Vorticity in atomic nuclei

V.O. Nesterenko

J. Kvasil, A. Repko

Institute of Particle and Nuclear Physics, Charles University,
Praha , Czech Republic

W. Kleinig
Laboratory of Theoretical Physics, JINR;
Technical University of Dresden, Institute for Analysis,
Dresden, Germany

P.-G. Reinhard
Institute of Theoretical Physics II, University of Erlangen,
Erlangen, Germany

BLTP, JINR, 05.12.2012

Introduction 1

Nuclei demonstrate both

- **irrotational flow**

$$\vec{w}(\vec{r}) = \vec{\nabla} \times \vec{v}(\vec{r}) = 0$$

examples: most of electric giant resonances (GR)

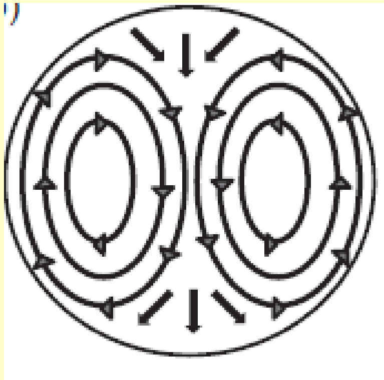
- **vortical flow**

$$\vec{w}(\vec{r}) = \vec{\nabla} \times \vec{v}(\vec{r}) \neq 0$$

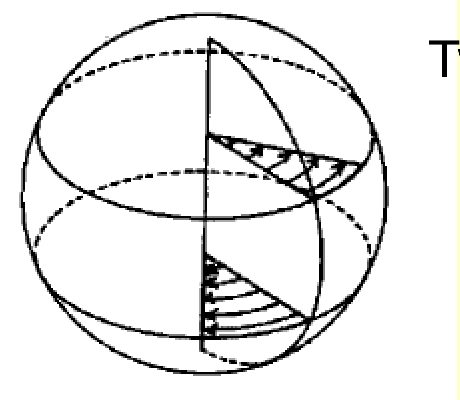
examples:

- **nuclear rotation of deformed nuclei,**
- **s-p excitations,**
- **toroidal E1 GR**
- **twist M2 GR**

- rotation-like oscillations



Toroidal E1



Twist M2

Introduction 2

Vorticity $\vec{w}(\vec{r})$ **is a fundamental quantity:**

- **does not contribute to the continuity equation,**
- **represents an independent part of charge-current distribution beyond the continuity equation.**

$$\dot{\rho} + \vec{\nabla} \cdot \vec{j}_{nuc} = 0$$

Vorticity is related to the exotic E1 modes of high interest:

- **toroidal**
- **compression**
- **pygmy**

second-order GR ⟺ ISGDR

N. Paar, D. Vretenar, E. Kyan, G. Colo, RPP, 70 691 (2007).

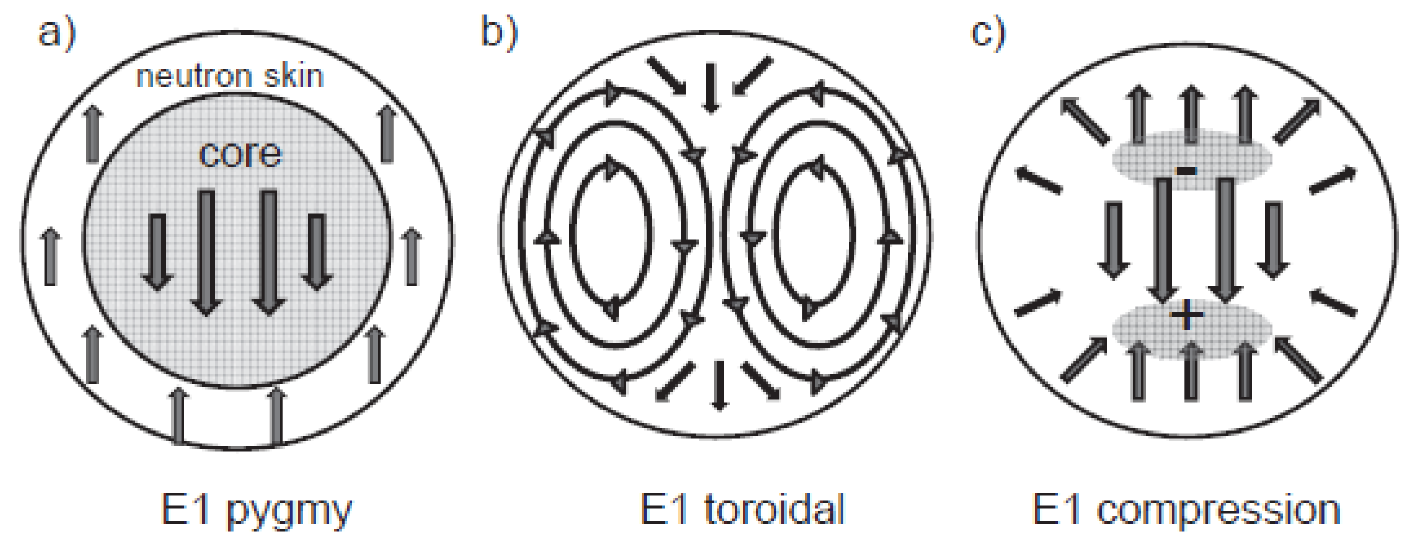
Beyond
long-wave approximation:

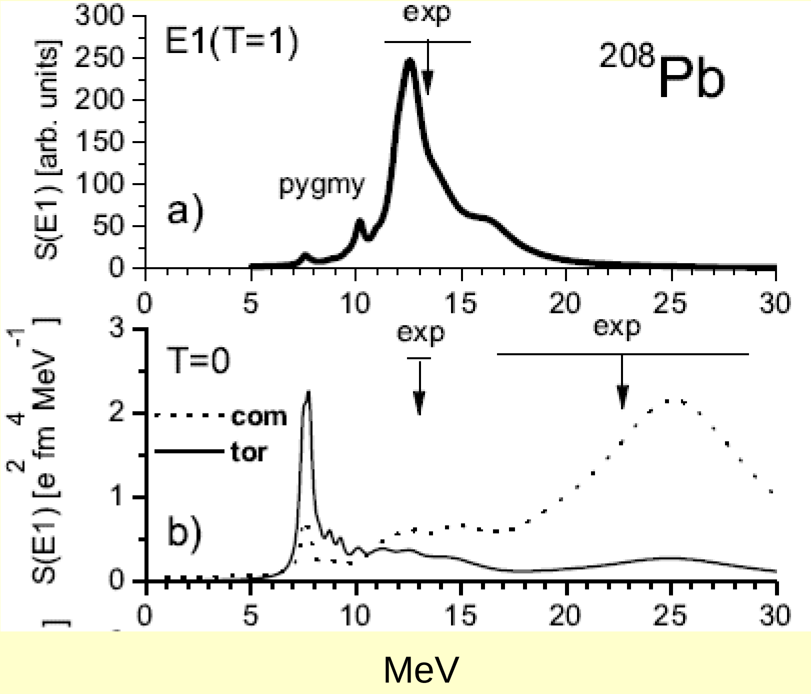
$$j_{\lambda}(kr) = \frac{(kr)^{\lambda}}{(2\lambda+1)!!}[1 - \frac{(kr)^{2}}{2(2\lambda+3)} + \ldots]$$

Leading dipole modes in T=0 channel !!!

Manifestation of nuclear elasticity (toroidal, twist, …)
May exist in other systems (atomic clusters, …)

VON et al, PRL, 85 , 3141 (2000)





Interplay of pygmy, toroidal, and compression flows in the PDR region?

Skyrme-RPA calculations

A. Repko, P.G. Reinhard, VON, J. Kvasil, to be submitted

Theoretical studies:

Many publications on toroidal and compressional (ISGDR) modes and manifestations of vorticity:

V.M. Dubovik and A.A. Cheshkov, SJPN 5, 318 (1975).
M.N. Harakeh et al, PRL 38, 676 (1977).
S.F. Semenko, SJNP 34 356 (1981).
J. Heisenberg, Adv. Nucl. Phys. 12, 61 (1981).
S. Stringari, PLB 108, 232 (1982).
E. Wust et al, NPA 406, 285 (1983).
E.E. Serr, T.S. Dumitrescu, T.Suzuki, NPA 404 359 (1983).
D.G.Raventhall, J.Wambach, NPA 475, 468 (1987).
E.B. Balbutsev and I.N. Mikhailov, JPG 14, 545 (1988).
S.I. Bastrukov, S. Misicu, A. Sushkov, NPA 562, 191 (1993).
I. Hamamoto, H.Sagawa, X.Z. Zang, PRC 53 765 (1996).
E.C.Caparelli, E.J.V.de Passos, JPG 25, 537 (1999).
N.Ryezayeva et al, PRL 89, 272502 (2002).
G.Colo, N.Van Giai, P.Bortignon, M.R.Quaglia, PLB 485, 362 (2000).
D. Vretenar, N. Paar, P. Ring,T. Nikshich, PRC 65, 021301(R) (2002).
V.Yu. Ponomarev, A.Richter, A.Shevchenko, S.Volz, J.Wambach, PRL 89, 272502 (2002).
J. Kvasil, N. Lo Iudice, Ch. Stoyanov, P. Alexa, JPG 29, 753 (2003).
A. Richter, NPA 731, 59 (2004).
X. Roca-Maza et al, PRC 85, 024601 (2012).
……….
N. Paar, D. Vretenar, E. Kyan, G. Colo, Rep. Prog. Phys. 70 691 (2007).

Recent
review

J. Kvasil, VON, W. Kleinig, P.-G. Reinhard, P. Vesely, PRC, 84, 034303 (2011)

Content

J. Kvasil, VON, W. Kleinig, P.-G. Reinhard,
P. Vesely, PRC, 84, 034303 (2011)
P.G. Reinhard, V.O. Nesterenko, J. Kvasl, A. Repko,
to be submitted

- Two definitions of vorticity in nuclear physics:
  - hydrodynamical (HD)
  - Rawenhall –Wambach (RW)

- Theory: derivation of operators for toroidal and compression modes

$$\hat{M}^{j+}_{vor}(E\lambda\mu) = \hat{M}_{tor}(E\lambda\mu) + \hat{M}_{com}(E\lambda\mu)$$

- Numerical results for vortical, toroidal and compression responses
  - strength functions, transition densities, velocity fields
  - role of convection and magnetization (spin) nuclear current

- Perspective to observe the vortical E1 GR in experiment?

-Twist M2 GR

May we use, in analogy to HD,

$$\vec{\nabla} \times \vec{j}(\vec{r})$$

$$\vec{j}(\vec{r}) = \rho_0 \vec{v}(\vec{r})$$

$$\vec{\nabla} \times \vec{j}(\vec{r}) = \rho_0 \vec{\nabla} \times \vec{v}(\vec{r})$$

as a measure of the nuclear vorticity?

No, because:

1) $\vec{\nabla} \times \vec{v}(\vec{r})$ and $\vec{\nabla} \times \vec{j}(\vec{r})$ are very different values

$$\delta\vec{v}(\vec{r}) = \frac{\delta\vec{j}_{nuc}(\vec{r})}{\rho_0(\vec{r})} \qquad \vec{\nabla} \times \delta\vec{v}(\vec{r}) = \frac{\rho_0(\vec{r})\vec{\nabla} \times \delta\vec{j}_{nuc}(\vec{r}) - \vec{\nabla}\rho_0(\vec{r}) \times \delta\vec{j}_{nuc}(\vec{r})}{\rho_0^2(\vec{r})}$$

2) Multipole electric operator

$$\hat{M}(Ek\lambda\mu) = \frac{(2\lambda+1)!!}{ck^{\lambda+1}} \sqrt{\frac{\lambda}{\lambda+1}} \int d\vec{r} \quad j_\lambda(kr)\vec{Y}_{\lambda\lambda\mu} \cdot [\vec{\nabla} \times \hat{\vec{j}}_{nuc}(\vec{r})]$$

If to use $\vec{\nabla} \times \vec{j}(\vec{r})$ as a measure of vorticity, then all the electric modes would be vortical. But this contradicts numerous exper. data:, e.g. for electric GR.

Two conceptions of vorticity in nuclear theory:

1. Hydrodynamical vorticity:

$$\vec{w}(\vec{r}) = \vec{\nabla} \times \vec{v}(\vec{r}) \qquad \delta\vec{v}(\vec{r}) = \frac{\delta\vec{j}_{nuc}(\vec{r})}{\rho_0(\vec{r})}$$

$$(\vec{\nabla} \times \delta\vec{j}_{nuc}) \rightarrow \rho_0(\vec{r})(\vec{\nabla} \times \delta\vec{v}) \rightarrow \rho_0(\vec{r})\,\vec{w}(\vec{r})$$

2. Wambach vorticity ⟷ j_+ vorticity

D.G.Raventhall, J.Wambach,
NPA 475, 468 (1987).

$$\dot{\rho} + \vec{\nabla} \cdot \vec{j}_{nuc} = 0$$

- **continuity equation**

$$\delta\vec{j}_{(fi)}(\vec{r}) = \left\langle j_f m_f \,|\, \hat{\vec{j}}_{nuc}(\vec{r}) \,|\, j_i m_i \right\rangle = \sum_{\lambda\mu} \frac{(j_i m_i \lambda\mu \,|\, j_f m_f)}{\sqrt{2j_f + 1}} [j^{(fi)}_{\lambda\lambda-1}(r)\vec{Y}^{\,*}_{\lambda\lambda-1\mu} + j^{(fi)}_{\lambda\lambda+1}(r)\vec{Y}^{\,*}_{\lambda\lambda+1\mu}]$$

$$\delta\vec{j}^{\,\nu}_{1\mu}(\vec{r}) = \left\langle \nu \,|\, \hat{\vec{j}}_{nuc}(\vec{r}) \,|\, 0 \right\rangle = -\frac{i}{\sqrt{3}} [\underbrace{j^{\nu}_{10}}_{j_-}(r)\vec{Y}^{\,*}_{10\mu} + \underbrace{j^{\nu}_{12}}_{j_+}(r)\vec{Y}^{\,*}_{12\mu}]$$

- **current transition density**

$$\boxed{j^{\nu}_{+}(r)}$$

- independent part of charge-current distribution, decoupled to CE
- may be the measure of the vorticity

HD and j+ prescriptions give opposite conclusions on CM vorticity!

Definition 2
(Ravenhall and Wambach)

D.G.Raventhall, J.Wambach,
NPA 475, 468 (1987).

$$\delta\vec{j}_{(fi)}(\vec{r}) = \left\langle j_f m_f \,|\, \hat{\vec{j}}_{nuc}(\vec{r}) \,|\, j_i m_i \right\rangle = \sum_{\lambda\mu} \frac{(j_i m_i \lambda\mu \,|\, j_f m_f)}{\sqrt{2j_f+1}} [j^{(fi)}_{\lambda\lambda-1}(r)\vec{Y}^{\,*}_{\lambda\lambda-1\mu} + j^{(fi)}_{\lambda\lambda+1}(r)\vec{Y}^{\,*}_{\lambda\lambda+1\mu}]$$

$$\delta\rho_{(fi)}(\vec{r}) = \left\langle j_f m_f \,|\, \hat{\rho}_{nuc}(\vec{r}) \,|\, j_i m_i \right\rangle = \sum_{\lambda\mu} \frac{(j_i m_i \lambda\mu \,|\, j_f m_f)}{\sqrt{2j_f+1}} \rho^{(fi)}_{\lambda}(r) Y^{*}_{\lambda\mu}$$

$$\dot{\rho} + \vec{\nabla}\cdot\vec{j}_{nuc} = 0 \qquad\qquad i\omega\delta\rho_{fi} + \vec{\nabla}\cdot\delta\vec{j}_{fi} = 0$$

$$i\omega\rho_\lambda(r) = \sqrt{\frac{\lambda}{2\lambda+1}}\left(\frac{d}{dr} - \frac{\lambda-1}{\lambda}\right) j_{\lambda\lambda-1}(r) - \sqrt{\frac{\lambda+1}{2\lambda+1}}\left(\frac{d}{dr} + \frac{\lambda+2}{\lambda}\right) j_{\lambda\lambda+1}(r)$$

$$\omega\gamma_\lambda = \omega\int_0^\infty dr\, r^{\lambda+2}\rho_\lambda(r) = \sqrt{\lambda(2\lambda+1)}\int_0^\infty dr\, r^{\lambda+1} j_{\lambda\lambda-1}(r)$$

$$\int_0^\infty dr\, \frac{d}{dr}\left(r^{\lambda-1} j_{\lambda\lambda+1}(r)\right) = \lim_{r\to\infty} r^{\lambda-1} j_{\lambda\lambda+1}(r)$$

So just $j^{(fi)}_{\lambda\lambda+1}(r)$
- is decoupled to CE
- has to be chosen as measure of vorticity

How to cure $\vec{\nabla}\times\delta\vec{j}_{(fi)}$ to make it indeed vortical?
How to decouple $\vec{\nabla}\times\delta\vec{j}_{(fi)}$ from the CE?

$$\vec{\nabla}\times\delta\vec{j}_{(fi)}(\vec{r}) = \sum_{\lambda\mu}\frac{(j_i m_i \lambda\mu \mid j_f m_f)}{\sqrt{2j_f+1}} T^{(fi)}_{\lambda\lambda}(r)\vec{Y}^{*}_{\lambda\lambda\mu}$$

$$T_{\lambda\lambda}(r) = \sqrt{\frac{\lambda+1}{2\lambda+1}}\left(\frac{d}{dr}-\frac{\lambda-1}{\lambda}\right)j_{\lambda\lambda-1}(r) - \sqrt{\frac{\lambda}{2\lambda+1}}\left(\frac{d}{dr}+\frac{\lambda+2}{\lambda}\right)j_{\lambda\lambda+1}(r)$$

$$\int_0^\infty dr\, r^{\lambda+2} T_{\lambda\lambda}(r) = \sqrt{\frac{\lambda+1}{\lambda}}\omega\gamma_\lambda \qquad \omega\gamma_\lambda = \omega\int_0^\infty dr\, r^{\lambda+2}\rho_\lambda(r)$$

$$\delta\varpi_{(fi)}(\vec{r}) = \vec{\nabla}\times\delta\vec{j}_{(fi)}(\vec{r}) - \vec{S}$$

$$\vec{S}_{(fi)}(\vec{r}) = \sum_{\lambda\mu}\frac{(j_i m_i \lambda\mu \mid j_f m_f)}{\sqrt{2j_f+1}}\sqrt{\frac{\lambda+1}{\lambda}}\omega\rho^{(fi)}_{\lambda}(r)\vec{Y}^{*}_{\lambda\lambda\mu}$$

$$\delta\vec{\varpi}_{(fi)}(\vec{r}) = \sum_{\lambda\mu}\frac{(j_i m_i \lambda\mu \mid j_f m_f)}{\sqrt{2j_f+1}}\varpi^{(fi)}_{\lambda\lambda}(r)\vec{Y}^{*}_{\lambda\lambda\mu}$$

Thus we get the vorticity transition density

$$\varpi^{(fi)}_{\lambda\lambda}(r) = T_{\lambda\lambda}(r) - \sqrt{\frac{\lambda+1}{\lambda}}\omega\rho^{(fi)}_{\lambda}(r)$$

$$\int_0^\infty dr\, r^{\lambda+2}\varpi_\lambda(r) = 0$$

Definition 2
(Ravenhall and Wambach)

D.G.Raventhall, J.Wambach,
NPA 475, 468 (1987).

One may construct the vorticity transition density

$$\varpi_{\lambda\lambda}(r) = \sqrt{\frac{2\lambda+1}{\lambda}}\left(\frac{d}{dr}+\frac{\lambda+2}{r}\right) j_{\lambda\lambda+1}(r)$$

and strength

$$\nu_{\lambda}^{(fi)} = \int_0^{\infty} r^{\lambda+4} \quad \varpi_{\lambda\lambda}^{(fi)}(r) \quad dr$$

expressed through the particular transverse current multipole $j_{\lambda\lambda+1}^{(fi)}(r)$ **, which, unlike** $j_{\lambda\lambda-1}^{(fi)}(r)$**, does not contribute to the continuity equation**

$$\dot{\rho} + \vec{\nabla} \cdot \vec{j}_{nuc} = 0$$

So, $j_{\lambda\lambda+1}^{(fi)}(r)$ **is an independent part of charge-current distribution.**

This approach does not use the vortical operator. However, such operator could be useful for the comparison of vortical, toroidal and compression flows.

$$\hat{M}_{tor}(E1\mu) = \frac{1}{10\sqrt{2}c}\int d\vec{r}\,[(r^3 - \frac{5}{3}r < r^2 >_0)]\,\vec{Y}_{11\mu}(\hat{\vec{r}})\cdot[\vec{\nabla}\times\vec{j}_{nuc}(\vec{r})]$$

$$\hat{M}_{tor}(E1\mu) = \frac{1}{20c}\int d\vec{r}\,\hat{\vec{j}}_{nuc}(\vec{r})\cdot[\vec{\nabla}\times(\vec{r}\times\vec{\nabla})\,(r^3 - \frac{5}{3}r < r^2 >_0)Y_{1\mu}]$$

$$\hat{M}'_{com}(E1\mu) = \int d\vec{r}\,\hat{\rho}(\vec{r})\,[r^3 - \frac{5}{3} < r^2 >_0 r]Y_{1\mu}$$

The toroidal and compression operators are related!

$$\vec{\nabla}$$

$\vec{v}_{com}(\vec{r}) \propto \vec{\nabla}(r^3 Y_{1\mu})$ - irrotational motion of CR $\vec{\nabla}\times\vec{v}_{com}(\vec{r}) = 0$

$\vec{v}_{tor}(\vec{r}) \propto \vec{\nabla}\times(\vec{r}\times\vec{\nabla})(r^3 Y_{1\mu})$ - vortical flow $\vec{\nabla}\times\vec{v}_{tor}(\vec{r}) \propto r\vec{Y}_{12\mu}$

How to get the relevant RW vortical operator and relate it to toroidal and compression operators?

Derivation of the vortical operator (Wambach)

Multipole electric operator

$$\delta\vec{j}_{nuc}(\vec{r}) = \rho_0(\vec{r})\delta\vec{v}(\vec{r})$$

$$\hat{M}(Ek\lambda\mu) = \frac{(2\lambda+1)!!}{ck^{\lambda+1}}\sqrt{\frac{\lambda}{\lambda+1}}\int d\vec{r} \quad j_\lambda(kr)\vec{Y}_{\lambda\lambda\mu}\cdot[\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})]$$

Main idea:

$$\rho_0(\vec{r})\vec{\nabla}\times\delta\vec{v}(\vec{r}) = \vec{\nabla}\times\delta\vec{j}_{nuc}(\vec{r}) - \vec{\nabla}\rho_0(\vec{r})\times\delta\vec{v}_{nuc}(\vec{r})$$

$$\vec{\nabla}\times\hat{\vec{\boldsymbol{j}}}_{\boldsymbol{nuc}}(\vec{\boldsymbol{r}}) \quad \Longrightarrow \quad \underbrace{\rho_0(\vec{r})\vec{\nabla}\times\vec{v}(\vec{r})}_{\text{truly vortical}} \to \vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r}) - i\frac{kc}{\lambda}\vec{\nabla}\rho_0(\vec{r})\times\hat{\vec{r}}$$

Then

$$\hat{M}_{vor}(Ek\lambda\mu) = \frac{(2\lambda+1)!!}{ck^{\lambda+1}}\sqrt{\frac{\lambda}{\lambda+1}}\int d\vec{r} \quad [j_\lambda(kr) \quad \vec{Y}_{\lambda\lambda\mu}(\vartheta,\varphi)]\cdot$$

$$\boxed{\begin{array}{l} \hat{\vec{v}} = \hat{\dot{\vec{r}}} = \dfrac{1}{i\hbar}[\vec{r},\hat{H}] = ikc\hat{\vec{r}}, \\ \hbar\omega_{if} = E_i - E_f = \hbar kc \end{array}}$$

$$\cdot\left[\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r}) \;-\; \frac{i}{\lambda}kc\vec{\nabla}\hat{\rho}(\vec{r})\times\hat{\vec{r}}\right] =$$

$$= \hat{M}(Ek\lambda\mu) - \hat{M}_S(k\lambda\mu)$$

Long-wave approximation:

$$j_{\lambda}(kr) = \frac{(kr)^{\lambda}}{(2\lambda+1)!!}[1 - \frac{(kr)^2}{2(2\lambda+3)} + \ldots]$$

⇓

The second order term gives:

- **toroidal operator**
- **compression operator**
- **vortical operator**

$$\hat{M}(Ek\lambda\mu) = \hat{M}(E\lambda\mu) + k\hat{M}_{tor}(E\lambda\mu) \qquad \hat{M}(E\lambda\mu) = \int d\vec{r}\,\rho(\vec{r})r^{\lambda}Y_{\lambda\mu}$$

$$\hat{M}_{tor}(E\lambda\mu) = -\frac{i}{2c}\sqrt{\frac{\lambda}{2\lambda+1}}\int d\vec{r}\;\;\hat{\vec{j}}_{nuc}(\vec{r})\cdot r^{\lambda+1}(\vec{Y}_{\lambda\lambda-1\mu} + \sqrt{\frac{\lambda}{\lambda+1}}\frac{2}{2\lambda+3}\vec{Y}_{\lambda\lambda+1\mu})$$

$$\hat{M}_{S}(Ek\lambda\mu) = \hat{M}(E\lambda\mu) - k\hat{M}_{com}(E\lambda\mu)$$

$$\hat{M}_{com}(E\lambda\mu) = \frac{i}{2c}\sqrt{\frac{\lambda}{2\lambda+1}}\int d\vec{r}\;\;\hat{\vec{j}}_{nuc}(\vec{r})\cdot r^{\lambda+1}(\vec{Y}_{\lambda\lambda-1\mu} - \sqrt{\frac{\lambda+1}{\lambda}}\frac{2}{2\lambda+3}\vec{Y}_{\lambda\lambda+1\mu})$$

$$\hat{M}_{com}(E\lambda\mu) = -k\hat{M}'_{com}(E\lambda\mu) \qquad \hat{M}'_{com}(E\lambda\mu) = \frac{1}{2(2\lambda+3)}\int d\vec{r}\,\hat{\rho}(\vec{r})\,r^{\lambda+2}Y_{\lambda\mu}$$

$$\hat{M}_{vor}(Ek\lambda\mu) = \hat{M}(Ek\lambda\mu) - \hat{M}_{S}(Ek\lambda\mu) = k\left[\hat{M}_{tor}(E\lambda\mu) + \hat{M}_{com}(E\lambda\mu)\right]$$

$$\hat{M}_{vor}(E\lambda\mu) = -\frac{i}{c(2\lambda+3)}\sqrt{\frac{\lambda+1}{2\lambda+1}}\int d\vec{r}\;\;\hat{\vec{j}}_{nuc}(\vec{r})\,r^{\lambda+1}\vec{Y}_{\lambda\lambda+1\mu}$$

$$\boxed{\hat{M}_{vor}(E\lambda\mu) = \hat{M}_{tor}(E\lambda\mu) + \hat{M}_{com}(E\lambda\mu)}$$

E1(T=0)

$$\langle \nu | \hat{M}^{j+}_{vor}(E1\mu) | 0 \rangle = -\frac{1}{5\sqrt{2}c}\int dr^3 \;\; r^2 \vec{Y}_{12\mu} \cdot \delta\vec{j}_{nuc,+}(\vec{r})$$

$$\langle \nu | \hat{M}_{com}(E1\mu) | 0 \rangle = -\frac{1}{10\sqrt{2}c}\int dr^3 \;\; [r^3 - \frac{5}{3} r \langle r^2 \rangle_0] Y_{1\mu} \, [\vec{\nabla} \cdot \delta\hat{\vec{j}}_{nuc}(\vec{r})]$$

$$\langle \nu | \hat{M}_{tor}(E1\mu) | 0 \rangle = -\frac{1}{10\sqrt{2}c}\int dr^3 \;\; [r^3 - \frac{5}{3} r \langle r^2 \rangle_0] \vec{Y}_{11\mu} \cdot [\vec{\nabla} \times \delta\hat{\vec{j}}_{nuc}(\vec{r})]$$

$$\langle \nu | \hat{M}^{HD}_{vor}(E1\mu) | 0 \rangle = -\frac{1}{10\sqrt{2}c}\int dr^3 \;\; [r^3 - \frac{5}{3} r \langle r^2 \rangle_0] \vec{Y}_{11\mu} \cdot \rho_0 [\vec{\nabla} \times \delta\vec{v}(\vec{r})]$$

$$\langle \nu / \hat{M}^{j+}_{vor}(E1\mu) / 0 \rangle = -\frac{1}{5\sqrt{2}c}\int dr\, r^4 \;\; j^{\nu}_{+}(r)$$

$$\langle \nu / \hat{M}_{com}(E1\mu) / 0 \rangle = -\frac{1}{6c}\int dr\, r^2 \; [\frac{2\sqrt{2}}{5} r^2 j^{\nu}_{+}(r) - (r^2 - \langle r^2 \rangle_0) j^{\nu}_{-}(r)]$$

$$\langle \nu / \hat{M}_{tor}(E1\mu) / 0 \rangle = -\frac{1}{6c}\int dr\, r^2 \; [\frac{\sqrt{2}}{5} r^2 j^{\nu}_{+}(r) + (r^2 - \langle r^2 \rangle_0) j^{\nu}_{-}(r)]$$

Wambach vorticity

- **Presence of** $j^{(fi)}_{\lambda\lambda+1}(r)$ **is decisive to make the flow vortical**

$$\hat{M}'_{com}(E1\mu) = \frac{1}{10}\int d\vec{r}\,\hat{\rho}(\vec{r})[r^3 - \frac{5}{3}r < r^2 >_0]Y_{1\mu}$$

$$\hat{M}_{com}(E1\mu) = -\frac{i}{2c\sqrt{3}}\int d\vec{r}\quad \hat{\vec{j}}_{nuc}(\vec{r})\ [r^2\frac{2\sqrt{2}}{5}\vec{Y}_{12\mu} + (r^2 - < r^2 >_0)\vec{Y}_{10\mu}]$$

CM involves $\vec{Y}_{12\mu}$ and so $\vec{j}_{12\mu}$. Hence CM is vortical despite its gradient flow !?

The reason of contradiction:
The Wambach vorticity $w_{\lambda\lambda}(r) \propto j_{\lambda\lambda+1}(r)$ was introduced mainly as a quantity fully unconstrained by the CE rather than the purely vortical value in the HD sense.

- Thus an essential difference between HD and Wambach vorticity.

The model for numerical calculations:

- self-consistent Skyrme RPA ,
- SLy6 force

^{208}Pb

Strength function

$$S(E1;\omega) = \sum_{\nu\neq 0} |< \Psi_{\nu} \,|\, \hat{M}_{E1} \,|\, 0 >|^{2} \, \varsigma(\omega-\omega_{\nu})$$

Toroidal, compressional, vortical operators

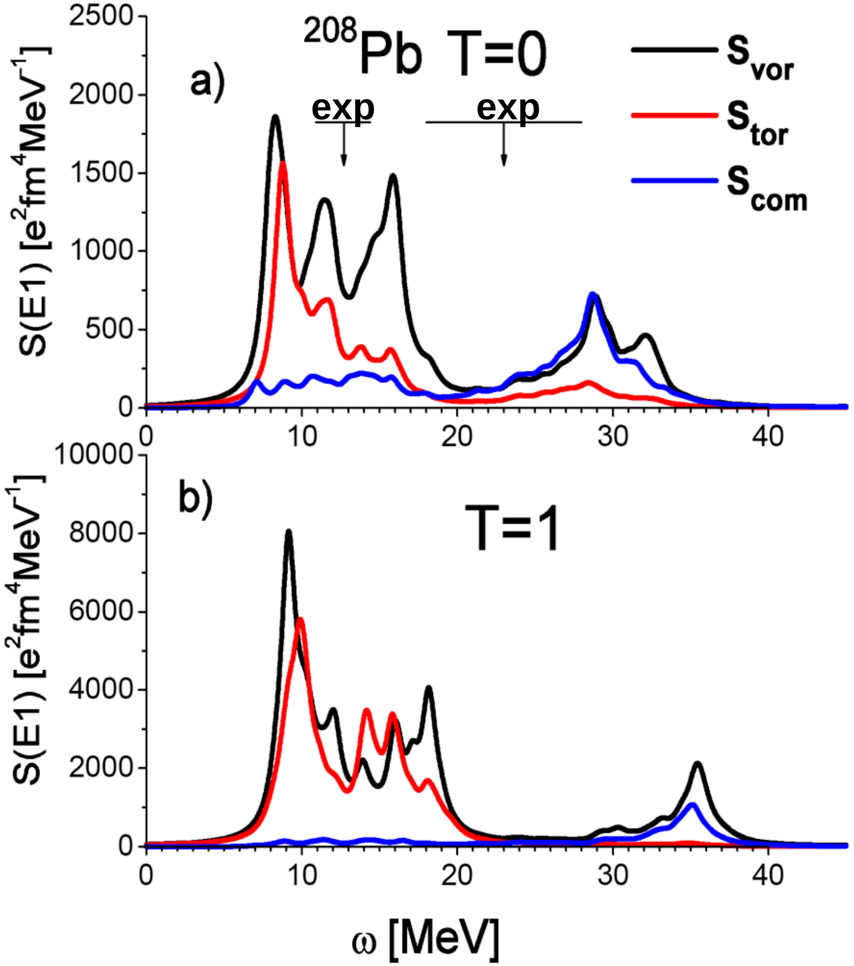
with the Lorentz weight

$$\varsigma(\omega-\omega_{\nu}) = \frac{1}{2\pi}\frac{\Delta}{[(\omega-\omega_{\nu})^{2}+\frac{\Delta^{2}}{4}]}$$

$$\Delta = 1 \ MeV$$

Comparison of VM, TM, and CM

J. Kvasil, VON, W. Kleinig, P.-G. Reinhard,
P. Vesely, PRC, 84, 034303 (2011)



- **Broad low-energy (LE) and high-energy (HE) bumps for VM, TM, and CM.**
- **LE strength is dominated by VM and TM**
- **HE strength is dominated by VM and CM**

- **General agreement for TM and CM with previous studies.**
- **Poor agreement with exper. of Ichida (like in previous studies).**

Uchida et al., 2003:

$E_1 = 12.7\,\text{MeV}, \quad \Gamma_1 = 3.5\,\text{MeV}$

$E_2 = 23.0\,\text{MeV}, \quad \Gamma_2 = 10.3\,\text{MeV}$

- Purely vortical VM does not coincide with partly vortical TM, especially at HE.

TM was previously considered as a typical example of the vortical flow.

- **Convection and magnetization (spin) parts of nuclear current,**
- **T=0 and T=1 channels**

$$\hat{\vec{j}}_{nuc}(\vec{r}) = \hat{\vec{j}}_{con}(\vec{r}) \ + \ \hat{\vec{j}}_{mag}(\vec{r}) = \frac{e\hbar}{m}\sum_{q=n,p}(\hat{\vec{j}}^{q}_{com}(\vec{r}) + \hat{\vec{j}}^{q}_{mag}(\vec{r}))$$

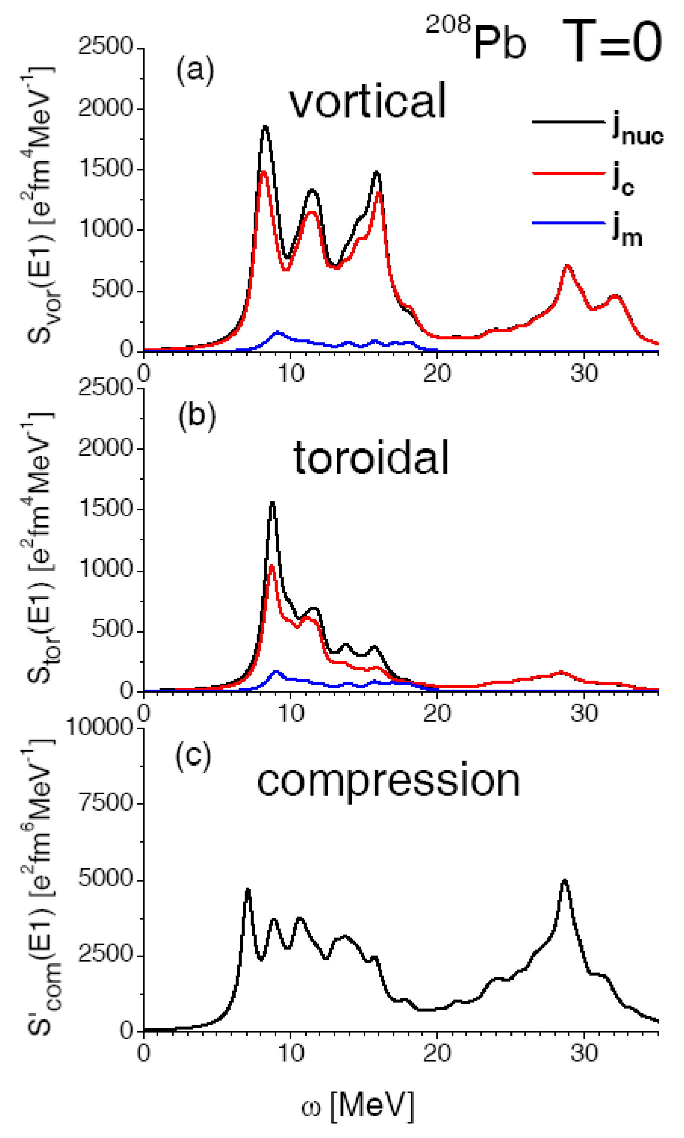
$$\hat{\vec{j}}^{q}_{con}(\vec{r}) = -ie^{q}_{eff}\sum_{k \ni q}\left(\delta(\vec{r}-\vec{r}_k)\right)\vec{\nabla}_k - \overleftarrow{\nabla}_k\delta(\vec{r}-\vec{r}_k))$$

$$\hat{\vec{j}}^{q}_{mag}(\vec{r}) = \frac{g_s}{2}\sum_{k \ni q}\vec{\nabla}_k \times \hat{\vec{s}}_{qk}\delta(\vec{r}-\vec{r}_k)$$

Vortical, toroidal, and compressional T=0 strength

SLy6

$\Delta = 1\text{MeV}$



- dominant contribution of j_{con} to VM and TM

-no j_{mag} contribution to:
- CM
- HE strength

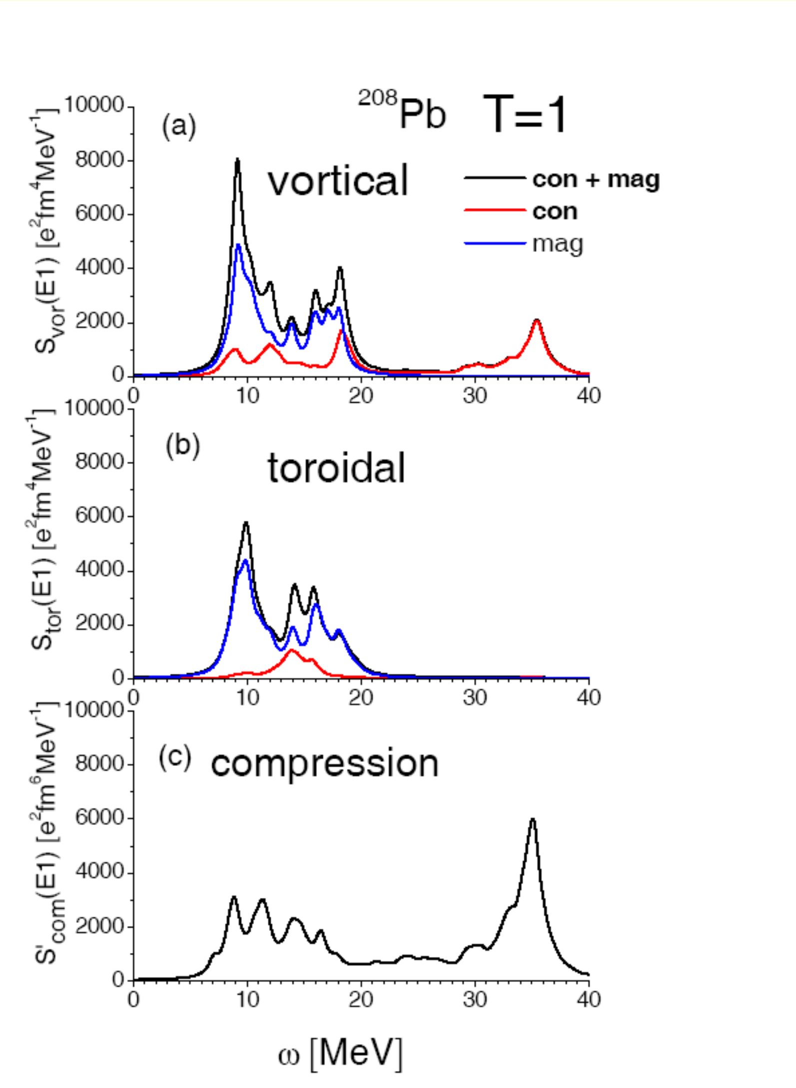
$$g_s^p = 5.58\varsigma, \quad g_s^n = -3.82\varsigma$$

$$g_s^{T=0} = \frac{1}{2}(g_s^p + g_s^n) = 0.88\varsigma$$

Small T=0 g-factors!

J. Kvasil, VON, W. Kleinig, P.-G. Reinhard, P. Vesely, PRC, 84, 034303 (2011)

Vortical, toroidal, and compressional **T=1** strength



SLy6

$$\Delta = 1\text{MeV}$$

VM and TM:

- dominant contribution of j_{mag} !!

$$g_s^p = 5.58\varsigma, \quad g_s^n = -3.82\varsigma,$$

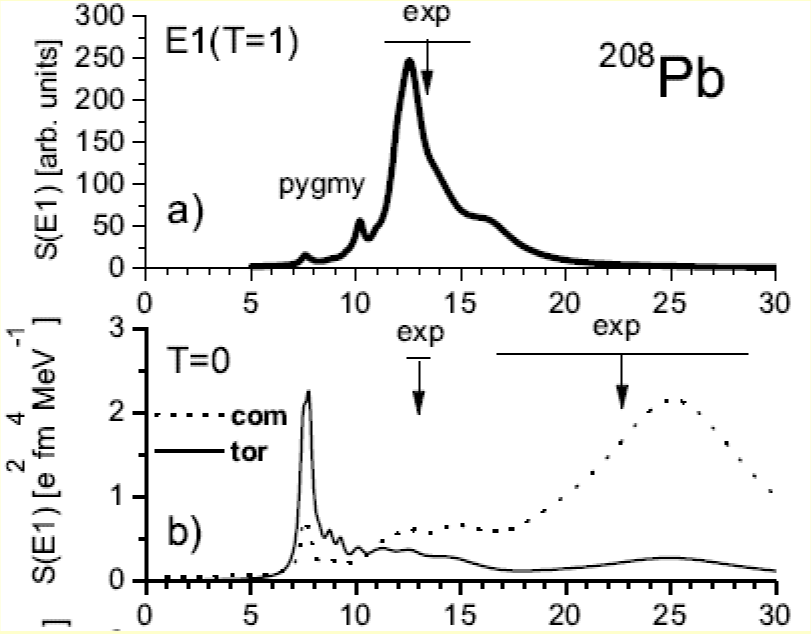
$$g_s^{T=1} = \frac{1}{2}(g_s^p - g_s^n) = 4.7\varsigma$$

Large T=1 g-factors!

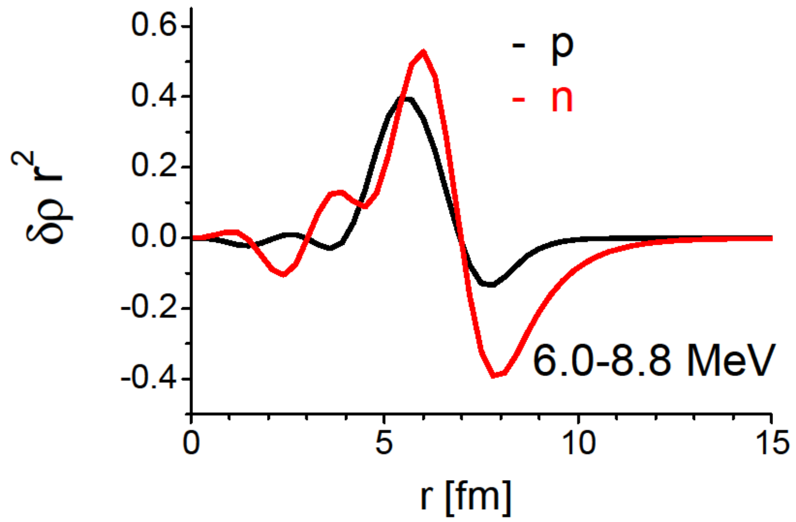
Vortical and toroidal modes in the T=1 channel are suitable to see the effect of j_{mag} in electric modes.

J. Kvasil, VON, W. Kleinig, P.-G. Reinhard, P. Vesely, PRC, 84, 034303 (2011)

A. Repko, P.G. Reinhard, VON, J. Kvasil,
to be submitted

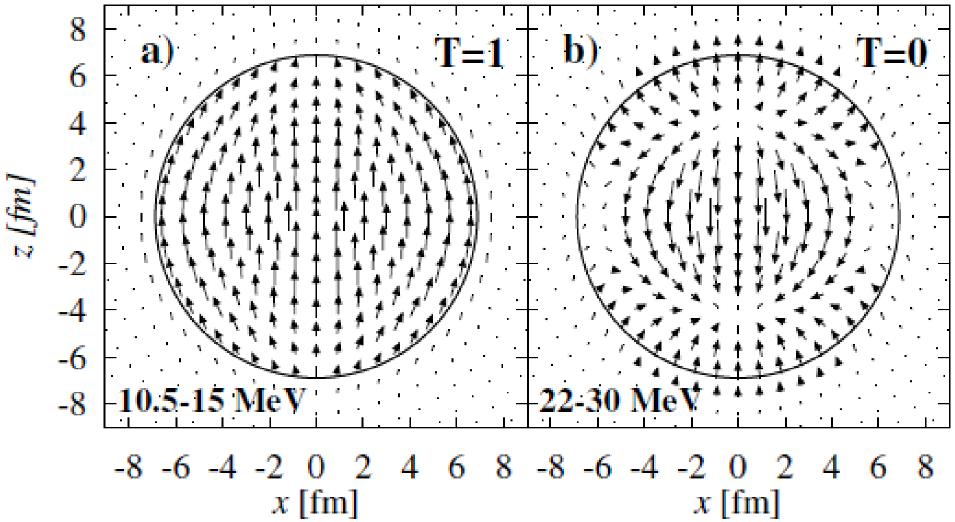


MeV

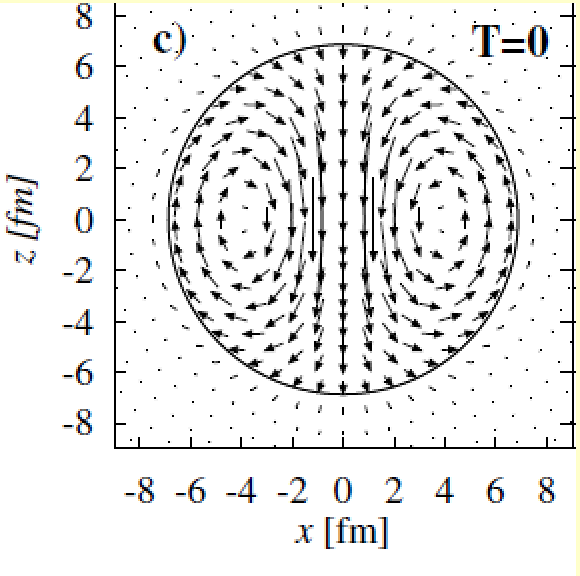


GDR

E1 compression



6-9 MeV



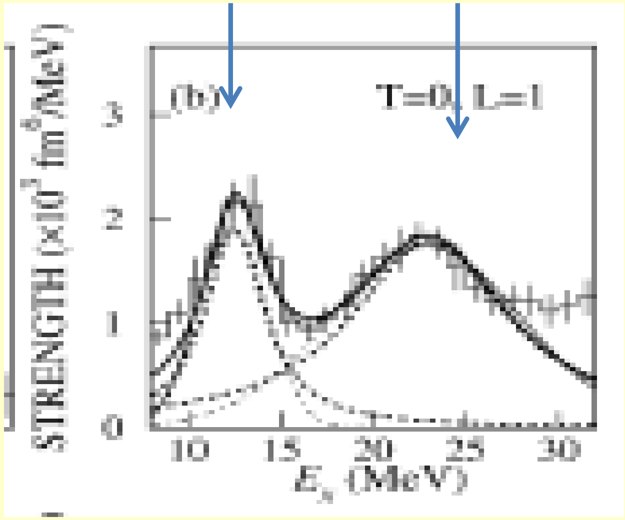
Is it possible to observe the vortical GR in experiment?

(α,α')

D.Y. Youngblood et al, 1977
H.P. Morsch et al, 1980
G.S. Adams et al, 1986
B.A. Devis et al, 1997
H.L. Clark et al, 2001
D.Y. Youngblood et al, 2004

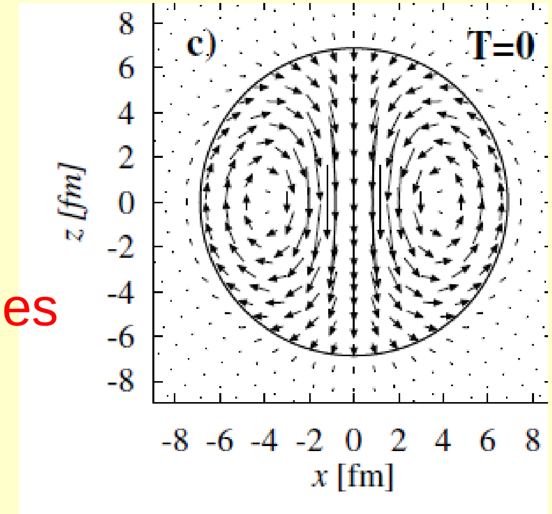
M.Uchida et al, PLB 557, 12 (2003),
PRC 69, 051301(R) (2004)

LE (toroidal)
HE (compression)



$$B(E1) \propto \int dr\; \underset{\downarrow}{r^2} \cdot \underset{\downarrow}{r^3} Y_{10} \cdot \delta\rho(r)$$

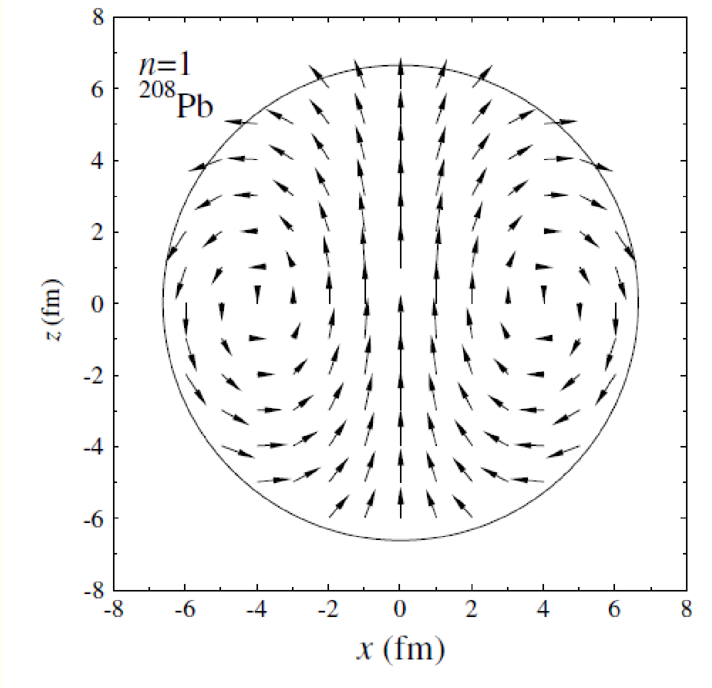
- radial factors make reaction very peripheral
- flow in the interior and thus the vorticity becomes invisible



Basics of E1 (T=0) toroidal and compression modes

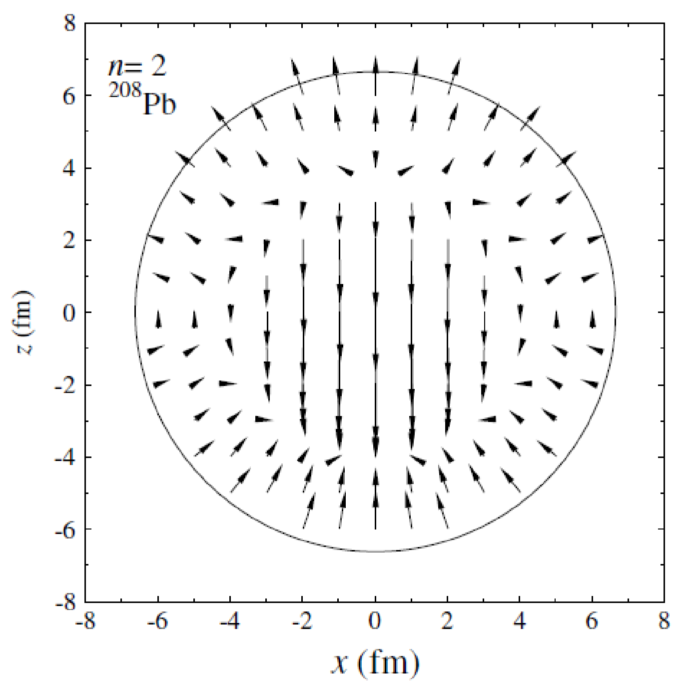
V.M. Dubovik (1975)
S.F. Semenko (1981)

TM



M.N. Harakeh (1977)
S. Stringari (1982)

CM



dependence on nuclear incompress.:	no	yes
energy:	$E = 68\,A^{-1/3} MeV$	$E = 132\,A^{-1/3} MeV$

S. Misiku, PRC, 73, 024301 (2006)

vorticity: - yes, vortical mode   - no, irrotational mode

Is it possible to observe the vortical electric GR in experiment?

$(\alpha,\alpha'\gamma')$

(γ,γ')

N.Ryezayeva et al, PRL 89, 272502 (2002).

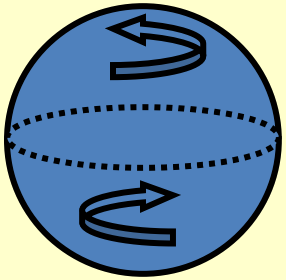
(e,e’)   - most promising!

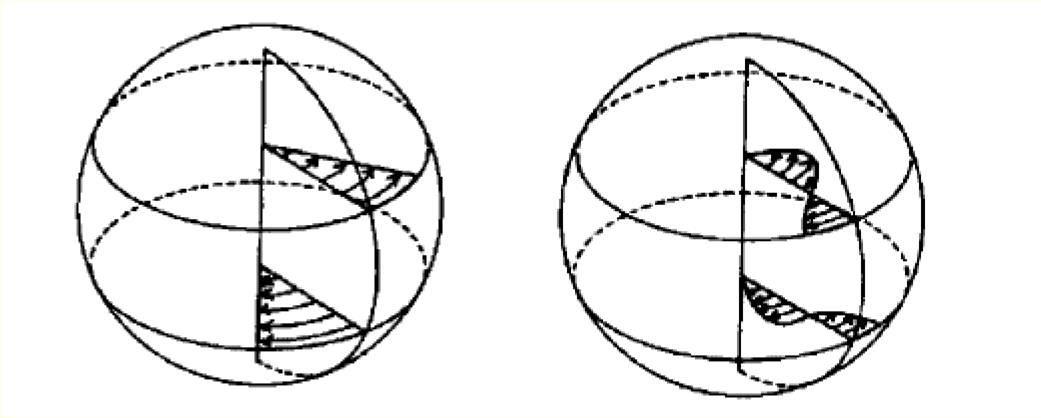
M2 twist GR (IV,IS)

$\vec{v} = (yz, -xz, 0)$ - velocity field

$$F(M2\mu) = \mu_b \sqrt{10} r [g_s \{Y_1 \hat{s}\}_{2\mu} + \frac{2}{3} g_l \{Y_1 \hat{l}\}_{2\mu}]$$

$z l_z \propto r (Y_{10} l_z)$ - external field to generate twist





- predicted: G. Holzwarth and G. Eckard, NPA, v.325, 1 (1979).
- observed: P. von Neumann-Cosel et al., PRL v.82, 1105 (1999).: **(e,e'), back scatt.**
- characterized by strong M2 transitions to the ground state
- orbital magnetic flow
- manifestation of nuclear elasticity
- may exist in other systems: atomic clusters, trapped fermi-gas, …

Conclusions

★ The vorticity is a fundamental property of nuclear motion.
The toroidal E1 resonance:
- one of the dominant modes in E1(T=0) channel,
- is one of the most interesting manifestations of the vorticity,
- very hot topic nowadays (correlation to CM, pygmy)

★ Still there are open problems:
- various definitions of vorticity,
- challenge to experiment to observe unambiguously a vortical flow in GR

Thank you for attention!