# DOMAIN WALL NETWORK AS QCD VACUUM: CONFINEMENT, CHIRAL SYMMETRY, HADRONIZATION

**Sergei Nedelko**

Bogoliubov Laboratory of Theoretical Physics

Vladimir Voronin

Bogoliubov Laboratory of Theoretical Physics & Dubna Uni.

**An overall task pursued by most of the approaches to QCD vacuum structure is an identification of the properties of nonperturbative gauge field configurations able to provide a coherent resolution of the confinement, the chiral symmetry breaking, the $U_{\rm A}(1)$ anomaly and the strong CP problems, both in terms of color-charged fields and colorless hadrons.**

**Finite classical action, pure gauge singularities** $\Leftrightarrow$ **Global minima of the effective quantum action**

**Instantons, monopoles, vortices, double-layer domain walls** $\Leftrightarrow$ **Defects in the initally homogeneous background**

- **Confinement of both static and dynamical quarks** $\longrightarrow W(C) = \langle \mathrm{Tr}\ \mathrm{P}\ e^{i\int_C dz_\mu \hat{A}_\mu}\rangle$
  $S(x,y) = \langle \psi(y)\bar{\psi}(x)\rangle$

- **Dynamical Breaking of chiral** $SU_L(N_f)\times SU_R(N_f)$ **symmetry** $\longrightarrow \langle\bar{\psi}(x)\psi(x)\rangle$

- $U_A(1)$ **Problem** $\longrightarrow \eta'$ ($\chi$, Axial Anomaly)

- **Strong CP Problem** $\longrightarrow Z(\theta)$

- **Colorless Hadron Formation:** $\longrightarrow$ Effective action for colorless collective modes:
  hadron masses, formfactors, scattering

  **Light** mesons and baryons, **Regge spectrum** of excited states of light hadrons, **heavy-light** hadrons, **heavy quarkonia**

**What would be a formalism for coherent simultaneous description of all these nonperturbative features of QCD?**

**QCD vacuum as a medium characterized by certain condensates,**
**quarks and gluons - elementary coloured excitations (confined),**
**mesons and baryons - collective colourless excitations (masses, form factors, etc)**

- Effective action of $SU(3)$ YM theory, global minima of the effective action.
- Gluon condensation, Weyl reflections, CP: kink-like gauge field configurations and domain wall network as QCD vacuum
- Confinement and the spectrum of charged field fluctuations, color charged quasi-particles
- Impact of the strong electromagnetic fields on the QCD vacuum structure
- Formulation of the domain model.
- Testing the model on the strandard set of problems of pure gluodynamics: $\sigma$, $\chi$, $\langle F^2\rangle$.
- Chirality of quark modes. Realisation of chiral symmetry and quark condensate: $U_A(1)$ and $SU_{\mathrm{L}}(N)\times SU_{\mathrm{R}}(N)$.
- $U_A(1)$ and the strong CP problem. Anomalous Ward Identities.
- Nonlocal effective meson Lagrangian: hadronization scheme and meson masses.

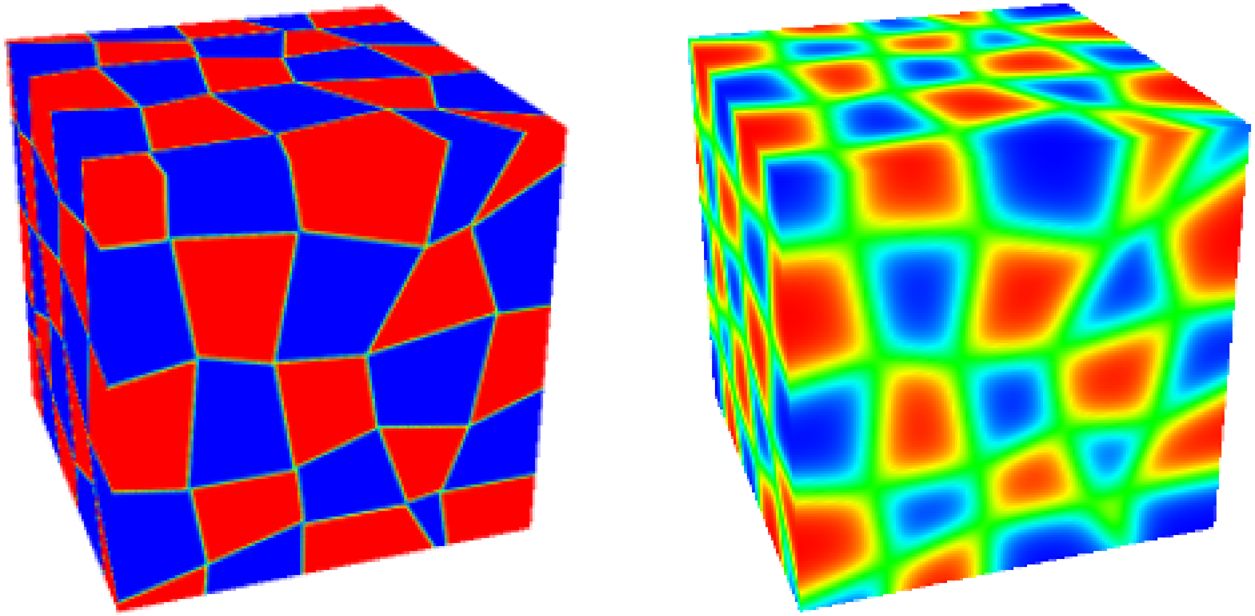
*P. Minkowski,* Nucl. Phys. B**177**, 203 (1981).
*H. Leutwyler,* Phys. Lett. B**96**, 154 (1980); Nucl. Phys. B**179**, 129 (1981).
*H. Pagels, and E. Tomboulis* // Nucl. Phys. B143. 1978.
*H. D. Trottier and R. M. Woloshyn*// Phys. Rev. Lett. 70. 1993.

*G.V. Efimov, Ja.V. Burdanov, B.V. Galilo, A.C. Kalloniatis, S.N. Nedelko , L. von Smekal, S.A. Solunin,* Phys. Rev. D 73 (2006), 71 (2005), 70 (2004), 69 (2004), 66 (2002), 51 (1995), 54 (1996).

*S.N. Nedelko, V.E. Voronin, in preparation* (2013)
*B. Galilo, S.N. Nedelko,* Phys. Rev D 84 (2011)
*B. Galilo, S.N. Nedelko,* PEPAN Lett. 8 (2011) [arXiv:1006.0248v2 ],
*J. M. Pawlowski, D. F. Litim, S. Nedelko and L. von Smekal*, Phys. Rev. Lett. **93** (2004)
*A. Eichhorn, H. Gies, J. M. Pawlowski,* Phys. Rev. D83 (2011) [arXiv:1010.2153 [hep-ph]]

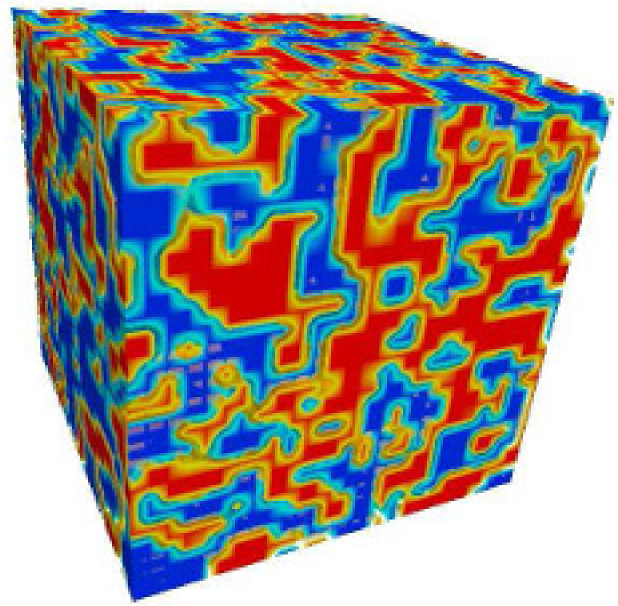
## Pure Yang-Mills vacuum (no quarks present):

$$\langle : g^2 F^2 : \rangle \neq 0, \quad \chi = \int d^4x \langle Q(x) Q(0) \rangle \neq 0, \quad \langle Q(x) \rangle = 0$$



**Topological charge density** $Q(x) = \frac{g^2}{32\pi^2} F^a_{\mu\nu}(x) \tilde{F}^a_{\mu\nu}(x)$

*Visualizations of Quantum Chromodynamics (QCD)Buried treasure in the sand of the QCD vacuum*
P.J. Moran,Derek B. Leinweber, Centre for the Subatomic Structure of Matter (CSSM), University of Adelaide, Australia arXiv:0805.4246v1 [hep-lat], 2008



**Topological charge density** $Q(x) = \frac{g^2}{32\pi^2} F^a_{\mu\nu}(x) \tilde{F}^a_{\mu\nu}(x)$

In Euclidean functional integral for YM theory **one has to allow the gluon condensate to be nonzero**:

$$Z = N \int\limits_{\mathcal{F}_B} DA \exp\{-S[A]\}$$

$$\mathcal{F}_B = \{A : \lim_{V\to\infty} \frac{1}{V} \int\limits_V d^4x g^2 F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) = B^2\}, \quad B^2 \neq 0.$$

**Separation of the long range modes $B^a_\mu$ and local fluctuations $Q^a_\mu$ in the background $B^a_\mu$**, background gauge fixing condition ($D(B)Q = 0$): $A^a_\mu = B^a_\mu + Q^a_\mu$

$$1 = \int\limits_{\mathcal{B}} DB\Phi[A,B] \int\limits_{\mathcal{Q}} DQ \int\limits_{\Omega} D\omega \delta[A^\omega - Q^\omega - B^\omega]\delta[D(B^\omega)Q^\omega]$$

$Q^a_\mu$ – local (perturbative) fluctuations of gluon field **with zero gluon condensate:** $Q \in \mathcal{Q}$;
$B^a_\mu$ are **long range field configurations with nonzero condensate:** $B \in \mathcal{B}$.

$$Z \;=\; N' \int\limits_{\mathcal{B}} DB \int\limits_{\mathcal{Q}} DQ \det[D(B)D(B+Q)]\delta[D(B)Q] \exp\{-S[B+Q]\}$$

**Self-consistency:** the character of long range fields has yet to be identified by the dynamics of fluctuations:

$$Z \;=\; N' \int\limits_{\mathcal{B}} DB \exp\{-S_{\rm eff}[B]\}.$$

Global minima of $S_{\rm eff}[B]$ – field configurations that are dominant in the thermodynamic limit $V \to \infty$.

L. D. Faddeev,“Mass in Quantum Yang-Mills Theory”, arcxiv:0911.1013v1[math-ph]

**The Abelian $\hat{B}_\mu(x)$ part of the gauge fields**

$$\hat{A}_\mu(x) = \hat{B}_\mu(x) + \hat{X}_\mu(x), \quad [\hat{B}_\mu(x), \hat{B}_\nu(x)] = 0$$

*L. D. Faddeev, A. J. Niemi* (2007); *Kei-Ichi Kondo, Toru Shinohara, Takeharu Murakami*( 2008); *Y.M. Cho* (1980, 1981); *S.V. Shabanov* (1989,1999)

**Covariantly constant Abelian (anti-)self-dual fields**

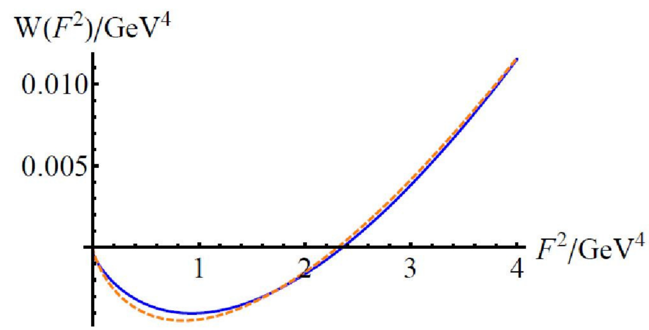
$$B^a_\mu = -\frac{1}{2} n^a B_{\mu\nu} x_\nu, \quad \tilde{B}_{\mu\nu} = \pm B_{\mu\nu}$$

are stable against local fluctuations $Q$. Explicit **one-loop effective action:**

$$S_{\rm eff}^{1-{\rm loop}} = B^2 \left[ \frac{11}{24\pi^2} \ln \frac{\lambda B}{\Lambda^2} + \varepsilon_0 \right]. \qquad (1)$$

*H. Leutwyler* (1980,1981); *P. Minkowski* (1981); *H. Pagels, and E. Tomboulis* (1978); *H. D. Trottier and R. M. Woloshyn* (1993).

**Effective action for covariantly constant Abelian (anti-)self-dual field within the Functional RG:**




A. Eichhorn, H. Gies, J. M. Pawlowski, Phys. Rev. D83 (2011) [arXiv:1010.2153 [hep-ph]]

## Effective Lagrangian

Consider the following Ginsburg-Landau effective Lagrangian for the soft gauge fields satisfying the requirements of invariance under the gauge group $SU(3)$ and space-time transformations,

$$L_{\rm eff} = -\frac{1}{4}\left(D^{ab}_{\nu}F^{b}_{\rho\mu}D^{ac}_{\nu}F^{c}_{\rho\mu} + D^{ab}_{\mu}F^{b}_{\mu\nu}D^{ac}_{\rho}F^{c}_{\rho\nu}\right) - U_{\rm eff}$$

$$U_{\rm eff} = \frac{1}{12}{\rm Tr}\left(C_1\hat{F}^2 + \frac{4}{3}C_2\hat{F}^4 - \frac{16}{9}C_3\hat{F}^6\right),$$

where

$$D^{ab}_{\mu} = \delta^{ab}\partial_{\mu} - i\hat{A}^{ab}_{\mu} = \partial_{\mu} - iA^{c}_{\mu}(T^{c})^{ab}, \quad F^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} - if^{abc}A^{b}_{\mu}A^{c}_{\nu},$$

$$\hat{F}_{\mu\nu} = F^{a}_{\mu\nu}T^{a}, \quad T^{a}_{bc} = -if^{abc} \quad {\rm Tr}\left(\hat{F}^2\right) = \hat{F}^{ab}_{\mu\nu}\hat{F}^{ba}_{\nu\mu} = -3F^{a}_{\mu\nu}F^{a}_{\mu\nu} \leq 0,$$

$$gA_{\mu} \to A_{\mu}.$$

$$\lim_{V\to\infty} V^{-1}\int\limits_{V} d^4x\langle F^2\rangle \neq 0 \longrightarrow C_1 > 0,\ C_2 > 0,\ C_3 > 0.$$

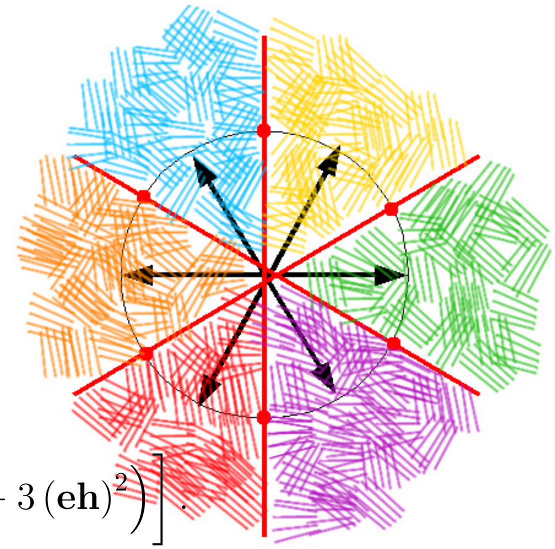
$$F^{a}_{\mu\nu}F^{a}_{\mu\nu} = 4b^2_{\rm vac}\Lambda^4 > 0,\ \ b^2_{\rm vac} = \left(\sqrt{C_2^2 + 3C_1C_3} - C_2\right)/3C_3.$$

Consider $A_\mu$ fields with the **Abelian field strength**

$$\hat{F}_{\mu\nu} = \hat{n}B_{\mu\nu},$$

where matrix $\hat{n}$ can be put into Cartan subalgebra

$$\hat{n} = T^3 \cos\xi + T^8 \sin\xi,\ 0 \le \xi < 2\pi.$$



It is convenient to introduce the following notation:

$$\hat{b}_{\mu\nu} = \hat{n}B_{\mu\nu}/\Lambda^2 = \hat{n}b_{\mu\nu},\ b_{\mu\nu}b_{\mu\nu} = 4b^2_{\rm vac},$$

$$e_i = b_{4i},\ h_i = \frac{1}{2}\varepsilon_{ijk}b_{jk},\ \mathbf{e}^2 + \mathbf{h}^2 = 2b^2_{\rm vac}.$$

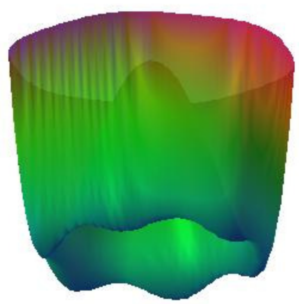
$$(\mathbf{eh}) = |\mathbf{e}|\,|\mathbf{h}|\cos\omega,\ \ (\mathbf{eh})^2 = \mathbf{h}^2\left(2b^2 - \mathbf{h}^2\right)\cos^2\omega.$$

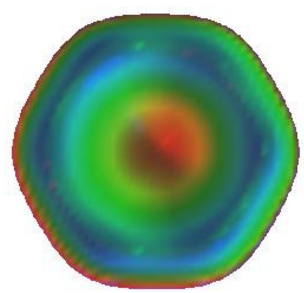
Hence the effective potential takes the form

$$U_{\rm eff} = \Lambda^4\left[-C_1 b^2_{\rm vac} + C_2\left(2b^4_{\rm vac} - (\mathbf{eh})^2\right) + \frac{1}{9}C_3 b^2\left(10 + \cos 6\xi\right)\left(4b^4_{\rm vac} - 3\,(\mathbf{eh})^2\right)\right].$$

There are twelve discrete global degenerated minima at the following values of the variables $h$, $\omega$ and $\xi$

$$\mathbf{h}^2 = b^2_{\rm vac} > 0,\ \ \omega = \pi k\ \ (k = 0, 1),\ \ \xi = \frac{\pi}{6}(2n+1)\,(n = 0, \ldots, 5).$$





# Kink-like configurations

Discrete minima mean that there exist kink-like field configurations interpolating between these minima. For instance, for the angle $\omega$

$$L_{\text{eff}} = -\frac{1}{2}\Lambda^2 b_{\text{vac}}^2 \partial_\mu\omega\partial_\mu\omega - b_{\text{vac}}^4\Lambda^4\left(C_2 + 3C_3 b_{\text{vac}}^2\right)\sin^2\omega,$$

with the sine-Gordon equation of motion

$$\partial^2\omega = m_\omega^2 \sin 2\omega, \quad m_\omega^2 = b_{\text{vac}}^2\Lambda^2\left(C_2 + 3C_3 b_{\text{vac}}^2\right),$$

with kink solution

$$\omega = 2\arctan\left(\exp\left(\sqrt{2}m_\omega x_1\right)\right),$$

which can be treated as domain wall.

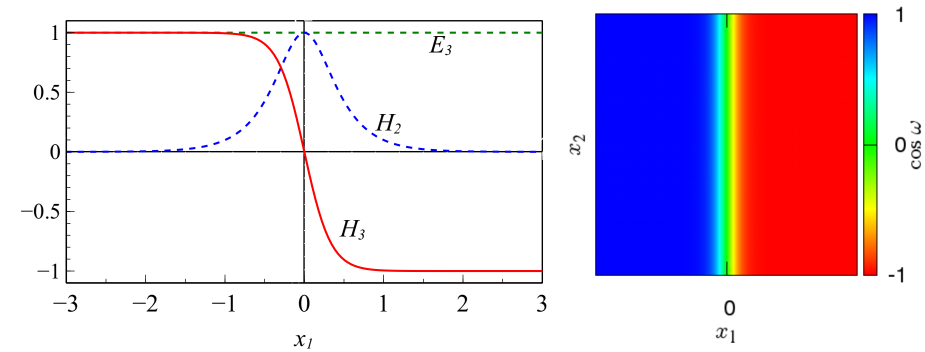



Figure 1: Kink profile in terms of the components of the chromomagnetic and chromoelectric field strength (left), and a two-dimensional slice for the topological charge density in the presence of a single kink measured in units of $g^2 F^b_{\alpha\beta}F^b_{\alpha\beta}$ (right). Chromomagnetic and chromoelectric fields are orthogonal to each other inside the wall (green color).

# Domain wall network

Denote the general kink configuration:

$$\zeta(\mu_i, \eta^i_\nu x_\nu - q^i) = \frac{2}{\pi} \arctan \exp(\mu_i(\eta^i_\nu x_\nu - q^i))$$

$\mu_i$ – inverse width of the kink, $\eta^i_\nu$ – a normal vector to the plane of the wall, $q^i = \eta^i_\nu x^i_\nu$ with $x^i_\nu$ - coordinates of the wall.

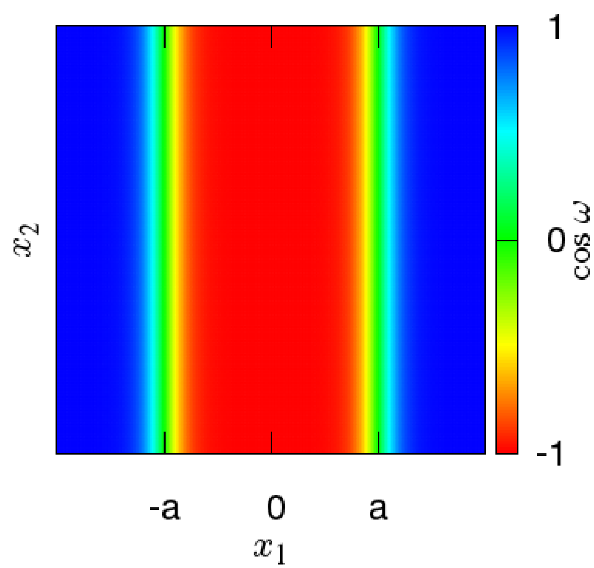



Figure 2: Two-dimentional slice of a multiplicative superposition of two kinks with normal vectors anti-parallel to each other $\omega(x_1) = \pi\zeta(\mu_1, x_1 - a_1)\zeta(\mu_2, -x_1 - a_2)$.

Additive superposition of infinitely many such pairs

$$\omega(x_1) = \pi \sum_{j=1}^{\infty} \zeta(\mu_j, x_1 - a_j)\zeta(\mu_{j+1}, -x_1 - a_{j+1})$$

gives a layered topological charge structure in $R^4$, Fig.3.

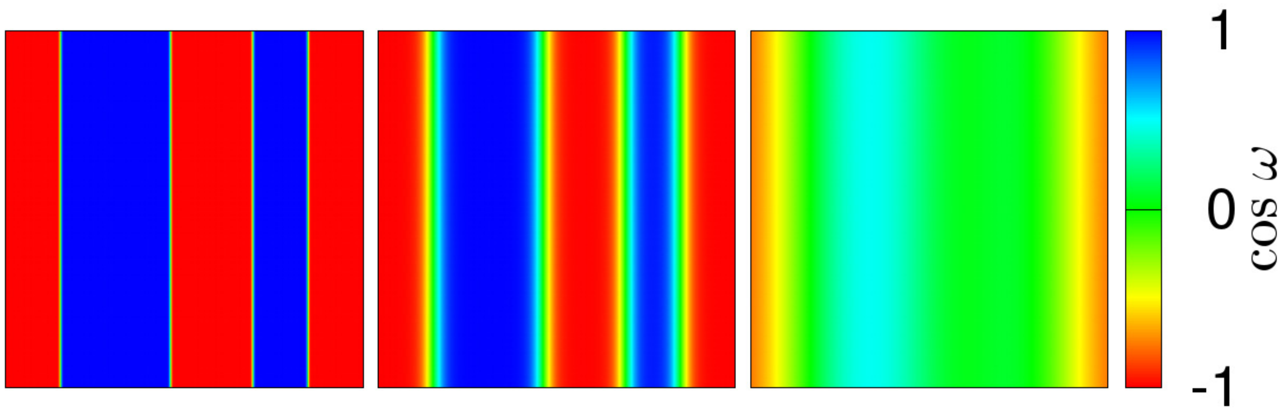



Figure 3: Two-dimentional slice of layered topological charge distribution in $R^4$ . The action density is equal to the same nonzero constant value for all three configurations. The LHS plot represents configuration with infinitely thin planar Bloch domain wall defects, which is Abelian homogeneous (anti-)self-dual field almost everywhere in $R^4$, characterized by the nonzero absolute value of topological charge density almost everywhere proportional to the value of the action density. The most RHS plot shows the opposite case of very thik kink network. Green color corresponds to the gauge field with infinitesimally small topological charge density. The most LHS configuration is confining (only colorless hadrons can be excited) while the most RHS one supports the color charged quasiparticles as elementary excitations.

One may go further and consider a product

$$\omega(x) = \pi \prod_{i=1}^{k} \zeta(\mu_i, \eta^i_\nu x_\nu - q^i). \qquad (2)$$

For an appropriate choice of normal vectors $\eta^i$ this superposition represents a lump of anti-selfdual field in the background of the selfdual one, in two, three and four dimensions for $k = 4, 6, 8$ respectively.

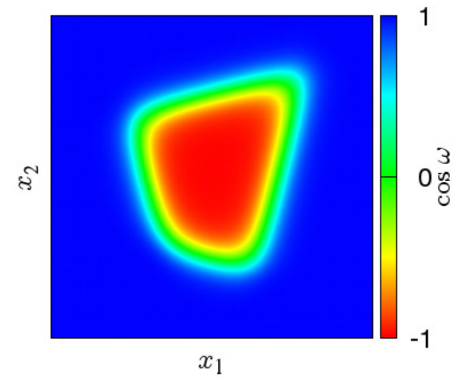



Figure 4: A two-dimensional slice of the four-dimentional lump of anti-selfdual field in the background of the self-dual configuration. The domain wall surrounding the lamp in the four-dimensional space is given by the multiplicative superposition of eight kinks as it is defined by Eq.(2).

$$\mathrm{div}\mathbf{H} \neq 0$$

## Gauge field parameterization

Cho-Faddeev-Niemi-Shabanov-Kondo: the Abelian part $\hat{V}_\mu(x)$ of the gauge field $\hat{A}_\mu(x)$ is separated manifestly,

$$\begin{aligned}
\hat{A}_\mu(x) &= \hat{V}_\mu(x) + \hat{X}_\mu(x),\\
\hat{V}_\mu(x) &= \hat{B}_\mu(x) + \hat{C}_\mu(x),\\
\hat{B}_\mu(x) &= [n^a A^a_\mu(x)]\hat{n}(x) = B_\mu(x)\hat{n}(x),\\
\hat{C}_\mu(x) &= g^{-1}\partial_\mu \hat{n}(x) \times \hat{n}(x),\\
\hat{X}_\mu(x) &= g^{-1}\hat{n}(x) \times \Big(\partial_\mu \hat{n}(x) + g\hat{A}_\mu(x) \times \hat{n}(x)\Big),
\end{aligned}$$

$\hat{A}_\mu(x) = A^a_\mu(x)t^a$, $\hat{n}(x) = n_a(x)t^a$, $n^a n^a = 1$

$$\partial_\mu \hat{n} \times \hat{n} = i f^{abc}\partial_\mu n^a n^b t^c,\ \ [t^a, t^b] = i f^{abc} t^c.$$

The field $\hat{V}_\mu$ is the Abelian field:

$$[\hat{V}_\mu(x), \hat{V}_\nu(x)] = 0$$

$$\hat{F}_{\mu\nu}(x) = \hat{n}(x)\left[\partial_\mu B_\nu - \partial_\nu B_\mu + i g^{-2} f^{abc} n^a \partial_\mu n^b(x) \partial_\nu n^c(x)\right]$$

*L. D. Faddeev, A. J. Niemi* // Nucl. Phys. B. 776. 2007
*Kri-Ichi Kondo, Toru Shinohara, Takeharu Murakami* // arXiv:0803.0176v2 [hep-th] 2008
*Y.M. Cho,* Phys. Rev. D 21, 1080(1980); Y.M. Cho, Phys. Rev. D 23, 2415(1981).
*S.V. Shabanov,* Phys. Lett. B 458, 322(1999); Phys. Lett. B 463, 263(1999),
**Teor. Mat. Fiz., Vol. 78, No. 3, pp. 411-421, 1989.**

The general kink network is then given by additive superposition of lumps (2)

$$\omega = \pi \sum_{j=1}^{\infty} \prod_{i=1}^{k} \zeta(\mu_{ij}, \eta_{\nu}^{ij} x_{\nu} - q^{ij}).$$

Correponding topological charge density is shown in Fig.5. The LHS plot in Figs.5 and 3 represents configuration with infinitely thin domain walls, that isAbelian homogeneous (anti-)self-dual field almost everywhere in $R^4$, characterized by the nonzero absolute value of topological charge density which is constant and proportional to the value of the action density almost everywhere.

The most RHS plots Figs.3 and 5 show the opposite case of the network composed of very thik kinks. Green color corresponds to the gauge field with infinitesimally small topological charge density. Study of the spectrum of clorless and color charged fluctuations indicates that the most LHS configuration is expected to be confining (only colorless hadrons can be excited as particles) while the most RHS one (crossed orthogonal field) supports the color charged quasi-particles as the dominant elementary excitations.

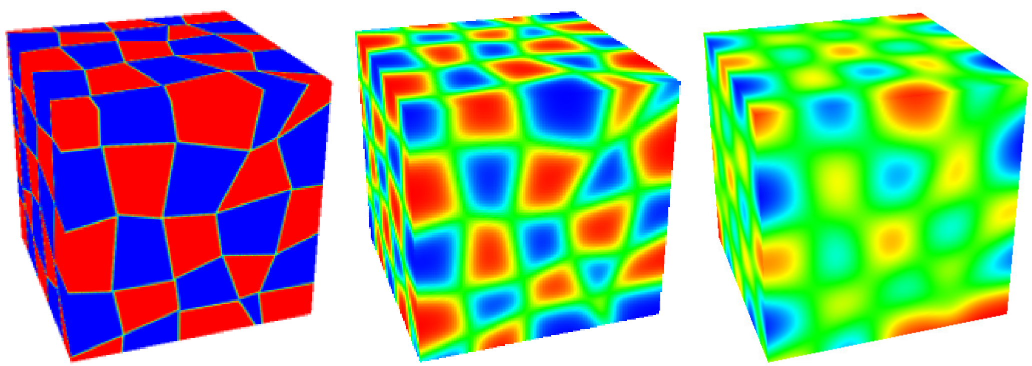


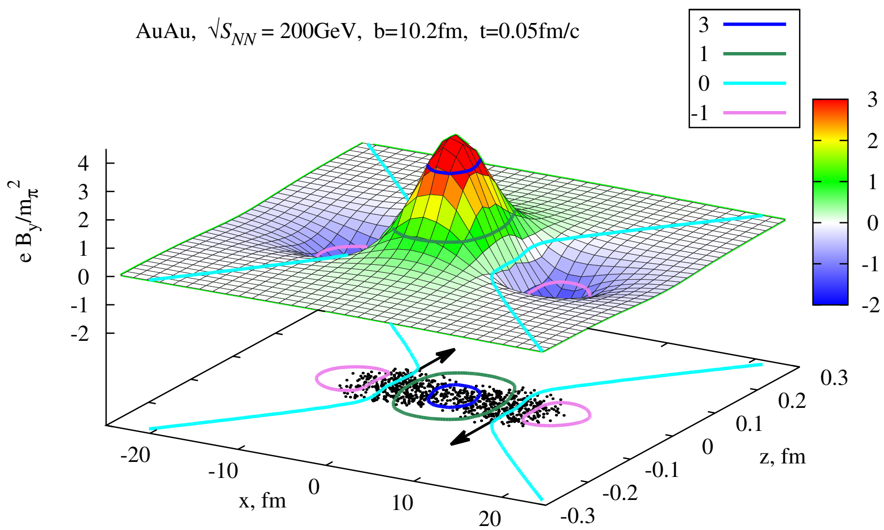
Figure 5: Three-dimensional slices of the kink network - additive superposition of numerous four-dimensional lamps.

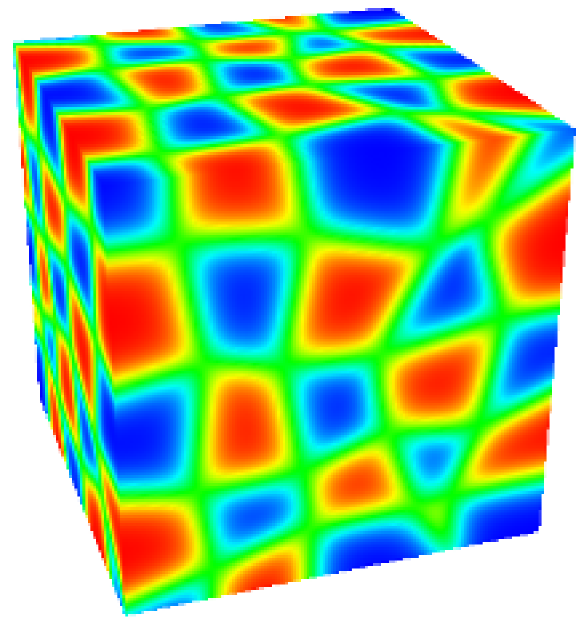
## *Impact of electromagnetic fields on "QCD vacuum".*

- **Relativistic heavy ion collisions - extremely strong electromagnetic fields**

*V. Voronyuk, V. D. Toneev, W. Cassing, E. L. Bratkovskaya,*

*V. P. Konchakovski and S. A. Voloshin, Phys. Rev C 84 (2011)*



# One-loop quark contribution to the effective potential in the presence of arbitrary homogenous Abelian fields

$$U_{\mathrm{eff}}(G) = -\frac{1}{V}\ln\frac{\det(i\not{D}-m)}{\det(i\not{\partial}-m)} = \frac{1}{V}\int\limits_V d^4x \,\mathtt{Tr}\int\limits_m^\infty dm'\,[S(x,x|m') - S_0(x,x|m')]\,|$$

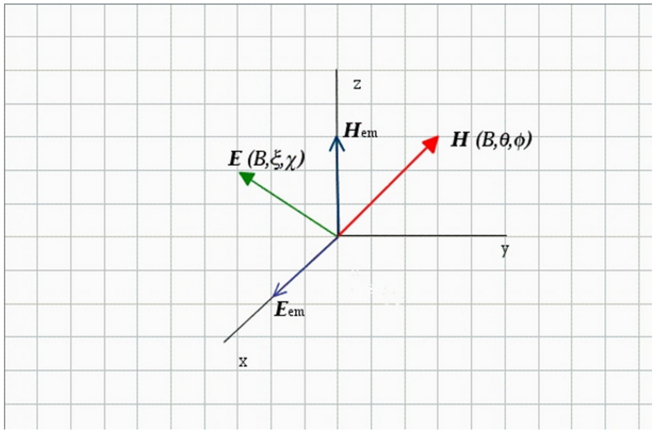
$$U^{\mathrm{ren}}_{\mathrm{eff}}(G) = \frac{B^2}{8\pi^2}\int\limits_0^\infty \frac{ds}{s^3}\mathtt{Tr}_n\left[s\varkappa_+\coth(s\varkappa_+)s\varkappa_-\coth(s\varkappa_-) - \mathbf{1} - \frac{s^2}{3}(\varkappa_+^2+\varkappa_-^2)\right]e^{-\frac{m^2}{B}s},$$

$$\varkappa_\pm = \frac{1}{2B}\sqrt{\mathcal{Q}\sigma_\pm} = \frac{1}{2B}\left(\sqrt{2(\mathcal{R}+\mathcal{Q})}\pm\sqrt{2(\mathcal{R}-\mathcal{Q})}\right),$$

$$\mathcal{R} = (H^2-E^2)/2 + \hat{n}^2B^2 + \hat{n}B(H\cos(\theta) + iE\cos(\chi)\sin(\xi))$$

$$\mathcal{Q} = \hat{n}BH\cos(\xi) + i\hat{n}BE\sin(\theta)\cos(\phi) + \hat{n}^2B^2(\sin(\theta)\sin(\xi)cos(\phi-\chi) + cos(\theta)cos(\xi))$$

Y. M. Cho and D. G. Pak, Phys.Rev. Lett., 6 (2001) 1047

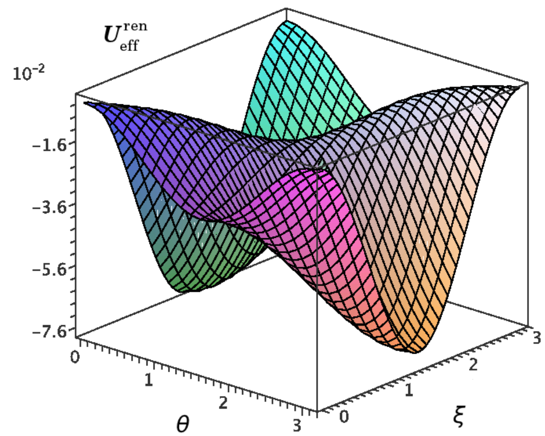


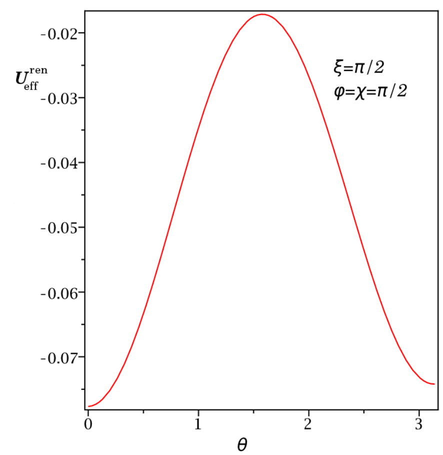

$$H_i = H\delta_{i3}, \ \ E_j = E\delta_{j1}, \ \ H^{\rm c} = \{B, \theta, \phi\}, \ \ E^{\rm c} = \{B, \xi, \chi\}$$

**$H \neq 0$, $E \neq 0$ and arbitrary gluon field**

$$\Im(U_{\rm eff}) = 0 \Longrightarrow \cos(\chi)\sin(\xi) = 0, \;\sin(\theta)\cos(\phi) = 0$$

Effective potential (in units of $B^2/8\pi^2$) for the electric $E = .5B$ and the magnetic $H = .9B$ fields as functions of angles $\theta$ and $\xi$ ($\phi = \chi = \pi/2$ )




Minimum is at $\theta = \pi$ and $\xi = \pi/2$:

orthogonal to each other chromomagnetic and chromoelectric fields: $\mathcal{Q} = 0$.

**Strong electro-magnetic field plays catalyzing role for deconfinement and anisotropies?!**

B.V. Galilo and S.N. Nedelko, Phys. Rev. D84 (2011) 094017.

M. D'Elia, M. Mariti and F. Negro, Phys. Rev. Lett. **110**, 082002 (2013)

G. S. Bali, F. Bruckmann, G. Endrodi, F. Gruber and A. Schaefer, JHEP **1304**, 130 (2013)

## Spectrum of the charged field in (anti-)selfdual background

Consider the eigenvalue equation of the charged field in the kink-like background

$$-\left(\partial_\mu - iB_\mu(x)\right)^2\phi = \lambda\phi.$$

The homogeneous (anti-)self-dual fields

$$B_\mu(x) = B_{\mu\nu}x_\nu,\ \ \tilde{B}_{\mu\nu} = \pm B_{\mu\nu},\ B_{\mu\alpha}B_{\nu\alpha} = B^2\delta_{\mu\nu},\ B = \Lambda^2 b_{\rm vac},$$

The eigenvalue equation can be rewritten as follows

$$\left[\beta^+_\pm\beta_\pm + \gamma^+_+\gamma_+ + 1\right]\phi = \frac{\lambda}{4B}\phi$$

where creation and annihilation operators $\beta_\pm$, $\beta^+_\pm$, $\gamma_\pm$, $\gamma^+_\pm$ are expressed in terms of the operators $\alpha^+$, $\alpha$,

$$\beta_\pm = \frac{1}{2}\left(\alpha_1 \mp i\alpha_2\right),\ \gamma_\pm = \frac{1}{2}\left(\alpha_3 \mp i\alpha_4\right),\ \alpha_\mu = \frac{1}{\sqrt{B}}(Bx_\mu + \partial_\mu),$$

$$\beta^+_\pm = \frac{1}{2}\left(\alpha^+_1 \pm i\alpha^+_2\right),\ \gamma^+_\pm = \frac{1}{2}\left(\alpha^+_3 \pm i\alpha^+_4\right),\ \alpha^+_\mu = \frac{1}{\sqrt{B}}(Bx_\mu - \partial_\mu).$$

The eigenvalues and the square integrable eigenfunctions are

$$\lambda_r = 4B\left(r+1\right),\ \ r = k+n\ (\text{for self}-\text{dual field}),\ \ r = l+n\ (\text{for anti}-\text{self}-\text{dual field}) \tag{3}$$

$$\phi_{nmkl}(x) = \frac{1}{\sqrt{n!m!k!l!\pi^2}}\left(\beta^+_+\right)^k\left(\beta^+_-\right)^l\left(\gamma^+_+\right)^n\left(\gamma^+_-\right)^m\phi_{0000}(x),\ \phi_{0000}(x) = e^{-\frac{1}{2}Bx^2}, \tag{4}$$

**Discrete spectrum. Absence of periodic solutions is treated as confinement of the charged field.**

$$D^2(x)G(x,y) = -\delta(x-y)\quad G(x,y) = e^{ixBy}H(x-y)\quad \tilde{H}(p^2) = \frac{1-e^{-p^2/B}}{p^2}$$

**Orthogonal to each other Euclidean chromoelectric and chromomagnetic fields**
**$\sim$ pure chromomagnetic field:**

$$B_2 = 0,\ B_1 = 2Bx_2,\ B_3 = 0,\ B_4 = 0 \quad (H_i = 2B\delta_{i2},\ E_i = -2B\delta_{i3})$$

$$\Phi(x) = \exp(-ip_4x_4 - ip_3x_3)\varphi(x_1, x_2)$$
$$\left[-\partial_2^2 + (p_1 + 2Bx_2)^2 + p_3^2 + p_4^2\right]\chi = \lambda\chi$$

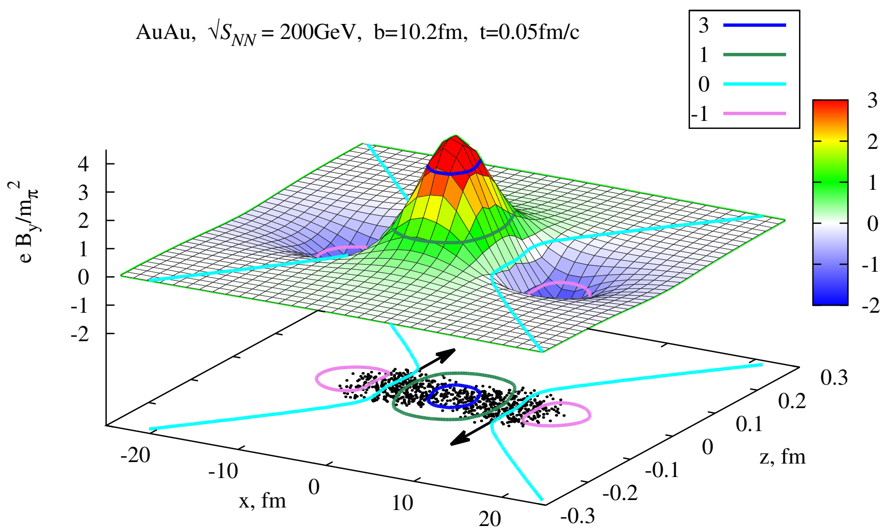
Square integrable over $x_1, x_2$ solution

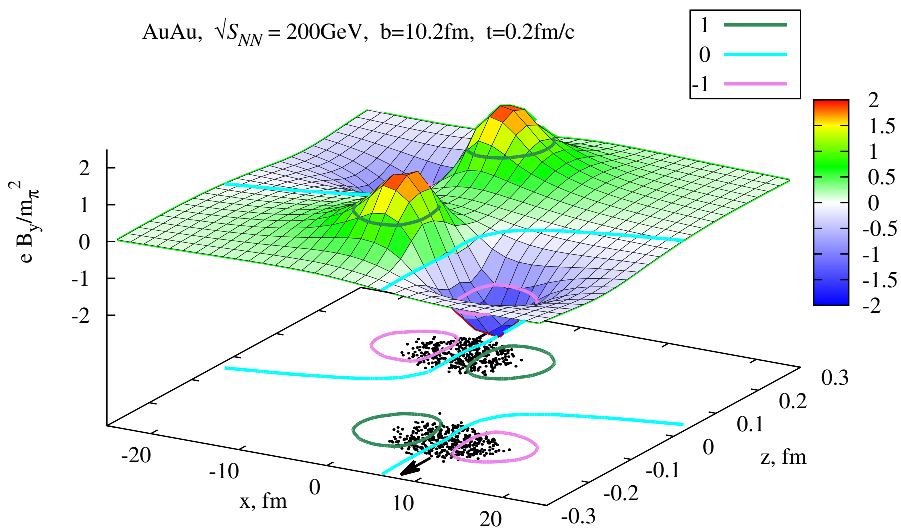
$$\varphi_n(x_1, x_2) = \int dp_1 f(p_1) e^{-ip_1x_1} \exp\left\{-B\left(x_2 + \frac{p_1}{2B}\right)^2\right\} H_n\left(\sqrt{2B}\left(x_2 + \frac{p_1}{2B}\right)\right)$$
$$\lambda_n(p_3^2, p_4^2) = 2B\left(2n + 1 + \frac{p_3^2}{2B} + \frac{p_4^2}{2B})\right)$$

**Continuous spectrum similar to Landau levels. Periodic eigenfunction can be treated as the presense of charged quasi-particles moving along the chromomagnetic field:**

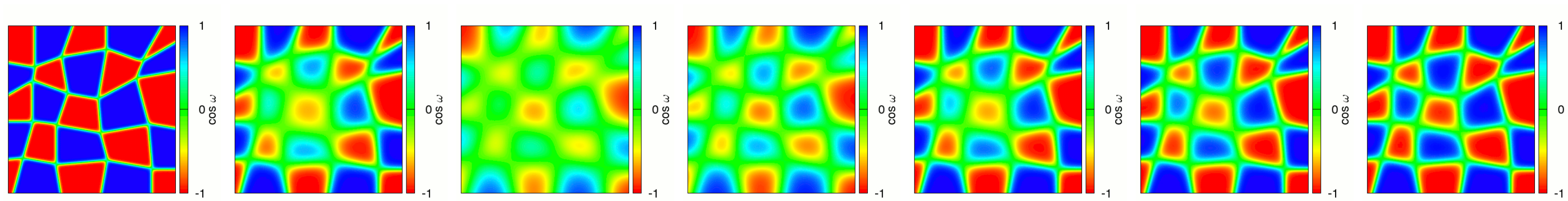
$$p_0^2 = p_3^2 + \mu_n^2, \qquad \mu_n^2 = 2B(2n+1)$$

*V. Voronyuk, V. D. Toneev, W. Cassing, E. L. Bratkovskaya,*
*V. P. Konchakovski and S. A. Voloshin, Phys. Rev C 84 (2011)*




Magnetic field $eB \gtrsim m_\pi^2$ in the region $5fm \times 5fm \times .2fm \times .2fm/c$



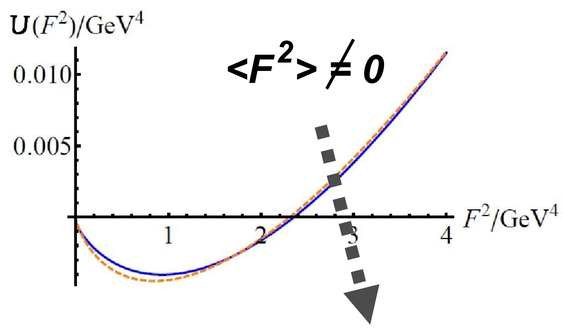

A bag filled by hundreds of color charged quasi-particles, azimuthal asymmetry

K. A. Bugaev, V. K. Petrov and G. M. Zinovjev, Phys. Atom. Nucl. **76** (2013) 341.

A. Eichorn, et al,
Phys.Rev.D83, 2011

## Weyl group, CP and the kink-like field configurations in the effective SU(3) gauge theory

**B. Galilo, S. Nedelko,**
***Phys.Part.Nucl.Lett.*, 8, 2011;**
***Phys. Rev. D* 84,2011.**

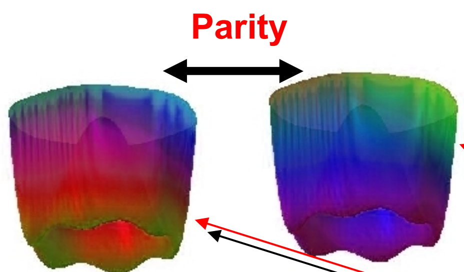


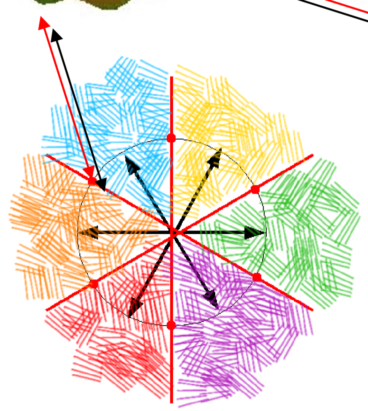

**Dynamical breaking of CP and colour gauge symmetry:**

***SU(3)*** ⟶ ***U(1) x U(1)*** ⟷ **Weyl reflections ?**

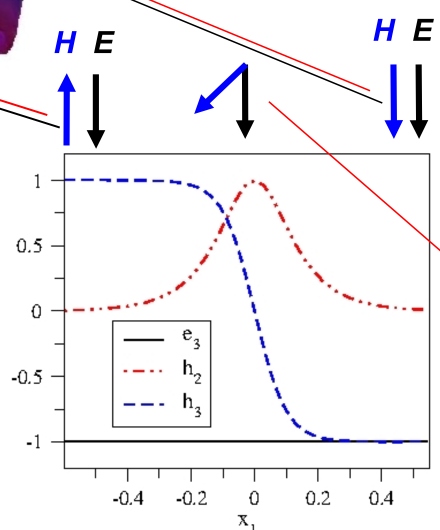
**Topological defects(domain walls and lower dim. deffects at their intersections) bring disorder into ensemble of vacuum gluon configurations — on average SU(3) and CP are not broken!**

**Ensemble of domain structured vacuum fields:**
**dynamical quark confinement, chiral symmetry breaking.**
**In bulk of domain color is confined, color charged quasi-particles are localized at the boundaries.**



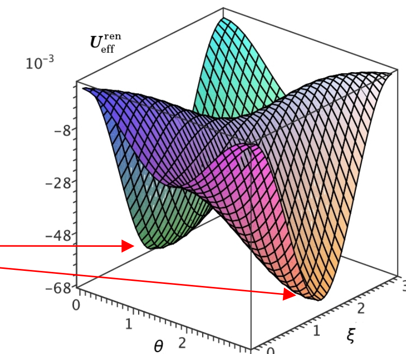



**Weyl reflections in the root space of *color su(3)* — kink between boundaries of Weyl chambers**




**Parity transformation - kink interpolates Between self- and antiself-dual Abelian gluon configurations**

**Strong crossed electromagnetic field creates relatively stable domain wall defect and thus triggers deconfinement of color charged particles in the space-time region of the relativistic heavy ion collision**




Quark contribution to QCD effective potential for Abelian gluon field in the presence of the strong crossed electromagnetic field

# Charged field fluctuations in the background of a domain wall
# A single planar domain wall of the Bloch type.

## Scalar color charged field

The quadratic part of the action for the scalar field in the background field of a planar kink with finite width placed at $x_1 = 0$ looks as

$$S[\Phi] = -\int d^4x (D_\mu\Phi)^\dagger(x) D_\mu\Phi(x) = \int d^4x \Phi^\dagger(x) D^2\Phi(x),$$

$$D_\mu = \partial_\mu + i\hat{B}_\mu,\ \hat{B}_\mu = -\hat{n}B_\mu(x).$$

Here $\hat{n}$ is a constant color matrix, $B_\mu$ is the vector potential for the planar Bloch domain wall.

$$B_1 = H_2(x_1)x_3 + H_3(x_1)x_2,$$

$$B_2 = B_3 = 0, \quad B_4 = -Bx_3,$$

$$H_2 = B\sin\omega(x_1),\ H_3 = -B\cos\omega(x_1),$$

$$\omega(x_1) = 2\,\mathrm{arctg}\exp\mu x_1.$$

A kink with finite width is a regular everywhere in $R^4$ function. However, there is a peculiarity related to the chosen gauge of the background field.

$$D^2 = \tilde{D}^2 + i\partial_\mu\hat{B}_\mu,$$

$$\tilde{D}^2 = \partial^2 + 2i\hat{B}_\mu\partial_\mu - i\hat{B}_\mu\hat{B}_\mu = (\partial_1 - i\hat{n}H_2(x_1)x_3 - i\hat{n}H_3(x_1)x_2)^2 + \partial_2^2 + \partial_3^2 + (\partial_4 + i\hat{n}Bx_3)^2 - i\partial_1 B_1$$

$$\partial_\mu\hat{B}_\mu = -\hat{n}H_2'(x_1)x_3 - \hat{n}H_3'(x_1)x_2.$$

The action can be written as

$$S[\Phi] = \int d^4x \Phi^\dagger(x)\tilde{D}^2\Phi(x) - i\int d^4x \Phi^\dagger(x)\hat{n}\Phi(x)\left[H_2'(x_1)x_3 + H_3'(x_1)x_2\right].$$

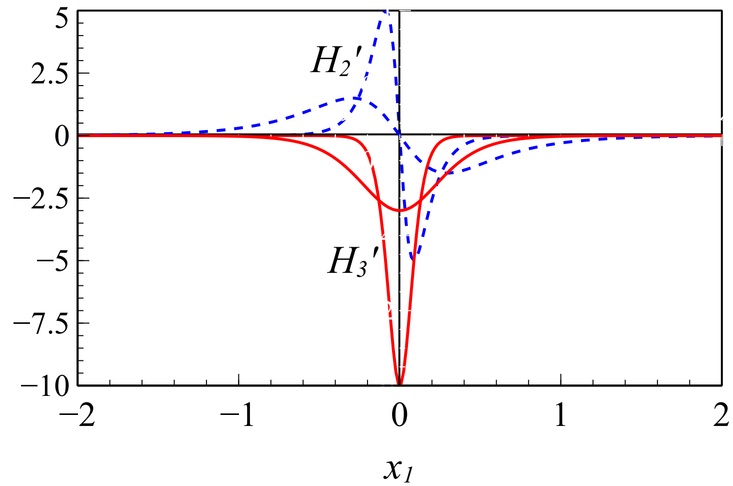



Figure 6: Derivatives of the components of the chromomagnetic field are plotted for two values of the width parameter $\mu/\sqrt{B} = 3, 10$.The coordinate $x_1$ is given in units of $1/\sqrt{B}$. In the limit of infinitely thin domain wall ($\mu/\sqrt{B} \to \infty$) the derivatives develop the delta-function singularities at the location of the wall.

It should be noted that the integral in the second line is equal to zero if $\Phi^\dagger(x)\hat{n}\Phi(x)$ is an even function of $x_2$ and $x_3$.
A continuity of the normal to the wall component of the total (through the whole hypersurface of the wall) charged current offers a reliable guiding principle for identification of the matching conditions. Continuity of the integral current means that the surface terms do not appear under integration by parts in the action

$$\lim_{\varepsilon\to 0}\left[J_1(\varepsilon) - J_1(-\varepsilon)\right] = 0,$$

$$J_\mu(x_1) = \int d^3x \Phi^\dagger(x) D_\mu \Phi(x),$$

$$d^3x = dx_2 dx_3 dx_4.$$

Moreover, this requirement restricts the form of the eigenfunctions in such a way that the surface terms associated with the gauge dependent delta-function singularuties in $\partial_\mu \hat{B}_\mu$, vanish as well.

## Confined fluctuations in the bulk: $x_1 \neq 0$

Consider the eigenvalue problem

$$-\tilde{D}^2\Phi = \lambda\Phi.$$

for the functions square integrable in $R^4$ and satisfying the integral current continuity condition .
For all $x_1 \neq 0$ the operator $\tilde{D}^2$ takes the form

$$\tilde{D}^2 = (\partial_1 \pm i\hat{n}Bx_2)^2 + \partial_2^2 + \partial_3^2 + (\partial_4 + i\hat{n}Bx_3)^2$$

"+" corresponds to the anti-selfdual configuration ($x_1 > 0$) and "-" is for the self-dual one ($x_1 < 0$). Respectively the square integrable solutions are

$$\Phi_{kl} = \int dp_1 dp_4 f(p_1) g(p_4) \exp\left\{\pm ip_1x_1 + ip_4x_4 - \frac{1}{2}|\hat{n}|B(x_2 + p_1/|\hat{n}|B)^2 - \frac{1}{2}|\hat{n}|B(x_3 + p_4/|\hat{n}|B)^2\right\}$$
$$\times H_k\left(\sqrt{|\hat{n}|B}\left[x_2 + \frac{p_1}{|\hat{n}|B}\right]\right) H_l\left(\sqrt{|\hat{n}|B}\left[x_3 + \frac{p_4}{|\hat{n}|B}\right]\right),$$

where $H_m$ are the Hermite polynomials. The eigenvalues are

$$\lambda_{kl} = 2|\hat{n}|B(k + l + 1),\ \ k, l = 0, 1, \dots.$$

**The amplitudes $f(p_1)$ and $g(p_4)$ have to provide the square integrability of the eigenfunctions in $x_1$ and $x_4$.** The integral current through the domain wall is continuous if **both $f$ and $H_k$ are odd or even functions simultaneously under the combined change $p_1 \to -p_1$ and $x_2 \to -x_2$**

$$f(-p_1)H_k(-z) = f(p_1)H_k(z).$$

This property also guarantees the absence of the gauge specific contribution to the action related to the derivative of $H_3$.
The eigenfunctions are of the bound state type with the purely discrete spectrum. Field fluctuations of this type can be seen as confined. The eigenvalues coincide with those for the purely homogeneous (anti-)selfdual Abelian field. In this sense domain wall defect does not destroy confinement of dynamical color charged fields. The eigenfunctions are restricted by the correlated evenness condition, while in the case of the the homogeneous field the properties of the amplitude $f(p_1)$ and the polynomial $H_k$ are mutually independent.

# Color charged quasi-particles on the wall: $x_1 = 0$

**On the wall the chromomagnetic and chromoelectric fields are orthogonal to each other (see Fig.6). In conformity with integral current continuitythe absence of the charged current off the infinitely thin domain wall requires**

$$\partial_1 \Phi|_{x_1=0} = 0,$$

and **the eigenvalue problem on the wall takes the form**

$$\left[-\partial_2^2 - \partial_3^2 + \hat{n}^2 B^2 x_3^2 + (i\partial_4 - \hat{n} B x_3)^2\right] \Phi = \lambda \Phi$$

**with the solution**

$$\Phi_{kp_2p_4}(x_2, x_3, x_4) = e^{ip_2x_2 + ip_4x_4} e^{-\frac{|\hat{n}|B}{\sqrt{2}}\left(x_3 - \frac{p_4}{2|\hat{n}|B}\right)^2} H_k\left[\sqrt{\sqrt{2}|\hat{n}|B}\left(x_3 - \frac{p_4}{2|\hat{n}|B}\right)\right],$$

$$\lambda_k(p_2^2, p_4^2) = \sqrt{2}|\hat{n}|B(2k+1) + \frac{p_4^2}{2} + \frac{p_2^2}{2},\ k = 0, 1, 2, \ldots$$

**The spectrum of the eigenmodes on the wall is continuous, it depends on the momentum $p_2$ longitudinal to the chromomagnetic field and Euclidean energy $p_4$, the corresponding eigenfunctions are oscillating in $x_2$ and $x_4$.**
In the transverse to chromomagnetic field direction $x_3$ the eigenfunctins are bounded and the eigenvalues display the Landau level structure.
**This can be treated as the lack of confinement - the color charged quasi-particles can be excited on the wall.**
The continuation $p_4 = -p_0$ leads to the dispersion relation for the quasi-particles with the masses $\mu_n$

$$p_0^2 = p_2^2 + \mu_k^2, \quad \mu_k^2 = 2\sqrt{2}(2k+1)|\hat{n}|B,\ k = 0, 1, 2, \ldots.$$

## Formulation of the Domain Model

**Euclidean partition function** is defined as

$$\mathcal{Z}(\theta) = \lim_{V,N\to\infty} \mathcal{N} \int\limits_{\Omega_{\alpha,\vec\beta}} d\Omega_{\alpha,\vec\beta} \prod_{i=1}^{N} \int\limits_{\mathcal{B}} dB_i \int\limits_{\mathcal{F}^i_\psi} \mathcal{D}\psi^{(i)}\mathcal{D}\bar\psi^{(i)} \int\limits_{\mathcal{F}^i_Q} \mathcal{D}\mu[Q^i] e^{-S^{\rm QCD}_{V_i}\left[Q^{(i)}+B^{(i)},\psi^{(i)},\bar\psi^{(i)}\right]-i\theta Q_{V_i}[Q^{(i)}+B^{(i)}]}$$

$$\mathcal{D}\mu = \delta[D(B^{(i)})Q^{(i)}]\Delta_{\rm FP}[B^{(i)},Q^{(i)}]$$

**The thermodynamic limit:** $v^{-1} = N/V$ = const, as $V, N \to \infty$. **Functional spaces** $\mathcal{F}^i_Q$ and $\mathcal{F}^i_\psi$ are specified by BCs at $(x-z_i)^2 = R^2$

$$\breve n_i Q^{(i)}(x) = 0, \quad i \not\eta_i(x) e^{i(\alpha+\beta^a\lambda^a/2)\gamma_5}\psi^{(i)}(x) = \psi^{(i)}(x), \quad \bar\psi^{(i)} e^{i(\alpha+\beta^a\lambda^a/2)\gamma_5} i \not\eta_i(x) = -\bar\psi^{(i)}(x),$$

$$\eta^\mu_i = \frac{(x-z_i)^\mu}{|x-z_i|}, \quad \breve n_i = n^a_i T^a, T^a\text{—adjoint representation.}$$

$\int\limits_\Sigma d\sigma_i \mathcal{Z}_i(\sigma) \Longrightarrow$ **Ensemble of** "domain-" or "cluster-like" structured **background fields** with the field strength tensor

$$F^a_{\mu\nu}(x) = \sum_{j=1}^N n^{(j)a} B^{(j)}_{\mu\nu}\theta(1-(x-z_j)^2/R^2), \quad B^{(j)}_{\mu\nu}B^{(j)}_{\mu\rho} = B^2\delta_{\nu\rho}, \quad B^2 = \text{const}$$

$$\tilde B^{(j)}_{\mu\nu} = \pm B^{(j)}_{\mu\nu}, \quad \hat n^{(j)} = t^3\cos\xi_j + t^8\sin\xi_j, \quad \xi_j \in \{\frac{\pi}{6}(2k+1),\ k=0,\dots,5\}$$

**Free parameters**: the field **strength** $B$ and the **radius** $R$. Domains are **hyperspherical, centered at random points** $z_j$.

$$\int\limits_{\mathcal{B}} dB_i \dots = \frac{1}{24\pi^2}\int\limits_V \frac{d^4 z_i}{V}\int\limits_0^{2\pi} d\varphi_i \int\limits_0^{\pi} d\theta_i \sin\theta_i \int\limits_0^{2\pi} d\xi_i \sum_{l=0,1,2}^{3,4,5}\delta(\xi_i - \frac{(2l+1)\pi}{6})\int\limits_0^{\pi} d\omega_i \sum_{k=0,1}\delta(\omega_i - \pi k)\dots$$

## Testing The Model In Pure Yang-Mills System.

• Mean **topological charge** is zero. At finite volume $V = vN$ the distribution of the topological charge $-Nq \leq Q \leq Nq$ is symmetric about $Q = 0$.

$$Q(x) = \frac{g^2}{32\pi^2}\tilde{F}(x)F(x), \quad \mathcal{P}_N(Q) = \frac{N!}{2^N\,(N/2 - Q/2q)!\,(N/2 + Q/2q)!},$$

$$Q = \int\limits_V d^4x Q(x) = q(N_+ - N_-), \quad q = \frac{B^2R^4}{16}, \quad N_+ + N_- = N$$

• **Gluon condensate**: $\langle : g^2 F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) : \rangle = 4B^2$

• **Topological susceptibility for pure YM** $\chi = \int d^4x \langle Q(x)Q(0)\rangle = B^4R^4/128\pi^2$

• **The area law**: Wilson loop for a circular contour with radius $L >> R$ for $N_c = 3$

$$W(L) = \lim_{V,N\to\infty} \prod_{j=1}^{N} \int d\sigma_j \frac{1}{N_c} \mathrm{Tr} e^{i\int_{S_L} d\sigma_{\mu\nu}(x)\hat{B}_{\mu\nu}(x)} = e^{-\sigma\pi L^2 + O(L)},$$

$$\sigma = Bf(\pi BR^2), \quad f(z) = \frac{2}{3z}\left(3 - \frac{\sqrt{3}}{2z}\int\limits_0^{2z/\sqrt{3}} \frac{dx}{x}\sin x - \frac{2\sqrt{3}}{z}\int\limits_0^{z/\sqrt{3}} \frac{dx}{x}\sin x\right)$$

Fitting $(B, R) : \sqrt{B} = 947\mathrm{MeV}, R = (760\mathrm{MeV})^{-1} = 0.26\mathrm{fm}$

$$\sigma = (420\mathrm{MeV})^2, \quad \chi = (197\mathrm{MeV})^4, \quad \frac{\alpha_s}{\pi}\langle F^2\rangle = .081\mathrm{GeV}^4 \ ; \quad q = 0.15, \ v^{-1} = 42\mathrm{fm}^{-4}$$

Here $q$ is a **fraction of top. charge per domain**, and $v^{-1}$ is the **density of domains** .

## Including Quark Fields

► **Eigen modes**

$$\psi(x) = \sum_n b_n \psi_n(x), \quad \bar\psi(x) = \sum_n \bar b_n \bar\psi_n(x)$$

$$\begin{aligned} &\not{D}\psi_n(x) = \lambda_n \psi_n(x), \\ &i\,\not{\eta}(x) e^{i\alpha\gamma_5}\psi(x) = \psi(x),\ x^2 = R^2 \\ &\bar\psi(x) e^{i\alpha\gamma_5} i\,\not{\eta}(x) = -\bar\psi(x),\ x^2 = R^2. \end{aligned}$$

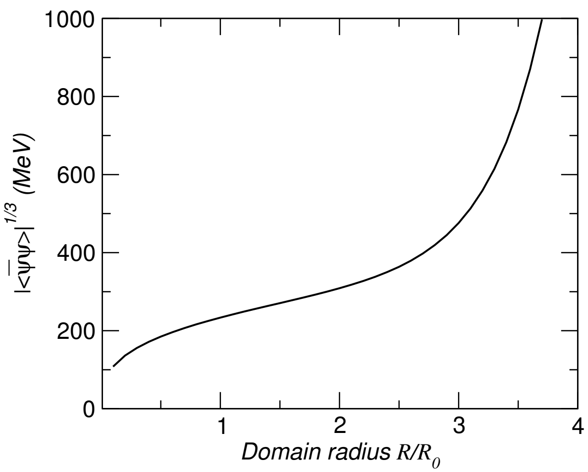
► **Quark determinant and realisation of** $U_{\rm A}(1)$ and $SU_{\rm L}(N_{\rm f}) \times SU_{\rm L}(N_{\rm f})$

Anomaly reduces $U_A(1)$ to a discrete subgroup. Unlike $U_A(1)$ flavour chiral symmetry is broken **spontaneously.**

$$\langle\bar\psi(x)\psi(x)\rangle = -\frac{1}{\pi^2 R^3}\mathrm{Tr}\sum_{k=1}^{\infty}\frac{k}{k+1}\left[M(1,k+2,z) - \frac{z}{k+2}M(1,k+3,-z) - 1\right]$$

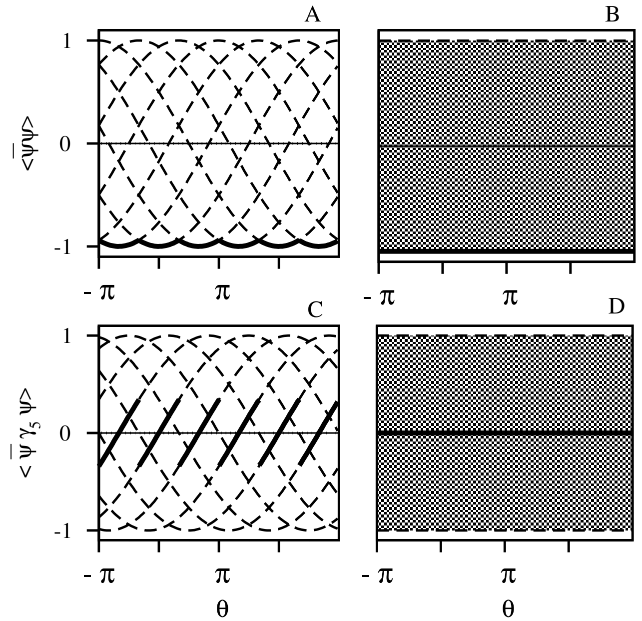
$z = \hat n B R^2/2$ , Tr – color trace (matrix $\hat n$ - diagonal).

With $B, R$ determined from pure gluodynamics $\langle\bar\psi\psi\rangle = -(237.8\mathrm{MeV})^3$




Absolute value of quark condensate as a function of domain radius in units of $R_0 = (760\mathrm{MeV})^{-1}$.

## Poincaré Recurrence Theorem and The Strong CP-problem

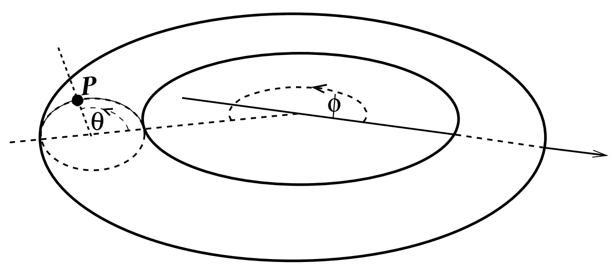



The scalar (A and B) and pseudoscalar (C and D) quark condensates as functions of $\theta$ for $N_f = 3$ in units of $\aleph$. The plots A and C are for rational $q = 0.15$, while B and D correspond to any irrational $q$, for instance to $q = \frac{3}{2.02\pi^2} = 0.15047\ldots$ which is numerically only slightly different from $0.15$. The dashed lines in A and C correspond to discrete minima of the free energy density which are degenerate for $m \equiv 0$. The solid bold lines denote the minimum which is chosen by an infinitesimally small mass term for a given $\theta$. Points on the solid line in A, where two dashed lines cross each other, correspond to critical values of $\theta$, at which CP is broken spontaneously. This is signalled by the discontinuity in the pseudoscalar condensate in C. For irrational $q$, as illustrated in B and D, the dashed lines densely cover the strip between $1$ and $-1$, and the set of critical values of $\theta$ is dense in $\mathbb{R}$.

**The set of critical values of the $\theta$ is dense in the interval $[-\pi, \pi]$:**

$$Z = \lim_{V\to\infty} Z_V(\theta) = \lim_{V\to\infty} Z_V(0), \quad \lim_{V\to\infty} \langle \mathbb{E} \rangle_V^{\theta} \equiv \lim_{V\to\infty} \langle \mathbb{E} \rangle_V^{\theta=0}, \quad \lim_{V\to\infty} \langle \mathbb{O} \rangle_{V,\theta} \equiv 0,$$

for **any CP-even and CP-odd operators** $\mathbb{E}$ and $\mathbb{O}$ respectively, which **resolves the problem of CP-violation.**




A simple example: the Poincaré theorem for uniform motion on a torus,

$$\dot{\theta} = b_\theta, \quad \dot{\phi} = b_\phi,$$

where $\theta$ and $\phi$ are the latitude and longitude of a point on the torus. If $b = b_\theta / b_\phi$ – **rational** number, trajectory is **closed** and can be characterised by an integer winding number. In the case of **irrational** $b$ the trajectory is **dense on the torus**.

## Mass relations

Simultaneously topological succeptibility in QCD with massive quarks

$$\chi^{\mathrm{QCD}} = -\lim_{V\to\infty}\frac{1}{V}\frac{\partial^2}{\partial\theta^2}Z_V(\theta) = -\frac{m\langle\bar\psi\psi\rangle}{N_f} + \mathcal{O}(m^2)$$

is nonzero, independent of $\theta$ and consistent (in Euclidean space) with the identity

$$N_f^2\chi^{QCD} = N_f m_\pi^2 F_\pi^2 + \mathcal{O}(m_\pi^4).$$

$$F_\pi^2 m_{\pi/\eta}^2 = -2m\langle\bar\psi\psi\rangle$$

which indicates a correct implementation of the $U_A(1)$ symmetry. In the chiral limit the mass of the $\eta'$ is expressed via the topological succeptibility $\chi^{\mathrm{YM}}$ of pure gluodynamics in agreement with the Witten-Veneziano formula.

$$m_{\eta'}^2 F_\pi^2 = 2N_f\chi^{YM} + m_\pi^2 F_\pi^2$$

The anomaly contribution to the free energy suppresses continuous axial $U(1)$ degeneracy in the ground state, leaving only a residual axial symmetry. This discrete symmetry and flavour $SU(N_f)_L \times SU(N_f)_R$ chiral symmetry in turn are spontaneously broken with a quark condensate arising due to the asymmetry of the spectrum of Dirac operator in the presense of domains.

## Hadronization: Effective meson action, meson spectrum

$$Z=\mathcal{N}\lim_{V\to\infty}\int D\Phi_{\mathcal{Q}}\exp\left\{-\frac{B}{2}\frac{h^2_{\mathcal{Q}}}{g^2C_{\mathcal{Q}}}\int dx\Phi^2_{\mathcal{Q}}(x)-\sum_k\frac{1}{k}W_k[\Phi]\right\},$$

$$C_{Jnl}=C_J\frac{l+1}{2^l n!(l+n)!},\quad C_{S/P}=\frac{1}{9},\quad C_{S/P}=\frac{1}{18}$$

$$1=\frac{g^2C_{\mathcal{Q}}}{B}\tilde\Gamma^{(2)}_{\mathcal{Q}\mathcal{Q}}(-M^2_{\mathcal{Q}}|B),\quad h^{-2}_{\mathcal{Q}}=\frac{d}{dp^2}\tilde\Gamma^{(2)}_{\mathcal{Q}\mathcal{Q}}(p^2)|_{p^2=-M^2_{\mathcal{Q}}}.$$

$$\begin{aligned}
W_k[\Phi] &= \sum_{\mathcal{Q}_1\dots\mathcal{Q}_k}h_{\mathcal{Q}_1}\dots h_{\mathcal{Q}_k}\int dx_1\dots\int dx_k\Phi_{\mathcal{Q}_1}(x_1)\dots\Phi_{\mathcal{Q}_k}(x_k)\Gamma^{(k)}_{\mathcal{Q}_1\dots\mathcal{Q}_k}(x_1,\dots,x_k|B),\\
\Gamma^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2} &= \overline{G^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2}(x_1,x_2)}-\Xi_2(x_1-x_2)\overline{G^{(1)}_{\mathcal{Q}_1}G^{(1)}_{\mathcal{Q}_2}},\\
\Gamma^{(3)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3} &= \overline{G^{(3)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3}(x_1,x_2,x_3)}-\frac{3}{2}\Xi_2(x_1-x_3)\overline{G^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2}(x_1,x_2)G^{(1)}_{\mathcal{Q}_3}(x_3)}\\
&+\frac{1}{2}\Xi_3(x_1,x_2,x_3)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1)G^{(1)}_{\mathcal{Q}_2}(x_2)G^{(1)}_{\mathcal{Q}_3}(x_3)},\\
\Gamma^{(4)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3\mathcal{Q}_4} &= \overline{G^{(4)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3\mathcal{Q}_4}(x_1,x_2,x_3,x_4)}-\frac{4}{3}\Xi_2(x_1-x_2)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1)G^{(3)}_{\mathcal{Q}_2\mathcal{Q}_3\mathcal{Q}_4}(x_2,x_3,x_4)}\\
&-\frac{1}{2}\Xi_2(x_1-x_3)\overline{G^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2}(x_1,x_2)G^{(2)}_{\mathcal{Q}_3\mathcal{Q}_4}(x_3,x_4)}\\
&+\Xi_3(x_1,x_2,x_3)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1)G^{(1)}_{\mathcal{Q}_2}(x_2)G^{(2)}_{\mathcal{Q}_3\mathcal{Q}_4}(x_3,x_4)}\\
&-\frac{1}{6}\Xi_4(x_1,x_2,x_3,x_4)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1)G^{(1)}_{\mathcal{Q}_2}(x_2)G^{(1)}_{\mathcal{Q}_3}(x_3)G^{(1)}_{\mathcal{Q}_4}(x_4)}.
\end{aligned}$$

The **vertices** $\Gamma^{(k)}$ are **expressed** ***via*** **quark loops** $G^{(n)}_{\mathcal{Q}}$ with $n$ quark-meson vertices

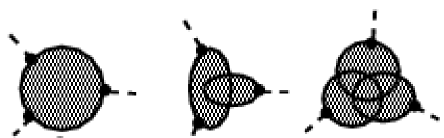
$$\overline{G^{(k)}_{\mathcal{Q}_1\dots\mathcal{Q}_k}(x_1,\dots,x_k)} = \int\limits_{\Sigma} d\sigma_j \mathrm{Tr} V_{\mathcal{Q}_1}(x_1|B^{(j)}) S(x_1,x_2|B^{(j)}) \dots V_{\mathcal{Q}_k}(x_k|B^{(j)}) S(x_k,x_1|B^{(j)})$$

$$\overline{G^{(l)}_{\mathcal{Q}_1\dots\mathcal{Q}_l}(x_1,\dots,x_l) G^{(k)}_{\mathcal{Q}_{l+1}\dots\mathcal{Q}_k}(x_{l+1},\dots,x_k)} = \int\limits_{\Sigma} d\sigma_j$$

$$\times \mathrm{Tr}\left\{ V_{\mathcal{Q}_1}(x_1|B^{(j)}) S(x_1,x_2|B^{(j)}) \dots V_{\mathcal{Q}_k}(x_l|B^{(j)}) S(x_l,x_1|B^{(j)}) \right\}$$

$$\times \mathrm{Tr}\left\{ V_{\mathcal{Q}_{l+1}}(x_{l+1}|B^{(j)}) S(x_{l+1},x_{l+2}|B^{(j)}) \dots V_{\mathcal{Q}_k}(x_k|B^{(j)}) S(x_k,x_{l+1}|B^{(j)}) \right\},$$

**bar** denotes **integration over** all configurations of the **background field with measure** $d\sigma_j$.




**All the elements of the effective action are fixed:** nonlocal meson-quark vertices $V_{\mathcal{Q}_1}(x|B)$ and quark propagators $S(x,y|B)$ and background field correlators $\Xi_n(x)$ are given in explicit analytical form.

**The quark propagator**

$$S(x,y) = \exp\left(-\frac{i}{2} x_\mu \hat{B}_{\mu\nu} y_\nu\right) H(x-y),$$

$$\tilde{H}(p) = \frac{1}{2v\Lambda^2} \int\limits_0^1 ds e^{-p^2/2v\Lambda^2} \left(\frac{1-s}{1+s}\right)^{m^2/4v\Lambda^2}$$

$$\times \left[ p_\alpha \gamma_\alpha \pm i s \gamma_5 \gamma_\alpha f_{\alpha\beta} p_\beta + m \left( P_\pm + P_\mp \frac{1+s^2}{1-s^2} - \frac{i}{2} \gamma_\alpha f_{\alpha\beta} \gamma_\beta \frac{s}{1-s^2} \right) \right].$$

in the presence of **the (anti-)self-dual homogeneous** field

$$\hat{B}_\mu(x) = -\frac{1}{2}\hat{n}B_{\mu\nu}x_\nu,\ \hat{B}_{\mu\nu}\hat{B}_{\mu\rho} = 4v^2\Lambda^4\delta_{\nu\rho},$$
$$f_{\alpha\beta} = \frac{\hat{n}}{v\Lambda^2}B_{\mu\nu},\ v = \mathrm{diag}(1/6, 1/6, 1/3),$$

$$\Lambda^2 = \frac{\sqrt{3}}{2}B.$$

**Quark-meson vertices**

$$V_{\mathcal{Q}} \propto \Gamma\lambda T^{(l)}(\overleftrightarrow{\nabla}/i\Lambda)F_{nl}(\overleftrightarrow{\nabla}^2/\Lambda^2)$$

$$F_{nl}(s) = \int_0^1 dt t^{l+n}e^{st},\ s = \overleftrightarrow{\nabla}^2/\Lambda^2,$$
$$\overleftrightarrow{\nabla}_{ff'} = \xi_f\overleftarrow{\nabla} - \xi_{f'}\overrightarrow{\nabla},\ \xi_f = m_f/(m_f + m_{f'}),$$
$$\overleftarrow{\nabla}_\mu = \overleftarrow{\partial}_\mu + iB_\mu,\ \overrightarrow{\nabla}_\mu = \overrightarrow{\partial}_\mu - iB_\mu.$$

$$\Xi_2(x-y) = \frac{N}{V}\int_V dz\theta(x-z)\theta(y-z) = \frac{2}{3\pi}\phi\left(\frac{(x-y)^2}{4R^2}\right),$$
$$\phi(\rho^2) = \left[\frac{3\pi}{2} - 3\arcsin(\rho) - 3\rho\sqrt{1-\rho^2} - 2\rho(1-\rho^2)\sqrt{1-\rho^2}\right].$$

| $m_u$ (MeV) | $m_d$ (MeV) | $m_s$ (MeV) | $m_c$ (MeV) | $m_b$ (MeV) | $\Lambda$ (MeV) | $g$ |
|---|---|---|---|---|---|---|
| 198.3 | 198.3 | 413 | 1650 | 4840 | 319.5 | 9.96 |

| Meson | $\ell$ | $j$ | $M$ | $M^{\rm exp}$ |
|---|---|---|---|---|
| $\pi$ | 0 | 0 | 140 | 140 |
| $b_1$ | 1 | 1 | 1252 | 1235 |
| $K$ | 0 | 0 | 496 | 496 |
| $K_1(1270)$ | 1 | 1 | 1263 | 1270 |
| $\rho$ | 0 | 1 | 770 | 770 |
| | 1 | 0 | 1238 | |
| $a_1$ | 1 | 1 | 1311 | 1260 |
| $a_2$ | 1 | 2 | 1364 | 1320 |
| $K^*$ | 0 | 1 | 890 | 890 |
| | 1 | 0 | 1274 | |
| $K_1(1400)$ | 1 | 1 | 1342 | 1400 |
| $K_2^*$ | 1 | 2 | 1388 | 1430 |

| Meson | $\pi$ | $\rho$ | $K$ | $K^*$ | $\omega$ | $\phi$ |
|---|---|---|---|---|---|---|
| $M$ | 140 | 770 | 496 | 890 | 770 | 1034 |
| $M^{\rm exp}$ | 140 | 770 | 496 | 890 | 786 | 1020 |
| $f_P$ | 126 | - | 145 | - | - | - |
| $f_P^{\rm exp}$ | 132 | - | 157 | - | - | - |
| $h$ | 6.51 | 4.16 | 7.25 | 4.48 | 4.16 | 4.94 |
| $M^*$ | 630 | 864 | 743 | 970 | 864 | 1087 |

| Meson | $D$ | $D^*$ | $D_s$ | $D_s^*$ | $B$ | $B^*$ | $B_s$ | $B_s^*$ |
|---|---|---|---|---|---|---|---|---|
| $M$ | 1766 | 1991 | 1910 | 2142 | 4965 | 5143 | 5092 | 5292 |
| $M^{\rm exp}$ | 1869 | 2010 | 1969 | 2110 | 5278 | 5324 | 5375 | 5422 |
| $f_P$ | 149 | - | 177 | - | 123 | - | 150 | - |

| Meson | $\eta_c$ | $J/\psi$ | $\chi_{c_0}$ | $\chi_{c_1}$ | $\chi_{c_2}$ | $\psi'$ | $\psi''$ |
|---|---|---|---|---|---|---|---|
| $n$ | 0 | 0 | 0 | 0 | 0 | 1 | 2 |
| $\ell$ | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| $j$ | 0 | 1 | 0 | 1 | 2 | 1 | 1 |
| $M$ (MeV) | 3000 | 3161 | 3452 | 3529 | 3531 | 3817 | 4120 |
| $M^{\rm exp}$ (MeV) | 2980 | 3096 | 3415 | 3510 | 3556 | 3770 | 4040 |

| Meson | $\Upsilon$ | $\chi_{b_0}$ | $\chi_{b_1}$ | $\chi_{b_2}$ | $\Upsilon'$ | $\chi'_{b_0}$ | $\chi'_{b_1}$ | $\chi'_{b_2}$ | $\Upsilon''$ |
|---|---|---|---|---|---|---|---|---|---|
| $n$ | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 2 |
| $\ell$ | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |
| $j$ | 1 | 0 | 1 | 2 | 1 | 0 | 1 | 2 | 1 |
| $M$ (MeV) | 9490 | 9767 | 9780 | 9780 | 10052 | 10212 | 10215 | 10215 | 10292 |
| $M^{\rm exp}$ (MeV) | 9460 | 9860 | 9892 | 9913 | 10230 | 10235 | 10255 | 10269 | 10355 |

$$M_\eta = 640~\text{MeV},~~M_{\eta'} = 950~\text{MeV},~~h_\eta = 4.72,~~h_{\eta'} = 2.55,~~\sqrt{B}R = 1.56.$$

Features of **the spectrum of light vector and pseudoscalar mesons** are driven by the chiral symmetries and are correctly reproduced by the model quantitatively.

$$B^a_\mu = n^a B_{\mu\nu} x_\nu, \quad \tilde{B}_{\mu\nu} = \pm B_{\mu\nu}, \quad B_{\mu\alpha} B_{\alpha\nu} = \delta_{\mu\nu} B^2, \quad B^2 = \text{const}$$

$$D^2(x) G(x,y) = -\delta(x-y) \quad G(x,y) = e^{ixBy} H(x-y) \quad \tilde{H}(p^2) = \frac{1 - e^{-p^2/B}}{p^2}$$

$$\tilde{H}_f(p \mid B) \to O\left(\exp\left\{\frac{p^2}{\Lambda^2}\right\}\right), \; F_{n\ell}\left(p^2\right) \to O\left(\exp\left\{\frac{p^2}{\Lambda^2}\right\}\right),$$

Regge behaviour of the spectrum is due to nonlocality of the vertices and propagators.

- $M^2_{aJ\ell n} = \frac{8}{3} \ln\left(\frac{5}{2}\right) \cdot \Lambda^2 \cdot n \; + O(\ln n)$ , for $n \gg \ell$, $\qquad M^2_{aJ\ell n} = \frac{4}{3} \ln 5 \cdot \Lambda^2 \cdot \ell \; + O(\ln \ell)$ , for $\ell \gg n$.

Heavy-light mesons and heavy quarkonia

- $m_Q \gg \Lambda, m_Q \gg m_q, \quad M_{Q\bar{q}} = m_Q + \Delta^{(J)}_{Q\bar{q}} + O(1/m_Q)$

- $m_Q \gg \Lambda, \quad M_{Q\bar{Q}} = 2m_Q - \Delta_{Q\bar{Q}}, \quad \Delta^{(P)}_{Q\bar{Q}} = 2\Delta^{(V)}_{Q\bar{Q}}$

# Summary

Starting with

$$\lim_{V\to\infty}\frac{1}{V}\int\limits_V d^4x g^2 F^a_{\mu\nu}(x)F^a_{\mu\nu}(x)\neq 0.$$

one arrives at the importance of the lumpy structured gluon configurations (almost everywhere homogeneous abelian (anti-)self-dual field) and correctly implemented:

- **Confinement of both static and dynamical quarks** $\longrightarrow W(C)=\langle \mathrm{Tr}\ \mathrm{P}\ e^{i\int_C dz_\mu \hat{A}_\mu}\rangle,$
$$S(x,y)=\langle\psi(y)\mathrm{P}e^{i\int_y^x dz_\mu\hat{A}_\mu}\bar{\psi}(x)\rangle$$
- **Dynamical Breaking of** $SU_L(N_f)\times SU_R(N_f)$ $\longrightarrow \langle\bar{\psi}(x)\psi(x)\rangle$
- $U_A(1)$ **Problem** $\longrightarrow \eta'$, $\chi$, Axial Anomaly
- **Strong CP Problem** $\longrightarrow Z(\theta)=Z(0)$
- **Colorless Hadron Formation:** $\longrightarrow$ Effective action for colorless collective modes: spectrum, formfactors (**Light** mesons and baryons, **Regge spectrum** of excited states of light hadrons, **heavy-light** hadrons, **heavy quarkonia**)
- **QCD vacuum is characterized as heterophase mixed state with corresponding phase transition mechanism.**

  **V. I. Yukalov and E. P. Yukalova, PoS ISHEPP 2012, 046 (2012) [arXiv:1301.6910 [hep-ph]]; Phys. Rep. 208 (1991) 395;**
- **Impact of a strong electromagnetic field as a trigger of deconfinement is indicated.**