

SKYRMIONS WITH VECTOR MESONS: SINGLE SKYRMION AND BARYONIC MATTER

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2013.6.26
SEMINAR @ BLTP, JINR

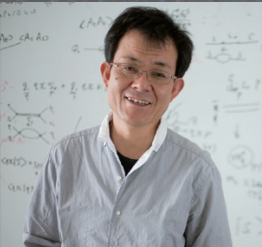
CONTENTS

- * Motivation
- * Approaches in the Skyrme model
 - Conventional approaches / short review on the earlier works
 - HLS with hQCD
 - Lagrangian up to $O(p^4)$
 - Role of vector mesons (ρ vs. ω) in Skyrmion properties
 - Nuclear matter
- * Outlook



Collaborators:

Mannque Rho
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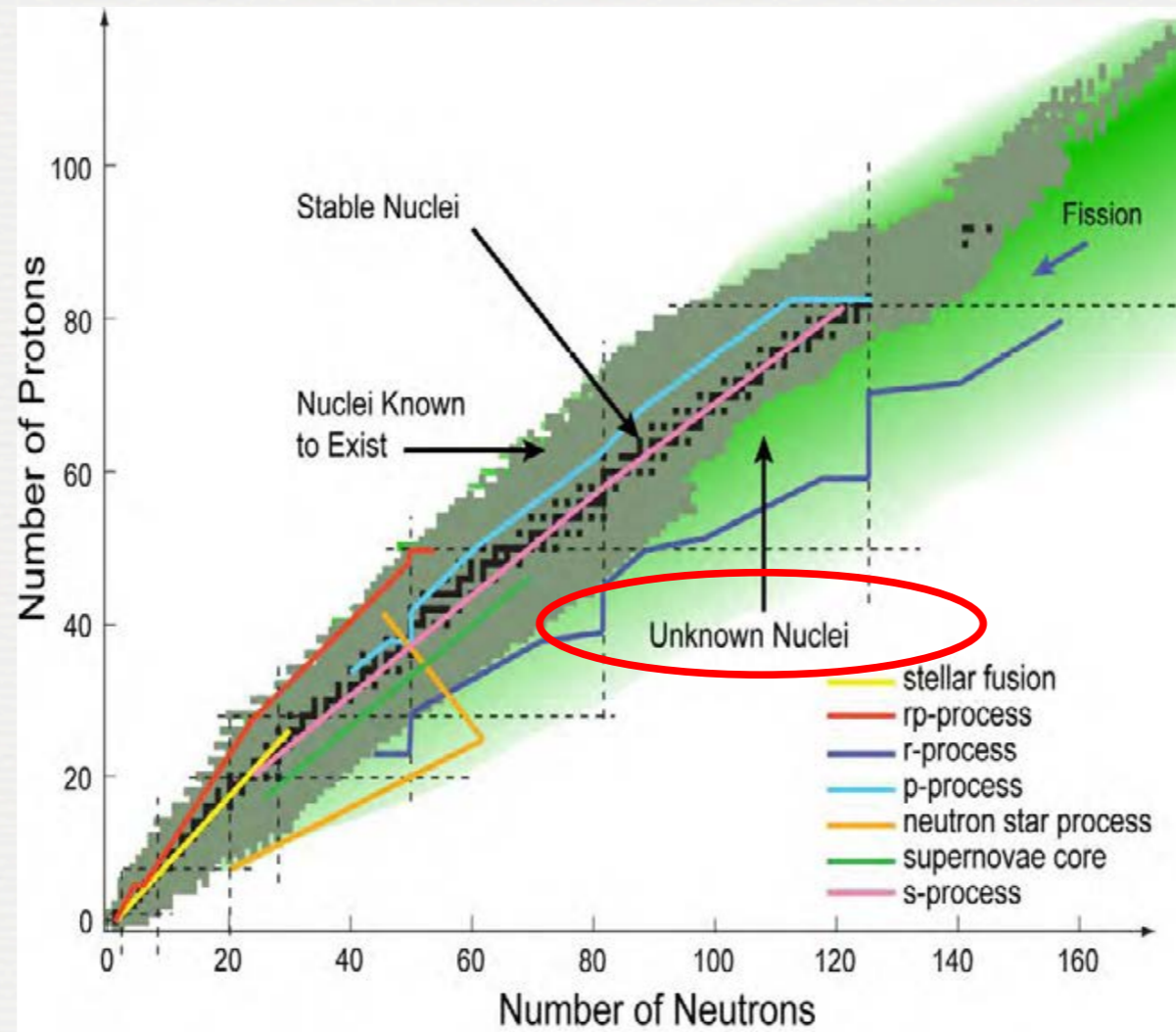
Hyun Kyu Lee
Byung-Yoon Park
Ghil-Seok Yang



References:

- Y.-L. Ma, Y. Oh, G.-S. Yang, M. Harada, H.K. Lee, B.-Y. Park, and M. Rho, *Hidden local symmetry and infinite tower of vector mesons for baryons*, **Phys. Rev. D****86**, 074025 (2012)
- Y.-L. Ma, G.-S. Yang, Y. Oh, and M. Harada, *Skyrmions with vector mesons in the hidden local symmetry approach*, **Phys. Rev. D****87**, 034023 (2013)
- Y.-L. Ma, M. Harada, H.K. Lee, Y. Oh, B.-Y. Park, and M. Rho, *Dense baryonic matter in hidden local symmetry approach: Half-Skyrmions and nucleon mass*, **arXiv:1304.5638 (submitted to Phys. Rev. D)**

MOTIVATION



Nuclear Physics → Hadron Physics → Nuclear Physics



IBS & RAON



The 7th BLTP JINR-APCTP Joint Workshop
"Modern problems in nuclear and elementary particle physics",
July 14-19, 2013

Russia, Irkutsk Region, Bolshiye Koty



July 17

9.00-9.40 Yongseok Oh (Kyngpook National University)
Skyrmions with vector mesons in hidden local symmetry

9.40-10.20 Byung-Yoon Park (Chungnam National University)
Dense Baryonic Matter in Hidden Local Symmetry

10.20-10.50 Coffee

SKYRME MODEL

1960s: T.H.R. Skyrme

Baryons are topological solitons within a nonlinear theory of pions.

$$\mathcal{L} = \frac{f_\pi^2}{4} \text{Tr} (\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2} \text{Tr} [U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2$$

f_π : pion decay constant

e : Skyrme parameter

Topological soliton

winding number = baryon number

$$B_\mu = \frac{1}{24\pi^2} \epsilon_{\mu\nu\alpha\beta} \text{Tr} (U^\dagger \partial_\nu U U^\dagger \partial_\alpha U U^\dagger \partial_\beta U)$$

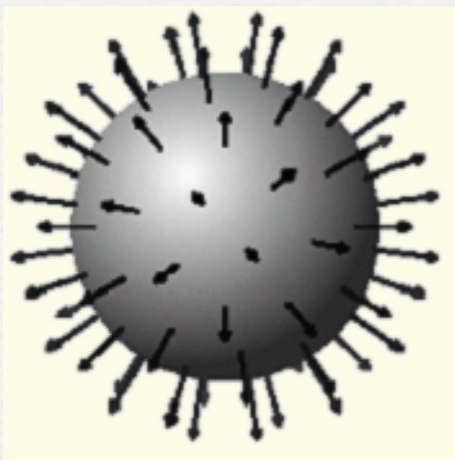
T.H.R. Skyrme: Proc. Roy. Soc. (London) 260, 127 (1961), Nucl. Phys. 31, 556 (1962)

REVIVAL

In large N_c , QCD \sim effective field theory of mesons and baryons may emerge as solitons in this theory.

E. Witten, 1980s

HEDGEHOG SOLUTION



$$U = \exp(iF(r)\boldsymbol{\tau} \cdot \hat{\mathbf{r}})$$

$$R \sim 1 \text{ fm}$$

$$M_{\text{sol}} \sim 146|B| \left(\frac{f_\pi}{2e}\right) \sim 1.2 \text{ GeV}$$

$$\text{for } B = 1$$



$$M_{\text{sol}} \sim 1.23 \times 12\pi^2|B| > \underline{12\pi^2|B|}$$

in the Skyrme unit: $f_\pi/(2e)$



Bogomolny bound

BARYON MASSES

- To give correct quantum numbers

- SU(2) collective coordinate quantization

$$U(t) = A(t)U_0A^\dagger(t)$$

- Mass formula: infinite tower of $I = J$

$$M = M_{\text{sol}} + \frac{1}{2\mathcal{I}}I(I + 1) \quad \mathcal{I} : \text{moment of inertia}$$

$$M_N = M_{\text{sol}} + \frac{3}{8\mathcal{I}}, \quad M_\Delta = M_{\text{sol}} + \frac{15}{8\mathcal{I}}$$

- Adjust f_π and e to reproduce the nucleon and Delta masses

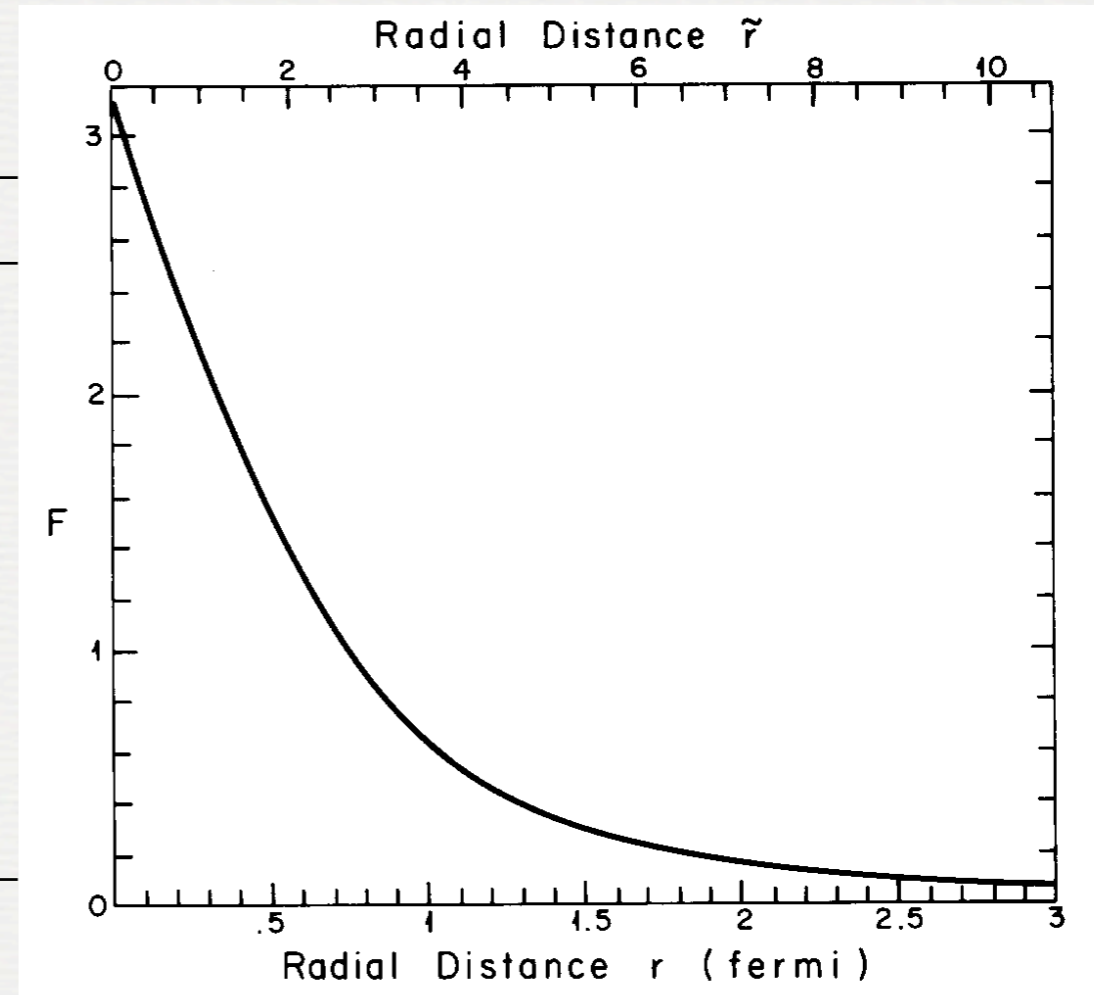
$$f_\pi = 64.5 \text{ MeV}, e = 5.45$$

$$\text{Empirically, } f_\pi = 93 \text{ MeV}, e = 5.85(?)$$

Skyrme model: results

■ Best-fitted results

Quantity	Prediction	Expt
M_N	input	939 MeV
M_Δ	input	1232 MeV
$\langle r^2 \rangle_{I=0}^{1/2}$	0.59 fm	0.72 fm
$\langle r^2 \rangle_{M,I=0}^{1/2}$	0.92 fm	0.81 fm
μ_p	1.87	2.79
μ_n	-1.31	-1.91
$ \mu_p/\mu_n $	1.43	1.46



G.S. Adkins, C.R. Nappi, and E. Witten, Nucl. Phys. B228, 552 (1983)

A.D. Jackson and M. Rho, Phys. Rev. Lett. 51, 751 (1983)

Skyrme model for Nuclear Physics

Single Baryon

Improvement of the model

- more degrees of freedom (mesons)
- $1/N_c$ corrections
- ChPT

Extension to other hadrons

- SU(3) extension to hyperons
- Heavy-quark baryons
- Hypernuclei & Exotic baryons

Nuclear Matter

Topics

- Properties of single baryon
- Equation of State
- Phase transition
- Application to nuclei

Approaches

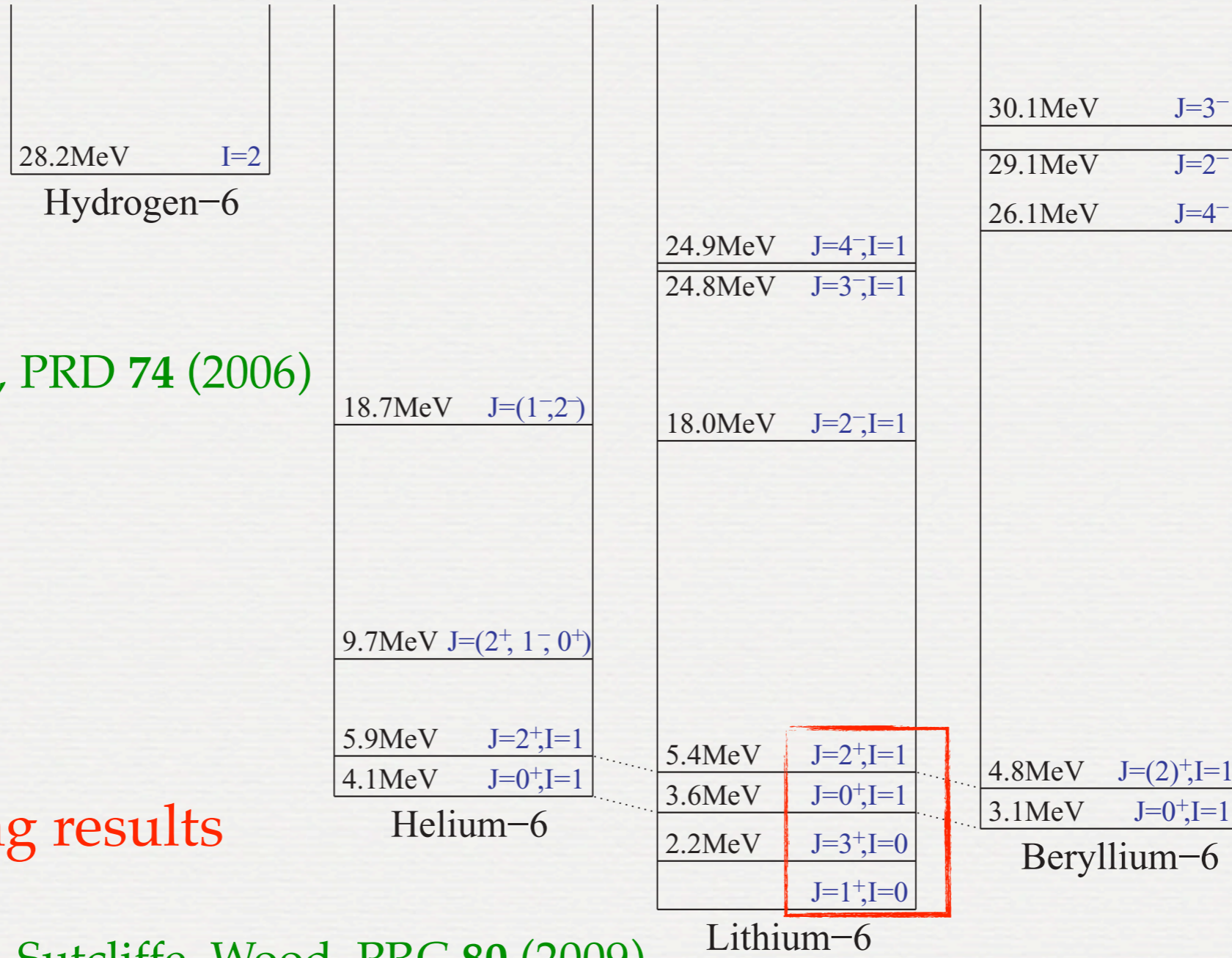
- Modified Effective Lagrangian
- Skyrminion Crystal
- Winding number n solutions



still there are many works to do

Nuclei

Light nuclei in the Skyrme model (e.g., mass number 6)



Manton, Wood, PRD 74 (2006)



encouraging results

Batty, Manton, Sutcliffe, Wood, PRC 80 (2009)

NUCLEI

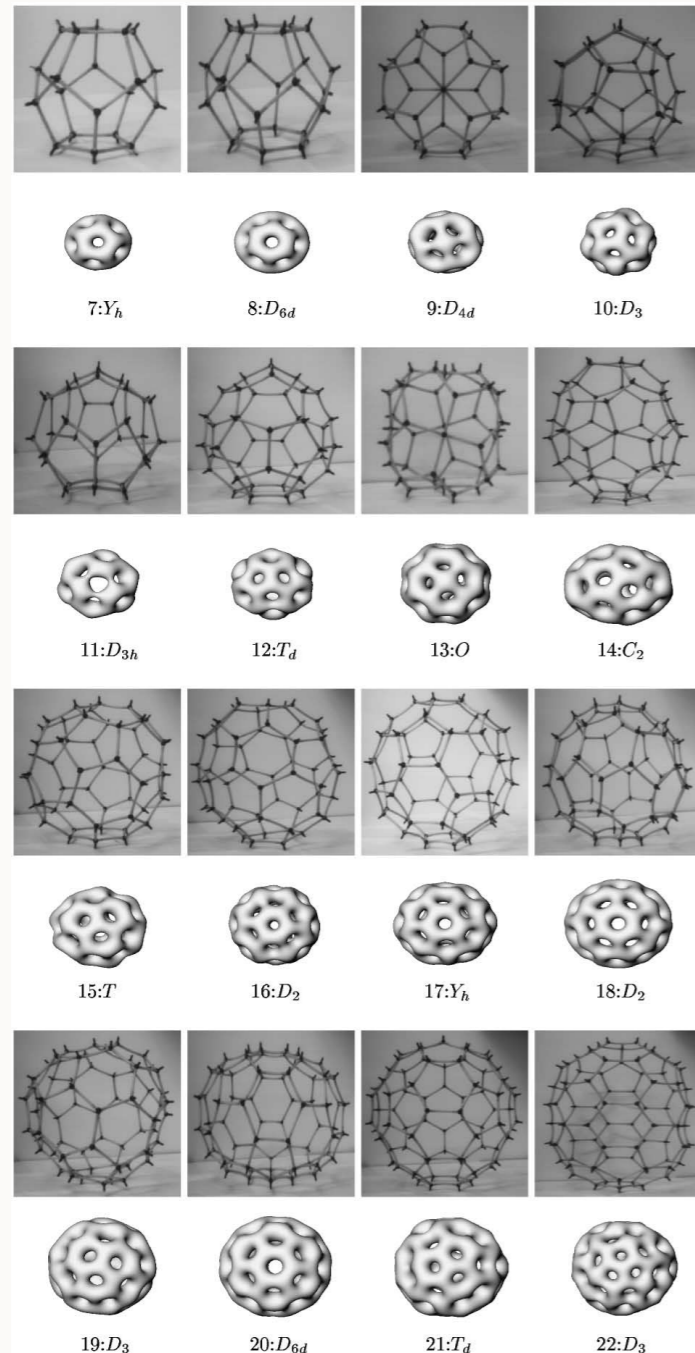
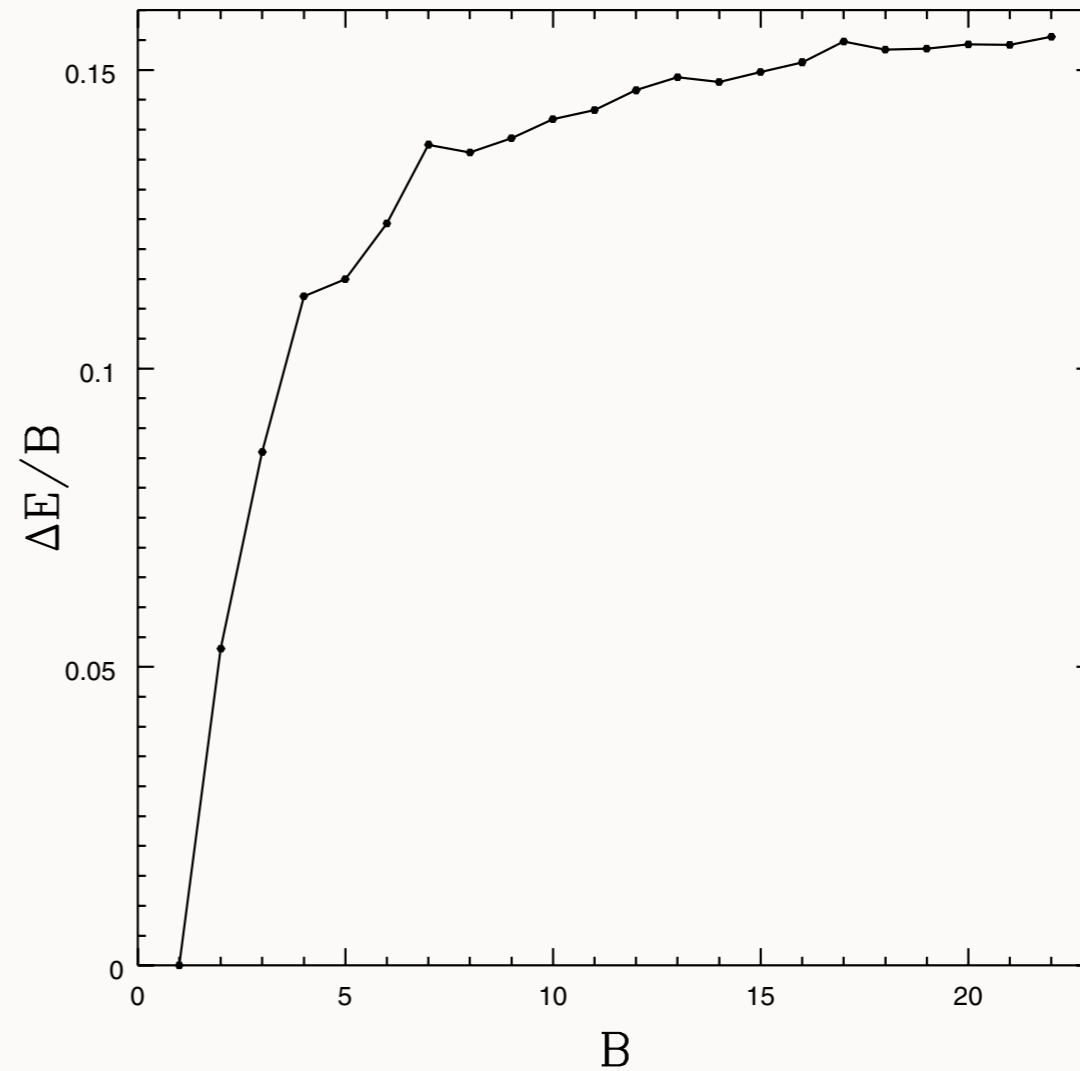


FIG. 1. The baryon density isosurfaces for the solutions which we have identified as the minima for $7 \leq B \leq 22$, and the associated polyhedral models. The isosurfaces correspond to $B = 0.035$ and are presented to scale, whereas the polyhedra are not to scale.

multi--baryon-number Skyrmion



Battye, Sutcliffe, PRL 86 (2001) 3989

Single Baryon (with Pion Mass)

- Adding the pion-mass term

Adkins, Nappi, NPB 233 (1984)

$$\mathcal{L}_{\text{pion}} = \frac{1}{2} m_{\pi}^2 f_{\pi}^2 (\text{Tr}(U) - 2)$$

Quantity	Prediction (massless pion)	Prediction (massive pion)	Expt
M_N	input	input	939 MeV
M_{Δ}	input	input	1232 MeV
f_{π}	64.5 MeV	54 MeV	93 MeV
$\langle r^2 \rangle_{I=0}^{1/2}$	0.59 fm	0.68 fm	0.72 fm
$\langle r^2 \rangle_{I=1}^{1/2}$	∞	1.04 fm	0.88 fm
$\langle r^2 \rangle_{M,I=0}^{1/2}$	0.92 fm	0.95 fm	0.81 fm
$\langle r^2 \rangle_{M,I=1}^{1/2}$	∞	1.04 fm	0.80 fm
μ_p	1.87	1.97	2.79
μ_n	-1.31	-1.24	-1.91
$ \mu_p/\mu_n $	1.43	1.59	1.46

Why vector mesons?

- Witten: QCD \sim weakly interacting mesons in large N_c
 - The lightest meson is the pion
 - The next low-lying mesons are vector mesons (ω and ρ) □

- Stability of the soliton

$$\mathcal{L} = \frac{f_\pi^2}{4} \text{Tr} (\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2} \text{Tr} [U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2$$

Skyrme terms

- Without the Skyrme term, the soliton collapses. [Derrick's Theorem](#)
- Vector mesons can stabilize the soliton without the Skyrme term.

Early Attempts to include VM

Including ω meson

$$\mathcal{L} = \mathcal{L}_{\text{pion}} + \mathcal{L}_{\omega} + \mathcal{L}_{\text{int}}$$

$$\mathcal{L}_{\text{pion}} = \frac{f_{\pi}^2}{4} \text{Tr}(\partial_{\mu} U \partial^{\mu} U^{\dagger}) + \frac{f_{\pi}^2}{2} m_{\pi}^2 (\text{Tr}(U) - 2),$$

$$\mathcal{L}_{\omega} = \frac{m_{\omega}^2}{2} \omega_{\mu} \omega^{\mu} - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu}, \quad \mathcal{L}_{\text{int}} = \beta \omega_{\mu} B^{\mu}$$

G.S. Adkins and C.R. Nappi, Phys. Lett. B137, 251 (1984)

Including ρ meson

$$\mathcal{L} = \mathcal{L}_{\text{pion}} + \mathcal{L}_{\rho} + \mathcal{L}_{\text{int}}$$

$$\mathcal{L}_{\text{int}} = \alpha \text{Tr}(\rho_{\mu\nu} \partial^{\mu} U^{\dagger} U \partial^{\nu} U^{\dagger}) \quad \rho\pi\pi \text{ interaction}$$

G.S. Adkins, Phys. Rev. D33, 193 (1986)

Early Attempts: results

Quantity	Skyrme (massive pion)	ω	ρ	Expt
M_N	input	input	input	939 MeV
M_Δ	input	input	input	1232 MeV
f_π	54 MeV	62 MeV	52.4 MeV	93 MeV
$\langle r^2 \rangle_{I=0}^{1/2}$	0.68 fm	0.74 fm	0.70 fm	0.72 fm
$\langle r^2 \rangle_{I=1}^{1/2}$	1.04 fm	1.06 fm	1.08 fm	0.88 fm
$\langle r^2 \rangle_{M,I=0}^{1/2}$	0.95 fm	0.92 fm	0.98 fm	0.81 fm
$\langle r^2 \rangle_{M,I=1}^{1/2}$	1.04 fm	1.02 fm	1.06 fm	0.80 fm
μ_p	1.97	2.34	2.16	2.79
μ_n	-1.24	-1.46	-1.38	-1.91
$ \mu_p/\mu_n $	1.59	1.60	1.56	1.46
$\mu_{I=0}$	0.365	0.44	0.39	0.44
$\mu_{I=1}$	1.605	1.9	1.77	2.35

SU(3) EXTENSION

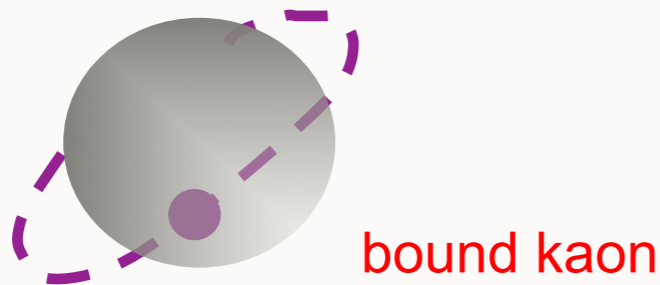
- Can we describe hyperons in the Skyrme model?
- Direct extension: SU(3) collective coordinate quantization
- New approaches
 - exact diagonalization methods *Yabu, Ando, NPB 301 (1988)*
Weigel et al., PRD 42 (1990)
 - bound state model
Callan, Klebanov, NPB 262 (1985)

BOUND STATE MODEL

- Starting point: flavor $SU(3)$ symmetry is badly broken
 - treat light flavors and strangeness on a different footing

$$SU(3) \rightarrow SU(2) \times U(1)$$

- Lagrangian $\mathcal{L} = \mathcal{L}_{SU(2)} + \mathcal{L}_{K/K^*}$
- The soliton provides a background potential that traps K/K^* (or heavy) mesons



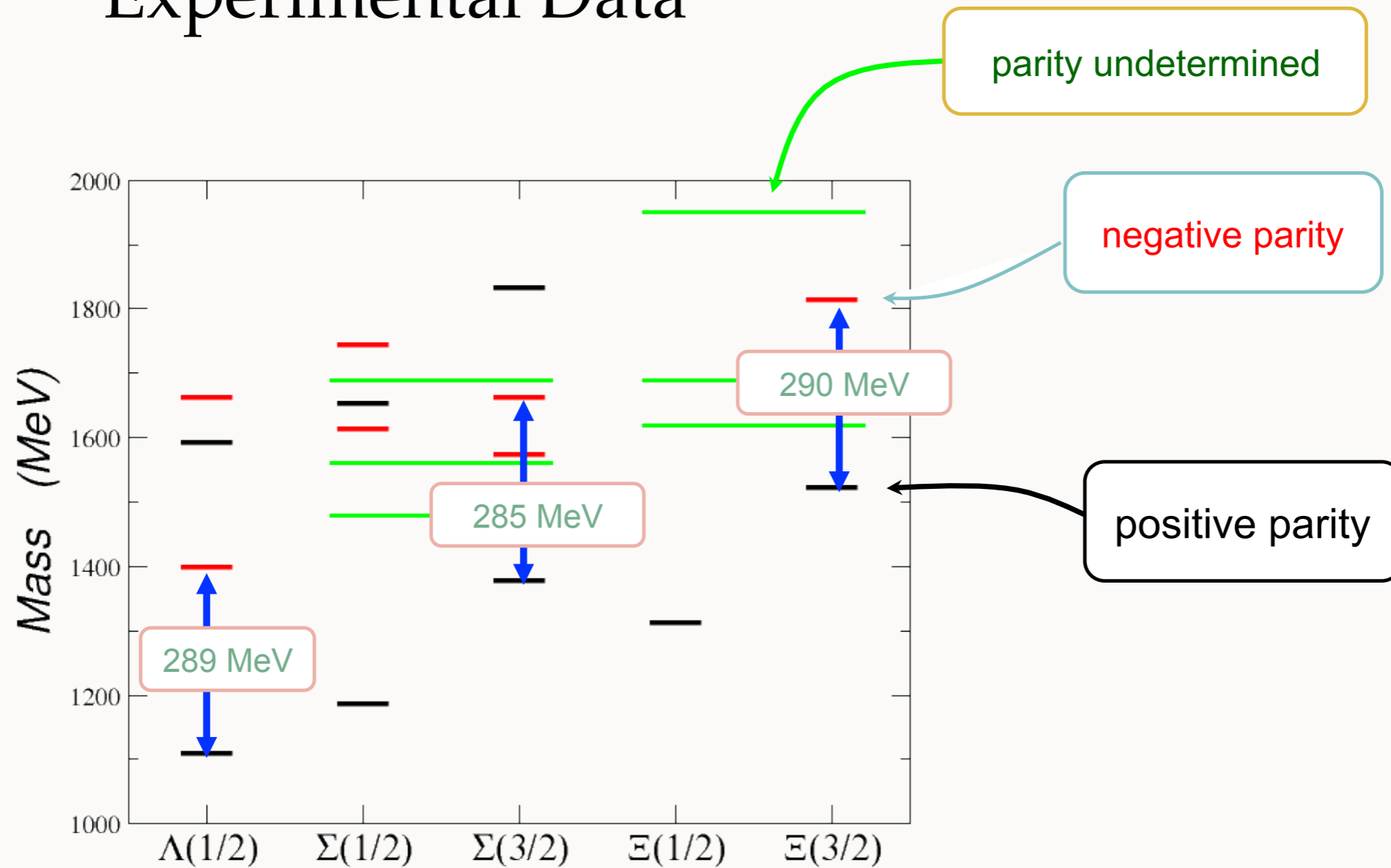
Callan, Klebanov, NPB 262 (1985)

BOUND STATE MODEL

- Anomalous Lagrangian
 - Pushes up the state of $S = +1$ states to the continuum \rightarrow no bound state
 - Pulls down the state of $S = -1$ states below the threshold \rightarrow makes bound states \rightarrow description of hyperons
- Renders two bound states with $S = -1$
 - the lowest state: p-wave \rightarrow gives (+) parity $\Lambda(1116)$
270 MeV energy difference
 - excited state: s-wave \rightarrow gives (-) parity $\Lambda(1405)$
after quantization

BOUND STATE MODEL

Experimental Data



BOUND STATE MODEL

- Mass sum rules

- modification to GMO and equal spacing rule

$$3\Lambda + \Sigma - 2(N + \Xi) = \Sigma^* - \Delta - (\Omega - \Xi^*)$$

$$(\Omega - \Xi^*) - (\Xi^* - \Sigma^*) = (\Xi^* - \Sigma^*) - (\Sigma^* - \Delta$$

- hyperfine relation

$$\Sigma^* - \Sigma + \frac{3}{2}(\Sigma - \Lambda) = \Delta - N$$

- The same relations hold for

$$\Lambda(1/2^-), \Sigma(1/2^-), \Sigma(3/2^-), \Xi(1/2^+), \Xi(3/2^+), \Omega(3/2^-)$$

BOUND STATE MODEL

■ Best-fitted results based on the derived mass formula

Particle	Prediction (MeV)	Expt
N	939*	N(939)
Δ	1232*	Δ (1232)
$\Lambda(1/2^+)$	1116*	Λ (1116)
$\Lambda(1/2^-)$	1405*	Λ (1405)
$\Sigma(1/2^+)$	1164	Σ (1193)
$\Sigma(3/2^+)$	1385	Σ (1385)
$\Sigma(1/2^-)$	1475	Σ (1480)?
$\Sigma(3/2^-)$	1663	Σ (1670)
$\Xi(1/2^+)$	1318*	Ξ (1318)
$\Xi(3/2^+)$	1539	Ξ (1530)
$\Xi(1/2^-)$	1658 (1660)	Ξ (1690)?
$\Xi(1/2^-)$	1616 (1614)	Ξ (1620)?
$\Xi(3/2^-)$	1820	Ξ (1820)
$\Xi(1/2^+)$	1932	Ξ (1950)?
$\Xi(3/2^+)$	2120*	Ξ (2120)
$\Omega(3/2^+)$	1694	Ω (1672)
$\Omega(1/2^-)$	1837	
$\Omega(3/2^-)$	1978	
$\Omega(1/2^+)$	2140	
$\Omega(3/2^+)$	2282	Ω (2250)?
$\Omega(3/2^-)$	2604	

Recently confirmed by COSY
PRL 96 (2006)

BaBar : the spin-parity of
 $\Xi(1690)$ is $1/2^-$
PRD 78 (2008)
NRQM predicts $1/2^+$

puzzle in QM

Unique prediction of this model.
The $\Xi(1620)$ should be there.
still one-star resonance

Ω 's would be discovered
in future.

YO, PRD 75 (2007)

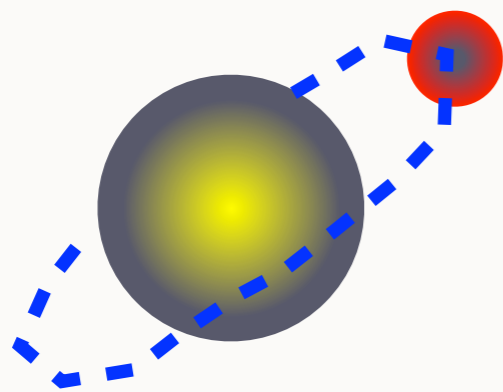
HEAVY QUARK BARYONS

- Replace the strangeness by the heavy-flavor

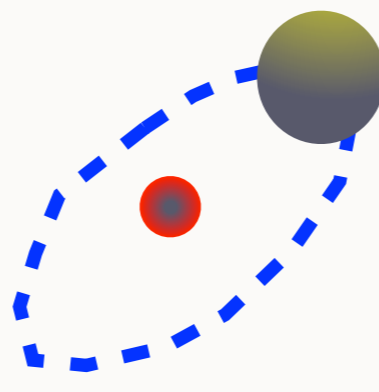
- $m_D/m_\pi \gg m_K/m_\pi$

- A dog wagging a tail?

large N_c vs. large m_Q



$$(m_Q < M_{\text{sol}})$$



$$(m_Q > M_{\text{sol}})$$

The two approaches converge only when both $N_c \rightarrow \infty$ and $m_Q \rightarrow \infty$

Heavy quark symmetry

HEAVY QUARK BARYONS

Soliton-fixed frame

Heavy-meson-fixed frame

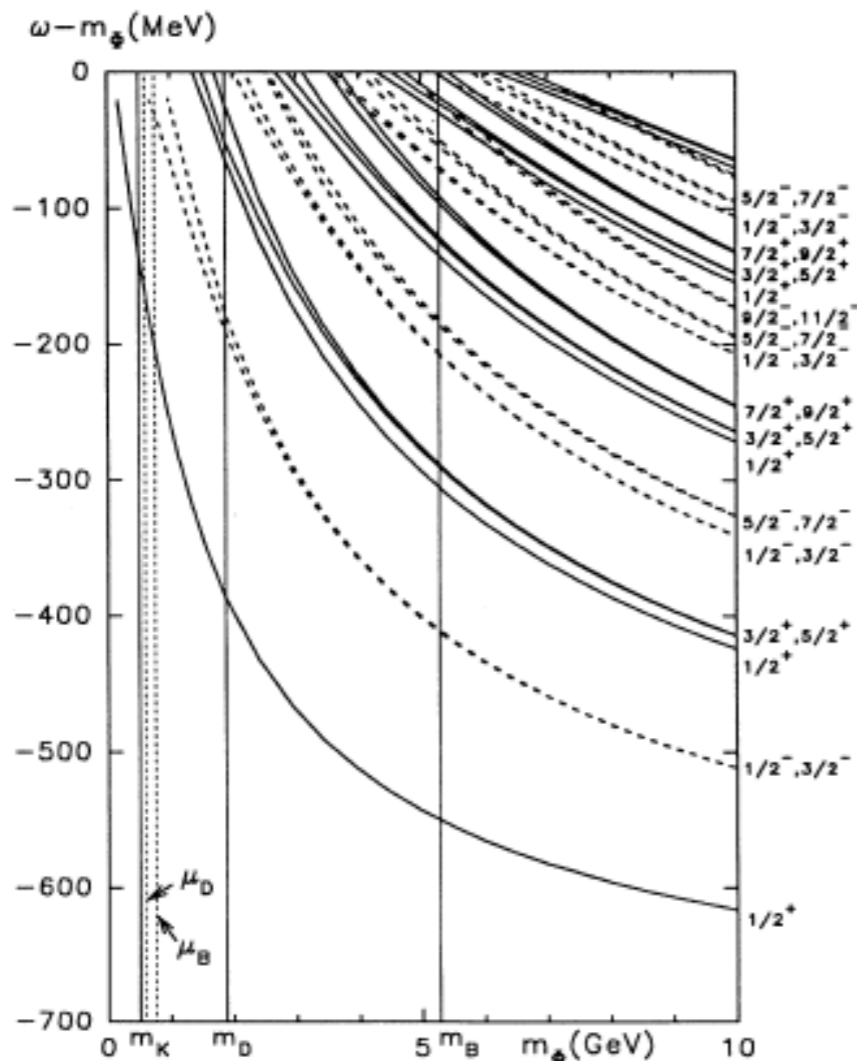


FIG. 4. Binding energies $\omega - m_\phi$ of the bound states with k^π as functions of the heavy-meson mass. Solid (dashed) lines denote the positive (negative) parity states.

Table 2. Numerical results on the bound states. Energies are given in MeV unit

(n, k_ℓ^π)	Set I	Set II	Set III	Set IV	Exp.
$(0, 0^+)$	-287	-461	-366	-588	-610
$(1, 0^+)$	-12	-62	-15	-79	-
$(0, 1^-)$	-89	-196	-113	-250	-320
$(0, 1^+)^a$	-17	-54	-21	-69	-

^a Bound state of soliton to antiflavored heavy meson

300 MeV

YO, B.Y. Park ZPA 359 (1997)

fewer bound states

YO, B.Y. Park PRD 51 (1995)

Vector Mesons

■ Systematic way to include vector mesons

- Massive Yang-Mills approach **Syracuse group**
- Hidden Local Symmetry **Nagoya group**
- Equivalence of the two approaches

■ Skyrmons in the HLS

- ρ meson stabilized model
Y. Igarashi et al., Nucl. Phys. B259, 721 (1985)
- ρ and ω meson stabilized model
U.-G. Meissner, N. Kaiser, and W. Weise, Nucl. Phys. A466, 685 (1987)
- ρ , ω and a_1 meson stabilized model
N. Kaiser and U.-G. Meissner, Nucl. Phys. A519, 671 (1990)
L. Zhang and N.C. Mukhopadhyay, Phys. Rev. D50, 4668 (1994)

Recent Works for Skyrmions with Vector Mesons

Holographic QCD: infinite tower of vector mesons
Solitons in hQCD

[D.K. Hong, M. Rho, H.-U. Yee, and P. Yi, Phys. Rev. D76, 061901 \(2007\); JHEP 0709, 063 \(2007\)](#)
[H. Hata, T. Sakai, S. Sugimoto, and S. Tamato, Prog. Theor. Phys. 117, 1157 \(2007\)](#)

HLS Lagrangian

$O(p^4)$ terms: [M. Tanabashi, Phys. Lett. B316, 534 \(1993\)](#)

$O(p^4)$ terms & hQCD: [M. Harada and K. Yamawaki, Phys. Rep. 381, 1 \(2003\)](#)

Skyrmions in HLS with ρ meson up to $O(p^4)$ terms with hQCD

[K. Nawa, H. Suganuma, and T. Kojo, Phys. Rev. D75, 086003 \(2007\)](#)

[K. Nawa, A. Hosaka, and H. Suganuma, Phys. Rev. D79, 126005 \(2009\)](#)

Earlier works

$O(p^2)$ Lagrangian with HLS

$$\mathcal{L}_\sigma = \frac{f_\pi^2}{4} \text{Tr}(\partial_\mu U \partial^\mu U^\dagger) \quad \text{with } U = \xi_L^\dagger \xi_R$$

Hidden Symmetry

$$\xi_{L,R}(x) \rightarrow h(x) \xi_{L,R}(x), \quad h \in \text{SU}(2)$$

$$V_\mu(x) \rightarrow i h(x) \partial_\mu h^\dagger(x) + h(x) V_\mu(x) h^\dagger(x)$$

Covariant derivative: $D_\mu \xi_{L,R} = \partial_\mu \xi_{L,R} - i V_\mu \xi_{L,R}$

$$\hat{\alpha}_{\mu\parallel} = \frac{1}{2i} (D_\mu \xi_L \xi_L^\dagger + D_\mu \xi_R \xi_R^\dagger)$$

$$\hat{\alpha}_{\mu\perp} = \frac{1}{2i} (D_\mu \xi_L \xi_L^\dagger - D_\mu \xi_R \xi_R^\dagger)$$

Unitary gauge: $\xi_L^\dagger = \xi_R = \xi$

HLS Lagrangian

$$\mathcal{L} = \mathcal{L}_A + a \mathcal{L}_V + \mathcal{L}_{\text{kin}}$$

$$\mathcal{L}_A = f_\pi^2 \text{Tr}(\hat{\alpha}_{\mu\perp}^2) = \mathcal{L}_\sigma, \quad \mathcal{L}_V = f_\pi^2 \text{Tr}(\hat{\alpha}_{\mu\parallel}^2)$$

$$\mathcal{L}_{\text{kin}} = -\frac{1}{2g^2} \text{Tr}(F_{\mu\nu}^2)$$

$$m_V^2 = a g^2 f_\pi^2$$

$$g_{\rho\pi\pi} = \frac{1}{2} a g$$

$a = 2$ gives KSRF relation and the universality of ρ coupling

M. Bando, T. Kugo, and K. Yamawaki, Phys. Rep. 164, 217 (1988)

ρ meson and the Skyrme term

As $a \rightarrow \infty$, i.e., as $m_V \rightarrow \infty$

$$\mathcal{L}_V \propto (\alpha_{\mu\parallel} - V_\mu)^2 = 0$$

$$\text{where } \alpha_{\mu\parallel} = \frac{1}{2i} (\partial_\mu \xi_L \xi_L^\dagger + \partial_\mu \xi_R \xi_R^\dagger)$$

\Rightarrow

$$\mathcal{L}_{\text{kin}} \rightarrow \frac{1}{32g^2} \text{Tr} [\partial_\mu U U^\dagger, \partial_\nu U U^\dagger]^2 = \mathcal{L}_{\text{Skyrme}}$$

Skyrmion in the HLS with the ρ meson

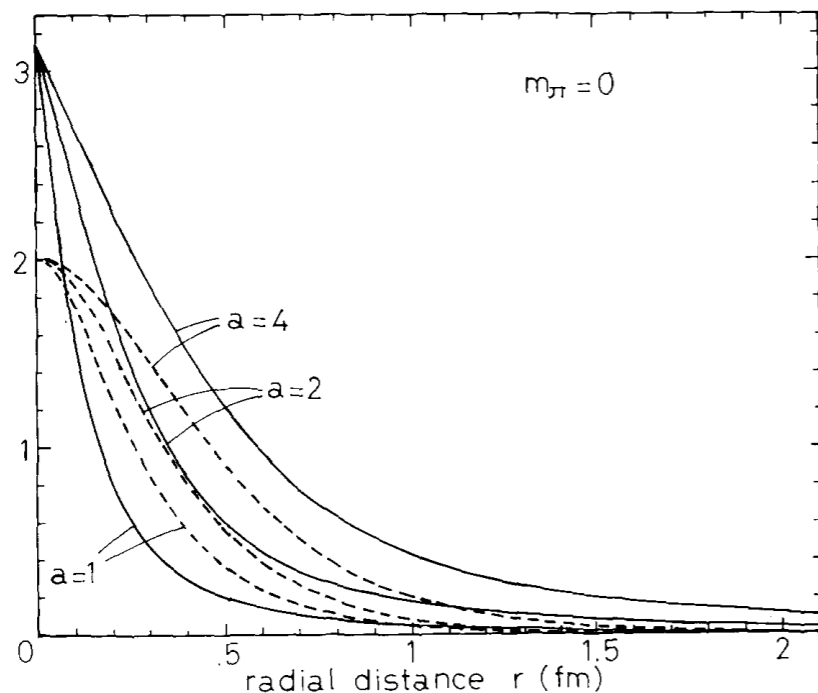


Fig. 1. $n=1$ solutions of $F(r)$ and $G(r)$ (dashed curves) for $m_\pi=0$ (chiral limit) and $a=1, 2$ and 4 fixing $ag^2f_\pi^2 = m_\rho^2$.

$$M_{\text{sol}} = (667 \sim 1575) \text{ MeV}$$

$$\text{for } 1 \leq a \leq 4$$

$$M_{\text{sol}} = 1045 \text{ MeV} \quad \text{for } a = 2$$

Y. Igarashi, M. Johmura, A. Kobayashi, H. Otsu, T. Sato, and S. Sawada, Nucl. Phys. B259, 721 (1985)

ρ and ω mesons

ω meson: introduced through HGS like the ρ meson

Anomalous Lagrangian: source of the ω meson

$$\begin{aligned} \mathcal{L}_{\text{an}} = & \frac{3}{8}gN_c(c_1 - c_2 - c_3)\omega_\mu B^\mu \\ & - \frac{g^3 N_c}{32\pi^2}(c_1 + c_2)\varepsilon^{\mu\nu\alpha\beta}\omega_\mu \text{tr} (a_\nu \bar{\rho}_\alpha \bar{\rho}_\beta) \\ & - \frac{gN_c}{8\pi^2}c_3\varepsilon^{\mu\nu\alpha\beta} \left\{ -\omega_\mu \text{tr} (a_\nu v_\alpha v_\beta) + \frac{ig}{4}\partial_\mu \omega_\nu \text{tr} (a_\alpha \rho_\beta - \rho_\alpha a_\beta) - \frac{ig}{4}\omega_\mu \text{tr} (\rho_{\nu\alpha} a_\beta) \right\}, \end{aligned}$$

Determination of parameters

T. Fujiwara, T. Kugo, H. Terao, S. Uehara, K. Yamawaki,
Prog. Theor. Phys., 73, 926 (1985)

Minimal model: $c_1 = \frac{2}{3}, c_2 = -\frac{2}{3}, c_3 = 0$ $\omega^\mu B_\mu$ term only

Vector Dominance: $c_1 = 1, c_2 = 0, c_3 = 1$ No $\omega\pi^3$ term

Or fit them to known phenomenology

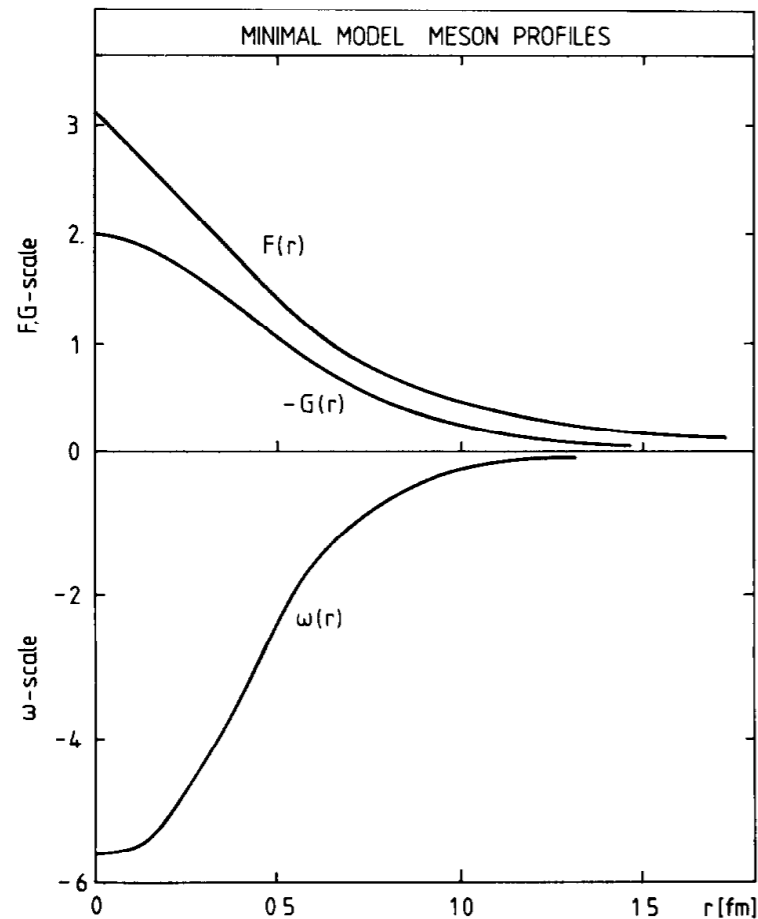
See, for example, P. Jain, U.-G. Meissner, N. Kaiser, H. Weigel, N.C. Mukhopadhyay, etc

U.-G. Meissner, N. Kaiser and W. Weise, Nucl. Phys. A466, 685 (1987)

ρ and ω mesons

minimal model results with $a = 2$, $f_\pi = 93$ MeV, $g = 5.85$

$$M_{sol} = 1475 \text{ MeV}$$



U.-G. Meissner et al. / Nucleons as Skyrme solitons

TABLE 1

Properties of the Skyrme soliton resulting from the lagrangians (2.11) or (2.19) with π , ρ and ω mesons

	Minimal model	Complete model	Following ref. ¹⁷⁾
M_H [MeV]	1474	1465	1057
r_H [fm]	0.50	0.48	0.27

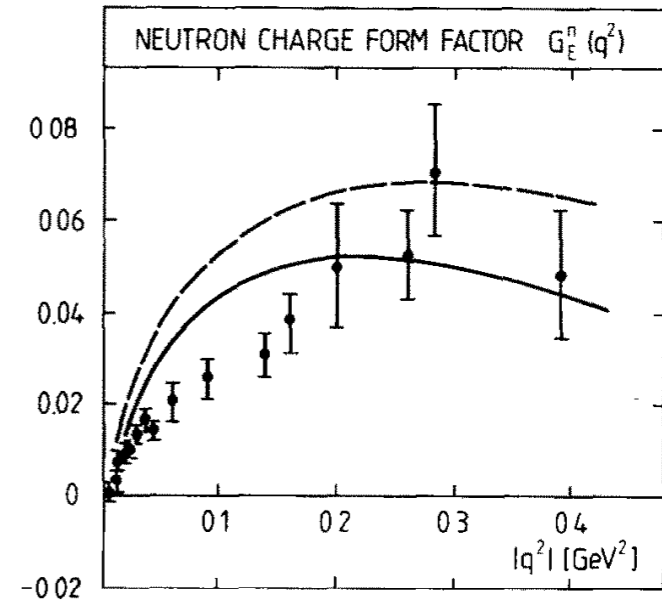
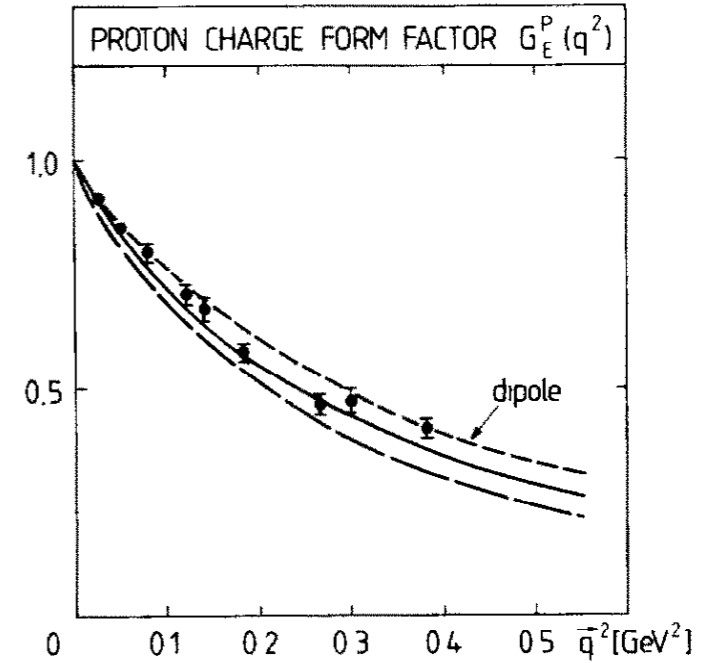
For comparison, the results of the model of ref. ¹⁷⁾ including pions and ρ mesons are also given. The parameters used are $m_\pi = 139$ MeV, $f_\pi = 93$ MeV, and $g = 5.85$. Here M_H is the static soliton mass, and r_H the baryonic r.m.s. radius.

ρ and ω mesons

U.-G. Meissner et al. / Nucleons as Skyrme solitons

TABLE 2
Baryon properties; parameters as in table 1

	Minimal model	Complete model	Experiment
Θ [fm]	0.82	0.68	
$M_\Delta - M_N$ [MeV]	359	437	293
M_N [MeV]	1564	1575	939
$r_H \equiv \langle r_B^2 \rangle^{1/2}$ [fm]	0.50	0.48	
$\langle r_E^2 \rangle_p^{1/2}$ [fm]	0.92	0.98	0.86 ± 0.01
$\langle r_E^2 \rangle_n$ [fm ²]	-0.22	-0.25	-0.119 ± 0.004
$\langle r_M^2 \rangle_p^{1/2}$ [fm]	0.84	0.94	0.86 ± 0.06
$\langle r_M^2 \rangle_n^{1/2}$ [fm]	0.85	0.93	0.88 ± 0.07
μ_p [n.m.]	3.36	2.77	2.79
μ_n [n.m.]	-2.57	-1.84	-1.91
$ \mu_p/\mu_n $	1.31	1.51	1.46



U.-G. Meissner, N. Kaiser and W. Weise, Nucl. Phys. A466, 685 (1987)

ρ , ω , and a_1 mesons

- Axial vector meson

$$U(x) = \xi_L^\dagger(x) \xi_M(x) \xi_R(x)$$

- 14 anomalous terms

cf. 6 independent terms in the $\pi\rho\omega$ system

- Hard to control the parameters

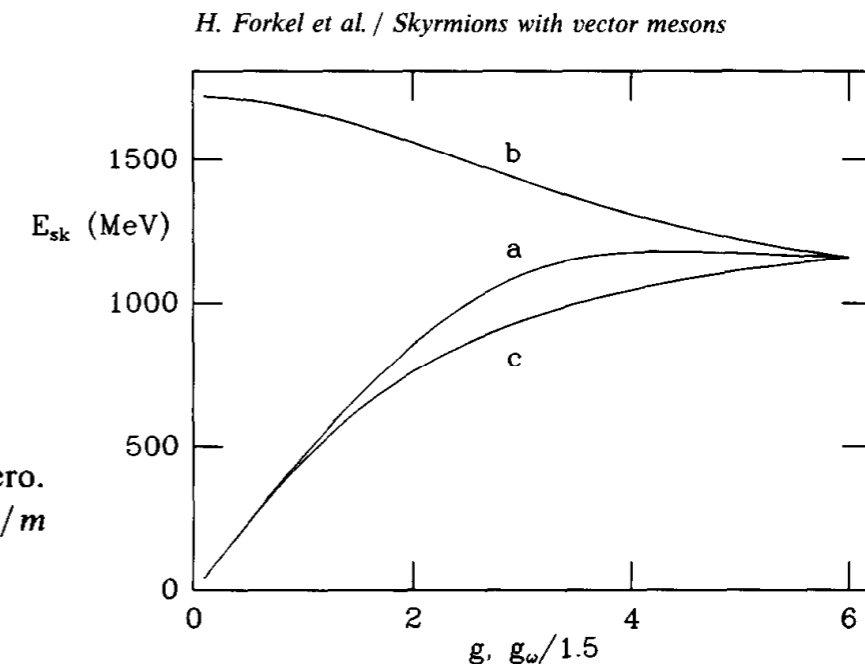
Results with $a = 2$, $f_\pi = 93$ MeV, $g = g_\omega/1.5 = 5.85$, $m_V = 770$ MeV

$$M_{sol} = 1002$$
 MeV

N. Kaiser and U.-G. Meissner,
Nucl. Phys. A519, 671 (1990)

L. Zhang and N.C. Mukhopadhyay,
Phys. Rev. D50, 4668 (1994)

Fig. 1. The behaviour of the skyrmion energy as the vector meson couplings and the masses go to zero. (a) $g = g_\omega/1.5 \rightarrow 0$, (b) $g \rightarrow 0$, $g_\omega/1.5 = 5.85$ (fixed), (c) $g_\omega \rightarrow 0$, $g = 5.85$ (fixed). In all cases the ratios g/m and $g_\omega/1.5m$ are kept constant at $5.85/770$ MeV.



H. Forkel, A.D. Jackson, and C. Weiss, Nucl. Phys. A526, 453 (1991)

ρ , ω , and a_1 mesons

The results are sensitive to the parameters.

TABLE VI. Nucleon observables in the $\pi\rho\omega a_1(f_1)$ chiral soliton model *without* the ϕ decay constraints (with all the energies in MeV). Here h_2 is calculated through Eq. (59) with $S_\omega > 0$.

Model	M_H	g_A	$g_{\pi NN}$	$\sigma_{\pi N}$
(1) $h_1 = 0.10$, $c'_i = 0, i = 2, \dots, 6, 8, Z = 0.9$	1403	0.70	10.56	29.8
(2) $h_1 = -0.10$, $c'_i = 0, i = 2, \dots, 6, 8, Z = 1.0$	1578	1.00	16.97	50.7
(3) $h_1 = -0.30$, $c'_i = 0, i = 2, \dots, 6, 8, Z = 1.0$	1725	1.25	23.19	70.6
(4) $h_1 = 0.10$, $c'_2 = -0.0020, c'_8 = -0.13$, $c'_i = 0, i = 3, \dots, 6, Z = 1.0$	1503	0.85	13.77	38.1
(5) $h_1 = 0.10, c'_2 = -0.012$, $c'_3 = 0.29, c'_4 = -0.42, c'_5 = 0.13$, $c'_6 = -0.015, c'_8 = -0.021, Z = 1.0$	1579	1.12	19.01	58.7
(6) $h_1 = 0.51, c'_2 = -0.019$, $c'_3 = -0.0022, c'_4 = -0.029, c'_5 = 0.53$, $c'_6 = -1.2, c'_8 = -0.094, Z = 1.0$	1379	0.90	13.30	43.5
The $\pi\rho\omega$ model ^a	1462	0.91	14.28	41.6
Expt.	939 ± 0	1.26 ± 0.01	13.45 ± 0.05	45 ± 10

^aReference [6].

TABLE III. Nucleon observables in the $\pi\rho\omega a_1(f_1)$ chiral soliton model with the ϕ decay constraints [Eq. (66)] put in (with all the energies in MeV).

Model	M_H	g_A	$g_{\pi NN}$	$\sigma_{\pi N}$
(Set A) $c'_2 = c'_3 = c'_4 = c'_5$, $c'_6 = c'_8 = 0, Z = 0$	704	0.18	1.38	3.4
(Set B) $c'_2 \approx 0.053, c'_3 \approx -0.029$, $c'_4 \approx 0.071, c'_5 \approx 0.72$, $c'_6 \approx -0.42, c'_8 \approx -0.24$, $Z = 0.4$	1070	0.59	6.74	23.0
The $\pi\rho\omega$ model ^a	1462	0.91	14.28	41.6
Expt.	939 ± 0	1.26 ± 0.01	13.45 ± 0.05	45 ± 10

^aReference [6].

L. Zhang and N.C. Mukhopadhyay, Phys. Rev. D50, 4668 (1994)

Summary of the earlier works

1. a dependence

- ambiguity in the value of a results in a large uncertainty in the soliton mass
(in free space, $a \sim 2$ and at high temperature / density $a \sim 1$)

2. Higher order terms

- $O(p^4)$ etc are at $O(N_c)$ like the $O(p^2)$ terms
- More complicated form of the Lagrangian
- Uncontrollably large number of low energy constants
E.g. 6 anomalous terms for the ω meson at $O(p^2)$
14 anomalous terms for the axial vector mesons at $O(p^2)$

3. In this work,

- $O(p^4)$ with ρ and ω mesons
- Fix the couplings by using hQCD

HLS Lagrangian up to $\mathcal{O}(p^4)$

$$\mathcal{L}_{\text{HGS}} = \mathcal{L}_{(2)} + \mathcal{L}_{(4)} + \mathcal{L}_{\text{anom}}$$

$$\mathcal{L}_{(2)} = f_\pi^2 \text{Tr} (\hat{a}_{\perp\mu} \hat{a}_\perp^\mu) + af_\pi^2 \text{Tr} (\hat{a}_{\parallel\mu} \hat{a}_\parallel^\mu) - \frac{1}{2g^2} \text{Tr} (V_{\mu\nu} V^{\mu\nu}),$$

$$\mathcal{L}_{(4)} = \mathcal{L}_{(4)y} + \mathcal{L}_{(4)z},$$

where

$$\begin{aligned} \mathcal{L}_{(4)y} = & y_1 \text{Tr} [\hat{a}_{\perp\mu} \hat{a}_\perp^\mu \hat{a}_{\perp\nu} \hat{a}_\perp^\nu] + y_2 \text{Tr} [\hat{a}_{\perp\mu} \hat{a}_{\perp\nu} \hat{a}_\perp^\mu \hat{a}_\perp^\nu] + y_3 \text{Tr} [\hat{a}_{\parallel\mu} \hat{a}_\parallel^\mu \hat{a}_{\parallel\nu} \hat{a}_\parallel^\nu] + y_4 \text{Tr} [\hat{a}_{\parallel\mu} \hat{a}_{\parallel\nu} \hat{a}_\parallel^\mu \hat{a}_\parallel^\nu] \\ & + y_5 \text{Tr} [\hat{a}_{\perp\mu} \hat{a}_\perp^\mu \hat{a}_{\parallel\nu} \hat{a}_\parallel^\nu] + y_6 \text{Tr} [\hat{a}_{\perp\mu} \hat{a}_{\perp\nu} \hat{a}_\parallel^\mu \hat{a}_\parallel^\nu] + y_7 \text{Tr} [\hat{a}_{\perp\mu} \hat{a}_{\perp\nu} \hat{a}_\parallel^\nu \hat{a}_\parallel^\mu] \\ & + y_8 \left\{ \text{Tr} [\hat{a}_{\perp\mu} \hat{a}_\parallel^\mu \hat{a}_{\perp\nu} \hat{a}_\parallel^\nu] + \text{Tr} [\hat{a}_{\perp\mu} \hat{a}_{\parallel\nu} \hat{a}_\perp^\nu \hat{a}_\parallel^\mu] \right\} + y_9 \text{Tr} [\hat{a}_{\perp\mu} \hat{a}_{\parallel\nu} \hat{a}_\perp^\mu \hat{a}_\parallel^\nu], \end{aligned}$$

$$\mathcal{L}_{(4)z} = iz_4 \text{Tr} [V_{\mu\nu} \hat{a}_\perp^\mu \hat{a}_\perp^\nu] + iz_5 \text{Tr} [V_{\mu\nu} \hat{a}_\parallel^\mu \hat{a}_\parallel^\nu].$$

$$\mathcal{L}_{\text{anom}} = \frac{N_c}{16\pi^2} \sum_{i=1}^3 c_i \mathcal{L}_i,$$

17 terms

where

$$\begin{aligned} \mathcal{L}_1 &= i \text{Tr} [\hat{\alpha}_L^3 \hat{\alpha}_R - \hat{\alpha}_R^3 \hat{\alpha}_L], \\ \mathcal{L}_2 &= i \text{Tr} [\hat{\alpha}_L \hat{\alpha}_R \hat{\alpha}_L \hat{\alpha}_R], \\ \mathcal{L}_3 &= \text{Tr} [F_V (\hat{\alpha}_L \hat{\alpha}_R - \hat{\alpha}_R \hat{\alpha}_L)], \end{aligned}$$

in the 1-form notation with

$$\begin{aligned} \hat{\alpha}_L &= \hat{\alpha}_\parallel - \hat{\alpha}_\perp, \\ \hat{\alpha}_R &= \hat{\alpha}_\parallel + \hat{\alpha}_\perp, \\ F_V &= dV - iV^2. \end{aligned}$$

M. Harada and K. Yamawaki, Phys. Rep. 381, 1 (2003)

HLS & hQCD

1. 5d action

$$S_5 = S_5^{\text{DBI}} + S_5^{\text{CS}}$$

$$S_5^{\text{DBI}} = N_c G_{\text{YM}} \int d^4 x dz \left\{ -\frac{1}{2} K_1(z) \text{Tr}[\mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu}] \right. \\ \left. + K_2(z) M_{KK}^2 \text{Tr}[\mathcal{F}_{\mu z} \mathcal{F}^{\mu z}] \right\},$$

$$S_5^{\text{CS}} = \frac{N_c}{24\pi^2} \int_{M^4 \times R} w_5(A).$$

$$w_5(A) = \text{Tr} \left[\mathcal{A} \mathcal{F}^2 + \frac{i}{2} \mathcal{A}^3 \mathcal{F} - \frac{1}{10} \mathcal{A}^5 \right].$$

2. induce the HLS Lagrangian from S_5 : integrate out the higher modes

$$A_\mu(x, z) \rightarrow A_\mu^{\text{integ}}(x, z) \\ = \hat{\alpha}_{\mu\perp}(x) \psi_0(z) + [\hat{\alpha}_{\mu\parallel}(x) + V_\mu(x)] \\ + \hat{\alpha}_{\mu\parallel}(x) \psi_1(z),$$

Determination of couplings

$$\begin{aligned}
 f_\pi^2 &= N_c G_{\text{YM}} M_{KK}^2 \int dz K_2(z) [\dot{\psi}_0(z)]^2, \\
 a f_\pi^2 &= N_c G_{\text{YM}} M_{KK}^2 \lambda_1 \langle \psi_1^2 \rangle, \\
 \frac{1}{g^2} &= N_c G_{\text{YM}} \langle \psi_1^2 \rangle, \\
 y_1 &= -y_2 = -N_c G_{\text{YM}} \langle (1 + \psi_1 - \psi_0^2)^2 \rangle, \\
 y_3 &= -y_4 = -N_c G_{\text{YM}} \langle \psi_1^2 (1 + \psi_1)^2 \rangle, \\
 y_5 &= 2y_8 = -y_9 = -2N_c G_{\text{YM}} \langle \psi_1^2 \psi_0^2 \rangle, \\
 y_6 &= -(y_5 + y_7), \\
 y_7 &= 2N_c G_{\text{YM}} \langle \psi_1 (1 + \psi_1) (1 + \psi_1 - \psi_0^2) \rangle, \\
 z_4 &= 2N_c G_{\text{YM}} \langle \psi_1 (1 + \psi_1 - \psi_0^2) \rangle, \\
 z_5 &= -2N_c G_{\text{YM}} \langle \psi_1^2 (1 + \psi_1) \rangle, \\
 c_1 &= \left\langle\left\langle \dot{\psi}_0 \psi_1 \left(\frac{1}{2} \psi_0^2 + \frac{1}{6} \psi_1^2 - \frac{1}{2} \right) \right\rangle\right\rangle, \\
 c_2 &= \left\langle\left\langle \dot{\psi}_0 \psi_1 \left(-\frac{1}{2} \psi_0^2 + \frac{1}{6} \psi_1^2 + \frac{1}{2} \psi_1 + \frac{1}{2} \right) \right\rangle\right\rangle, \\
 c_3 &= \left\langle\left\langle \frac{1}{2} \dot{\psi}_0 \psi_1^2 \right\rangle\right\rangle,
 \end{aligned}$$

a is still undetermined

where λ_1 is the smallest (non-zero) eigenvalue of the eigenvalue equation given in Eq. (34), and $\langle \rangle$ and $\langle\langle \rangle\rangle$ are defined as

$$\begin{aligned}
 \langle A \rangle &\equiv \int_{-\infty}^{\infty} dz K_1(z) A(z), \\
 \langle\langle A \rangle\rangle &\equiv \int_{-\infty}^{\infty} dz A(z)
 \end{aligned} \tag{36}$$

$K_1(z)$, $K_2(z)$: metric functions

$$K_1(z) = K^{-1/3}(z), \quad K_2(z) = K(z)$$

with $K(z) = 1 + z^2$

in the Sakai-Sugimoto model

Two parameters

KK MASS

'T HOOFT COUPLING

$$m_\rho = 776 \text{ MeV}$$

$$f_\pi = 92.4 \text{ MeV}$$

TABLE I. Low energy constants of the HLS Lagrangian at $O(p^4)$ with $a = 2$.

Model	y_1	y_3	y_5	y_6	z_4	z_5	c_1	c_2	c_3
SS model	-0.001096	-0.002830	-0.015917	+0.013712	0.010795	-0.007325	+0.381653	-0.129602	0.767374
BPS model	-0.071910	-0.153511	-0.012286	-0.196545	0.090338	-0.130778	-0.206992	+3.031734	1.470210

Comparison with the Skyrme Lagrangian

Original Skyrme Lagrangian

$$\mathcal{L}_{\text{Sk}} = \frac{f_\pi^2}{4} \text{Tr} (\partial_\mu U \partial^\mu U^\dagger) + \frac{1}{32e^2} \text{Tr} [\partial_\mu U U^\dagger, \partial_\nu U U^\dagger]^2, \quad (55)$$

After integrating out VM in HLS

$$\begin{aligned} \mathcal{L}_{\text{ChPT}} = & f_\pi^2 \text{Tr} [\alpha_{\perp\mu} \alpha_{\perp}^\mu] \\ & + \left(\frac{1}{2g^2} - \frac{z_4}{2} - \frac{y_1 - y_2}{4} \right) \text{Tr} [\alpha_{\perp\mu}, \alpha_{\perp\nu}]^2 \\ & + \frac{y_1 + y_2}{4} \text{Tr} \{ \alpha_{\perp\mu}, \alpha_{\perp\nu} \}^2, \end{aligned} \quad (56)$$

$$\frac{1}{2e^2} = \frac{1}{2g^2} - \frac{z_4}{2} - \frac{y_1 - y_2}{4}.$$

$$e \simeq 7.31$$

in the SS model

Three models

- HLS(π, ρ, ω) model:
full $O(p^4)$ Lagrangian with hWZ terms
- HLS(π, ρ) model:
without hQZ terms, the ω meson decouples
- HLS(π) model:
integrates out VMs
same as the Skyrme Lagrangian but e is fixed by the HLS

Soliton Wave Functions

Classical Solution

$$\xi(\mathbf{r}) = \exp \left[i\boldsymbol{\tau} \cdot \hat{\mathbf{r}} \frac{F(r)}{2} \right]$$

$$\omega_\mu = W(r) \delta_{0\mu},$$

$$\rho_0 = 0, \quad \boldsymbol{\rho} = \frac{G(r)}{gr} (\hat{\mathbf{r}} \times \boldsymbol{\tau})$$

Boundary Conditions

$$\begin{aligned} F(0) &= \pi, & F(\infty) &= 0, \\ G(0) &= -2, & G(\infty) &= 0, \\ W'(0) &= 0, & W(\infty) &= 0. \end{aligned}$$

FOR B=1 SOLITON

Collective Quantization

$$\begin{aligned} \xi(\mathbf{r}) &\rightarrow \xi(\mathbf{r}, t) = A(t) \xi(\mathbf{r}) A^\dagger(t), \\ V_\mu(\mathbf{r}) &\rightarrow V_\mu(\mathbf{r}, t) = A(t) V_\mu(\mathbf{r}) A^\dagger(t), \end{aligned}$$

$$i\boldsymbol{\tau} \cdot \boldsymbol{\Omega} \equiv A^\dagger(t) \partial_0 A(t).$$

$$\rho^0(\mathbf{r}, t) = A(t) \frac{2}{g} [\boldsymbol{\tau} \cdot \boldsymbol{\Omega} \xi_1(r) + \hat{\boldsymbol{\tau}} \cdot \hat{\mathbf{r}} \boldsymbol{\Omega} \cdot \hat{\mathbf{r}} \xi_2(r)] A^\dagger(t),$$

$$\omega^i(\mathbf{r}, t) = \frac{\varphi(r)}{r} (\boldsymbol{\Omega} \times \hat{\mathbf{r}})^i, \quad (21)$$

Boundary Conditions

$$\xi_1'(0) = \xi_1(\infty) = 0,$$

$$\xi_2'(0) = \xi_2(\infty) = 0,$$

$$\varphi(0) = \varphi(\infty) = 0,$$

$$M_{baryon}(I, J) = M_{sol} + \frac{I^2}{2\mathcal{I}} = M_{sol} + \frac{J^2}{2\mathcal{I}}$$

Soliton mass

$$M_{\text{sol}} = 4\pi \int dr [M_{(2)}(r) + M_{(4)}(r) + M_{\text{anom}}(r)], \quad (\text{A1})$$

where $M_{(2)}$, $M_{(4)}$, and M_{anom} are from $\mathcal{L}_{(2)}$, $\mathcal{L}_{(4)y} + \mathcal{L}_{(4)z}$, and $\mathcal{L}_{\text{anom}}$, respectively. Their explicit forms are

$$M_{(2)}(r) = \frac{f_\pi^2}{2} (F'^2 r^2 + 2 \sin^2 F) - \frac{ag^2 f_\pi^2}{2} W^2 r^2 + af_\pi^2 \left(G + 2 \sin^2 \frac{F}{2} \right)^2 - \frac{W'^2 r^2}{2} + \frac{G'^2}{g^2} + \frac{G^2}{2g^2 r^2} (G + 2)^2, \quad (\text{A2})$$

$$\begin{aligned} M_{(4)}(r) = & -y_1 \frac{r^2}{8} \left(F'^2 + \frac{2}{r^2} \sin^2 F \right)^2 - y_2 \frac{r^2}{8} F'^2 \left(F'^2 - \frac{4}{r^2} \sin^2 F \right) - y_3 \frac{r^2}{2} \left[\frac{g^2 W^2}{2} - \frac{1}{r^2} \left(G + 2 \sin^2 \frac{F}{2} \right)^2 \right]^2 \\ & - y_4 \frac{g^2 W^2 r^2}{2} \left\{ \frac{g^2 W^2}{4} - \frac{1}{r^2} \left(G + 2 \sin^2 \frac{F}{2} \right)^2 \right\} + \frac{y_5}{4} (r^2 F'^2 + 2 \sin^2 F) \left[\frac{g^2 W^2}{2} - \frac{1}{r^2} \left(G + 2 \sin^2 \frac{F}{2} \right)^2 \right] \\ & + \left(y_8 - \frac{y_7}{2} \right) \frac{\sin^2 F}{r^2} \left(G + 2 \sin^2 \frac{F}{2} \right)^2 + y_9 \left\{ \frac{g^2 W^2 r^2}{8} \left(F'^2 + \frac{2}{r^2} \sin^2 F \right) + \frac{F'^2}{4} \left(G + 2 \sin^2 \frac{F}{2} \right)^2 \right\} \\ & + z_4 \left\{ G' F' \sin F + \frac{\sin^2 F}{2r^2} G(G + 2) \right\} + \frac{z_5}{2r^2} G(G + 2) \left(G + 2 \sin^2 \frac{F}{2} \right)^2, \end{aligned} \quad (\text{A3})$$

$$\begin{aligned} M_{\text{anom}}(r) = & \alpha_1 F' W \sin^2 F + \alpha_2 W F' \left(G + 2 \sin^2 \frac{F}{2} \right)^2 \\ & - \alpha_3 \left\{ G(G + 2) W F' + 2 \sin F \left[W G' - W' \left(G + 2 \sin^2 \frac{F}{2} \right) \right] \right\}, \end{aligned} \quad (\text{A4})$$

where

$$\alpha_1 = \frac{3gN_c}{16\pi^2} (c_1 - c_2), \quad \alpha_2 = \frac{gN_c}{16\pi^2} (c_1 + c_2), \quad \alpha_3 = \frac{gN_c}{16\pi^2} c_3. \quad (\text{A5})$$

Moment of Inertia

$$L = -M_{\text{sol}} + I \text{Tr}(\dot{A}\dot{A}^\dagger), \quad I = 4\pi \int dr [I_{(2)}(r) + I_{(4)}(r) + I_{\text{anom}}(r)].$$

$$I_2(r) = \frac{2}{3} f_\pi^2 r^2 \sin^2 F + \frac{1}{3} a f_\pi^2 r^2 \left[(\xi_1 + \xi_2)^2 + 2 \left(\xi_1 - 2 \sin^2 \frac{F}{2} \right)^2 \right] - \frac{1}{6} a g^2 f_\pi^2 \varphi^2 - \frac{1}{6} \left(\varphi'^2 + \frac{2\varphi^2}{r^2} \right) + \frac{r^2}{3g^2} (3\xi_1'^2 + 2\xi_1'\xi_2' + \xi_2'^2) + \frac{4}{3g^2} G^2 (\xi_1 - 1)(\xi_1 + \xi_2 - 1) + \frac{2}{3g^2} (G^2 + 2G + 2) \xi_2^2.$$

$$I_{(4)} = \sum_i y_i I_{y_i} + \sum_i z_i I_{z_i}.$$

$$I_{y_1}(r) = -\frac{1}{3} r^2 \sin^2 F \left(F'^2 + \frac{2}{r^2} \sin^2 F \right),$$

$$I_{y_2}(r) = \frac{1}{3} r^2 \sin^2 F F'^2,$$

$$I_{y_3}(r) = -\frac{1}{12} g^2 \varphi^2 \left[g^2 W^2 - \frac{4}{r^2} \left(G + 2 \sin^2 \frac{F}{2} \right)^2 \right] + \frac{2}{3} g^2 W \varphi \left(G + 2 \sin^2 \frac{F}{2} \right) \left(\xi_1 - 2 \sin^2 \frac{F}{2} \right) + \left[\frac{1}{2} r^2 g^2 W^2 - \frac{1}{3} \left(G + 2 \sin^2 \frac{F}{2} \right)^2 \right] \left[(\xi_1 + \xi_2)^2 + 2 \left(\xi_1 - 2 \sin^2 \frac{F}{2} \right)^2 \right],$$

$$I_{y_4}(r) = \frac{r^2}{2} g^2 W^2 \left[(\xi_1 + \xi_2)^2 + 2 \left(\xi_1 - 2 \sin^2 \frac{F}{2} \right)^2 \right] - \frac{1}{12} g^2 W \varphi \left[g^2 W \varphi - 8 \left(G + 2 \sin^2 \frac{F}{2} \right) \left(\xi_1 - 2 \sin^2 \frac{F}{2} \right) \right] + \frac{1}{3} \left(G + 2 \sin^2 \frac{F}{2} \right)^2 \left[\frac{g^2 \varphi^2}{r^2} + (\xi_1 + \xi_2)^2 \right],$$

$$I_{y_5}(r) = \frac{1}{6} \sin^2 F \left[r^2 g^2 W^2 - 2 \left(G + 2 \sin^2 \frac{F}{2} \right)^2 \right] - \frac{r^2}{12} \left(F'^2 + \frac{2}{r^2} \sin^2 F \right) \left[2 \left(\xi_1 - 2 \sin^2 \frac{F}{2} \right)^2 + (\xi_1 + \xi_2)^2 - \frac{g^2 \varphi^2}{2r^2} \right],$$

$$I_{y_6}(r) = \frac{1}{6} \sin^2 F \left(r g W - \frac{g \varphi}{2r} \right)^2,$$

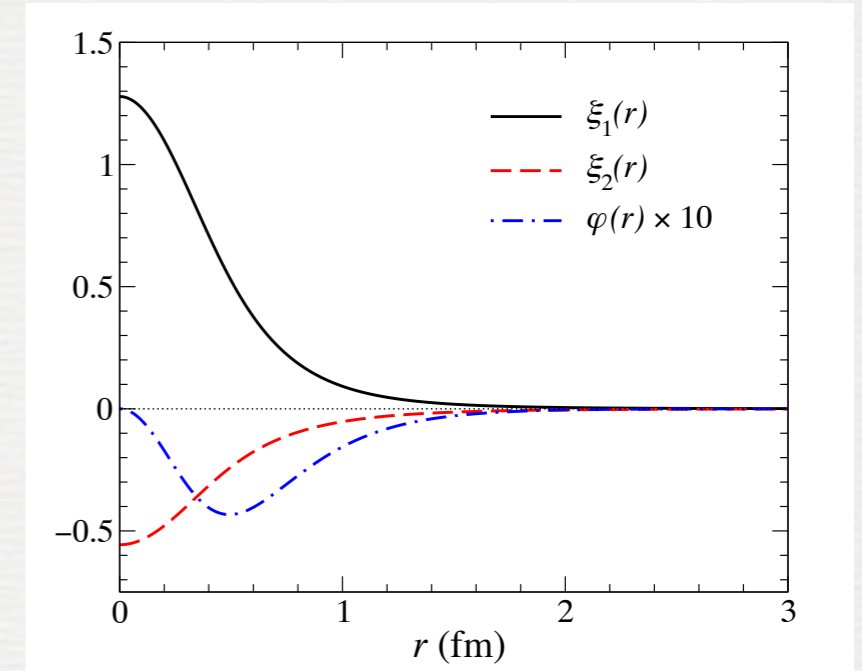
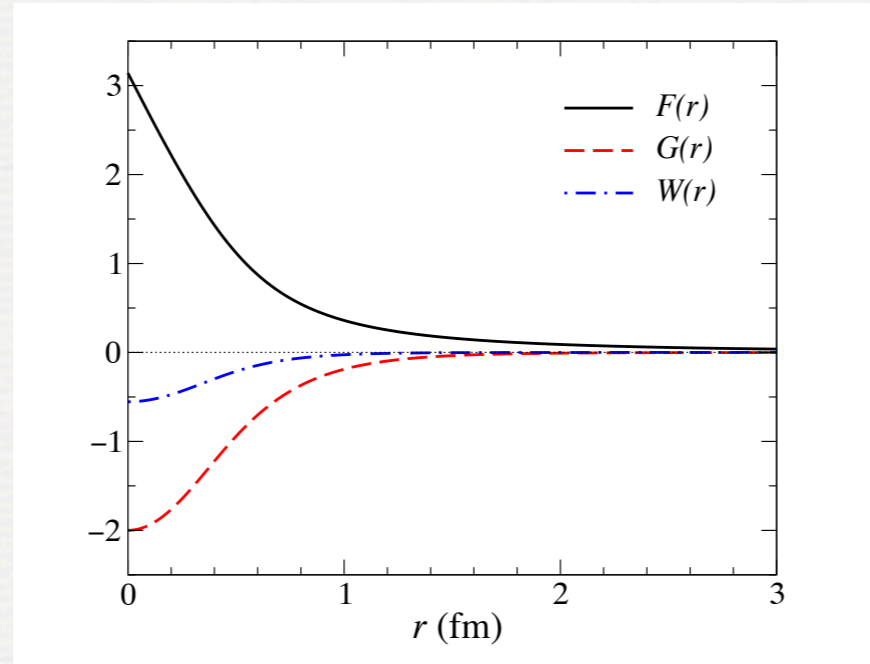
$$I_{y_7}(r) = \frac{1}{6} \sin^2 F \left[\left(r g W - \frac{g \varphi}{2r} \right)^2 + 4 \left(G + 2 \sin^2 \frac{F}{2} \right) \left(\xi_1 - 2 \sin^2 \frac{F}{2} \right) \right],$$

$$I_{y_8}(r) = \frac{1}{3} \sin^2 F \left[\left(r g W - \frac{g \varphi}{2r} \right)^2 - 4 \left(G + 2 \sin^2 \frac{F}{2} \right) \left(\xi_1 - 2 \sin^2 \frac{F}{2} \right) \right],$$

$$I_{y_9}(r) = \frac{r^2}{6} g^2 W^2 \sin^2 F + \frac{r^2}{6} F'^2 \left(\xi_1 - 2 \sin^2 \frac{F}{2} \right)^2 - \frac{r^2}{12} \left(F'^2 - \frac{2}{r^2} \sin^2 F \right) (\xi_1 + \xi_2)^2 + \frac{1}{24} g^2 \varphi^2 \left(F'^2 + \frac{2}{r^2} \sin^2 F \right),$$

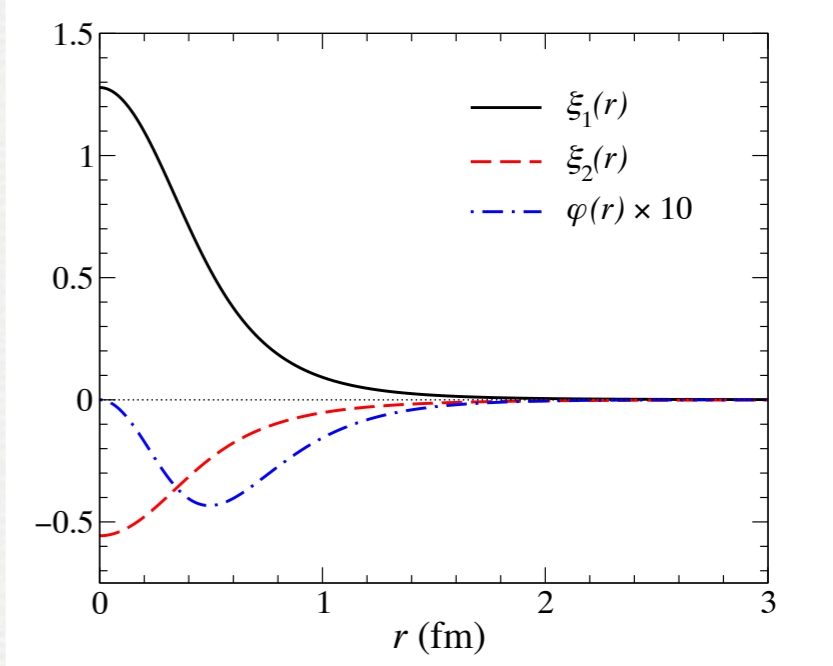
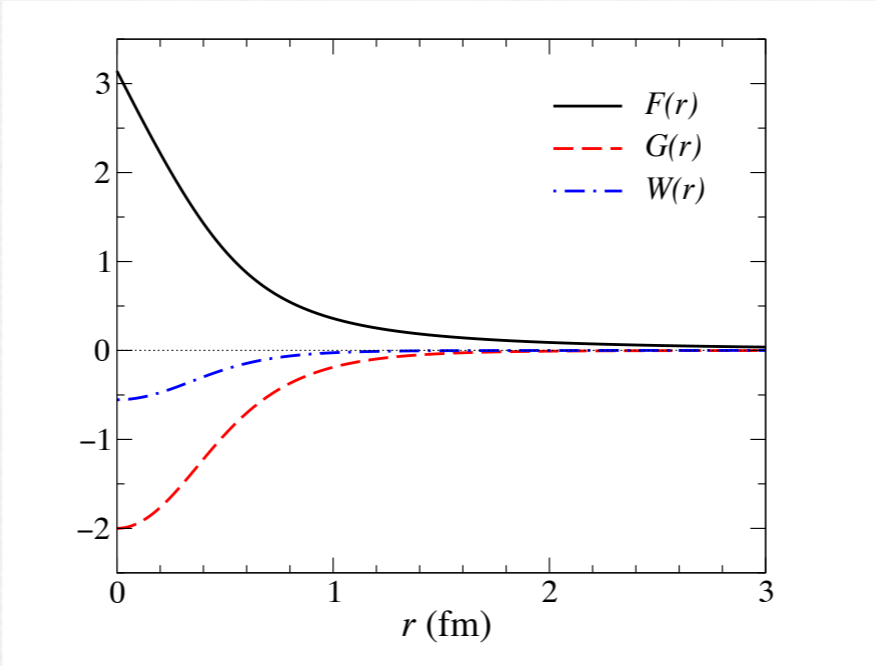
Solutions

HLS(π , ρ , ω) model

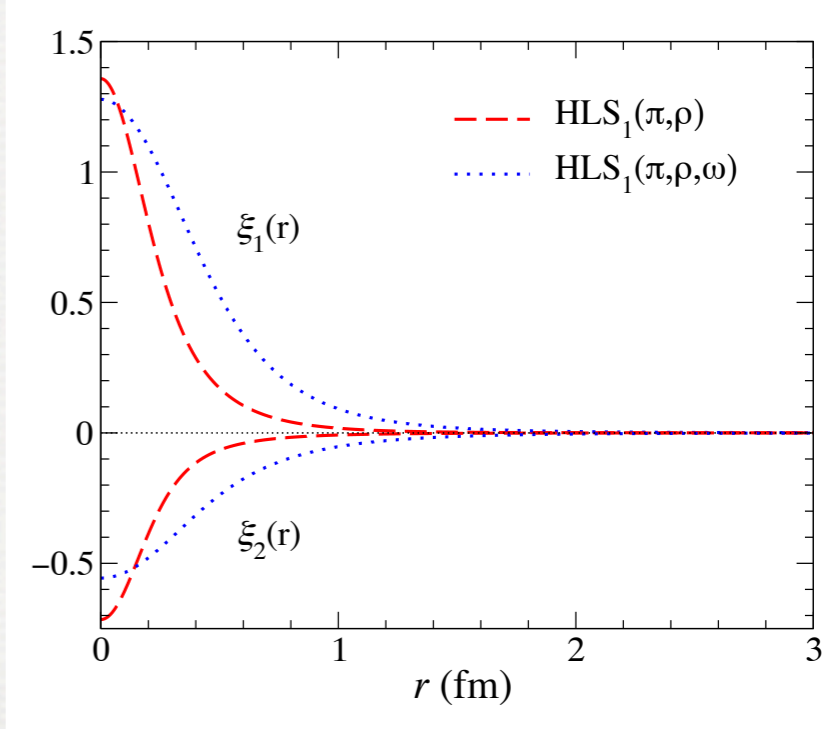
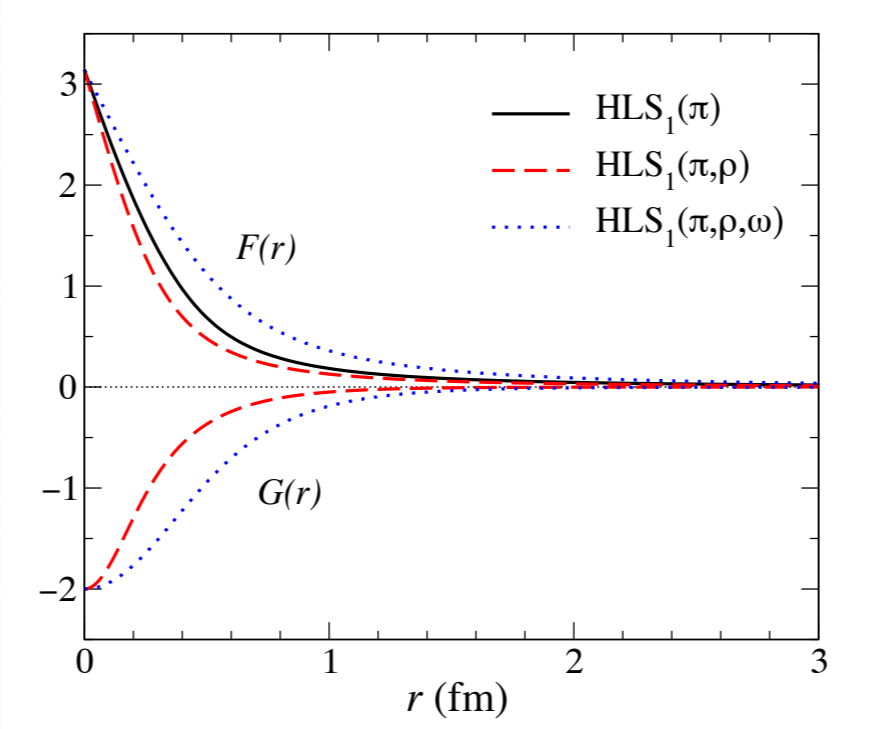


Solutions

HLS(π, ρ, ω) model



Comparison of the three models



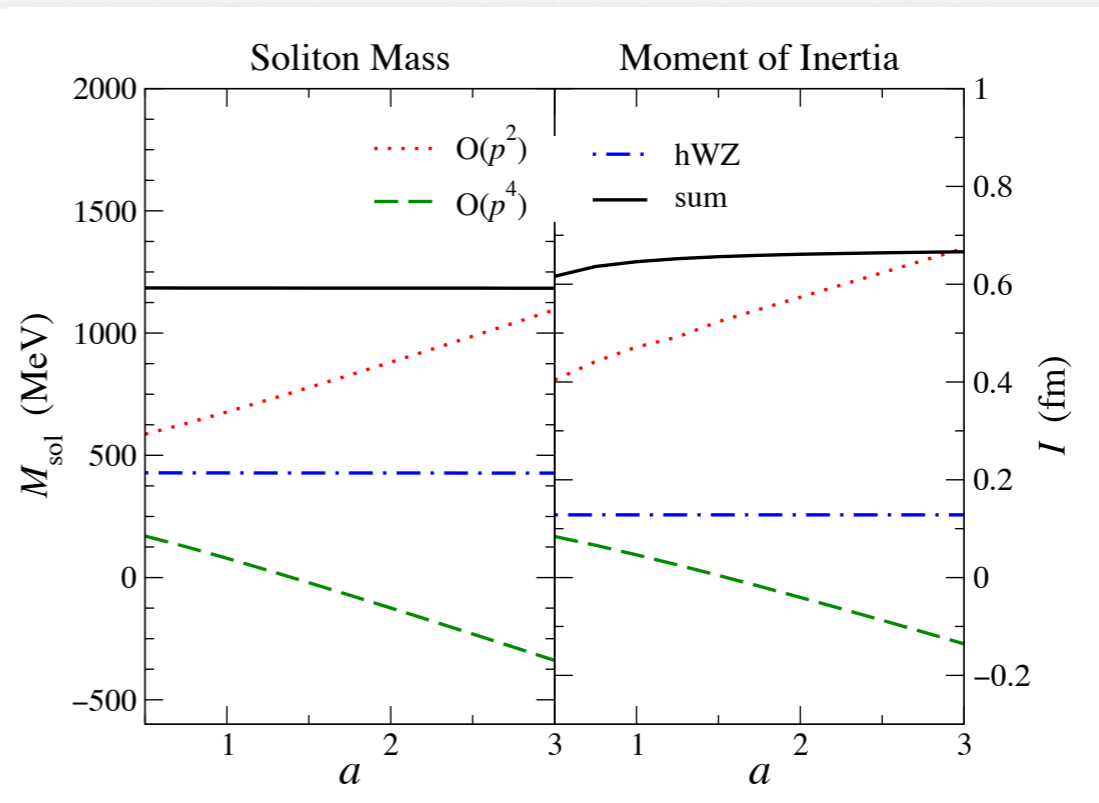
Results

TABLE II. Skyrmion mass and size calculated in the HLS with the SS and BPS models with $a = 2$. The soliton mass M_{sol} and the Δ -N mass difference Δ_M are in unit of MeV while $\sqrt{\langle r^2 \rangle_W}$ and $\sqrt{\langle r^2 \rangle_E}$ are in unit of fm. The column of $O(p^2) + \omega_\mu B^\mu$ is “the minimal model” of Ref. [20] and that of $O(p^2)$ corresponds to the model of Ref. [19]. See the text for more details.

	HLS ₁ (π, ρ, ω)	HLS ₁ (π, ρ)	HLS ₁ (π)	BPS(π, ρ, ω)	BPS(π, ρ)	BPS(π)	$O(p^2) + \omega_\mu B^\mu$ [20]	$O(p^2)$ [19]
M_{sol}	1184	834	922	1162	577	672	1407	1026
Δ_M	448	1707	1014	456	4541	2613	259	1131
$\sqrt{\langle r^2 \rangle_W}$	0.433	0.247	0.309	0.415	0.164	0.225	0.540	0.278
$\sqrt{\langle r^2 \rangle_E}$	0.608	0.371	0.417	0.598	0.271	0.306	0.725	0.422

$$\Delta_M \equiv M_\Delta - M_N$$

a independence of the Skyrmion properties



Discussions

1. The role of ρ meson

- reduction of the soliton mass: from 922 MeV to 834 MeV
- increase of the Δ -N mass difference: from 1014 MeV to 1707 MeV
- shrink the soliton profile: from 0.417 fm to 0.371 fm

2. The role of ω meson

- increase of the soliton mass: from 834 MeV to 1184 MeV
- decrease of the Δ -N mass difference: from 1707 MeV to 448 MeV
- expand the soliton profile: from 0.371 fm to 0.608 fm

3. Without ω meson

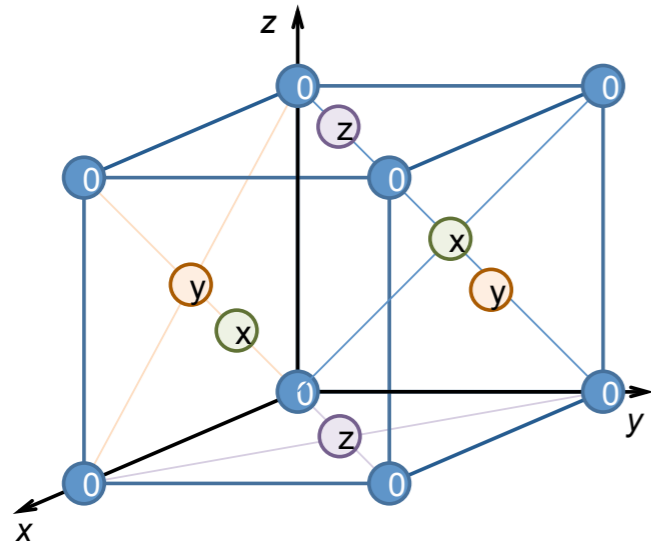
- the Δ -N mass difference of $O(1/N_c) >$ the soliton mass of $O(N_c)$

4. The independence of a

- Direct consequence from hQCD

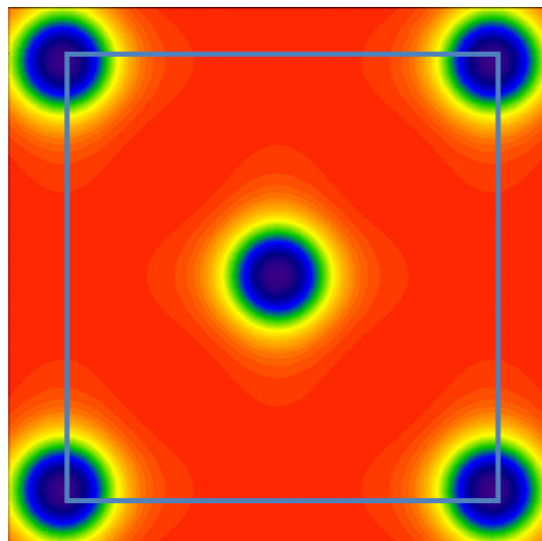
Nuclear Matter: Skyrme Crystal

Skyrme Crystal (FCC)

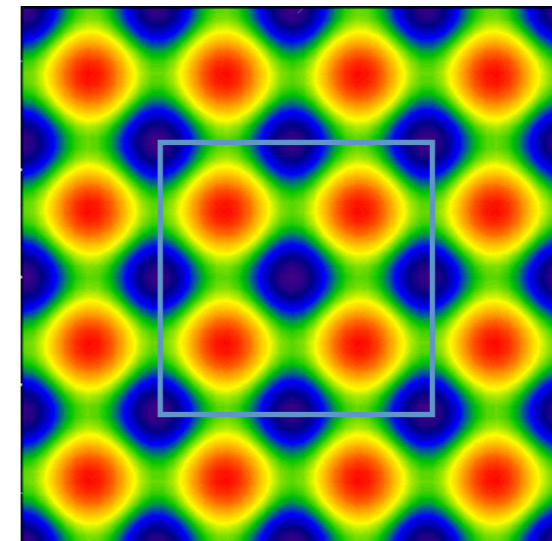


I. Klebanov, Nucl. Phys. B262, 133 (1985)
M. Kugler et al., Phys. Lett. B208, 491 (1988)
H.-J. Lee, B.-Y. Park, D.-P. Min, M. Rho, and U.
Dento, Nucl. Phys. A723, 427 (2003)

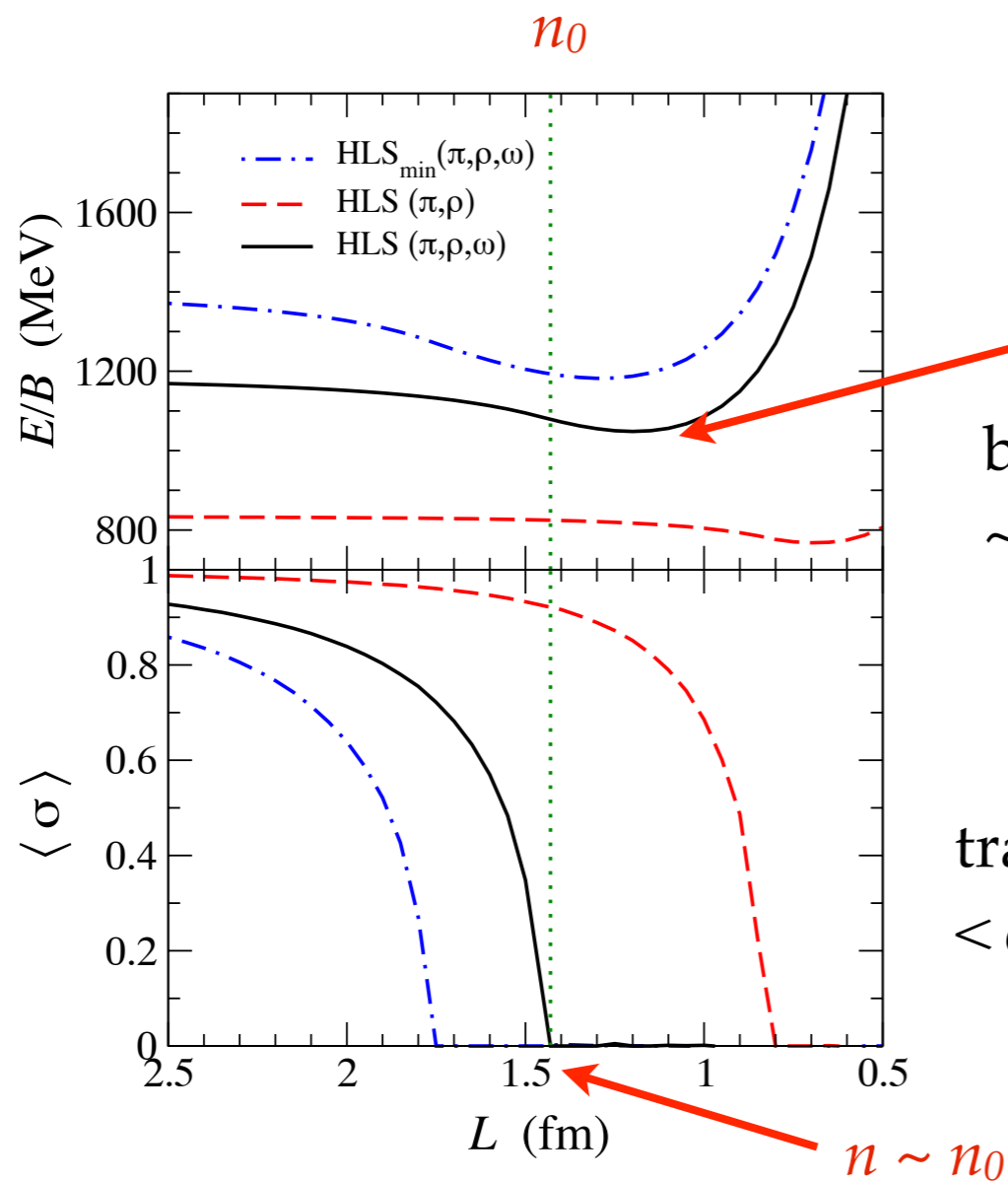
Half-Skyrmion Phase



increasing density



Skyrme Crystal

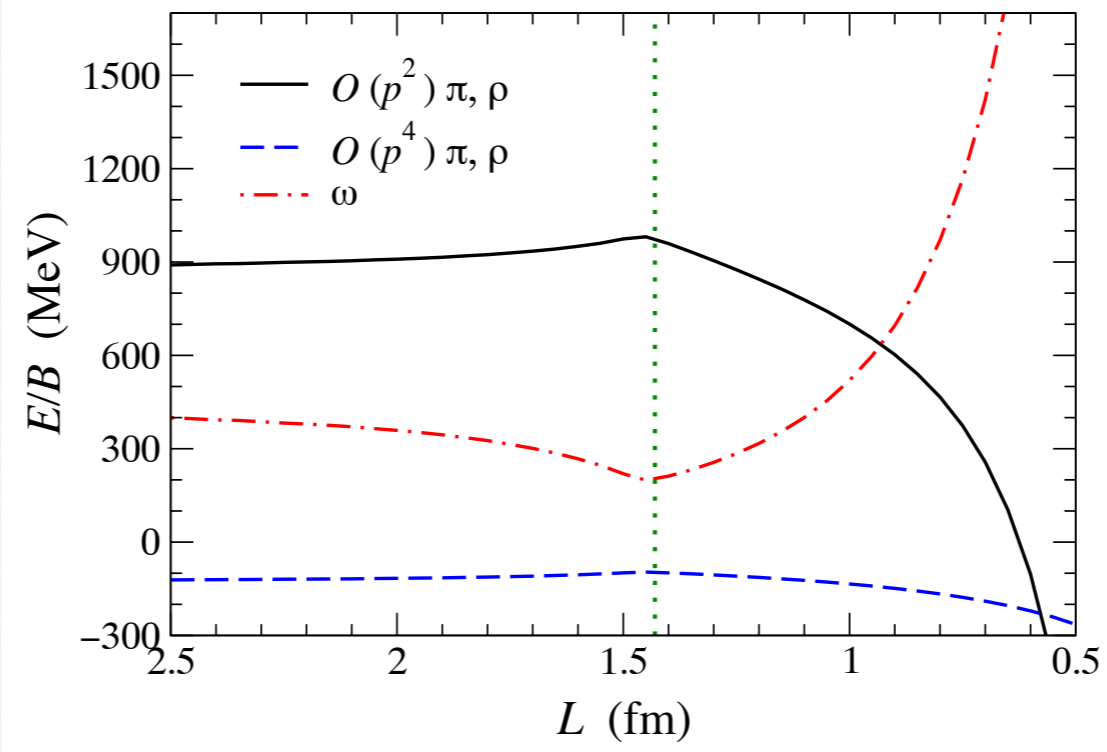


- Skymion number density $n = 1 / (2L^3)$
- normal nuclear density $n_0 : 0.17 \text{ fm}^{-3}$ corresponds to $L \sim 1.43 \text{ fm}$

$n \sim 2n_0$

binding energy per baryon
 $\sim 150 \text{ MeV}, \sim 100 \text{ MeV}, \sim 50 \text{ MeV}$ (too big!)

transition to the half-Skymion phase
 $\langle \sigma \rangle = 0$



Change of Meson Properties

$$\mathcal{L} = \frac{f_\pi^2}{4} \text{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2} \text{Tr} [U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2$$

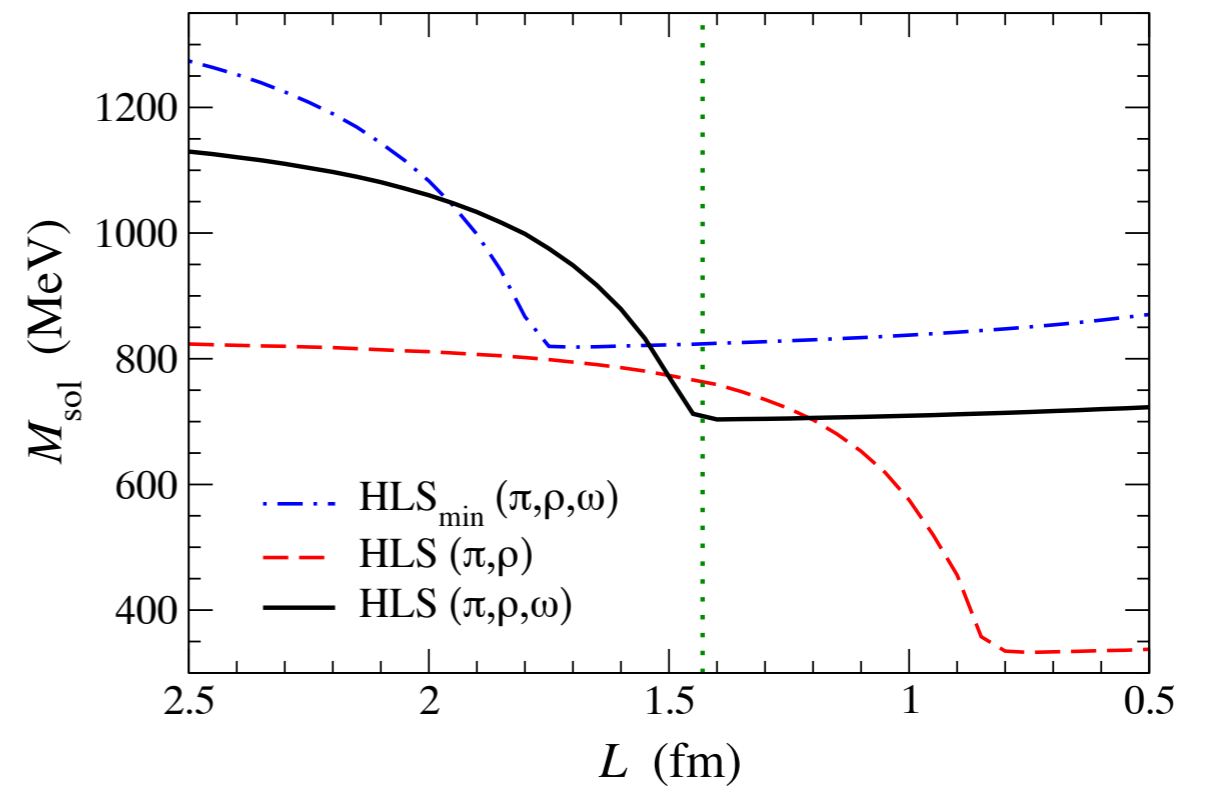
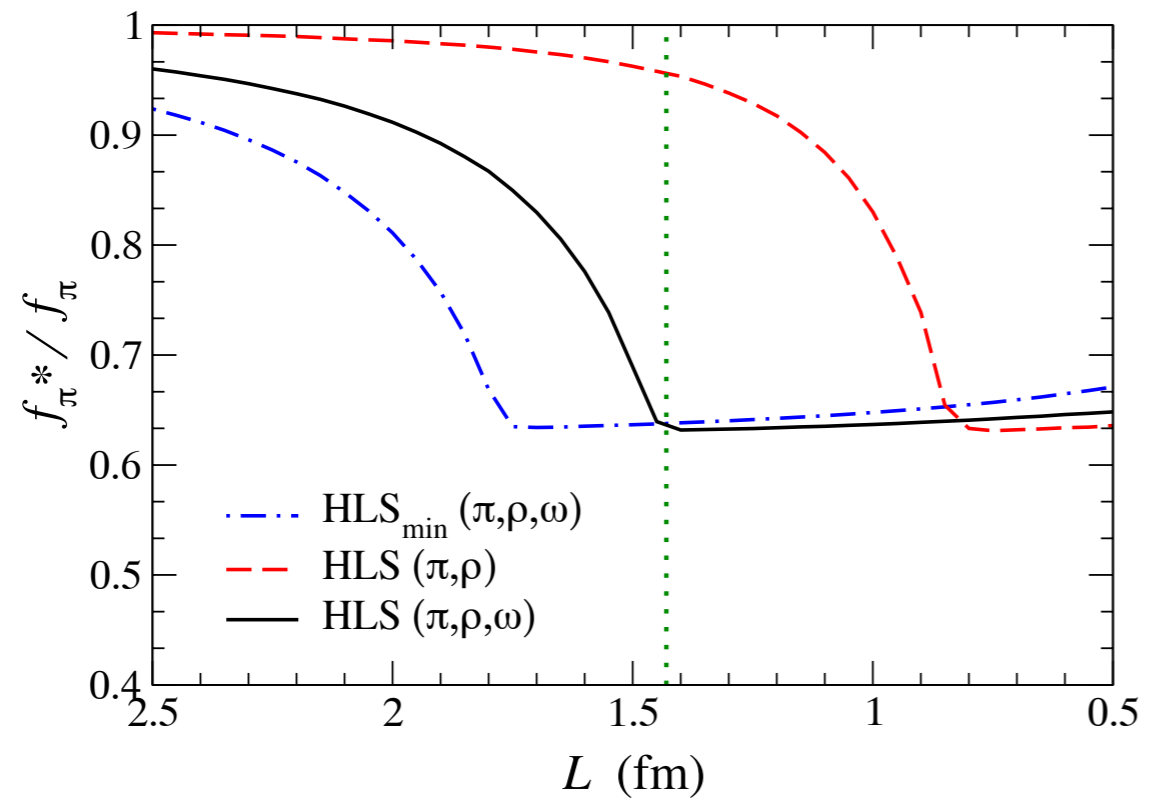
$$U_\pi = \exp(i\vec{\tau} \cdot \vec{\pi})$$

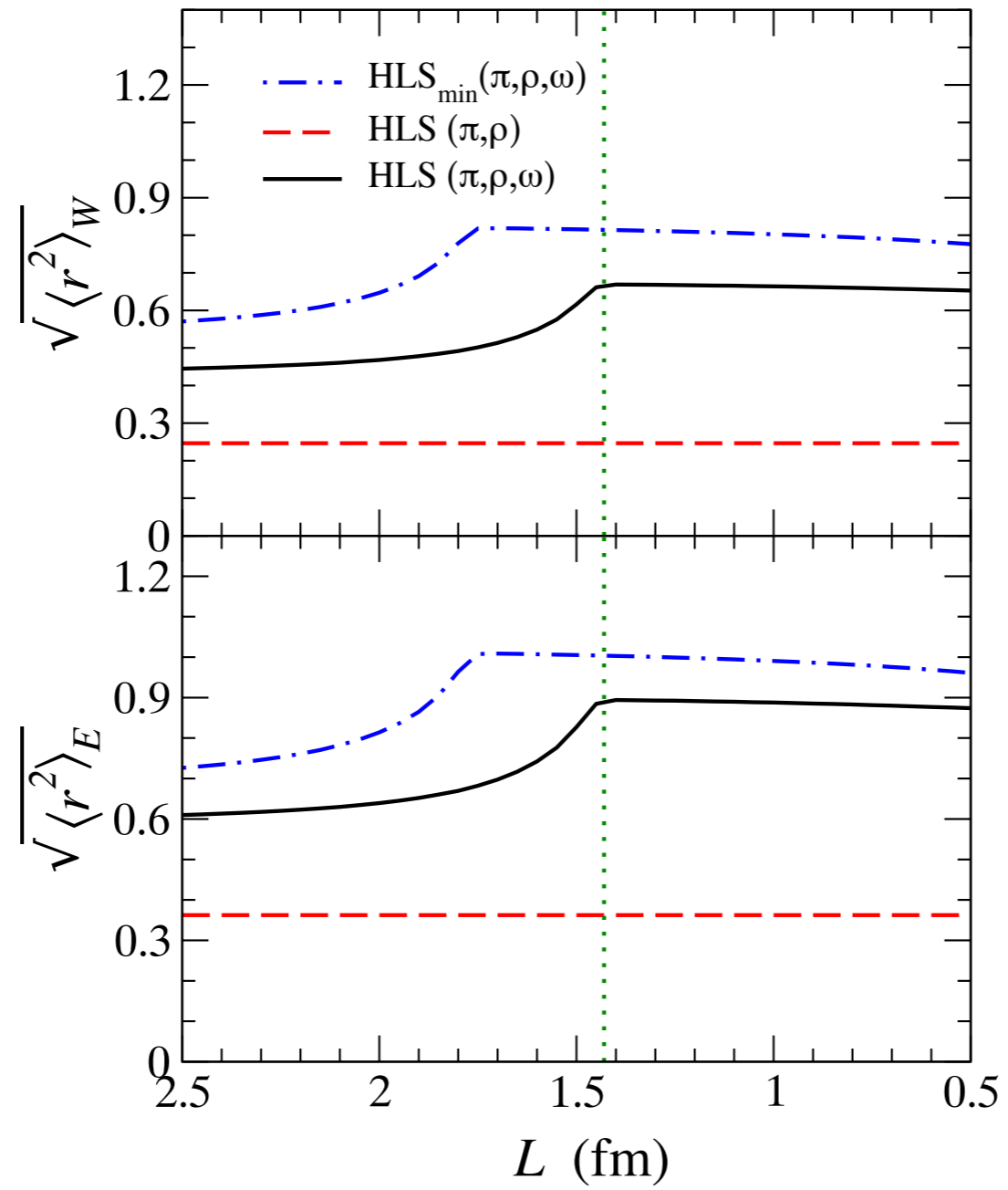
$$\mathcal{L} = \frac{1}{2} \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \dots$$

$$U = \sqrt{U_\pi} U_B \sqrt{U_\pi}$$

$$\mathcal{L} = \frac{1}{2} G_{ab}(U_B) \partial_\mu \pi^a \partial^\mu \pi^b + \dots$$

$$\frac{f_\pi^*}{f_\pi} = \sqrt{\langle G_{aa}(U_B) \rangle}$$





Summary

1. The nontrivial role of the ω meson
2. The presence of topological change from Skyrmions to half-Skyrmions at slightly above the normal nuclear density of higher density
3. Problems: minimal energy occurs at too high density with too high binding energy
4. The structure may be robust, but the numbers?
1/Nc corrections: Casimir energy

F. Meier and H. Walliser, Phys. Rep. 289, 383 (1997)

Table 4.1
Tree and one-loop contribution to various quantities for parameter set A ($e = 4.25$, $g_\omega = 0$)

	Tree	One-loop	Σ	Exp.
M (MeV)	1629	-683	946	939
σ (MeV)	54	-22	32	45 ± 7
$\langle r^2 \rangle^S$ (fm ²)	1.0	+0.3	1.3	1.6 ± 0.3
g_A	0.91	-0.25	0.66	1.26
$\langle r^2 \rangle_A$ (fm ²)	0.45	-0.04	0.41	$0.42^{+0.18}_{-0.08}$
$\langle r^2 \rangle_E^S$ (fm ²)	0.62	-0.11	0.51	0.59
μ^V	1.62	+0.62	2.24	2.35
$\langle r^2 \rangle_M^V$ (fm) ²	0.77	-0.13	0.64	0.73
α (10 ⁻⁴ fm ³)	17.8	-8.0	9.8	9.5 ± 5

Outlook

1. The role of vector mesons

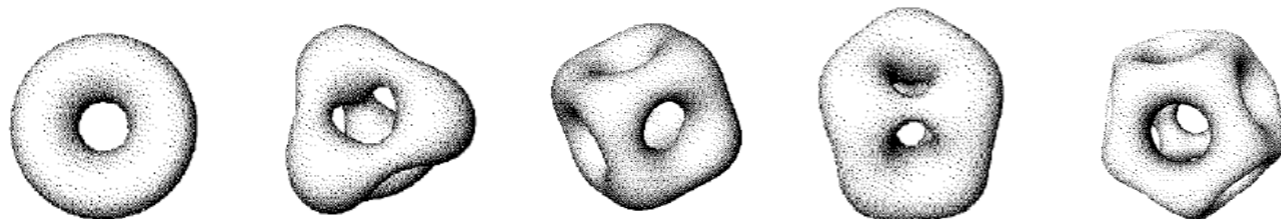
- previous works: more VMs lead to the Bogomolny bound
- the inclusion of the ρ meson confirms it
- but, the ω meson has the opposite role:
important from both the theoretical and phenomenological views

2. Issues

- next order corrections: $O(N_c^0)$ pion fluctuation
- next order terms in the HLS: in N_c and in p

3. Final goal

- few-nucleon systems \Rightarrow semi-empirical mass formula?
- nuclear matter, Skyrmion crystal
- equation of state, nuclear symmetric energy



*Thank you very much
for your attention*