On the origin of mixed inhomogeneous phase in vortical gluon plasma

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in collaboration with

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based on arXiv:2411.15085 [hep-lat]

JOINT INSTITUTE
FOR NUCLEAR RESEARCH

Outline

- Introduction
- Formulation of QCD in rotating frames: mechanical and magnetic coupling, lattice setup
- Mixed inhomogeneous phase in rotating gluodynamics and local critical temperature
- Decomposition of rotating action
- Approximation of local thermalization and TE law
- Mixed phase in rotating QCD: first results
- Conclusions

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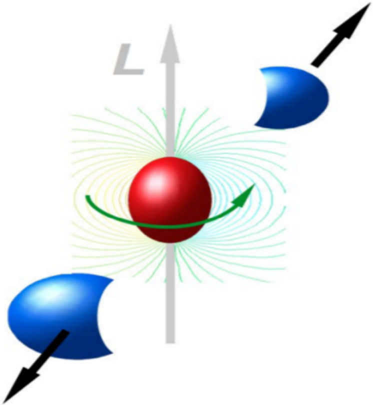
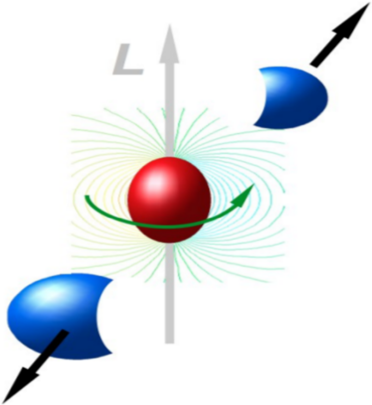
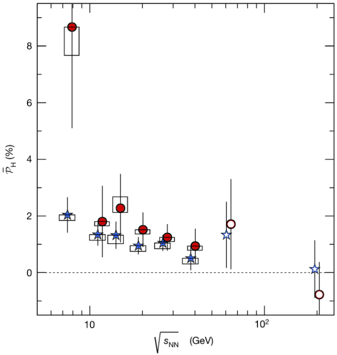
- In non-central heavy ion collisions the creation of QGP with angular momentum is expected.



- In non-central heavy ion collisions the creation of QGP with angular momentum is expected.
- The rotation occurs with relativistic velocities.





[L. Adamczyk et al. (STAR), Nature **548**, 62–65 (2017), arXiv:1701.06657 [nucl-ex]]

$\langle\omega\rangle \sim 7$ MeV $\quad(\sqrt{s_{NN}}$-averaged)

Theoretical study of rotating QCD properties

Various properties of rotating QCD are actively studied (thermodynamics, phase diagram, etc.):

- E. Siri and N. Sadooghi, (2024), arXiv:2411.12581 [hep-ph]
- P. Singha, V. E. Ambrus, and M. N. Chernodub, Phys. Rev. D **110**, 094053 (2024), arXiv:2407.07828 [hep-ph]
- E. Siri and N. Sadooghi, Phys. Rev. D **110**, 036016 (2024), arXiv:2405.09481 [hep-ph]
- Y. Chen, X. Chen, D. Li, and M. Huang, (2024), arXiv:2405.06386 [hep-ph]
- F. Sun et al., Phys. Rev. D **109**, 116017 (2024), arXiv:2402.16595 [hep-ph]
- Y. Jiang, Phys. Lett. B **853**, 138655 (2024), arXiv:2312.06166 [hep-th]
- D. N. Voskresensky, Phys. Rev. D **109**, 034030 (2024), arXiv:2311.06804 [nucl-th]
- K. Mameda and K. Takizawa, Phys. Lett. B **847**, 138317 (2023), arXiv:2308.07310 [hep-ph]
- F. Sun, K. Xu, and M. Huang, Phys. Rev. D **108**, 096007 (2023), arXiv:2307.14402 [hep-ph]
- H.-L. Chen, Z.-B. Zhu, and X.-G. Huang, Phys. Rev. D **108**, 054006 (2023), arXiv:2306.08362 [hep-ph]
- and many others . . .

Mostly a "global", bulk-averaged T_c is measured, but action is spatially inhomogeneous.

Mixed phase in rotating QCD and QCD-like systems:

- M. N. Chernodub, Phys. Rev. D **103**, 054027 (2021), arXiv:2012.04924 [hep-ph]
- N. R. F. Braga and O. C. Junqueira, Phys. Lett. B **848**, 138330 (2024), arXiv:2306.08653 [hep-th]
- S. Chen, K. Fukushima, and Y. Shimada, Phys. Lett. B **859**, 139107 (2024), arXiv:2404.00965 [hep-ph]
- Y. Jiang, Phys. Rev. D **110**, 054047 (2024), arXiv:2406.03311 [nucl-th]

Lattice study of rotating QCD properties

Formulation of rotating QCD on the lattice

- A. Yamamoto and Y. Hirono, Phys. Rev. Lett. **111**, 081601 (2013), arXiv:1303.6292 [hep-lat]

Bulk-averaged critical temperature in rotating gluodynamics: T_c increases with Ω:

- V. Braguta, A. Kotov, D. Kuznedelev, and A. Roenko, JETP Lett. **112**, 6–12 (2020)
- V. Braguta, A. Kotov, D. Kuznedelev, and A. Roenko, Phys. Rev. D **103**, 094515 (2021), arXiv:2102.05084 [hep-lat]

Bulk-averaged critical temperature in rotating QCD: $T_c^{(\mathrm{conf})}$ and $T_c^{(\mathrm{chiral})}$ both increase with rotation; fermions and gluons have opposite influence on T_c, but contribution from gluons dominate.

- V. Braguta, A. Kotov, A. Roenko, and D. Sychev, PoS **LATTICE2022**, 190 (2023), arXiv:2212.03224 [hep-lat]
- J.-C. Yang and X.-G. Huang, (2023), arXiv:2307.05755 [hep-lat]

Thermodynamical properties and moment of inertia of rotating gluon plasma: $I < 0$ below $T_s \simeq 1.5 T_c$, instability? negative spin-vortical coupling? NBE?

- V. V. Braguta, M. N. Chernodub, A. A. Roenko, and D. A. Sychev, Phys. Lett. B **852**, 138604 (2024), arXiv:2303.03147 [hep-lat]
- V. V. Braguta et al., JETP Lett. **117**, 639–644 (2023)
- V. V. Braguta et al., Phys. Rev. D **110**, 014511 (2024), arXiv:2310.16036 [hep-ph]

Mixed inhomogeneous phase in rotating gluon plasma: local $T_c(r, \Omega)$ increases with Ω and r, deconfinement in the center and confinement at the periphery

- V. V. Braguta, M. N. Chernodub, and A. A. Roenko, Phys. Lett. B **855**, 138783 (2024), arXiv:2312.13994 [hep-lat]
- V. V. Braguta, M. N. Chernodub, Y. A. Gershtein, and A. A. Roenko, (2024), arXiv:2411.15085 [hep-lat]

Outline

The coordinates in the laboratory frame $x^\alpha_{\text{lab}} \equiv \bar{x}^\alpha = (\bar{t}, \bar{x}, \bar{y}, \bar{z}) = (\bar{t}, \bar{r}\cos\bar{\varphi}, \bar{r}\sin\bar{\varphi}, \bar{z})$ and in the rotating frame $x^\alpha = (t, x, y, z) = (t, r\cos\varphi, r\sin\varphi, z)$ are connected by the transformation (rotation around z-axis)

$$\varphi = [\bar{\varphi} - \Omega t]_{2\pi}\,, \quad t = \bar{t}, \quad z = \bar{z}, \quad r = \bar{r}, \tag{1}$$

which leads to the metric tensor

$$g_{\mu\nu} = \frac{\partial \bar{x}^\alpha}{\partial x^\mu}\frac{\partial \bar{x}^\beta}{\partial x^\nu}\eta_{\alpha\beta} = \begin{pmatrix} 1 - r^2\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{2}$$

where $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$, and the verbein is $\eta_{mn} = g_{\mu\nu} e^\mu_m e^\nu_n$,

$$e^t_0 = e^x_1 = e^y_2 = e^z_3 = 1\,, \quad e^x_0 = y\Omega\,, \quad e^y_0 = -x\Omega\,. \tag{3}$$

The theory is formulated in the rotating reference frame, $\Omega = \partial\bar{\varphi}/\partial t$.
The causality restriction is $\Omega R < 1$.

The Dirac Lagrangian in curved space is given by

$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu}(\partial_{\mu} + \Gamma_{\mu}) - m\right)\psi \tag{4}$$

where

$$\gamma^{\mu} = \gamma^{i} e_{i}^{\mu}, \qquad \Gamma_{\mu} = -\frac{i}{4}\omega_{\mu ij}\sigma^{ij}, \qquad \omega_{\mu ij} = g_{\alpha\beta} e_{i}^{\alpha}\left(\partial_{\mu} e_{j}^{\beta} + \Gamma^{\beta}_{\nu\mu} e_{j}^{\nu}\right), \qquad \sigma^{ij} = \frac{i}{2}\left(\gamma^{i}\gamma^{j} - \gamma^{j}\gamma^{i}\right), \tag{5}$$

The Dirac Lagrangian in curved space is given by

$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu}(\partial_{\mu} + \Gamma_{\mu}) - m\right)\psi = \mathcal{L}_{\psi}^{(0)} + \mathcal{L}_{\psi}^{(1)} \equiv \mathcal{L}_{\psi,\mathrm{lab}} + \mathcal{L}_{\psi,\mathrm{mech}}\,, \tag{4}$$

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and

$$\mathcal{L}_{\psi,\mathrm{lab}} \equiv \mathcal{L}_{\psi}^{(0)} = \bar{\psi}\left(i\gamma^{i}\partial_{i} - m\right)\psi\,, \tag{6}$$

$$\mathcal{L}_{\psi,\mathrm{mech}} \equiv \mathcal{L}_{\psi}^{(1)} = \bar{\psi}(\boldsymbol{\Omega}\cdot\hat{\boldsymbol{J}})\psi = \bar{\psi}\gamma^{0}\left(-i\Omega(-y\partial_{x} + x\partial_{y}) + \frac{1}{2}\Omega\sigma^{12}\right)\psi\,. \tag{7}$$

The angular momentum operator $\hat{\boldsymbol{J}}$ contains orbital and spin terms.

For the fermions, the linear “mechanical-vortical coupling” is realized.

Lagrangian of rotating gluons

The Lagrangian of Yang-Mills theory in the Minkowski curved spacetime has the following form:

$$\mathcal{L}_G = -\frac{1}{4g_{YM}^2} g^{\mu\nu} g^{\alpha\beta} F^a_{\mu\alpha} F^a_{\nu\beta} \tag{8}$$

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where

$$\mathcal{L}_{G,\mathrm{lab}} \equiv \mathcal{L}_G^{(0)} = -\frac{1}{4g_{YM}^2} \eta^{\mu\nu} \eta^{\alpha\beta} F^a_{\mu\alpha} F^a_{\nu\beta}\,, \tag{9}$$

$$\mathcal{L}_{G,\mathrm{mech}} \equiv \mathcal{L}_G^{(1)} = \boldsymbol{\Omega} \cdot \boldsymbol{J}_G = \boldsymbol{\Omega} \cdot \left(\frac{1}{g_{YM}^2} \boldsymbol{r} \times (\boldsymbol{E}^a \times \boldsymbol{B}^a) \right), \tag{10}$$

$$\mathcal{L}_{G,\mathrm{magn}} \equiv \mathcal{L}_G^{(2)} = \frac{1}{2g_{YM}^2} \Big[\Omega^2 (\boldsymbol{B}^a \cdot \boldsymbol{r})^2 + r^2 (\boldsymbol{B}^a \cdot \boldsymbol{\Omega})^2 \Big]. \tag{11}$$

The angular momentum of gluons $\boldsymbol{J}_G$ (in the non-rotating limit) is $J_{G,i} = \frac{1}{2}\varepsilon_{ijk} J_G^{jk}$, $J_G^{ij} = x^i T_G^{jt} - x^j T_G^{it}$.

For rotating gluons, there are two contributions: linear “mechanical-vortical coupling” and quadratic “magneto-vortical coupling”.

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Note that the Lagrangian is written in terms of fields in rotating frames, and $[J_{G,z}]_{\mathrm{lab}} \to [J_{G,z} + \Omega \cdot (\dots)]_{\mathrm{rot}}$

Lattice simulation is a powerful method to study strong-interacting systems:

- Based on the *path-integral* representation of the partition function:

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[A]\mathcal{D}[\psi, \bar{\psi}]\, e^{-S_E(\psi,\bar{\psi},A)}\, \mathcal{O}(\psi, \bar{\psi}, A)\,, \qquad \mathcal{Z} = \int \mathcal{D}[A]\mathcal{D}[\psi, \bar{\psi}]\, e^{-S_E(\psi,\bar{\psi},A)}$$

- The spacetime is discretized on the hypercubic Euclidean lattice with finite spacing a.
- The integrals are computed using Monte-Carlo algorithms. HPC is needed.
- The continuum limit $a \to 0$ should be taken.
- Statistical and systematic uncertainties are under control.

QCD on the lattice

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We will focus on the properties of the rotating gluon plasma

We study rotating gluodynamics at thermodynamic equilibrium.
Observables are calculated on the lattice from first principles:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \, e^{-S_G[U]} \, \mathcal{O}(U) \,, \qquad \text{where} \qquad Z = \mathrm{Tr}\left[e^{-\hat{H}/T_0} \right] = \int \mathcal{D}U \, e^{-S_G[U]} \,. \tag{12}$$

The Euclidean action S_G in co-rotating reference frame is formulated in curved space,

$$g^E_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & -y\Omega_I \\ 0 & 1 & 0 & x\Omega_I \\ 0 & 0 & 1 & 0 \\ -y\Omega_I & x\Omega_I & 0 & 1 + r^2\Omega_I^2 \end{pmatrix}, \tag{13}$$

where $r^2 = x^2 + y^2$, and the angular velocity is imaginary, $\Omega_I = \partial_\tau \bar{\varphi} = -i\partial_t \bar{\varphi} = -i\Omega$, to avoid the sign problem.

The inverse temperature $1/T_0$ sets the system length in τ-direction.

Ehrenfest–Tolman (TE) law: the local temperature is $T(r)\sqrt{g_{00}} = T(r)\sqrt{1 - r^2\Omega^2} = T(r)\sqrt{1 + r^2\Omega_I^2} = T_0$.

We denote by $T \equiv T_0$ the temperature at the rotation axis ($r = 0$).

Lattice formulation of rotating gluodynamics

The gluon action is a quadratic function in angular velocity

$$S = \frac{1}{4g_{YM}^2}\int d^4x \sqrt{g_E}\, g_E^{\mu\nu} g_E^{\alpha\beta} F^a_{\mu\alpha} F^a_{\nu\beta} \equiv S_0 + S_1 \Omega_I + S_2 \frac{\Omega_I^2}{2}\,, \tag{14}$$

where

$$S_0 = \frac{1}{4g_{YM}^2}\int d^4x F^a_{\mu\nu} F^a_{\mu\nu}\,, \tag{15}$$

$$S_1 = \frac{1}{g_{YM}^2}\int d^4x \left[-yF^a_{xy}F^a_{y\tau} - yF^a_{xz}F^a_{z\tau} + xF^a_{yx}F^a_{x\tau} + xF^a_{yz}F^a_{z\tau}\right], \tag{16}$$

$$S_2 = \frac{1}{g_{YM}^2}\int d^4x \left[r^2 (F^a_{xy})^2 + y^2 (F^a_{xz})^2 + x^2 (F^a_{yz})^2 + 2xyF^a_{xz}F^a_{zy}\right], \tag{17}$$

Sign problem

- The sign problem is due to the linear terms ($S_1 \neq 0$)
- The Monte–Carlo simulation is conducted with imaginary angular velocity $\Omega_I = -i\Omega$
- The results are analytically continued to real angular velocity, $\Omega^2 \leftrightarrow -\Omega_I^2$

Causality restriction

- Analytic continuation is allowed only for bounded system with $\Omega r < 1$, i.e. $v_I^2 = (\Omega_I R)^2 < 1/2$
- Boundary conditions are important! (two different types of b.c.: open/periodic)

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Observables

The Polyakov loop is an order parameter,

$$L(x,y) = \frac{1}{N_z} \sum_z \mathrm{Tr} \left[\prod_{\tau=0}^{N_t-1} U_4(\vec{r},\tau) \right], \qquad L = \frac{1}{N_s^2} \sum_{x,y} L(x,y) \,. \tag{18}$$

In confinement $\langle L \rangle = 0$; in deconfinement $\langle L \rangle \neq 0$. $\langle L \rangle = e^{-F_Q/T}$
The local critical temperature is associated with the peak of the local Polyakov loop susceptibility

$$\chi_L(r) = \langle |L(r)|^2 \rangle - \langle |L(r)| \rangle^2 \,. \tag{19}$$

We use tree-level improved (Symanzik) lattice action; lattice size $N_t \times N_z \times N_s^2$; $R \equiv a(N_s - 1)/2$;
The temperature is $T = 1/N_t a$. It coincides with the temperature on the rotation axis T_0.

Inhomogeneous phases for imaginary rotation

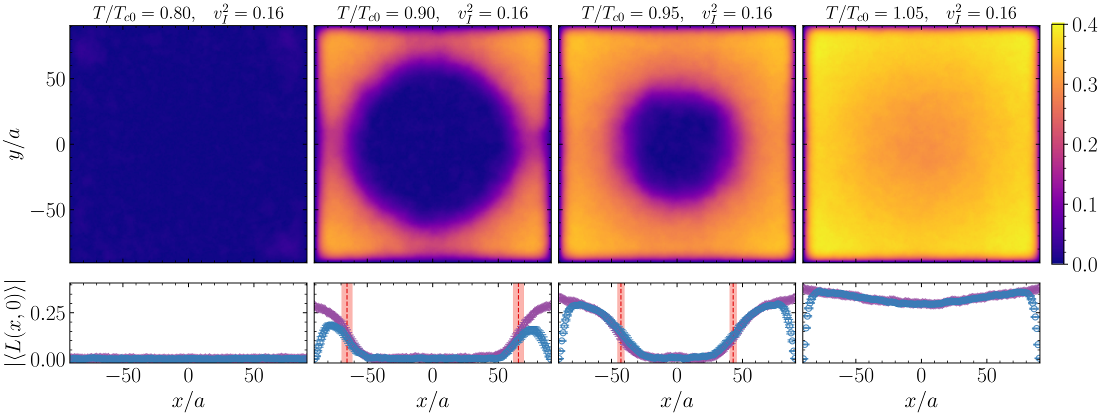


Figure: The distribution of the local Polyakov loop in x, y-plane for the lattice of size $5 \times 30 \times 181^2$ at the fixed imaginary velocity at the boundary $v_I^2 \equiv (\Omega_I R)^2 = 0.16$ and different on-axis temperatures, $T = 1/N_t a$.

- As the (on-axis) temperature increases, the radius of the inner confining region shrinks.
- Boundary is screened.
- Local thermalization takes place; Phase transition occurs as a vortex evolution.

Inhomogeneous phases for imaginary rotation

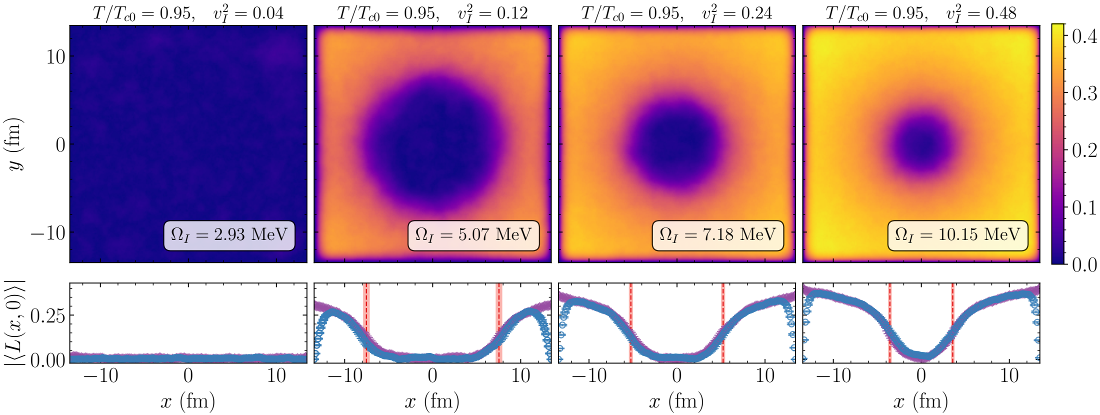


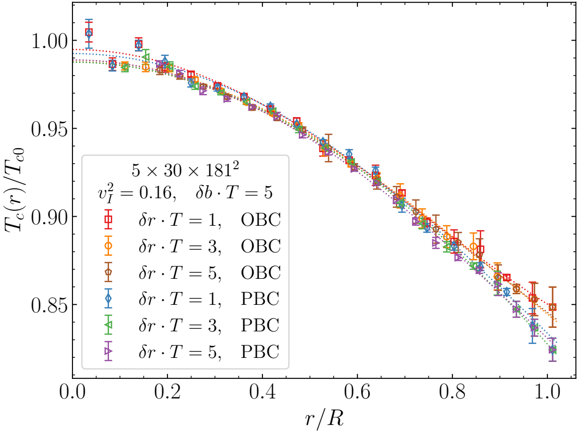
Figure: The distribution of the local Polyakov loop in x, y-plane for the lattice of size $5 \times 30 \times 181^2$ at the fixed temperature $T = 0.95\,T_{c0}$ and different Ω_I; System size $R = 13.5$ fm.

- Mixed inhomogeneous phase may be observed for $T \lesssim T_{c0}$. For imaginary rotation, deconfinement appears at the periphery; confinement is in the central regions.
- The confinement region shrinks with the increase in Ω_I;

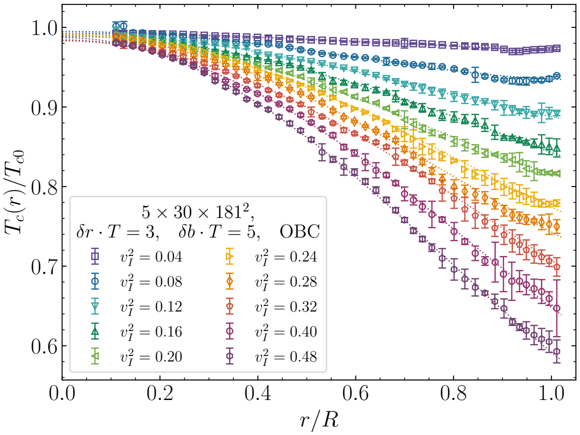
Local critical temperature

The local critical temperature $T_c(r)$ is the temperature at the rotation axis when the phase transition occurs at radius r.

- Technical details: We split the system into thin cylinders of width δr and measure $T_c(r)$.



- Results for different $\delta r \cdot T = 1, \ldots, 5$ are in agreement.
- δb is a width of ignored boundary layer
- Minor difference on b.c. appears at $r/R \sim 1$

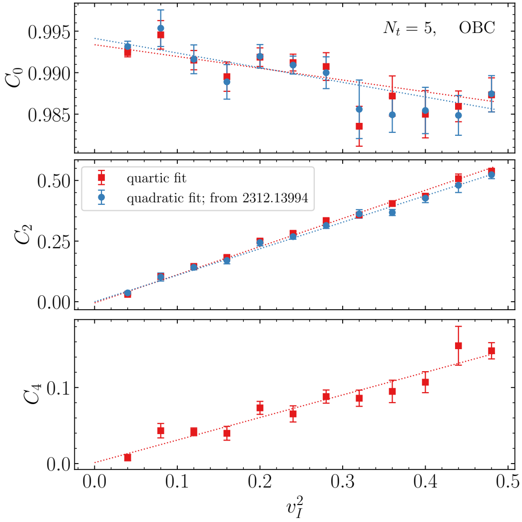


- The results in the whole region are well described by the quartic formula

$$\frac{T_c(r)}{T_{c0}} = C_0 - C_2\left(\frac{r}{R}\right)^2 + C_4\left(\frac{r}{R}\right)^4. \quad (20)$$

- In the bulk, $r/R \lesssim 0.5$, quadratic fit is sufficient ($C_4 = 0$).

Local critical temperature



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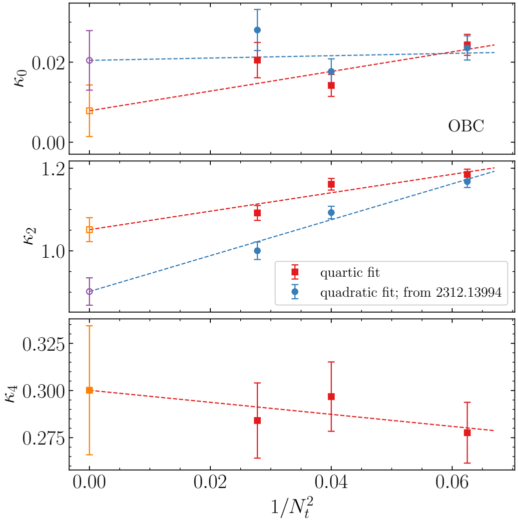
- In the bulk, $r/R \lesssim 0.5$, quadratic fit is sufficient ($C_4 = 0$).

- We found numerically that

$$C_i(v_I^2) = a_i + \kappa_i v_I^2 . \quad (21)$$

- $T_c(0) \approx T_{c0}$ with few percent accuracy:
  - Effects of finite radius R.
  - Effects of averaging in layers of width δr.



- Results: The local critical temperature decreases with imaginary angular velocity.

$$\frac{T_c(r,\Omega_I)}{T_{c0}} = 1 - (\Omega_I r)^2 \left(\kappa_2 - \kappa_4 \left(\frac{r}{R}\right)^2\right). \quad (22)$$

- The vortical curvature in continuum limit from quadratic fit ($r/R \lesssim 0.5$) is universal

$$\kappa_2 = 0.902(33)\,, \quad (23)$$

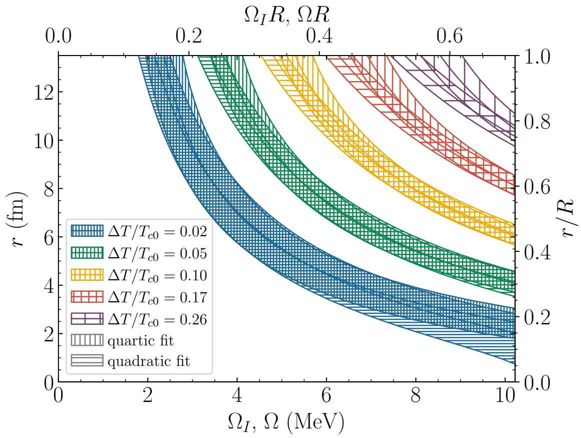
- And from quartic fit (for OBC) there is

$$\kappa_2 = 1.051(29)\,, \qquad \kappa_4 = 0.300(34)\,, \quad (24)$$

where κ_4 term is a finite volume correction;

- We can not distinguish $\sim \Omega^4$ term.

Critical radius and phase diagram



The critical distance r may be found from the following conditions:

- for imaginary angular velocity

$$T_{c0} - \Delta T = T_c(r, \Omega_I)\,,$$

(confinement in the center;
deconfinement at the periphery)

- for real angular velocity

$$T_{c0} + \Delta T = T_c(r, \Omega)\,.$$

(deconfinement in the center;
confinement at the periphery)

The diagram has the same shape for a given $\Delta T > 0$ (plot for $R = 13.5$ fm).

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Decomposition of rotating action

The action of rotating gluons is a quadratic function in Ω_I,

$$S_G = S_0 + \quad S_1\Omega_I + \quad S_2\Omega_I^2\,, \tag{25}$$

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where we introduce switching factors λ_1, λ_2.

- $S_1 \equiv S_{\mathrm{mech}}$ is an angular momentum of gluons (in laboratory frame) – "mechanical" coupling.
- $S_2 = S_{\mathrm{magn}}$ is related to the chromomagnetic fields F_{ij}^2 – "chromomagnetic" coupling.

The following regimes of the rotation are possible:

Im1) $\lambda_1 = 1, \quad \lambda_2 = 0; \quad \Omega_I^2 > 0$

Im2) $\lambda_1 = 0, \quad \lambda_2 = 1; \quad \Omega_I^2 > 0$

Im12) $\lambda_1 = 1, \quad \lambda_2 = 1; \quad \Omega_I^2 > 0$ (physical regime; it is already considered above)

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$$S_G = S_0 + \lambda_1 S_1 \Omega_I + \lambda_2 S_2 \Omega_I^2 \,, \tag{25}$$

where we introduce switching factors λ_1, λ_2.

- $S_1 \equiv S_{\text{mech}}$ is an angular momentum of gluons (in laboratory frame) – "mechanical" coupling.
- $S_2 = S_{\text{magn}}$ is related to the chromomagnetic fields F_{ij}^2 – "chromomagnetic" coupling.

The following regimes of the rotation are possible:

Im1) $\lambda_1 = 1, \quad \lambda_2 = 0; \quad \Omega_I^2 > 0$

Im2) $\lambda_1 = 0, \quad \lambda_2 = 1; \quad \Omega_I^2 > 0$

Im12) $\lambda_1 = 1, \quad \lambda_2 = 1; \quad \Omega_I^2 > 0$ (physical regime; it is already considered above)

Note that in case Im2 there is, actually, no sign problem:

Re2) $\lambda_1 = 0, \quad \lambda_2 = -1; \quad \Omega_I^2 < 0$ (real rotation)

Imaginary vs real rotation for different regimes

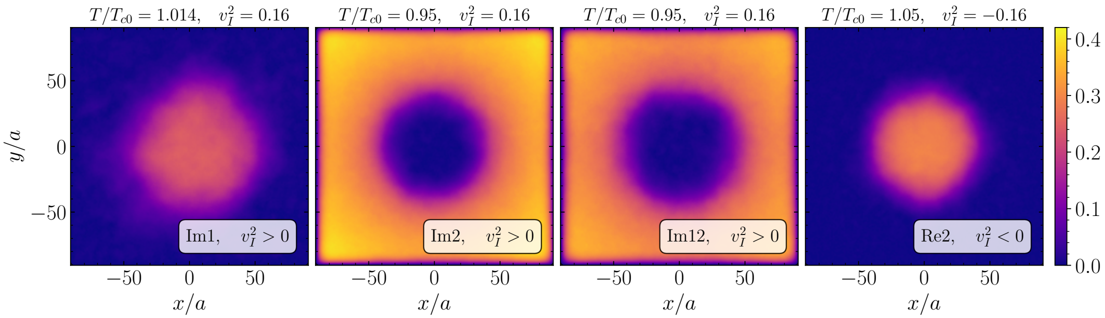
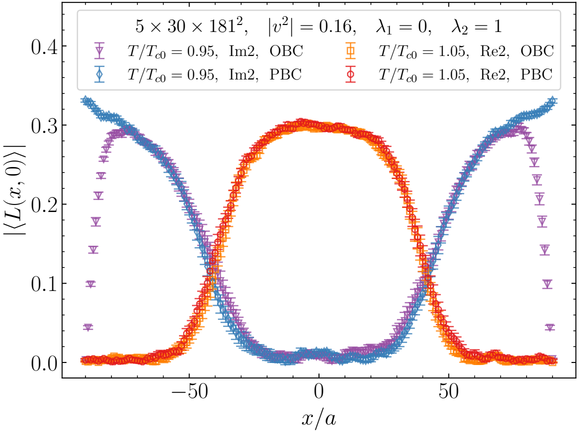


Figure: The distribution of the local Polyakov loop in x, y-plane for lattice size $5 \times 30 \times 181^2$, open boundary conditions (OBC) at fixed velocity $|v_I^2| = 0.16$ and different regimes. Temperature was chosen to see mixed phase.

- In the regimes Im1 and Re2, the rotation produces confinement phase in the outer region at $T > T_{c0}$. Regime Re2 realizes real rotation for S_2 system.
- Phase arrangement is the same in Im2- and Im12-regimes. The radius of the inner region in regime Im2 is slightly smaller, than in regime Im12.

Imaginary vs real rotation for different regimes

The distributions of the Polyakov loop for real and imaginary rotation (S_1 term is omitted).

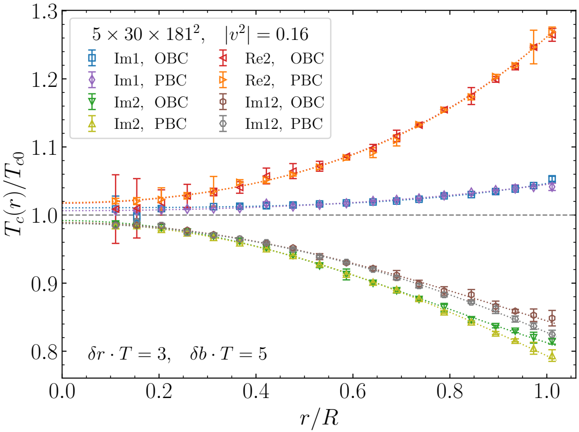


- Re2: $T = T_{c0} + \Delta T$
  for real rotation $v^2 = 0.16$
- Im2: $T = T_{c0} - \Delta T$
  for imaginary rotation $v_I^2 = 0.16$

Confinement ↔ deconfinement
with approximately the same boundary.

Imaginary vs real rotation for different regimes

The local critical temperature in these regimes has different behaviour.



- In the Im1-regime, $T_c(r) \nearrow$
- In the Im2-regime, $T_c(r) \searrow$
  the vortical curvature $\kappa_2^{(\mathrm{Im2})} > \kappa_2^{(\mathrm{Im12})}$
- The Re2-regime is in agreement with a.c. of the Im2-results in a bulk
- Contribution from $S_2 \equiv S_{\mathrm{magn}}$ dominates.

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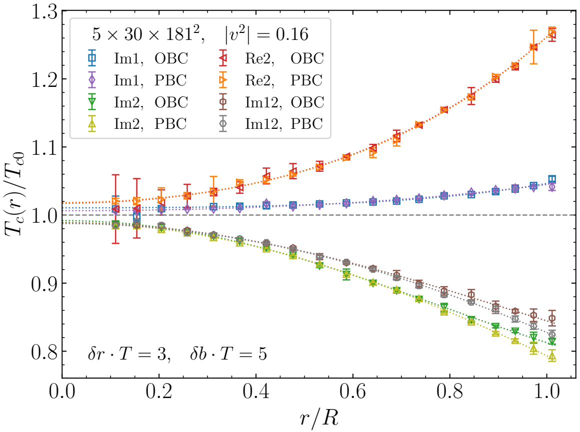

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  The results resemble the decomposition of I from [V. V. Braguta et al., Phys. Rev. D **110**, 014511 (2024), arXiv:2310.16036 [hep-ph]]

Equation of State and Moment of Inertia

A mechanical response of a thermodynamic ensemble to rigid rotation $\boldsymbol{\Omega} = \Omega \boldsymbol{e}$ is described in terms of the total angular momentum $\boldsymbol{J}$. The energy in co-rotating reference frame is

$$E = E^{(lab)} - \boldsymbol{J} \cdot \boldsymbol{\Omega}\,, \qquad F = E - TS\,, \qquad dF = -SdT - \boldsymbol{J} \cdot d\boldsymbol{\Omega} + \ldots\,,$$

The moment of inertia is a scalar quantity, $\boldsymbol{J} = I(T,\Omega)\boldsymbol{\Omega}$,

$$I(T,\Omega) = \frac{J(T,\Omega)}{\Omega} = -\frac{1}{\Omega}\left(\frac{\partial F}{\partial \Omega}\right)_T,$$

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For a classical system with characteristic radius R the moment of inertia is given by

$$I(T,\Omega) = \int_V d^3x\, x_\perp^2 \rho(T, x_\perp, \Omega) \simeq \alpha\, \rho_0(T) V R^2\,,$$

The free energy may be represented as a series in angular velocity (or linear velocity $v_R = \Omega R$)

$$F(T,V,\Omega) = F_0(T,V) - \frac{F_2(T,V)}{2}\Omega^2 + \mathcal{O}(\Omega^4) \equiv F_0(T,V) - \frac{\mathsf{i}_2(T)}{2} V v_R^2 + \mathcal{O}(v_R^4)\,,$$

where $F_2(T,V) = I(T,V,\Omega = 0) \equiv \mathsf{i}_2(T) V R^2$, and $\mathsf{i}_2(T)$ is a *specific* moment of inertia.

Taking the derivative at $\Omega = 0$, we obtain:

$$I = F_2 = T \frac{\partial^2 \log Z}{\partial \Omega^2}\bigg|_{\Omega=0} = T\left(\langle\langle S_1^2 \rangle\rangle_T + \langle\langle S_2 \rangle\rangle_T\right),$$

where $\langle\langle \mathcal{O} \rangle\rangle_T = \langle \mathcal{O} \rangle_T - \langle \mathcal{O} \rangle_{T=0}$.
Using the exact forms of S_1, S_2, we get

$$I = I_{\mathrm{mech}} + I_{\mathrm{magn}}$$

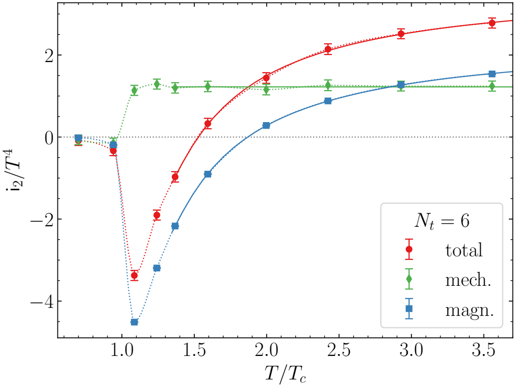
where ($\langle J \rangle = 0$ for any T) and

$$I_{\mathrm{mech}} = \frac{1}{T}\left(\langle\langle J^2 \rangle\rangle_T - \langle\langle J \rangle\rangle_T^2\right) \geq 0,$$

$$I_{\mathrm{magn}} = \frac{1}{3}\int_V d^3x\, x_\perp^2 \langle\langle (F_{ij}^a)^2 \rangle\rangle_T = \frac{\alpha}{3} V R^2 \langle\langle (G_{\mathrm{magn}})^2 \rangle\rangle_T \,.$$

J is the total angular momentum of gluon field.

- Mass density $\rho_0(T) \leftrightarrow \langle\langle (G_{\mathrm{magn}})^2 \rangle\rangle_T / 3$.



[V. V. Braguta et al., Phys. Rev. D **110**, 014511 (2024), arXiv:2310.16036 [hep-ph]]

Negative moment of inertia and magnetic gluon condensate

Taking the derivative at $\Omega = 0$, we obtain:

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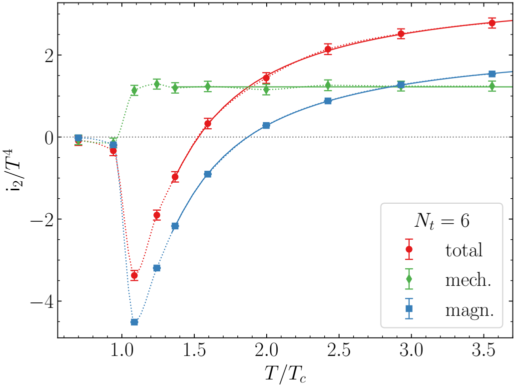

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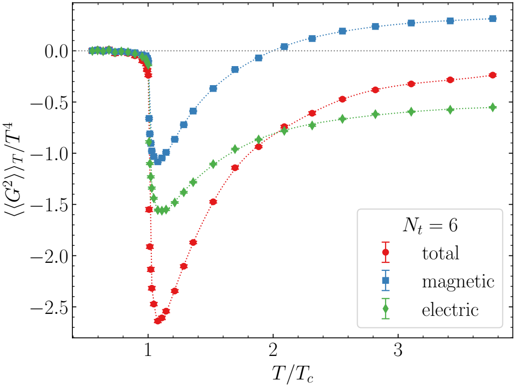
J is the total angular momentum of gluon field.

- Mass density $\rho_0(T) \leftrightarrow \langle\langle (G_{\mathrm{magn}})^2 \rangle\rangle_T / 3$.
- Magnetic gluon condensate reverse its sign at $\sim 2T_c$.



[V. V. Braguta et al., Phys. Rev. D **110**, 014511 (2024), arXiv:2310.16036 [hep-ph]]

Interpretation of the results: negative Barnett effect

Total angular momentum $\boldsymbol{J} = I\boldsymbol{\Omega}$ is a sum of the orbital and spin parts:

$$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}\,, \tag{26}$$

and $I < 0$. The possible physical picture: instability, or *negative* Barnett effect for gluon.

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and $I < 0$. The possible physical picture: instability, or *negative* Barnett effect for gluon.

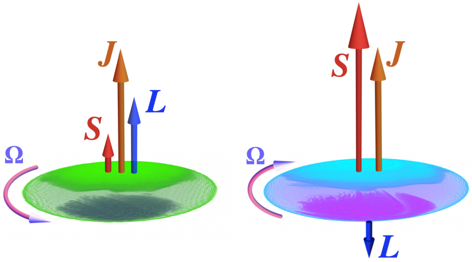
In the temperature range $T_c \lesssim T < T_s \simeq 1.5T_c$:

(i) a sizable fraction of the total angular momentum $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$ is accumulated in the spin of gluons $\boldsymbol{S}$;

(ii) the spin polarization $\boldsymbol{S}$ is parallel to the total angular momentum $\boldsymbol{J}$ and anti-parallel to the orbital angular momentum of plasma $\boldsymbol{L}$.

Let's introduce $\boldsymbol{L} = I_L\boldsymbol{\Omega}$, $\boldsymbol{S} = I_S\boldsymbol{\Omega}$, therefore

$$I_L > 0\,, \quad I_S < 0\,, \quad I = I_L + I_S < 0\,.$$



(left) usual Barnett effect
(right) negative Barnett effect
[V. V. Braguta et al., Phys. Rev. D **110**, 014511 (2024), arXiv:2310.16036 [hep-ph]]

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- Conclusions

Local approximation for inhomogeneous action

Two major effects of rotation:

- Inhomogeneity: coefficients depend on coordinates (x, y).
- Anisotropy: chromoelectric and chromomagnetic components are affected differently by rotation.

The approximation of *local thermalization*: We consider a small subsystem at distance r_0 from the rotation axis. In the vicinity of the point $(x, y) = (r_0, 0)$ the coefficients in action is approximately constant.

The homogeneous local action is

$$S_G = \frac{1}{2g^2} \int d^4x \left[F^a_{x\tau} F^a_{x\tau} + F^a_{y\tau} F^a_{y\tau} + F^a_{z\tau} F^a_{z\tau} + F^a_{xz} F^a_{xz} + \right.$$

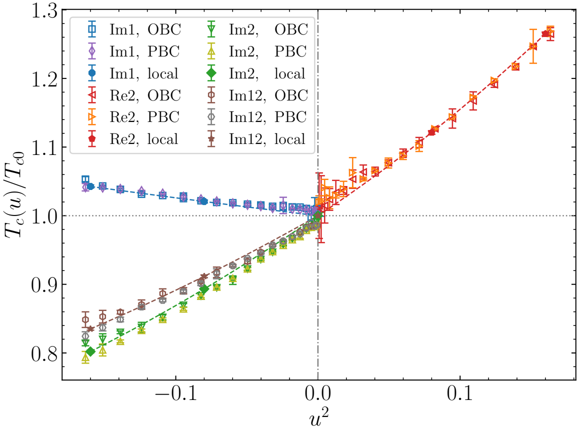
$$\left. + \left(1 + u_I^2\right) F^a_{yz} F^a_{yz} + \left(1 + u_I^2\right) F^a_{xy} F^a_{xy} + 2u_I \left(F^a_{yx} F^a_{x\tau} + F^a_{yz} F^a_{z\tau}\right) \right], \quad (27)$$

where $u_I = \Omega_I r_0$ is a local velocity.

Local thermalization approximation

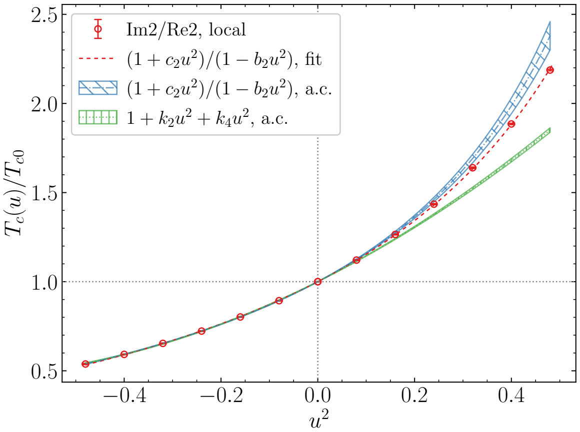
- The system (27) is simulated using standard lattice methods with PBC.
- Local approximation is free from the effects of finite R and the influence of boundary conditions.

Results for local action



- The results for *local* action and for full system are in a good agreement with each other in all regimes.
- The data are well described by the polynomial:

$$\frac{T_c(u_I)}{T_{c0}} = 1 + k_2 u^2 + k_4 u^4 \,, \qquad (28)$$



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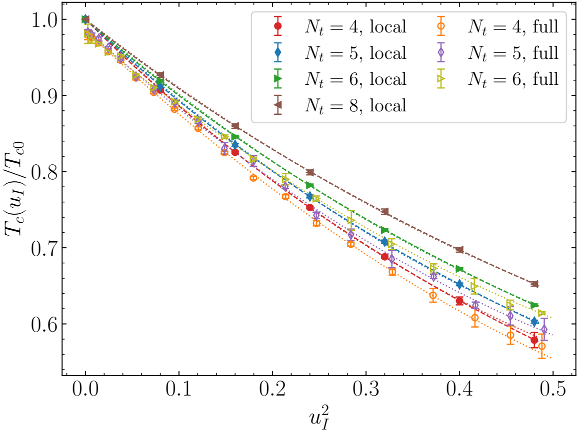
- Or, by the rational function:

$$\frac{T_c(u)}{T_{c0}} = \frac{1+c_2u^2}{1-b_2u^2}\,. \qquad (29)$$

- The function (29) better describe all data from regimes Im2/Re2.

Results for local action



- The data are well described by the polynomial:

$$\frac{T_c(u_I)}{T_{c0}} = 1 + k_2 u^2 + k_4 u^4 \,, \qquad (30)$$

- And by the rational function:

$$\frac{T_c(u)}{T_{c0}} = \frac{1 + c_2 u^2}{1 - b_2 u^2} \,. \qquad (31)$$

- In continuum limit the coefficients are

$$k_2 = 0.869(31) \,, \qquad k_4 = 0.388(53) \,. \qquad (32)$$

$$c_2 = 0.206(66) \,, \qquad b_2 = 0.694(101) \,. \qquad (33)$$

- The local critical temperature increases with real velocity $u = \Omega r$.

Ehrenfest-Tolman effect: In gravitational field the temperature isn't a constant in space at thermal equilibrium, $T(r)\sqrt{g_{00}} = T_0 = \text{const}$. In the co-rotating reference frame:

$$T(r) = \frac{T_0}{\sqrt{1-\Omega^2 r^2}} = \frac{T_0}{\sqrt{1+\Omega_I^2 r^2}} \,. \tag{34}$$

TE law suggests that the rotation effectively heats the periphery. Let's derive $T_c^{TE}(u)$ from an assumption $T(r) = T_{c0}$, then the local critical temperature decreases:

$$\frac{T_c^{TE}(u)}{T_{c0}} = \sqrt{1-u^2} \approx 1 - 0.5u^2 + \dots\,, \tag{35}$$

In the result, TE predicts confinement in the center and deconfinement at the periphery (for *real* rotation):

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- Holography: N. R. F. Braga and O. C. Junqueira, Phys. Lett. B **848**, 138330 (2024), arXiv:2306.08653 [hep-th]

Ehrenfest-Tolman effect in rotating (Q)GP

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Lattice simulation gives opposite arrangement of the phases. Qualitatively consistent results:

- S. Chen, K. Fukushima, and Y. Shimada, Phys. Lett. B **859**, 139107 (2024), arXiv:2404.00965 [hep-ph]
- Y. Jiang, Phys. Rev. D **110**, 054047 (2024), arXiv:2406.03311 [nucl-th]

Asymmetry in local action

The local action without the linear term is

$$S_G = \int d^4x \left[\beta\left((F^a_{x\tau})^2 + (F^a_{y\tau})^2 + (F^a_{z\tau})^2 + (F^a_{xz})^2\right) + \tilde{\beta}\left((F^a_{yz})^2 + (F^a_{xy})^2\right)\right], \quad (36)$$

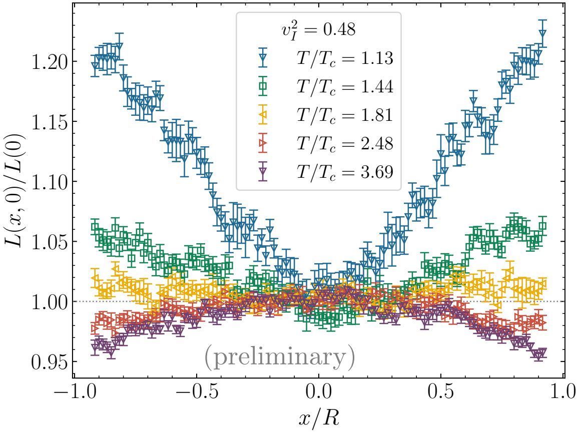
where $\beta = \frac{1}{2}g^2_{YM}$ and $\tilde{\beta} = \left(1 - (\Omega r_0)^2\right)\beta \equiv \left(1 + (\Omega_I r_0)^2\right)\beta$. This action corresponds to Im2/Re2-regimes.

External gravitational field generates asymmetry in the coupling constants of different components of the fields $(F_{\mu\nu})^2$, which influences the dynamics of gluons:

- $\tilde{\beta}/\beta > 1$ (imaginary rotation) $\Rightarrow$ T_c decreases.
- $\tilde{\beta}/\beta < 1$ (real rotation) $\Rightarrow$ T_c increases.

This mechanism can not be accounted for by TE. Asymmetry arises from "magentovortical" coupling. System with "mechanical" coupling only has mixed phase structure which is in agreement with TE.

Local Polyakov loop at high temperatures



- At temperatures $T > T_s \simeq 1.5T_{c0}$ the moment of inertia is positive.
- At temperatures $T \gtrsim 2T_{c0}$ the thermal gluon condensate becomes positive.
- Local Polyakov loop decreases with r at high temperatures $T \gtrsim 2T_{c0}$

  (local temperature from TE decreases with r for imaginary Ω_I)

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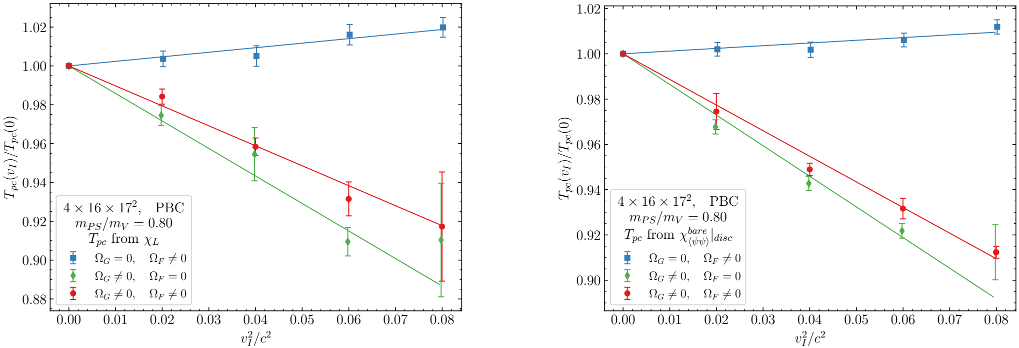


Figure: The (bulk-averaged) pseudo-critical temperature as a function of imaginary linear velocity on the boundary for various rotation regimes (full, only gluons, only fermions). [V. Braguta, A. Kotov, A. Roenko, and D. Sychev, PoS **LATTICE2022**, 190 (2023), arXiv:2212.03224 [hep-lat]]

QCD action: $S = S_G(\Omega_G) + S_F(\Omega_F)$

Rotation in fermionic and gluonic sectors have different influence on the critical temperature.

Inhomogeneous phase in QCD (preliminary)

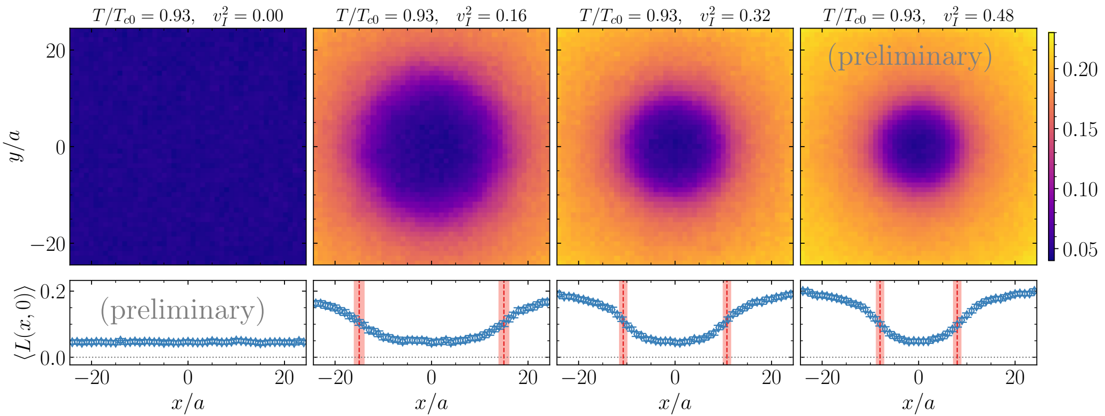


Figure: The distribution of the local Polyakov loop in x, y-plane for the lattice of size $4 \times 20 \times 49^2$ at the fixed temperature $T = 0.93\,T_{c0}$ and different v_I; QCD with Wilson fermions (Iwasaki action), $m_\pi/m_\rho = 0.80$.

- Mixed inhomogeneous phase takes place also in QCD! (work in progress . . .)

Conclusions

- Using lattice simulation, we found the mixed confinement-deconfinement phase in rotating SU(3) gluodynamics at thermal equilibrium. It takes place for $T > T_{c0}$ with deconfinement phase in the center and confinement at the periphery.
- The local critical temperature increases for real rotation, and in a bulk it is determined by the local velocity of rotation $u = \Omega r$:

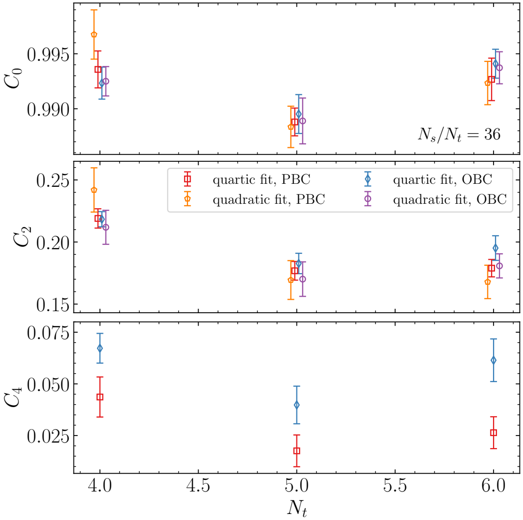
$$\frac{T_c(r,\Omega)}{T_{c0}} = 1 + \kappa_2 (\Omega r)^2 \qquad [\text{bulk of full rotating system}], \qquad (37)$$

$$\frac{T_c(u)}{T_{c0}} = 1 + k_2 u^2 + k_4 u^4, \quad \text{or} \quad \frac{T_c(u)}{T_{c0}} = \frac{1 + c_2 u^2}{1 - b_2 u^4}, \qquad [\text{local action}], \qquad (38)$$

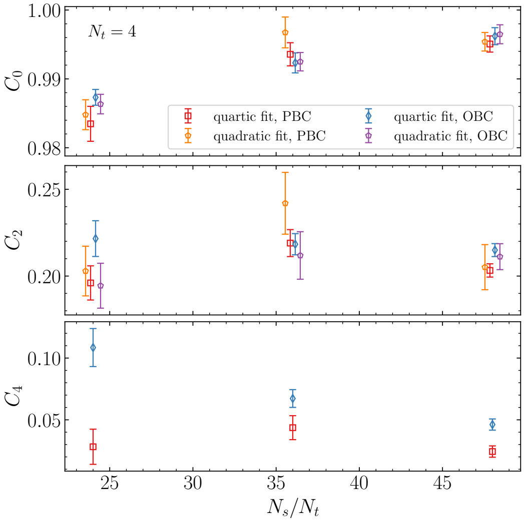
  The approximation of local thermalization gives consistent results.
- The local critical temperature $T_c(0)$ on the axis of rotation is T_{c0} with a few percent accuracy.
- The magnetovortical coupling generates asymmetry in the action for chromomagnetic fields. Linear coupling play subleading role. This mechanism can not be accounted for by TE.
- We demonstrate the validity of analytic continuation using Im2/Re2-regimes.
  The results in different regimes resemble decomposition of $I = I_{\text{mech}} + I_{\text{magn}}$.
- Phase transition occurs as a evolution of vortex of new phase. We expect similar picture for QCD.

Thank you for your attention!

Backup



C_0
0.995
0.990
$N_s/N_t = 36$
C_2
0.25
0.20
0.15
quartic fit, PBC
quadratic fit, PBC
quartic fit, OBC
quadratic fit, OBC
C_4
0.075
0.050
0.025
4.0
4.5
5.0
5.5
6.0
N_t



1.00
$N_t = 4$
C_0
0.99
0.98
quartic fit, PBC
quadratic fit, PBC
quartic fit, OBC
quadratic fit, OBC
C_2
0.25
0.20
C_4
0.10
0.05
25
30
35
40
45
N_s/N_t