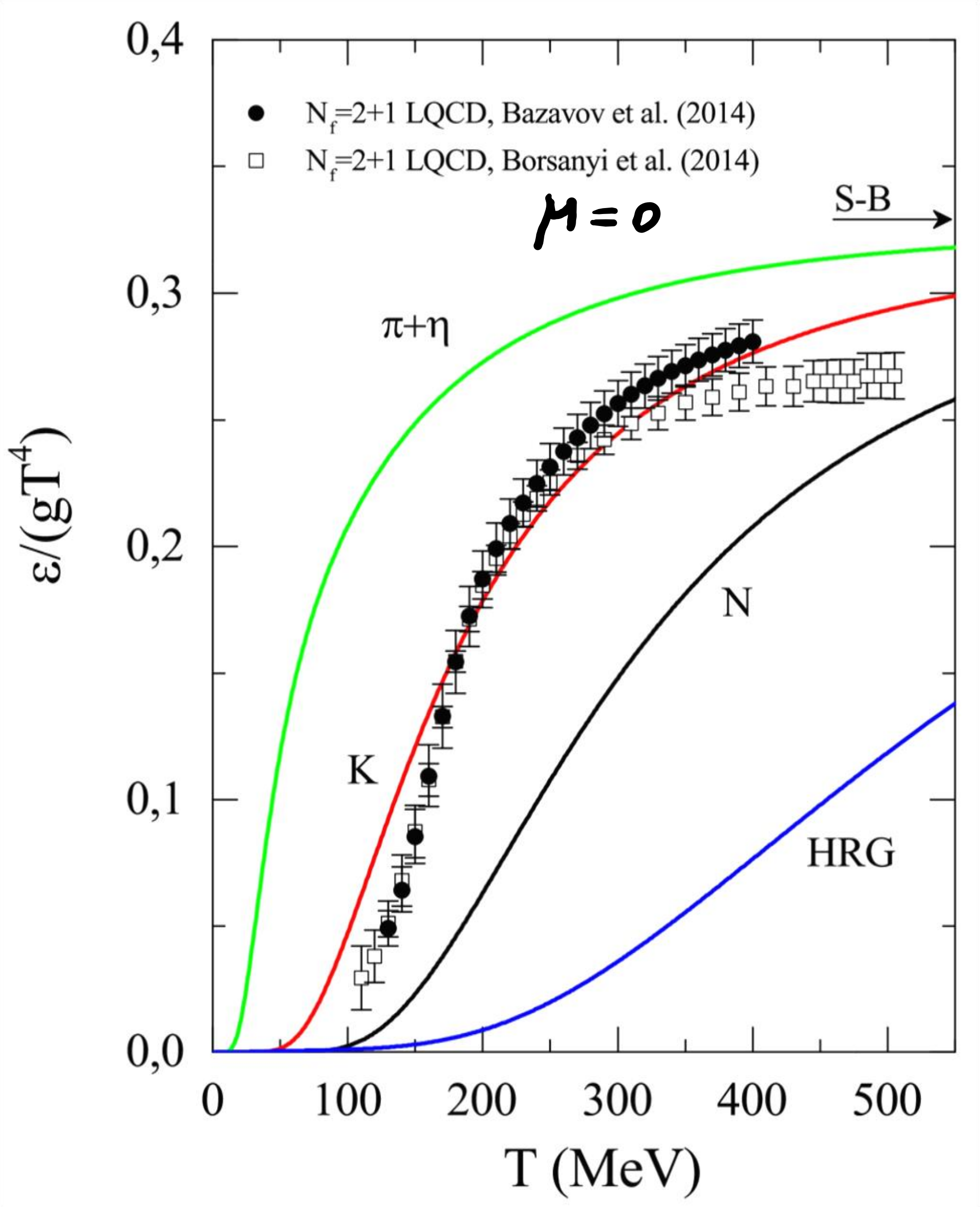
SCALED QUARK-HADRON DUALITY

- Let us scale the energy densities of the HRG model and the lattice QCD by their own effective degeneracy factors
- The quark-hadron duality of the HRG model disappears
- However, the new quark-hadron duality in the scaled variables appears for the kaon-antikaon ideal gas
- The curve of the ideal gas of kaons and antikaons has similar qualitative behaviour as the lattice QCD data and approaches them in the deconfinement region at temperatures higher than 170 MeV

Fig. 1 represents the dependence of the scaled energy density ε/g on the temperature T for the HRG model ($g_{HRG} = 1251$), the ideal gas of nucleons and antinucleons $p, n, \bar{p}, \bar{n}$ ($g_N = 7$), kaons and antikaons $K^{\pm}, K^0, \bar{K}^0$ ($g_K = 4$), light unflavored mesons $\pi^{\pm}, \pi^0, \eta$ ($g_{\pi+\eta} = 4$) and for the lattice QCD with $N_f = 2 + 1$ flavors ($g_{QCD} = 47.5$)



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VOLUME DEPENDENCE OF ENERGY DENSITY

- For the small values of volume V = (1.2 fm)^3 and at high temperatures T > 250 MeV, the scaled energy density of the ideal gas of kaons and antikaons describes the lattice QCD data obtained by the HotQCD Collaboration very well
- For large values of the volume V = (10 fm)^3, the scaled energy density of the ideal gas of kaons and antikaons becomes closer to the lattice QCD equation of state in the region of small temperatures T and describes it very well in the region of high temperatures at T > 170 MeV
- The behavior of the scaled energy density of the ideal gas of kaons and antikaons in a large volume is the same as the behavior of the scaled energy density of the lattice QCD

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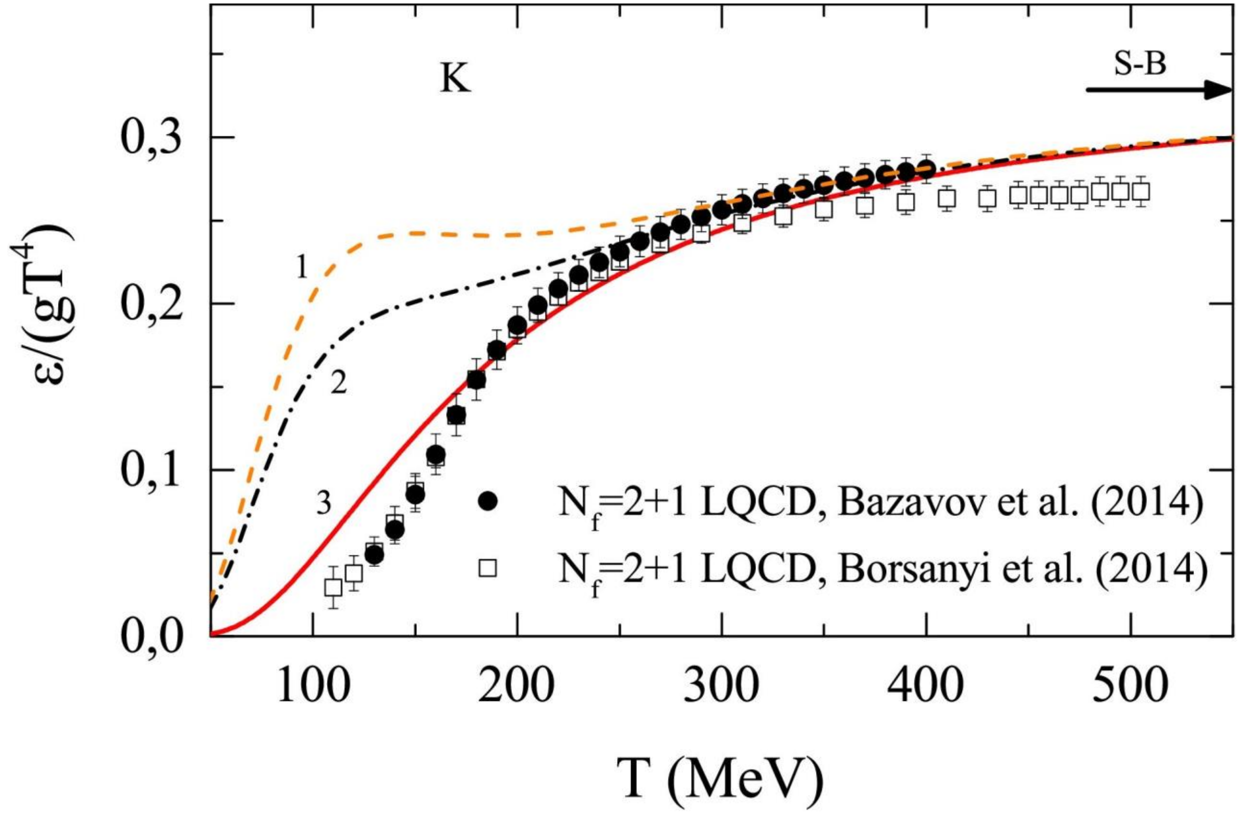

Fig. 2 The scaled energy density, the scaled pressure and the scaled trace anomaly as functions of temperature T for the ideal gas of kaons ($K^{\pm}$, K^0, $\bar{K}^0$) and for the lattice QCD with $N_f = 2 + 1$ flavors [2,3]. The lines 1, 2 and 3 are calculations for the ideal gas in the volume $V = 1.1^3$, 1.2^3 and 10^3 fm^3, respectively, at $\mu = 0$. The arrows represent the Stefan–Boltzmann limit for the scaled energy density, $\pi^2/30$, and the scaled pressure, $\pi^2/90$. Symbols represent the lattice QCD data

VOLUME DEPENDENCE OF PRESSURE

- For small values of the volume V = (1.2 fm)^3 and at T < 150 and T > 400 MeV, the scaled pressure of the ideal gas of kaons and antikaons is very close to the lattice QCD data.
- For large values of the volume V = (10 fm)^3, the scaled pressure of the ideal gas of kaons and antikaons describes the lattice QCD scaled pressure in the region of high temperatures at T > 200 MeV very well and does not differ essentially from it in the region of small temperatures T
- The behavior of the scaled pressure of the ideal gas of kaons and antikaons in a large volume is the same as the behavior of the scaled pressure of the lattice QCD

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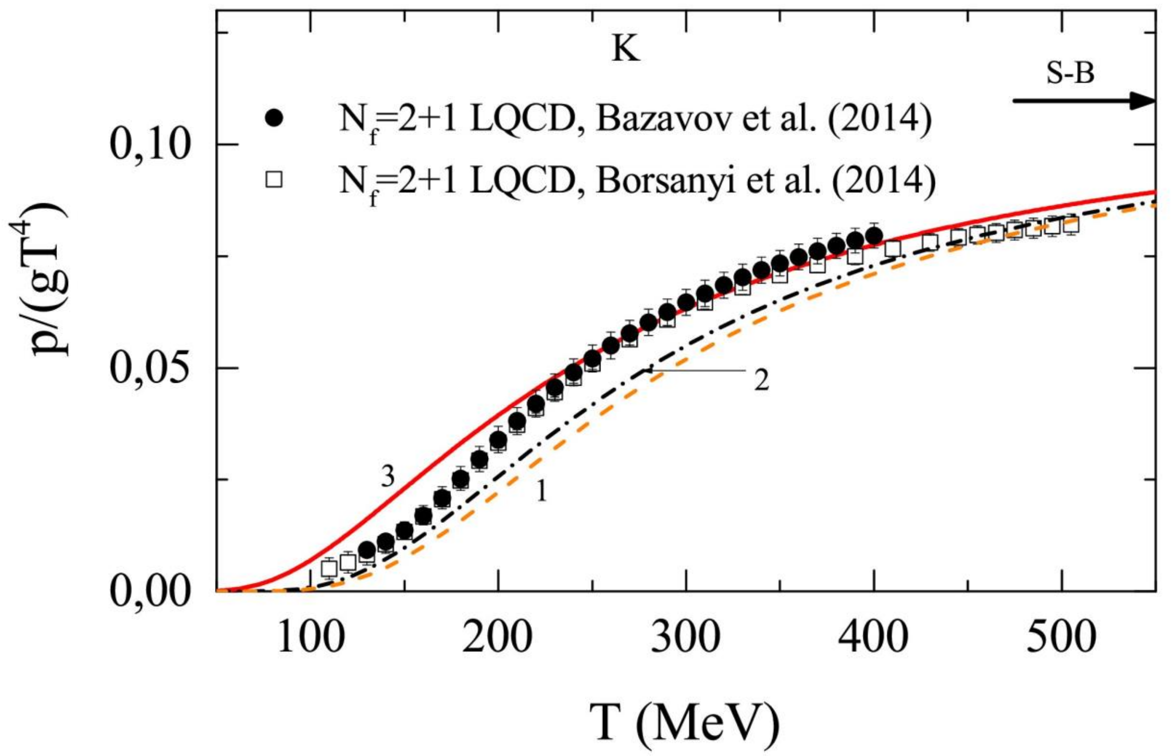

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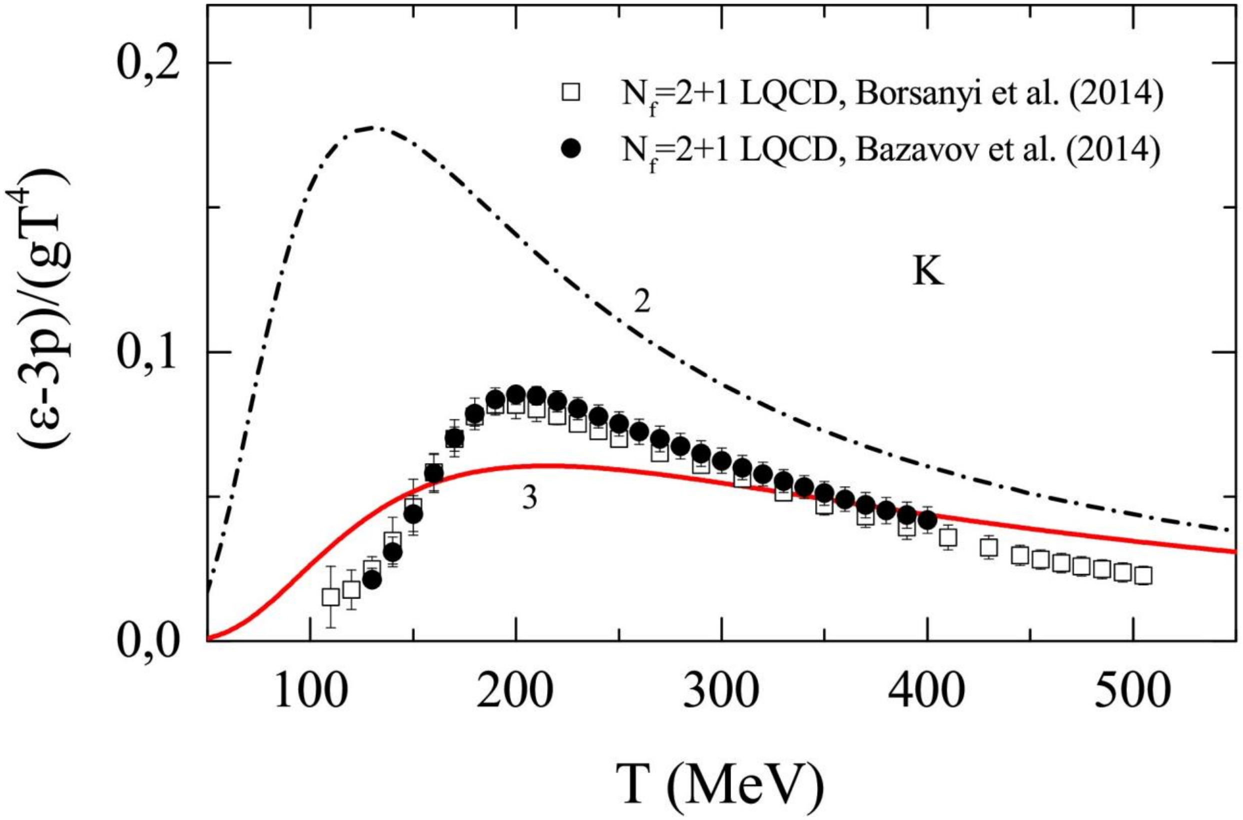
VOLUME DEPENDENCE OF TRACE ANOMALY

- The behavior of the scaled trace anomaly of the ideal gas of kaons and antikaons in both large and small volumes is similar to the behavior of the scaled trace anomaly of the lattice QCD.
- However, the curves of the scaled trace anomaly of the ideal gas differ quantitatively from the results of the lattice QCD. The scaled trace anomaly of the ideal gas of kaons and antikaons decreases with the volume V.



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VOLUME DEPENDENCE OF ENTROPY DENSITY

- For small values of the volume V = (1.2 fm)^3 and at T > 250 MeV, the scaled entropy density of the ideal gas of kaons and antikaons describes the lattice QCD scaled results obtained by the HotQCD Collaboration very well
- For large values of the volume V = (10 fm)^3, the scaled entropy density of the ideal gas of kaons and antikaons becomes closer to the lattice QCD scaled entropy density in the region of small temperatures T and describes it very well in the region of high temperatures at T > 170 MeV
- The behavior of the scaled entropy density of the ideal gas of kaons and antikaons in a large volume is the same as the behavior of the scaled entropy density of the lattice QCD and they are very close to each other

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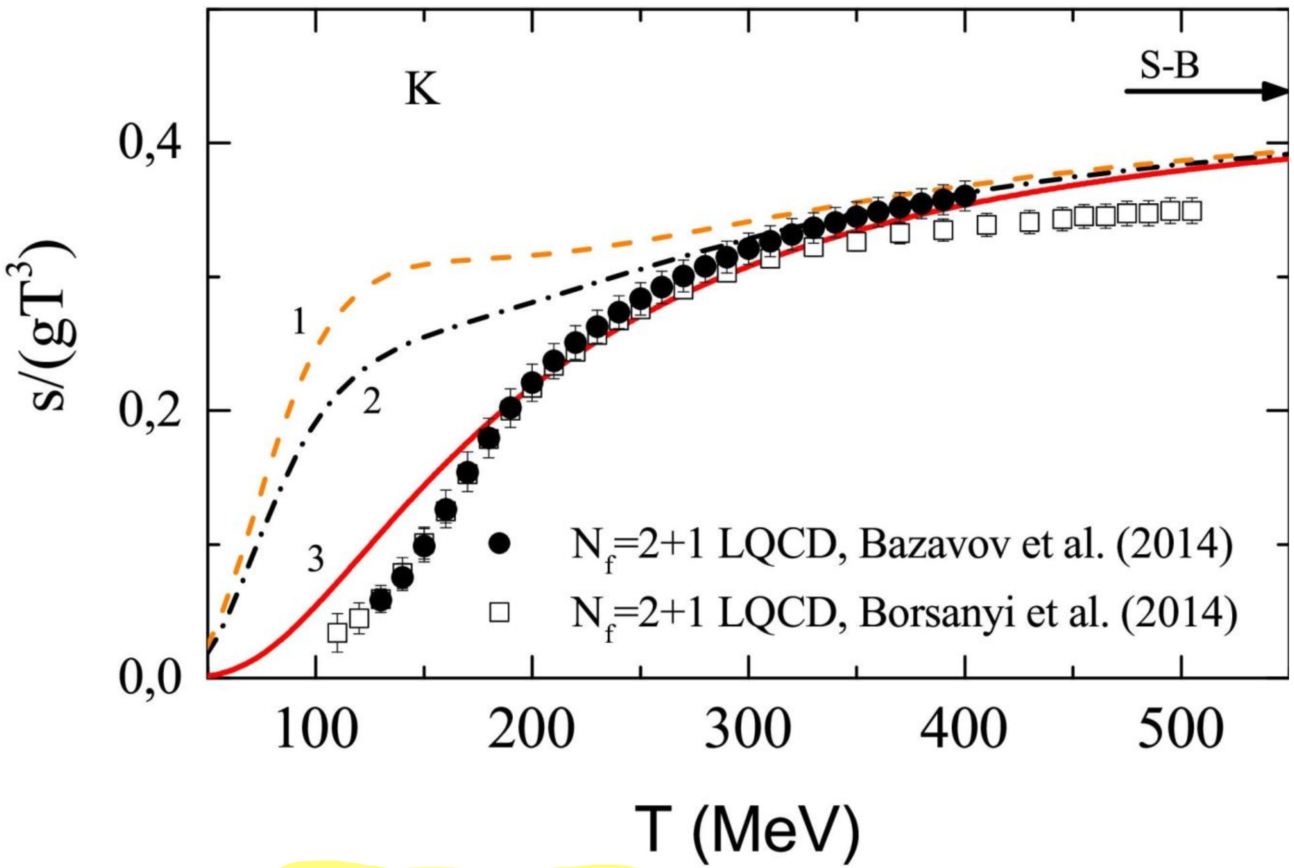

Fig. 3 The scaled entropy density and the scaled heat capacity as functions of temperature T for the ideal gas of kaons ($K^{\pm}$, K^0, $\bar{K}^0$) and for the lattice QCD with $N_f = 2 + 1$ flavors [2,3]. The lines 1, 2 and 3 are calculations for the ideal gas in the volume $V = 1.1^3$, 1.2^3 and 10^3 fm^3, respectively, at $\mu = 0$. The arrows represent the Stefan–Boltzmann limit for the scaled entropy density, $2\pi^2/45$, and the scaled heat capacity, $2\pi^2/15$. Symbols represent the lattice QCD data

VOLUME DEPENDENCE OF HEAT CAPACITY

- The high temperature plateau in the scaled heat capacity of the lattice QCD at T > 170 MeV is well described by the ideal gas of kaons and antikaons in the volume V = (1.1 fm)^3
- For large values of the volume V = (10 fm)^3, the scaled heat capacity of the ideal gas of kaons and antikaons is compatible with the lattice QCD results in the region of high temperatures at T > 250 MeV
- In general, the behavior of the scaled heat capacity of the ideal gas of kaons and antikaons in a large volume is similar to the behavior of the scaled heat capacity of the lattice QCD

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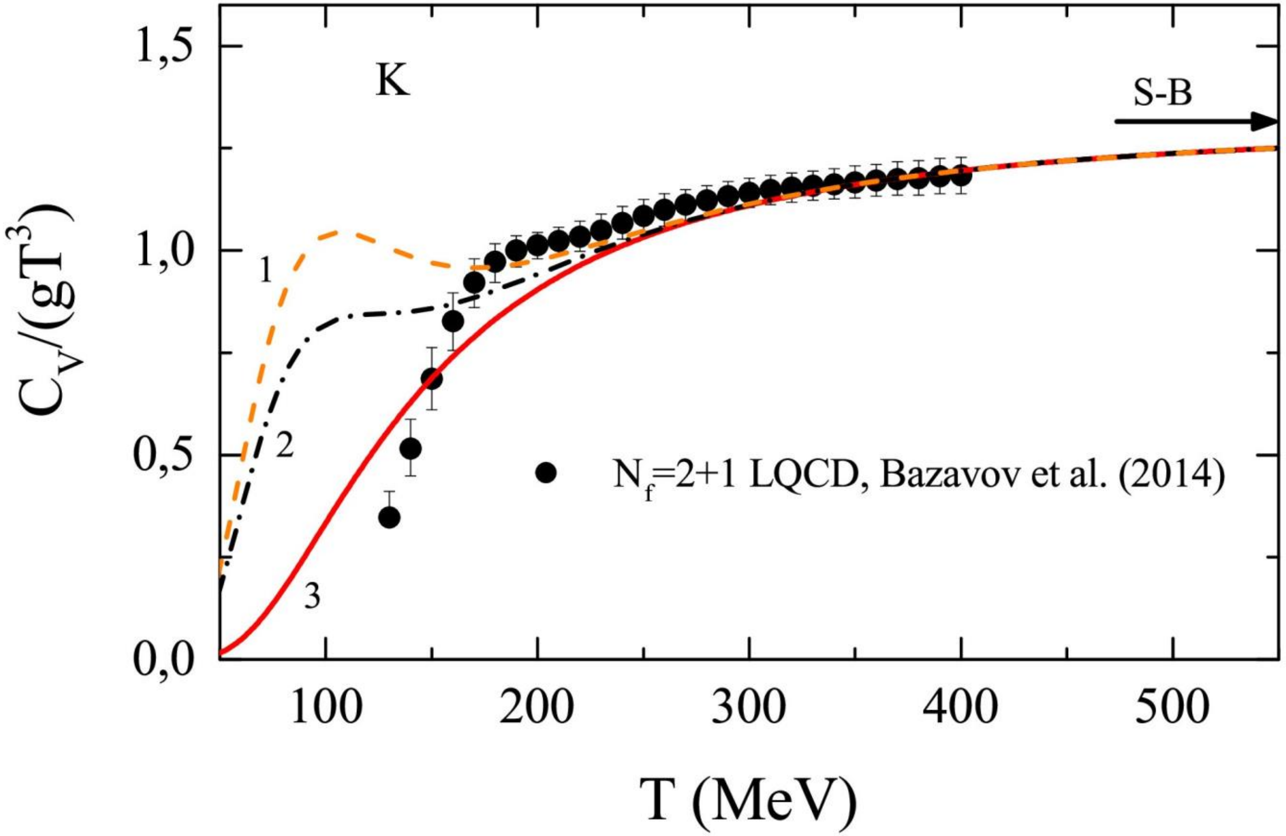

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CONCLUSIONS

1. The thermodynamic quantities of the ideal gas of hadrons and the (2 + 1)-flavor lattice QCD scaled by the effective degeneracy factors of the corresponding models were compared.

2. We have found that in terms of the scaled variables the quark-hadron duality of the lattice QCD and the hadron resonance gas (HRG) model disappears.

3. However, we have revealed that in the scaled variables the quark-hadron duality of the lattice QCD and the quantum ideal gas of kaons and antikaons appears. Namely, it appears in the ideal gas of those hadrons that contain all the three quarks u, d, s and their antiquarks.

4. Satisfactory agreement between the scaled results of the kaon ideal gas and the lattice QCD data is achieved at large values of the volume in the entire temperature range.

5. In the ideal gas of kaons there is not any phase transition. Nevertheless, in our calculations the scaled thermodynamic quantities of the ideal gas and the lattice QCD follow the same qualitative behavior and are consistent with each other especially at high temperatures in the perturbative region and the Stefan–Boltzmann limit.

THANK YOU FOR YOUR ATTENTION