Study of QCD Phase Diagram by Heavy Ion Experiments and Lattice QCD

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"Theory of Hadronic Matter under Extreme Conditions", Lab. of Theor. Phys. , Moscow
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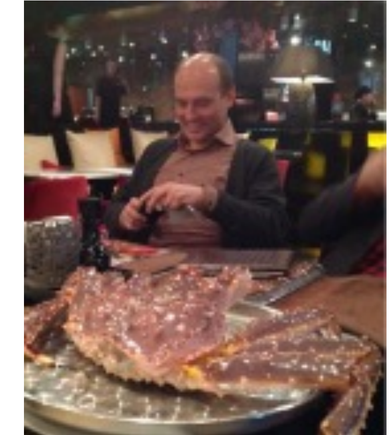
Plan of the Talk

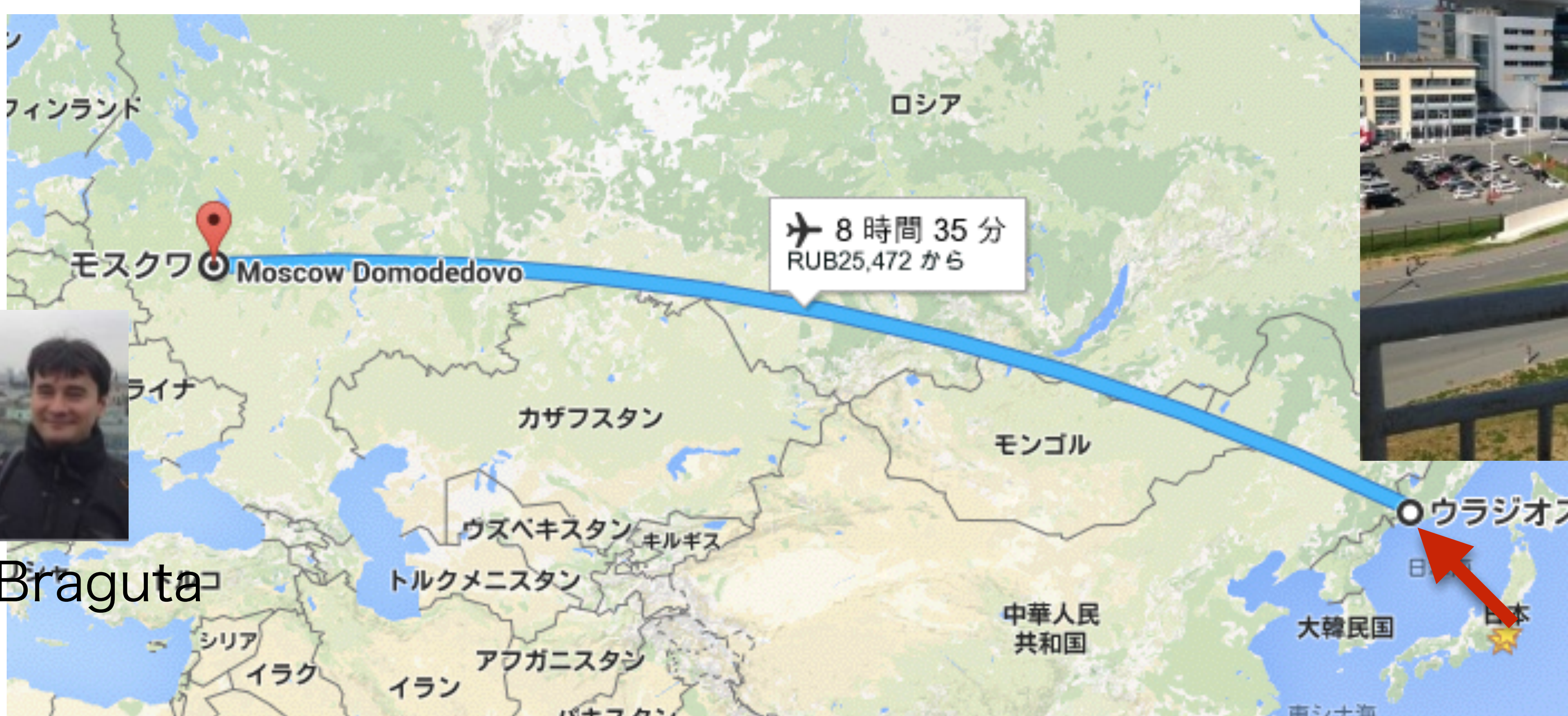
- FEFU Group, Zn Collaboration
- Transport Coefficients
  - Motivation
  - Formulation
  - Difficult Points
- Finite Density
  - Sign Problem
  - Canonical Approach
    - Zn collabartion and FEFU collaboration
  - ~~Experimental Data~~ → One summary Slied
  - Lee-Yang Zeros ← If the time allows
- Summary

FEFU
FAR EASTERN FEDERAL UNIVERSITY



in Collaboration with
V. Bornyakov, D. Boyda, M. Chernodub, V. Goy, A. Molochkov,
A. Nikolaev and V. I. Zakharov

Our GPU machine

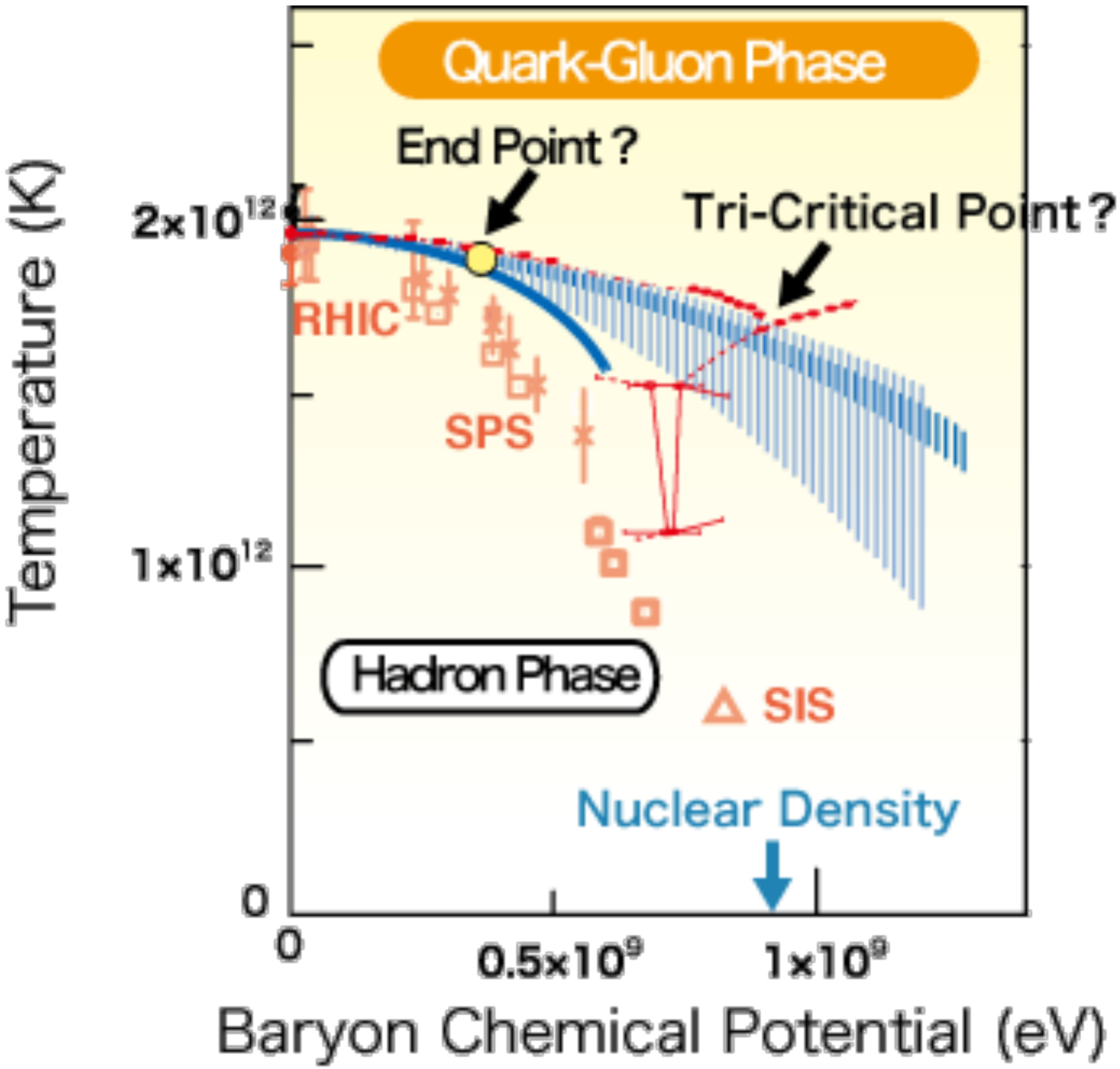


V.Braguta

Zn Collaboration

R.Fukuda (Tokyo), S. Oka(Rikkyo),
S.Sakai (Kyoto), A.Suzuki, Y. Taniguchi (Tsukuba)
and A.N.

JHEP02(2016)054 (arXiv:1504.04471)
arXiv:1504.06351



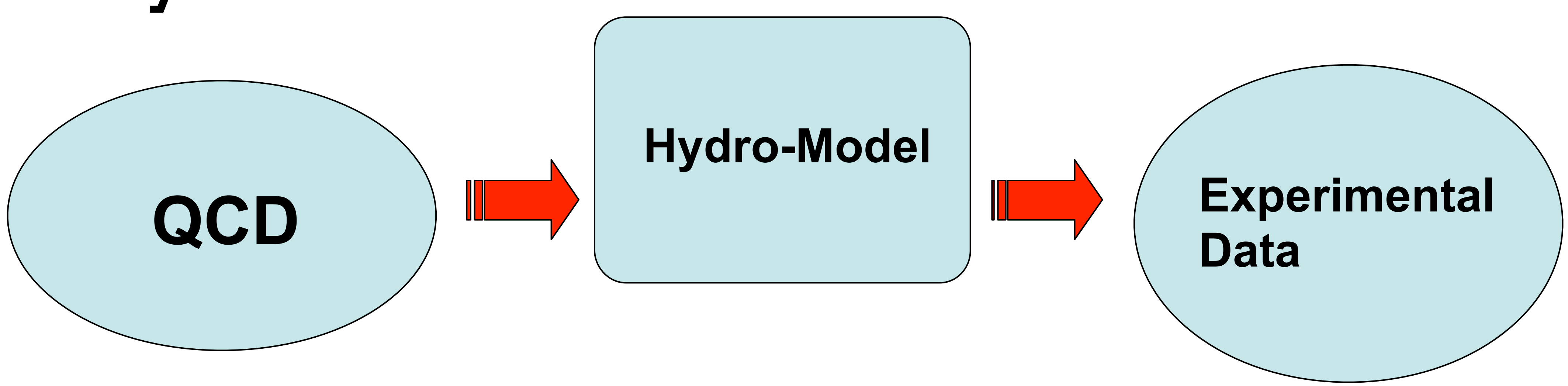
- Experiments
- Lattice
- Neutron Star

Story of Transport Coefficients

- Fighting against
  - Noise
  - Determine Spectral Functions

Transport Coefficients

- A Step towards Gluon Dynamical Behavior.
- They are (in principle) calculable by a well established formula (Linear Response Theory).
- They are important to understand QGP which is realized in Heavy Ion Collisions and early Universe.



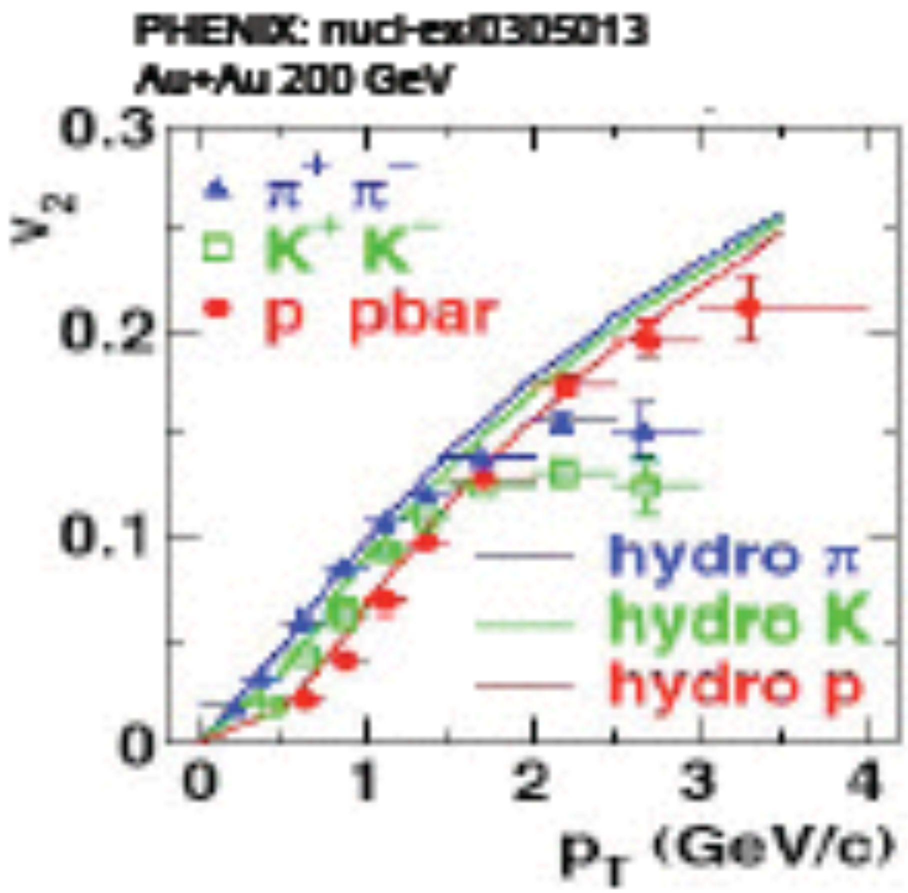
RHIC-data → *Big Surprise !*

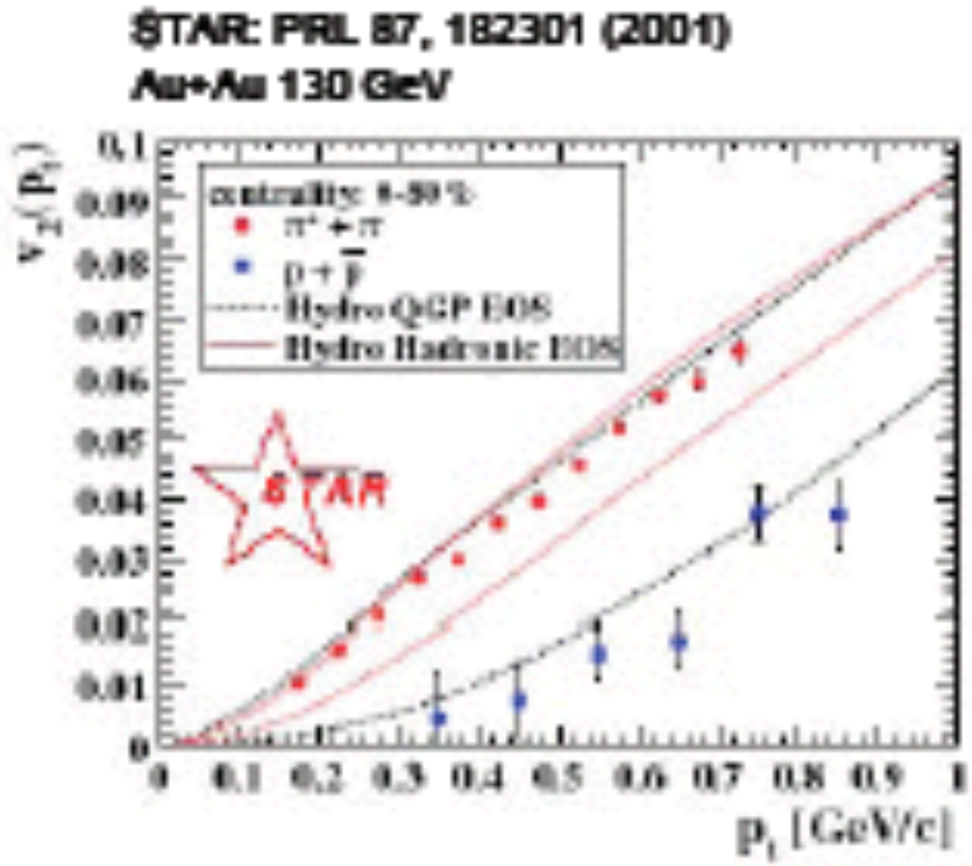
Hydro-dynamical
Model describes
RHIC data well !

At SPS, the Hydro describes well one-particle distributions,

HBT etc., but fails for the elliptic flow.

Hydro describes well v2





Hydrodynamical calculations are based on Ideal Fluid, i.e., zero shear viscosity.

Or not so surprise …

- E. Fermi, Prog. Theor. Phys. 5 (1950) 570
  - Statistical Model
- S.Z.Belen'skji and L.D.Landau, Nuovo.Cimento Suppl. 3 (1956) 15
  - Criticism of Fermi Model

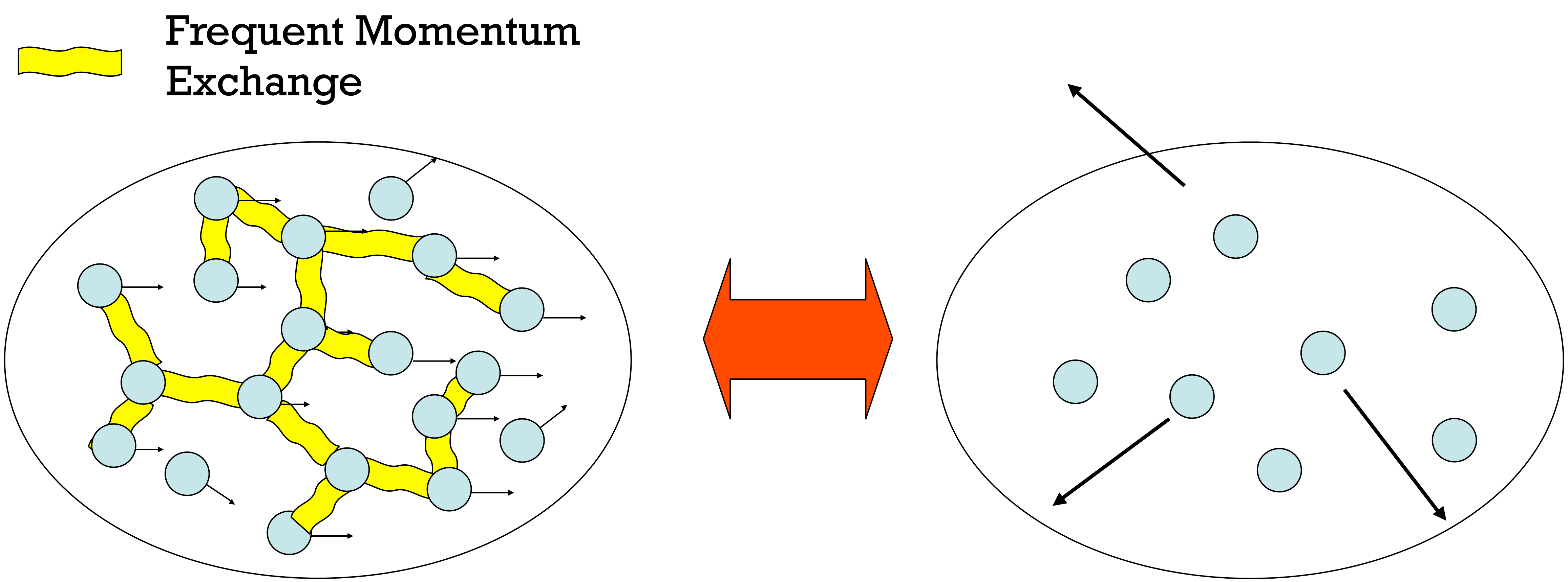
    "Owing to high density of the particles and to strong interaction between them, one cannot really speak of their number."

Hagedorn, Suppl. Nuovo Cim. 3 (1956) 147. Limiting Temperature

Another Big Surprise !

- The Hydrodynamical model assumes zero viscosity, i.e., Perfect Fluid.
- Phenomenological Analyses suggest also small viscosity.

Liquid or Gas ?

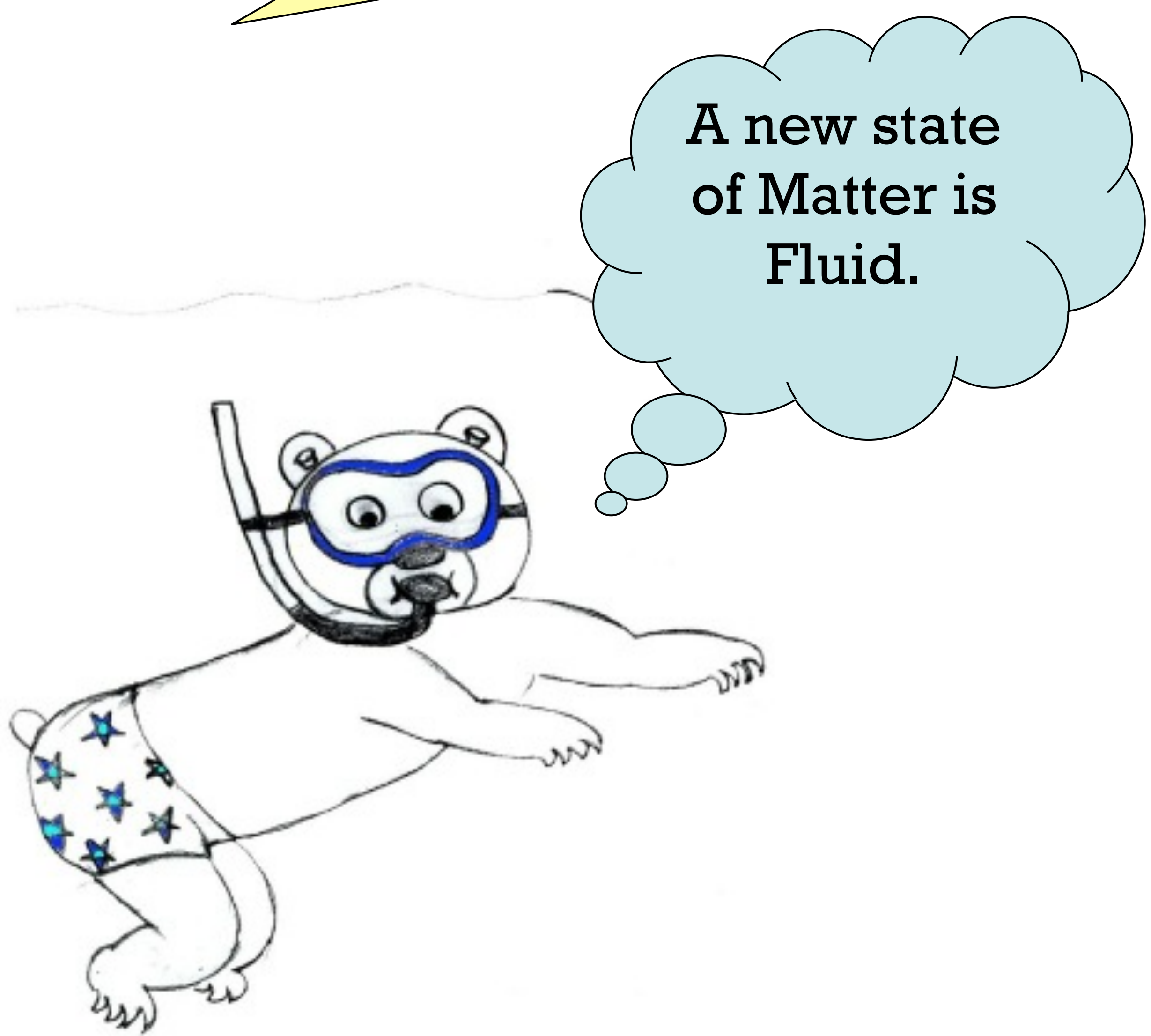


Perfect fluid

Opposite
Situation

Ideal Gas

If produced matter at RHIC is (perfect) Fluid, not Free Gas, what does it matter ?



Lowest Perturbation
(Illustration purpose only)

Pressure

$$P = \frac{\pi^2}{90} T^4 \left(1 - \frac{15}{8} \left(\frac{g}{\pi}\right)^2 + \cdots \right)$$

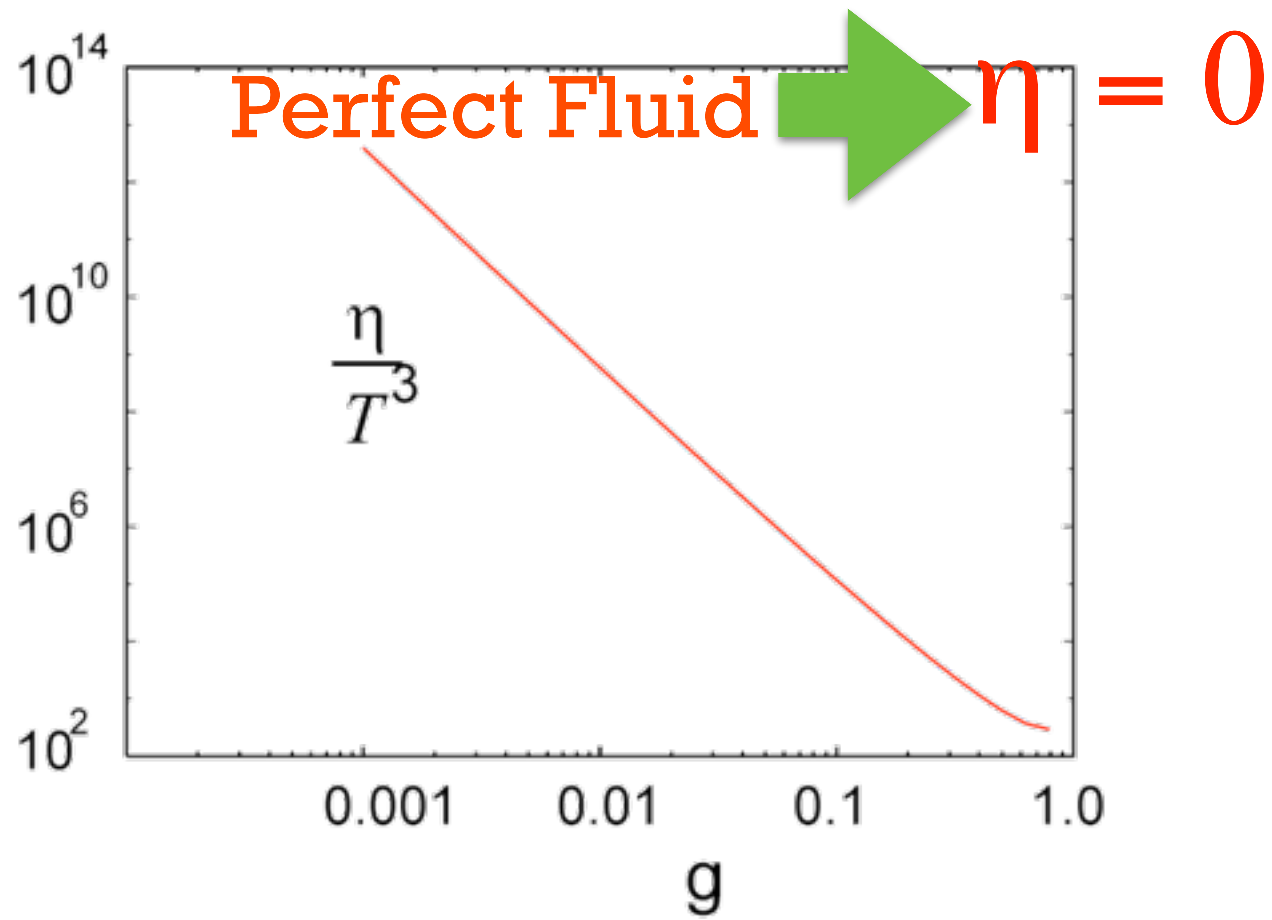
Ideal Free Gas

Viscosity

$$\eta = \kappa \frac{T^3}{g^4 \ln g^{-1}}$$

$$\kappa = 27.126 (N_f = 0), \quad 86.473 (N_f = 2)$$

- At weak coupling, it increases.



Karsch and Wyld (1987)

Masuda, Nakamura and Sakai (Lattice 95)

Sakai, Nakamura, Saito(QM97,Lattice 98)
(Improved Action)

Aarts and Martinez-Resco (2002)

Sakai, Nakamura (2004) Anisotropic Lattice
caliburation for improved gauge actions

Nakamura and Sakai (2005) η/s

Aarts, Allton, Foley, Hands, Kim (2007)

Meyer (2007) Luescher-Weiz 2-level

20th Century

21st Century

Linear Response Theory

- Zubarev
  “Non-Equilibrium Statistical Thermo-dynamics”
- Kubo, Toda and Saito
  “Statistical Mechanics”

$$\eta = -\int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1 - t)} \int_{-\infty}^{t_1} dt' < T_{12}(x,t) T_{12}(x',t') >_{ret}$$

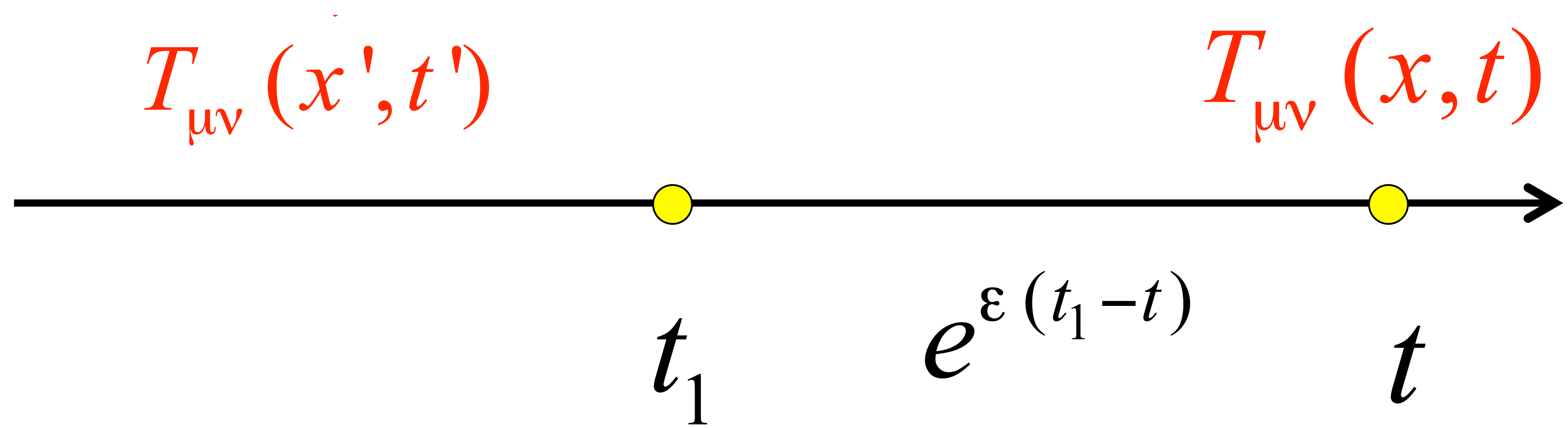
$$\frac{4}{3}\eta + \varsigma = -\int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1 - t)} \int_{-\infty}^{t_1} dt' < T_{11}(x,t) T_{11}(x',t') >_{ret}$$

$$\chi = -\frac{1}{T}\int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1 - t)} \int_{-\infty}^{t_1} dt' < T_{01}(x,t) T_{01}(x',t') >_{ret}$$

η **: Shear Viscosity** ς **: Bulk Viscosity**

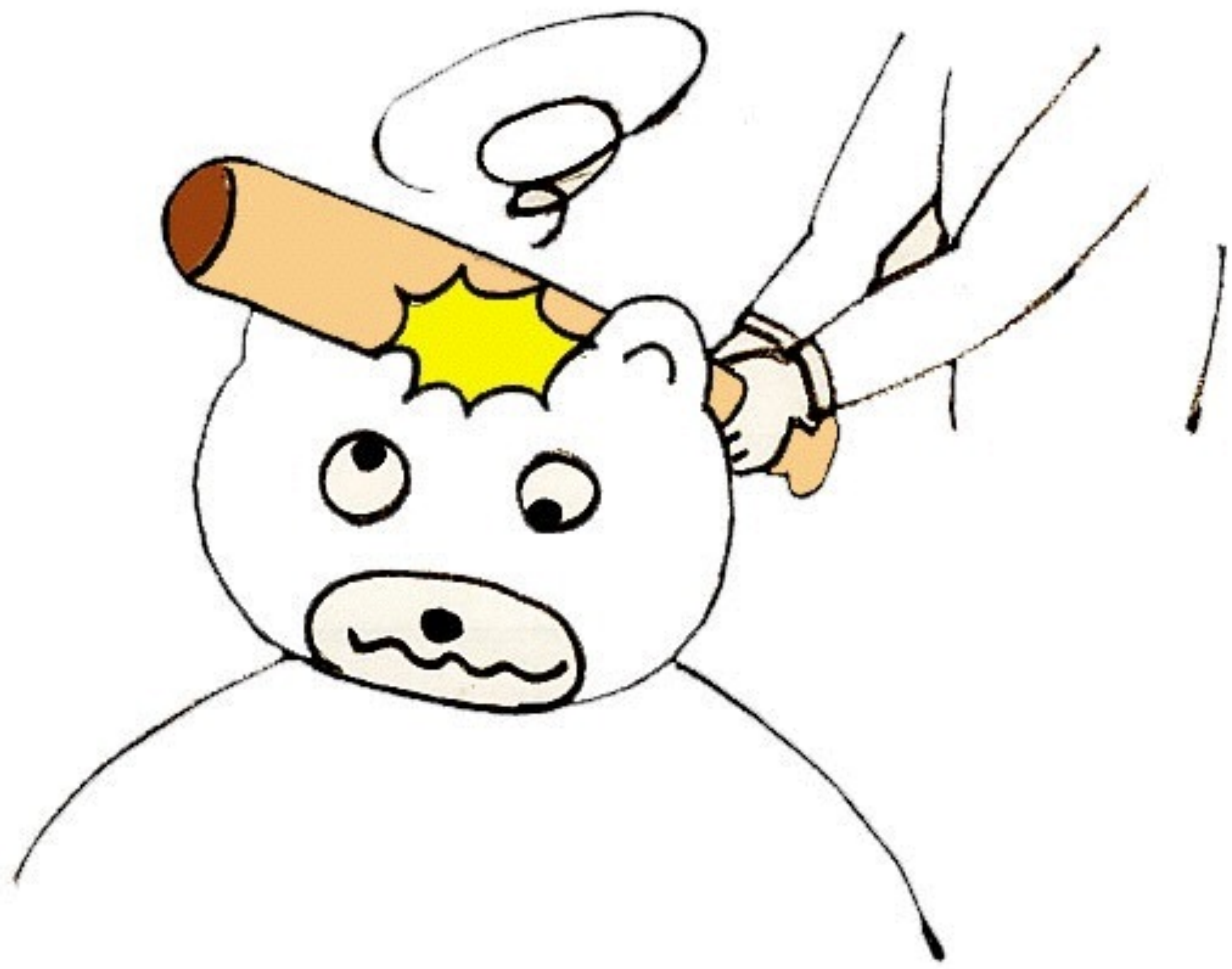
χ **: Heat Conductivity**

we do not consider this in Quench simulations.



$$-\infty < t' < t_1 < t$$

Green Functions in the above formula
are Retarded, but on Lattice you measure
Temperature Green Functions !



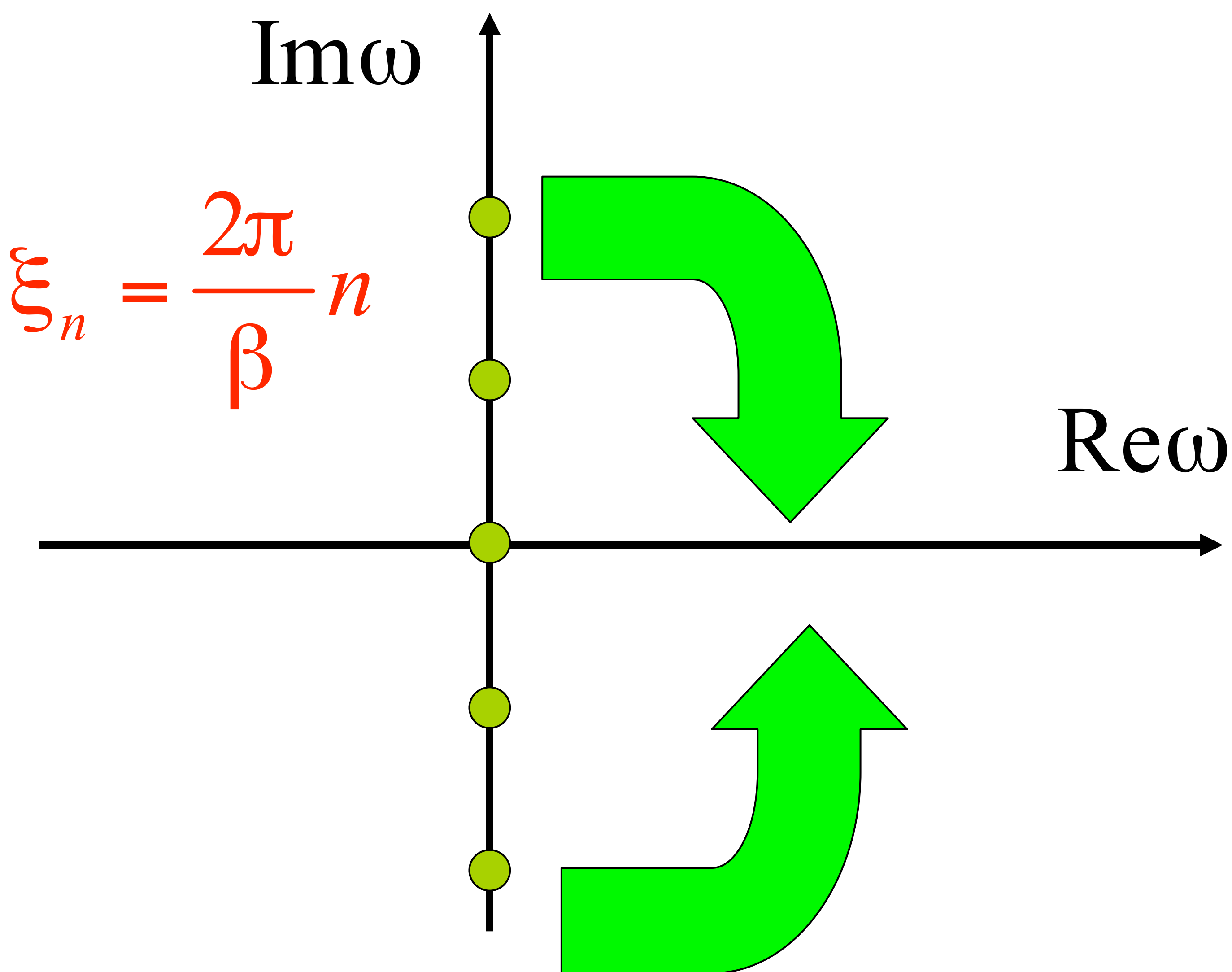
Abrikosov-Gorkov-Dzyalosinski-Fradkin Theorem

Kernel

Green Function → $$G(t) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau, \omega)\rho(\omega)$$

Spectral Function

$$\hat{K}_\beta(\xi_n) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\Lambda(\omega)}{\omega - i\xi_n} = iK_\beta(i\xi_n)$$



On the lattice, we measure Temperature Green function at

$$\omega = \xi_n$$

We must reconstruct Advance or Retarded Green function.

Transport Coefficients of QGP

We measure Correlations of Energy-Momentum tensors

$$< T_{\mu\nu}(0) T_{\mu\nu}(\tau) >$$

Convert them (Matsubara Green Functions) to Retarded ones (real time).

Transport Coefficients (Shear Viscosity, Bulk Viscosity and Heat Conductivity)

Still difficult to determine Spectral Function from Lattice data
Ansatz for the Spectral Functions

We measure Matsubara Green Function on Lattice (in coordinate space).

$$\langle T_{\mu,\nu}(t,\vec{x})T_{\mu,\nu}(0,\vec{0})\rangle = G_\beta(t,\vec{x}) = F.T.G_\beta(\omega_n,\vec{p})$$

$$G_\beta(\vec{p},i\omega_n) = \int d\omega \frac{\rho(\vec{p},\omega)}{i\omega_n - \omega}$$

We assume (Karsch-Wyld)

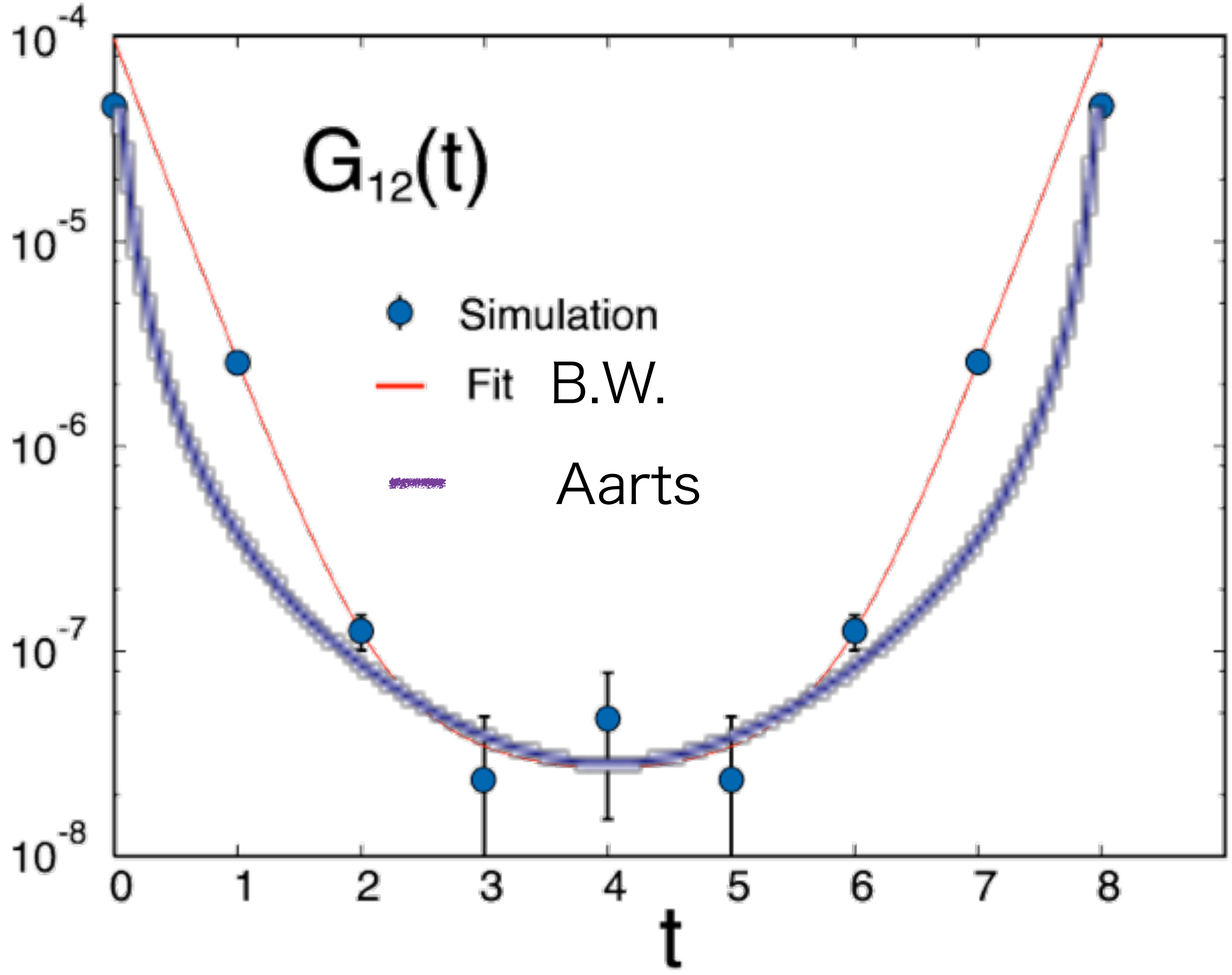
$$\rho = \frac{A}{\pi}\left(\frac{\gamma}{(m-\omega)^2+\gamma^2} + \frac{\gamma}{(m+\omega)^2+\gamma^2}\right)$$

and determine three parameters, A, m, γ.

Spectral Functions at Market

- Breit-Wigner ⟵ We use this
- Weak coupling
  - Aarts and Resco, JHEP 053,(2002) (hep-ph/0203177)
- Holography
  - Teaney, Phys. Rev. D74 (2006) 045025 (hep-ph0602044)
  - Myers, Starinetsa and Thomsona, JHEP 0711:091,2007(hep-th0706.0162)
  -)

Nt=8



Lattice and Statistics

Iwasaki Improved Action

$16^3 \times 8$

β=3.05 : 1.3M sweeps → β=3.05 : 3.0M sweeps
β=3.20 : 1.2M sweeps → β=3.20 : 2.5M sweeps
β=3.30 : 1.3M sweeps → β=3.30 : 2.0M sweeps

Crazy !

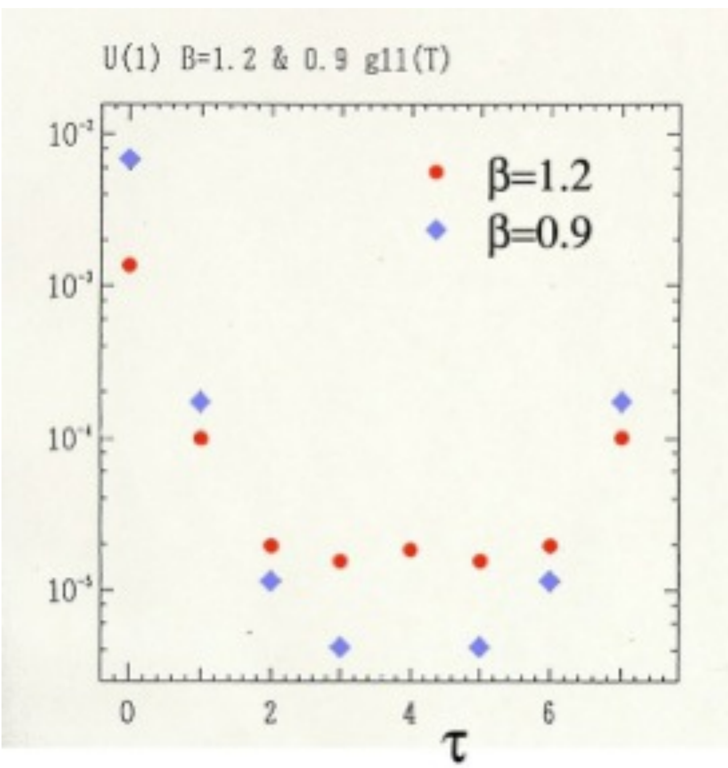
$24^3 \times 8$

β=3.05 : 0.6M sweeps → β=3.05 : 6.0M sweeps
β=3.30 : 0.8M sweeps → β=3.30 : 6.0M sweeps

Quench

History

1995



U(1)
Coulomb and Confinement Phases

1995

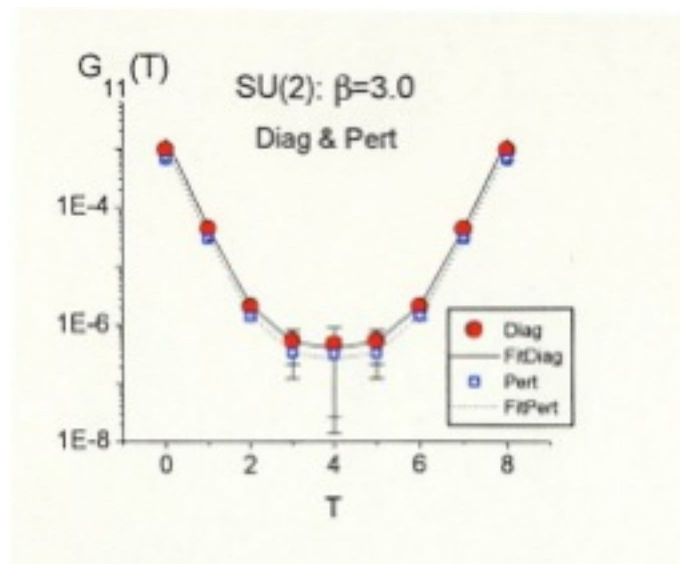


Figure 3. Fit of $G_{11}(T)$ by the parameters of spectral function for SU(2) at $\beta = 3.0$

SU(2)

Two Definitions:

F=log U

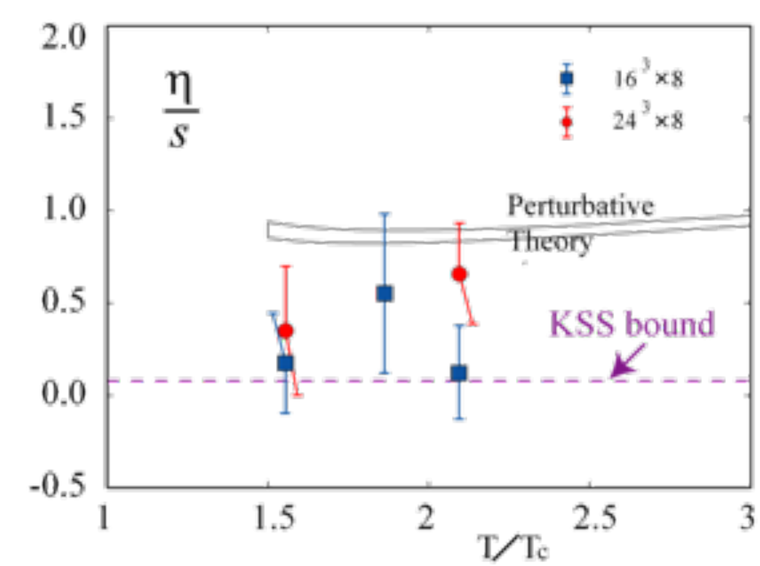
F=U-1

1998

SU(3)

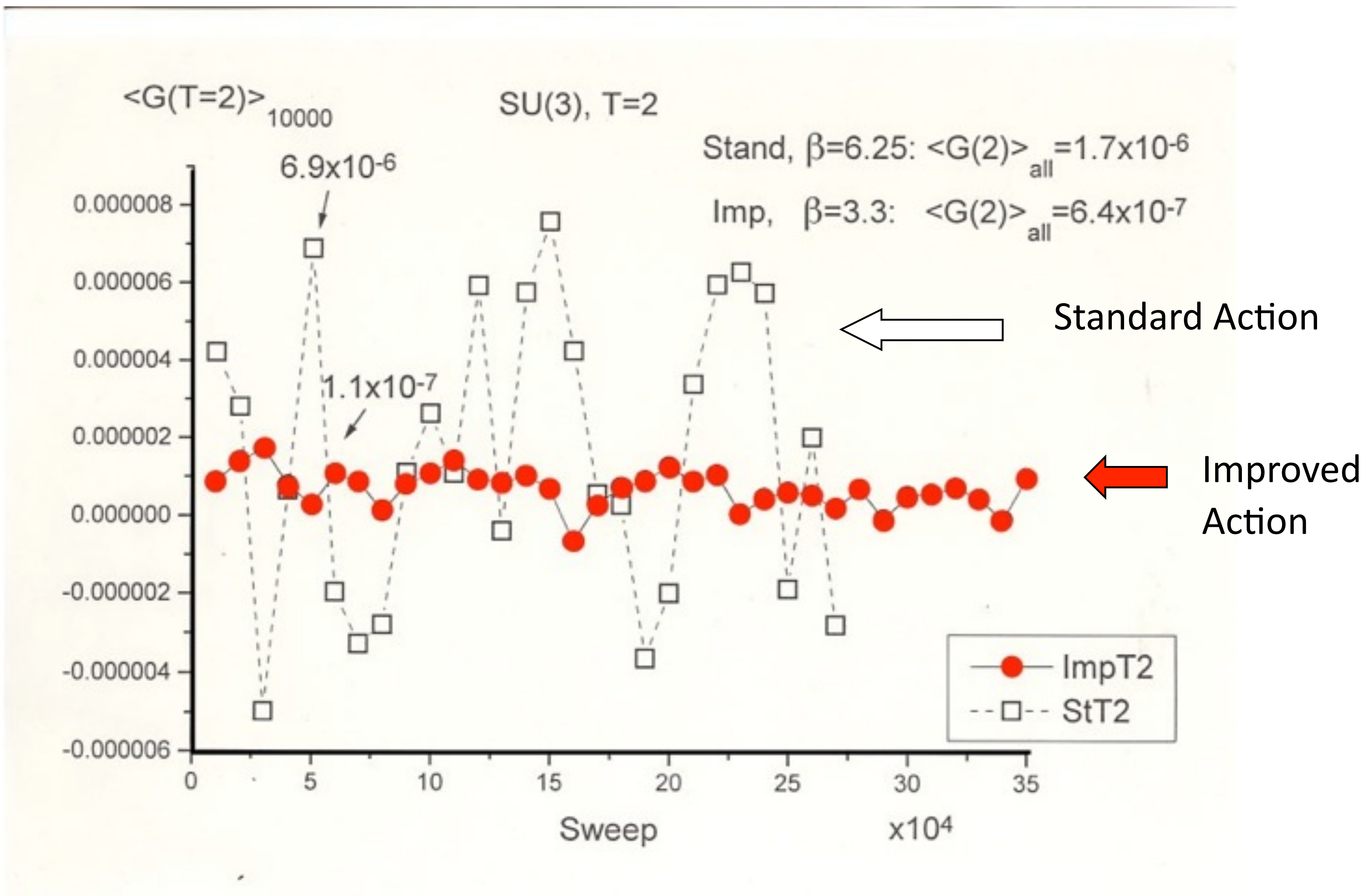
Improved Action

2005

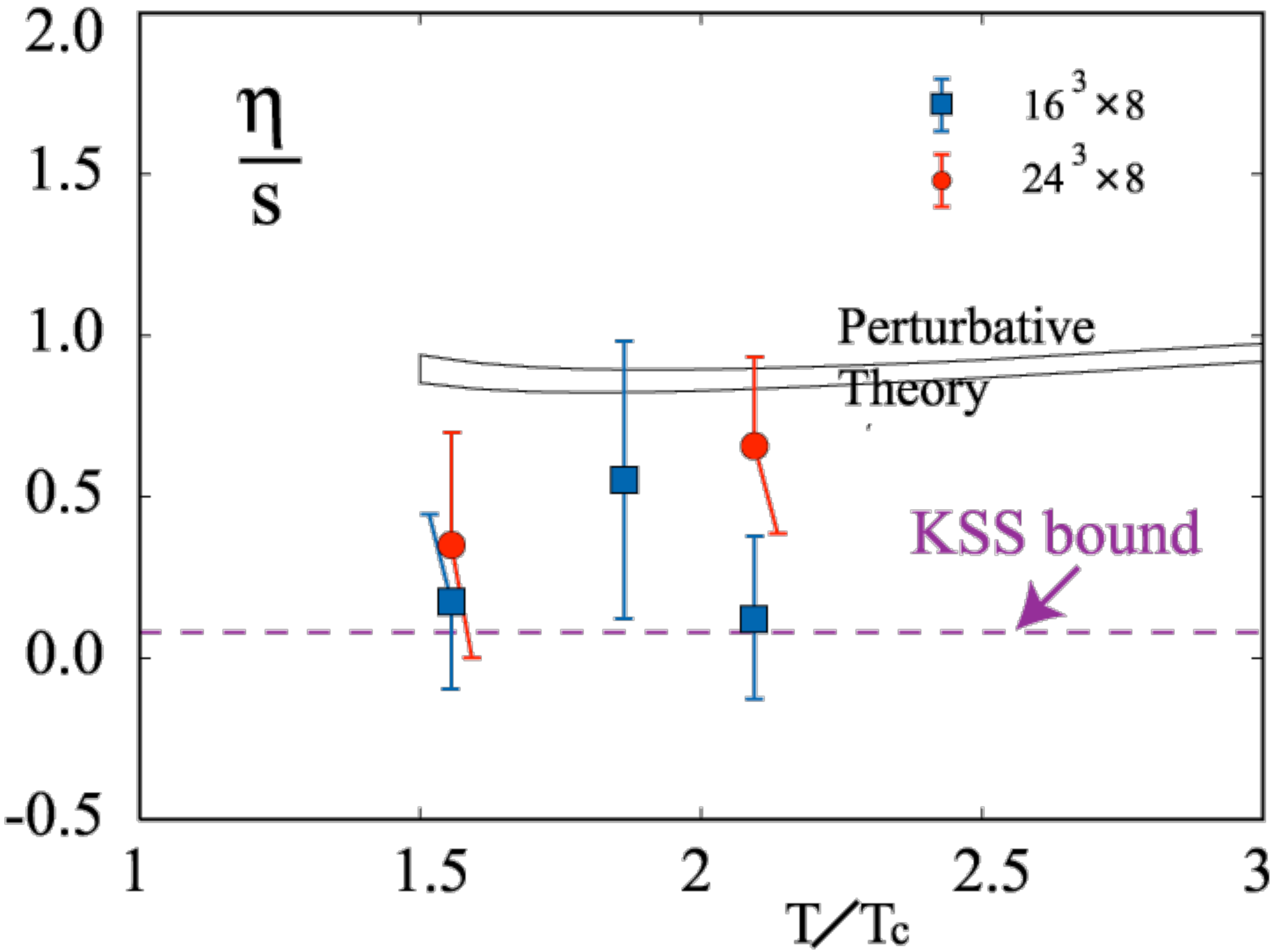


The first calculation of eta/s on the lattice, which is consistent with KSS bound

Fluctuations in MC sweeps

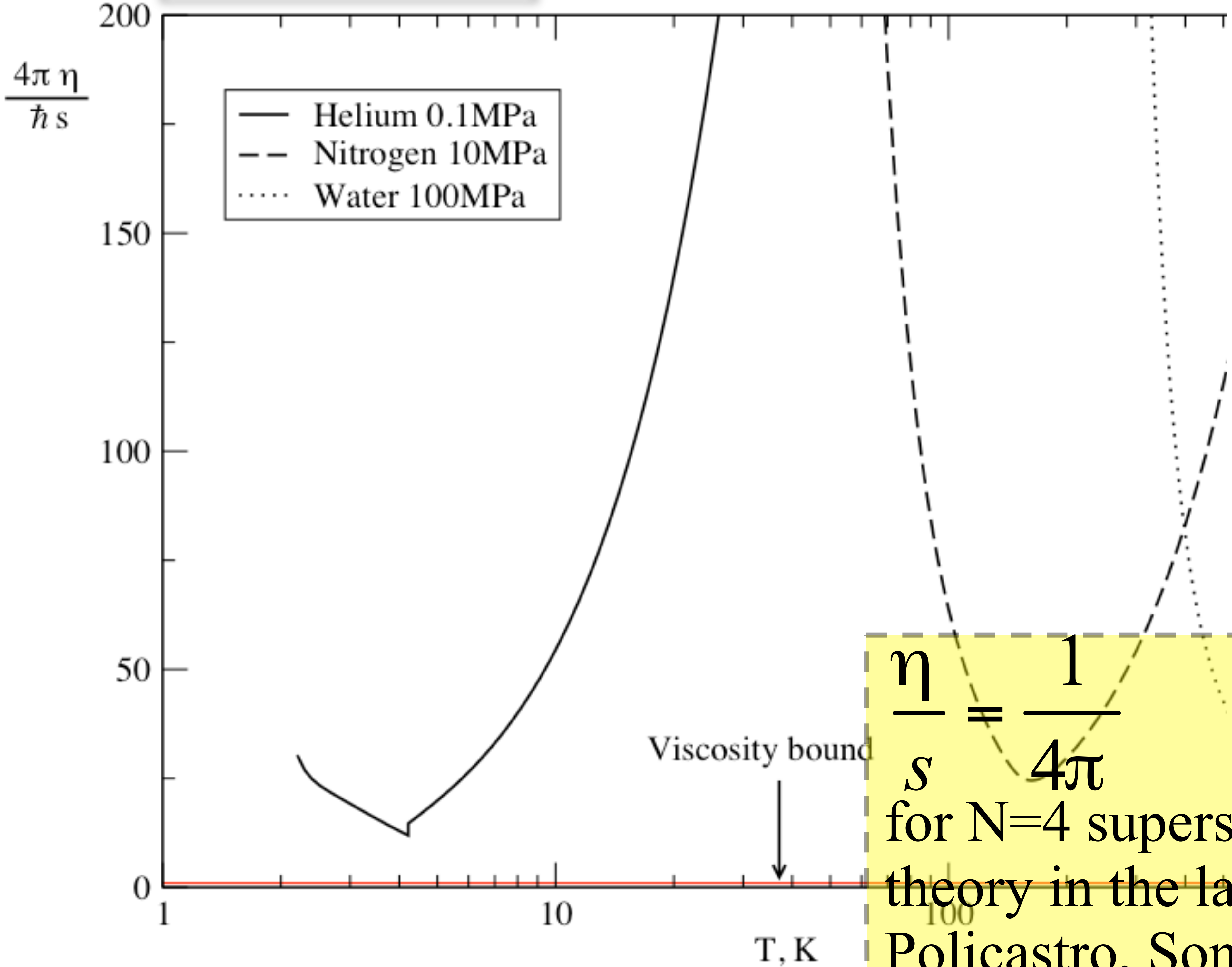


Nakamura and Sakai, 2005



$$\frac{\eta}{s} \geq \frac{1}{4\pi} \;!$$

Kovtun, Son and Starinets, hep-th/0405231



$$\frac{\eta}{s} = \frac{1}{4\pi}$$

for N=4 supersymmetric Yang-Mills theory in the large N.

Policastro, Son and Starinets, Phys. Rev. Lett. 87 (2001) 081601

How to reduce Noise ?

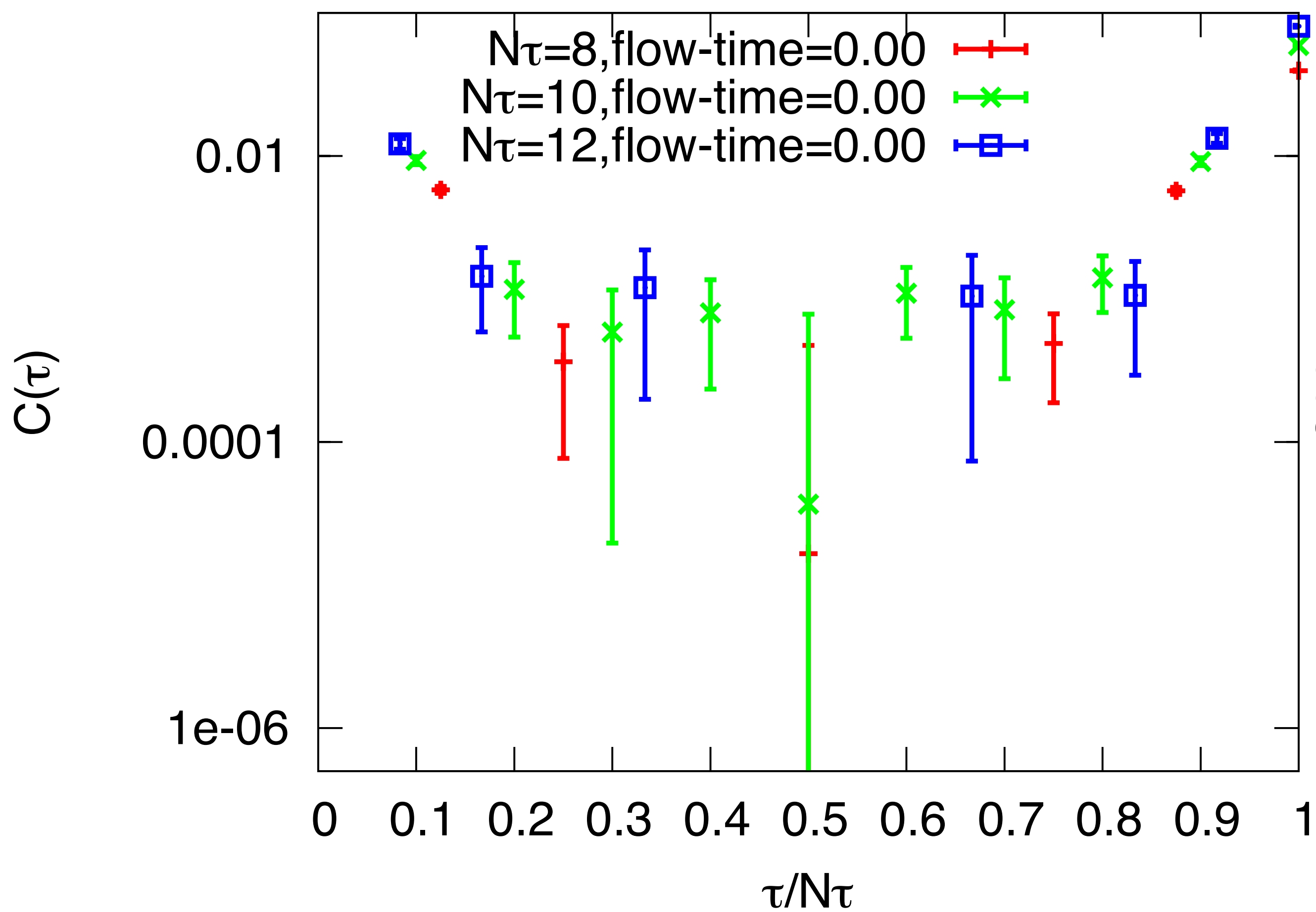
- Improved Actions
- Multi-hit (Luescher-Weiss)
- Source method (Parisi)
- Gradient Flow (Luescher)

E.Itou
Talk at FEFU

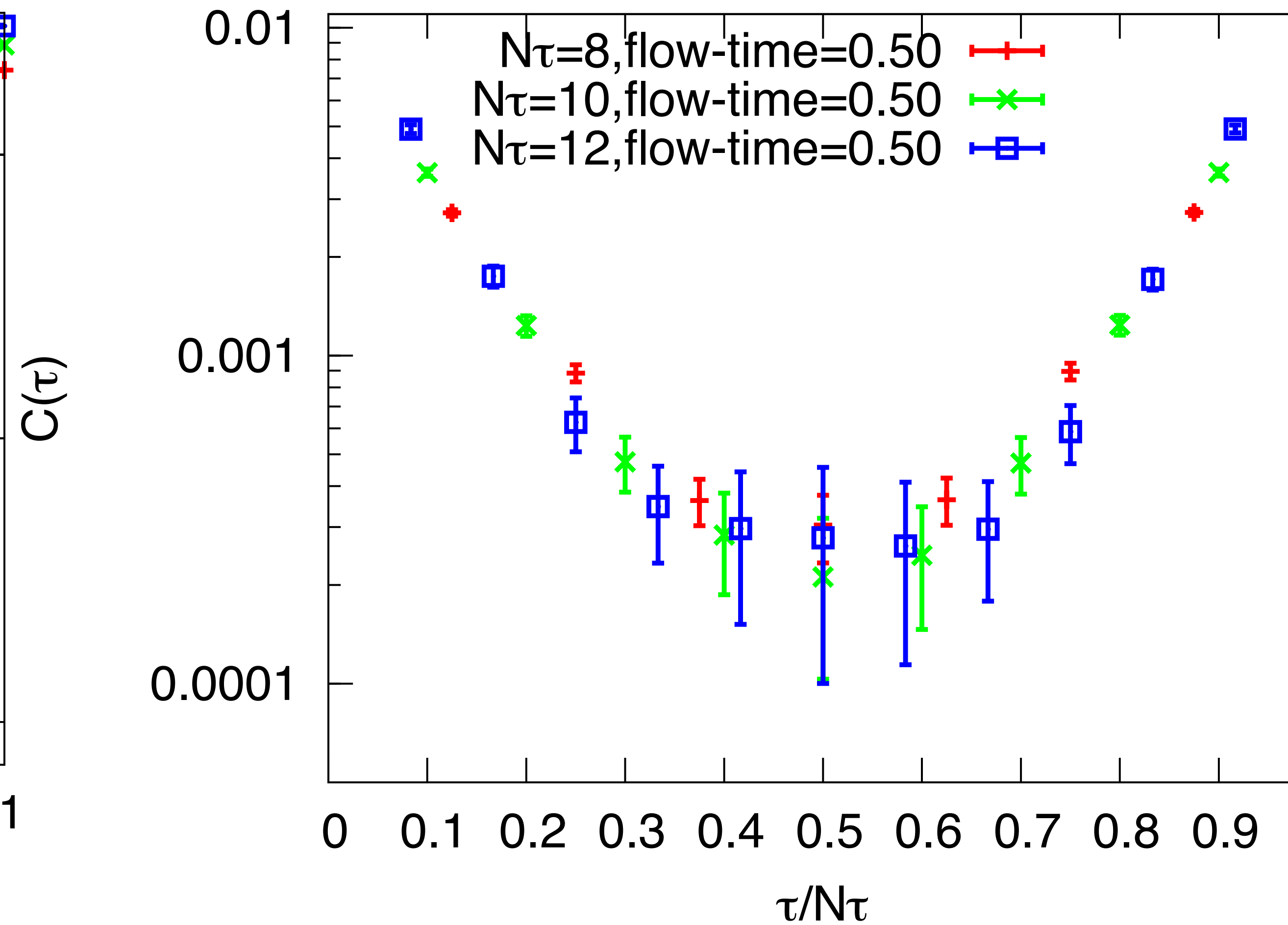
lattice raw data

$$C(\tau) = \langle \frac{1}{N_s^3} \sum_{\vec{x}} U_{12}(\vec{x}, \tau) \frac{1}{N_s^3} \sum_{\vec{y}} U_{12}(\vec{y}, 0) \rangle$$

flow-time=0



flow-time t/a^2=0.50



fixed smeared length in lattice unit

beta=6.40,Nt=8, 2,000 conf.

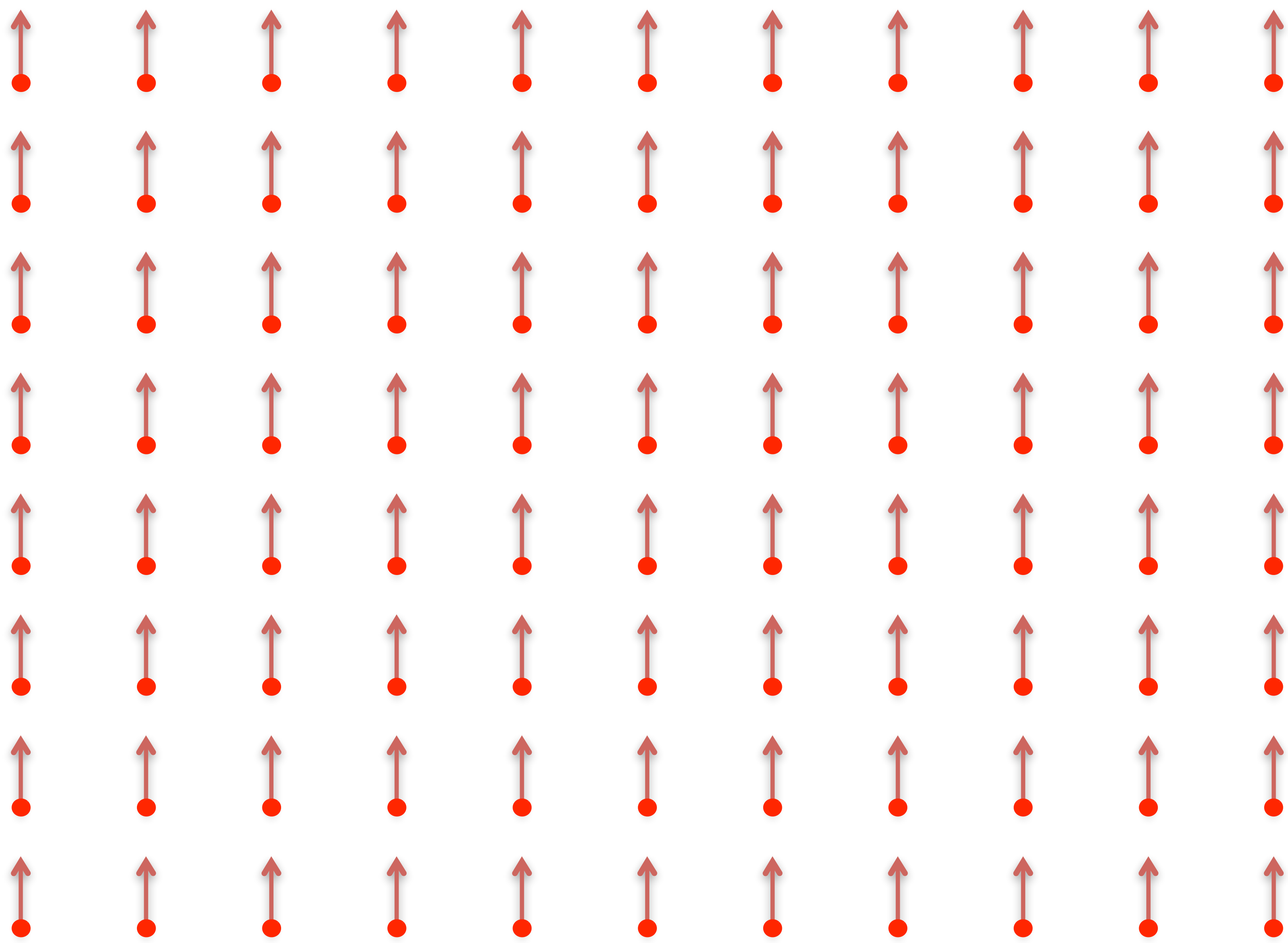
Magnetic Degrees of Freedom

- G.t'Hooft, Nucl.Phys. B190 (1981) 455
- H Shiba and T Suzuki. Phys. Lett. B (1994) 461
- A. Di Giacomo and G. Paffuti, Phys.Rev.D56,6816 (1997)
- Kei-ichi Kondo, Phys.Rev.D58,105019 (1998)
- …..
- J. Liao and E. Shuryak, Phys.Rev.Lett.,101, 162302 (2008)
- M.N. Chernodub and V.I. Zakharov, Phys. Rev. Lett.98, 082002 (2007)
- M.N. Chernodub, A. Nakamura and V.I. Zakharov Phys.Rev.D78:074021,2008
- M.N. Chernodub and V.I. Zakharov, Phys.Atom.Nucl. 72:2136-2145,2009 (arXiv:0806.2874)

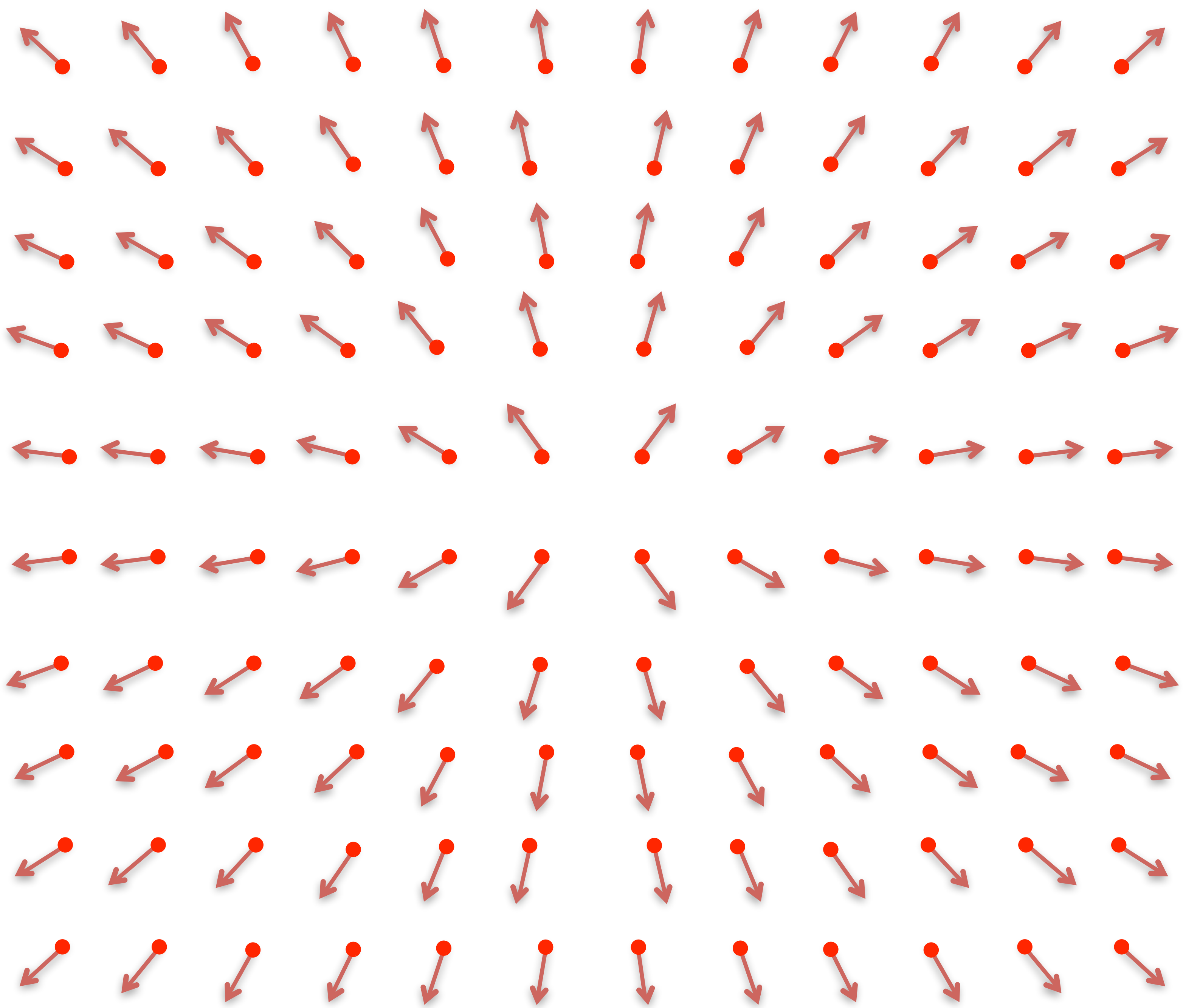
Who has seen the Mag. Monopole ?

Neither I nor you

Spin System



Singular Configuration, or Vortex



Center Projection

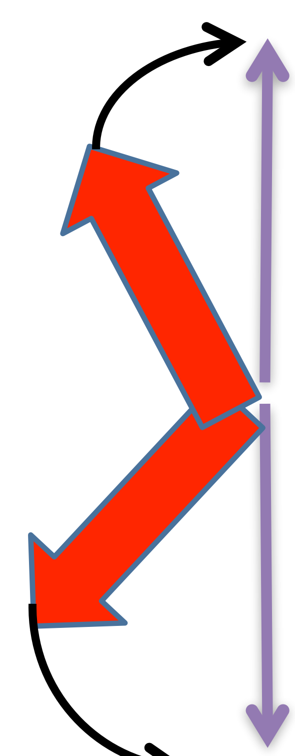
Del Debbio, Faber, Giedt, Greensite, Olejnik
Phys.Rev. D58, 1998, 094501

$$\mathrm{Max} \sum_{x,\mu} \mathrm{Tr}\, U_\mu(x) \;\Rightarrow\; \mathrm{Max} \sum \mathrm{Tr} \left(U_\mu\right)^2$$

↑

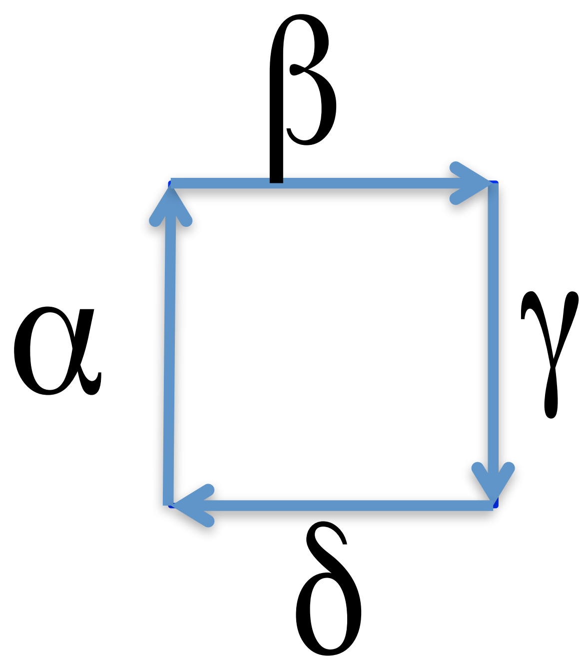
Landau gauge or
Coulomb Gauge

$$Z_\mu(x) \equiv \mathrm{sign}\, \mathrm{Tr}\, U_\mu(x) = +1 \text{ or } -1$$



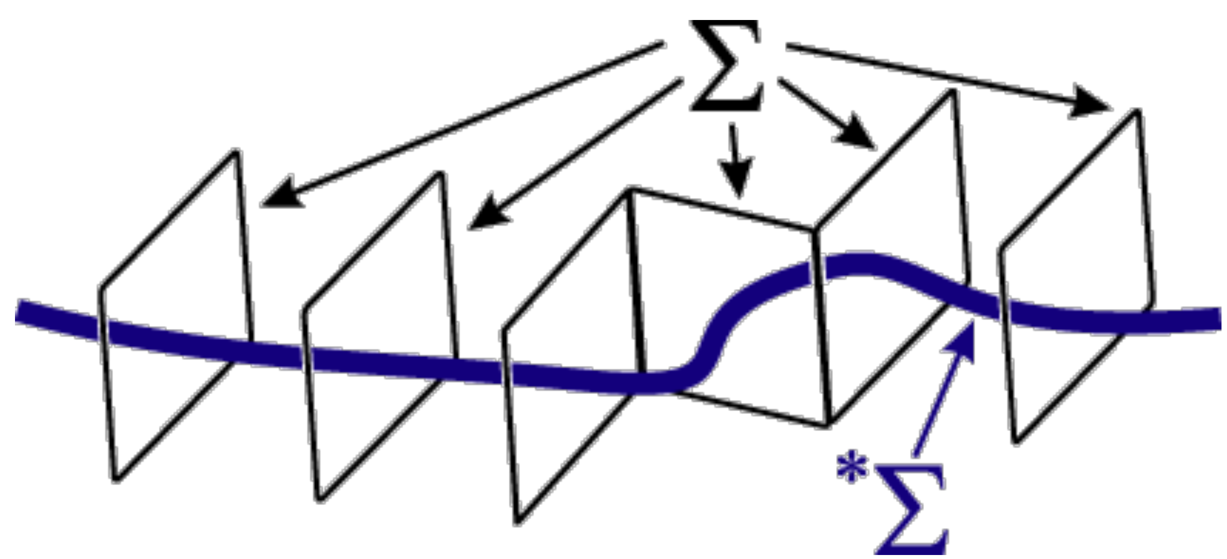
Gauge Rotation.
Therefore non-local

Plaquette pierced by a Vortex



$$\text{If } Z_\alpha \times Z_\beta \times Z_\gamma \times Z_\delta = -1$$

A Vortex pierces the Plaquette.



1-d Object (Charge) → Wilson Loop

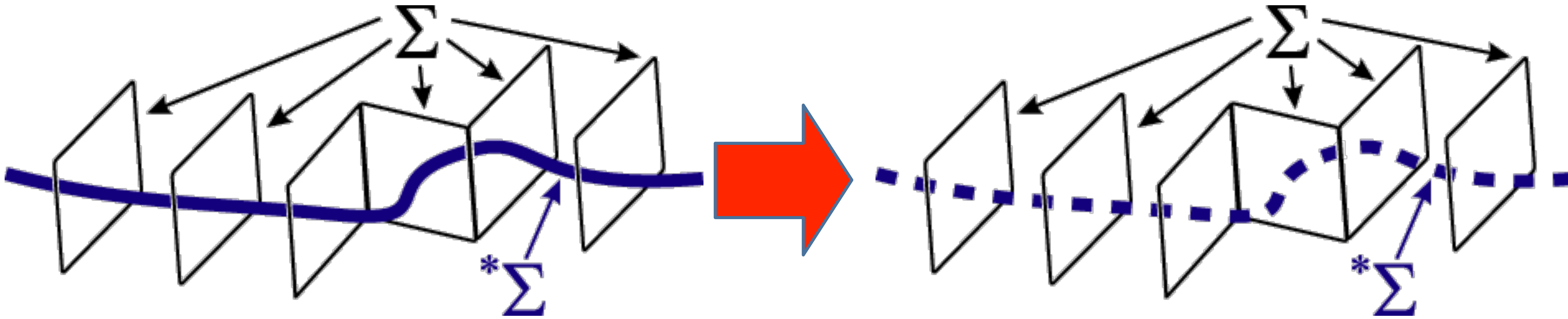
2-d Object (Vortex Line) → Wilson Loop

Vortex Removing

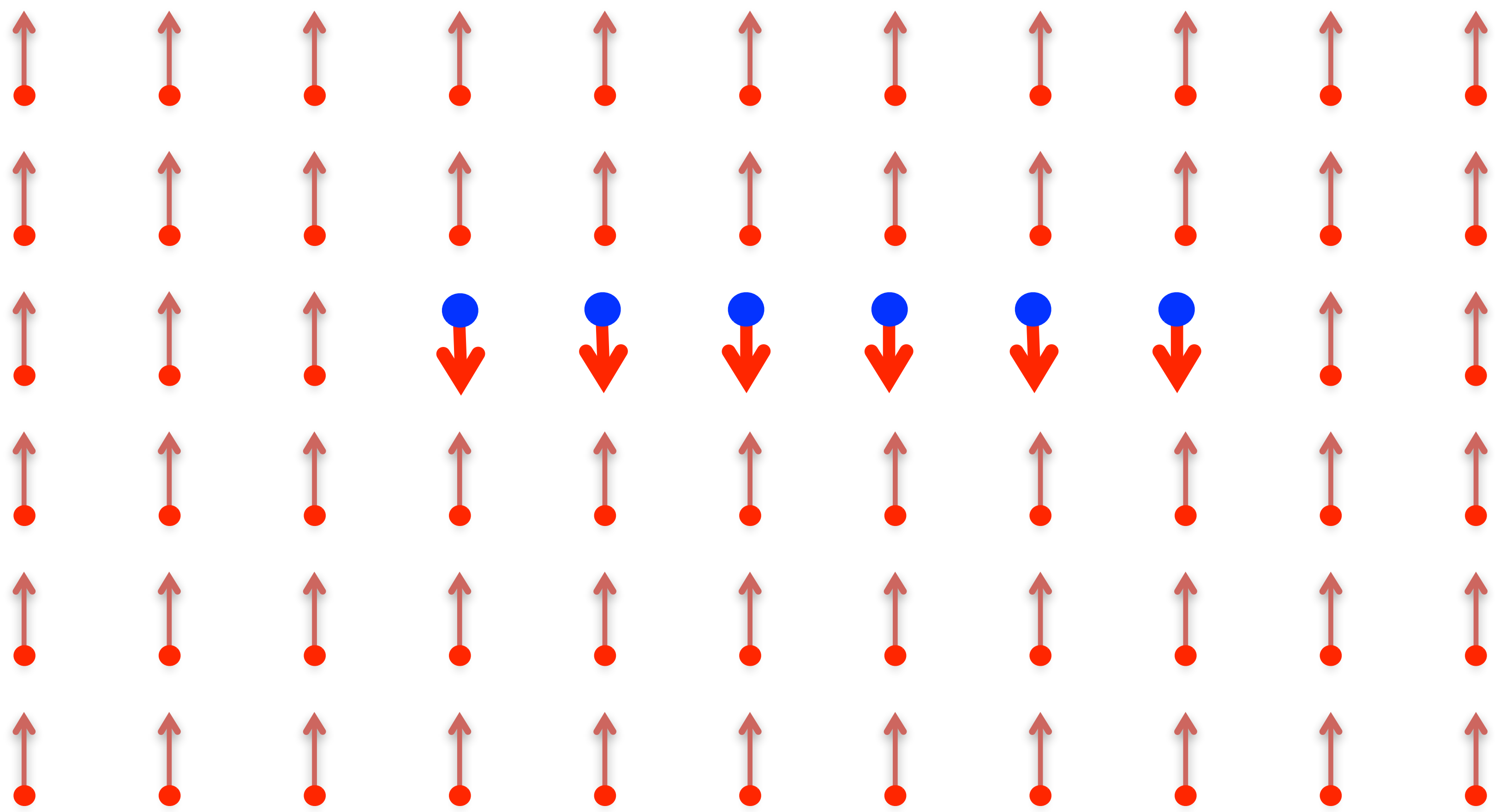
$$U_\mu(x) \Rightarrow Z_\mu(x) \times U_\mu(x)$$

Remember that $Z_\mu(x) \equiv \mathrm{sign}\,\mathrm{Tr}\,U_\mu(x)$

By definition, now $\mathrm{sign}\,\mathrm{Tr}\,U_\mu(x) = +1$ for all links.

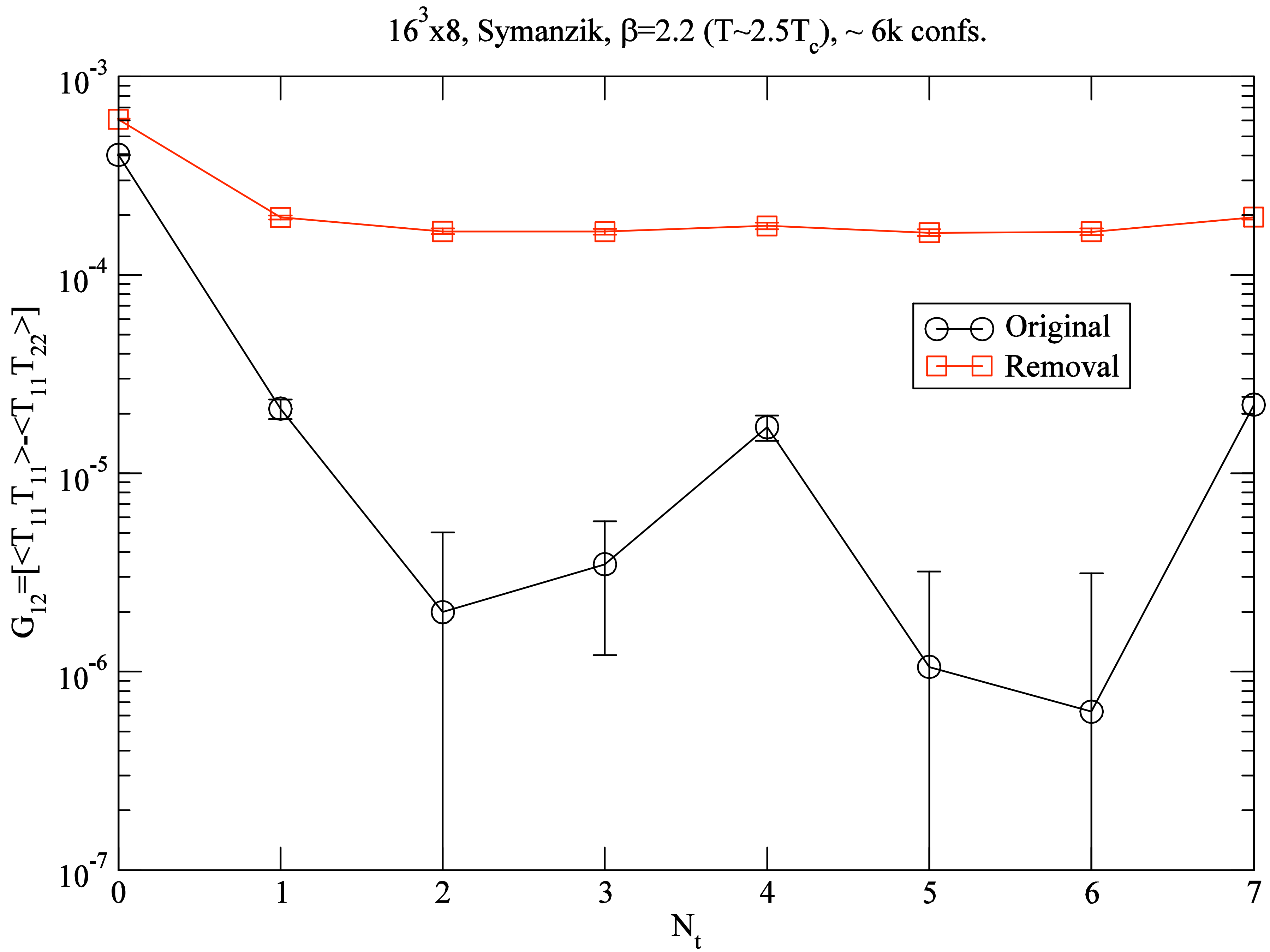


All vortices are fading out (by definition).



$$\langle T_{12}(0)T_{12}(t)\rangle$$

Improve Action
(Symanzik)



$\beta = 2.2 \;\; (T/T_c \approx 2.5)$

$16^3 \times 8$

$SU(2)$

60000×100 Sweeps

Finite Density LatticeQCD

Brief History

- 1984 SU(2)
  A.Nakamura, Phys. Lett. 149B (1984) 391
- 2001Taylor Expansion
  QCD-TARO Collaboration: S. Choe, Ph. de Forcrand, M. Garcia Perez, S. Hioki, Y. Liu,H. Matsufuru, O. Miyamura, A. Nakamura, I. -O. Stamatescu, T. Takaishi, T. Umeda,Phys. Rev. D65, 054501 (2002)
- 2002 Multi-Parameter Reweighting
  Z. Fodor, S. D. Katz, JHEP 0203 (2002) 014, (hep-lat/0106002).
- 2002 Multi-Parameter Reweighting+Taylor Expantion
  C. R. Allton, S. Ejiri, S. J. Hands, O. Kaczmarek, F. Karsch, E. Laermann, Ch. Schmidt, L. Scorzato (Bielefeld-Swansea), Phys. Rev. D66 074507 (2002), (hep-lat/0204010).
- 2002 Imaginary Chemical Potential
  M. D'Elia, M. P. Lombardo, Proceedings of the GISELDA Meeting held in Frascati, Italy, 14-18 January 2002, hep-lat/0205022.
- 2002 Imaginary Chemical Potential
  Ph. de Forcrand, O. Philipsen, Nucl. Phys. B642 290 (2002), hep-lat/0205016.

Sign Problem

QCD at finite density

$$Z = \mathrm{Tr}\, e^{-\beta(H-\mu N)} = \int \mathcal{D}U \mathcal{D}\bar{\psi} \mathcal{D}\psi \, e^{-\beta S_G - \bar{\psi}\Delta\psi}$$

$$= \int \mathcal{D}U \prod_f \det \Delta(m_f)\, e^{-\beta S_G}$$

$$\Delta(\mu) = D_\nu \gamma_\nu + m + \mu\gamma_0$$

$$\Delta(\mu)^\dagger = -D_\nu \gamma_\nu + m + \mu^* \gamma_0 = \gamma_5 \Delta(-\mu^*)\gamma_5$$

$$\Rightarrow (\det \Delta(\mu))^* = \det \Delta(\mu)^\dagger = \det \Delta(-\mu^*)$$

$$(\det \Delta(\mu))^* = \det \Delta(\mu)^\dagger = \det \Delta(-\mu^*)$$

For $\mu = 0$

$$(\det \Delta(0))^* = \det \Delta(0)$$

$\det \Delta$ → ***Real***

For $\mu \neq 0$ **(in general)**

$\det \Delta$ → ***Complex***

$$Z = \int \mathcal{D}U \prod_f \det \Delta(m_f, \mu_f) e^{-\beta S_G}$$

↑

Complex → **Sign Problem**

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}U \, O \, \det \Delta \, e^{-\beta S_G}$$

In Monte Carlo simulation, configurations are generated according to the Probability:

$$\det \Delta \, e^{-\beta S_G} / Z$$

$\det \Delta$: *Complex*

Monte Carlo Simulations very difficult !

$$\langle O \rangle = \frac{\int DU\, O\, \det\Delta e^{-S_G}}{\int DU\, \det\Delta e^{-S_G}}$$

$$\det\Delta = |\det\Delta| e^{i\theta}$$

$$\langle O \rangle = \frac{\int DU\, O |\det\Delta| e^{i\theta} e^{-S_G}}{\int DU |\det\Delta| e^{-S_G}} \times \frac{\int DU |\det\Delta| e^{-S_G}}{\int DU |\det\Delta| e^{i\theta} e^{-S_G}}$$

$$= \frac{\langle O e^{i\theta} \rangle_{|\det|}}{\langle e^{i\theta} \rangle_{|\det|}}$$

Pion-Condensation Problem

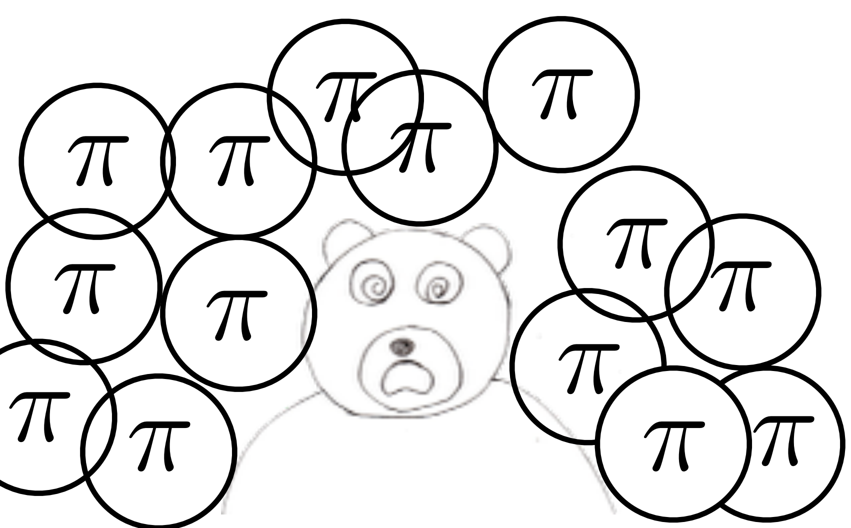
Phase Quench = Finite-Isospin

$$\int |\det \Delta(\mu)|^2 e^{-S_G} = \int \det \Delta(\mu) \det \Delta(\mu)^* e^{-S_G}$$

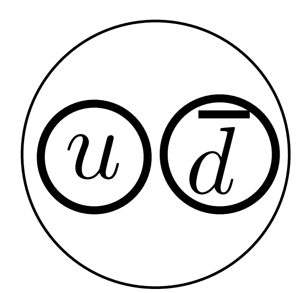
$$= \int \det \Delta(\mu) \det \Delta(-\mu) e^{-S_G}$$

$$= \int \det \Delta(\mu_u) \det \Delta(\mu_d) e^{-S_G}$$

$$\mu_u = \mu, \quad \mu_d = -\mu$$



For $\mu > \dfrac{m_\pi}{2}$



π^+ is created by μ

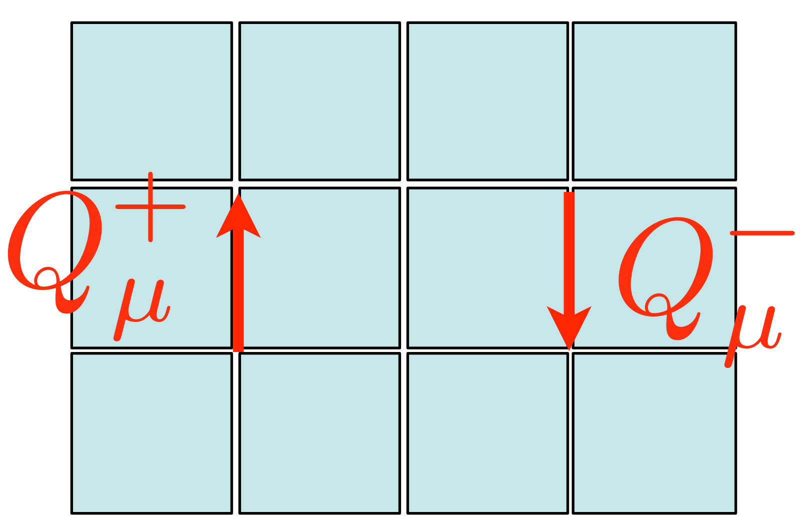
Origin of the Sign Problem

Wilson Fermions $\Delta = I - \kappa Q$

KS(Staggered) Fermions $\Delta = m - Q'$

$$= m(I - \frac{1}{m}Q)$$

$$Q = \sum_{i=1}^{3} \left(Q_i^+ + Q_i^-\right) + \left(e^{+\mu} Q_4^+ + e^{-\mu} Q_4^-\right)$$



$$Q_\mu^+ = **U_\mu(x)\delta_{x',x+\hat{\mu}}$$

$$Q_\mu^- = **U_\mu^\dagger(x')\delta_{x',x-\hat{\mu}}$$

$$\det \Delta = e^{\mathrm{Tr}\log \Delta} = e^{\mathrm{Tr}\log(I - \kappa Q)}$$

$$= e^{-\sum_n \frac{1}{n}\kappa^n \mathrm{Tr} Q^n}$$

Hopping Parameter expansion or 1/(Large Mass) expansion

Only closed loops remain.

The lowest μ dependent terms

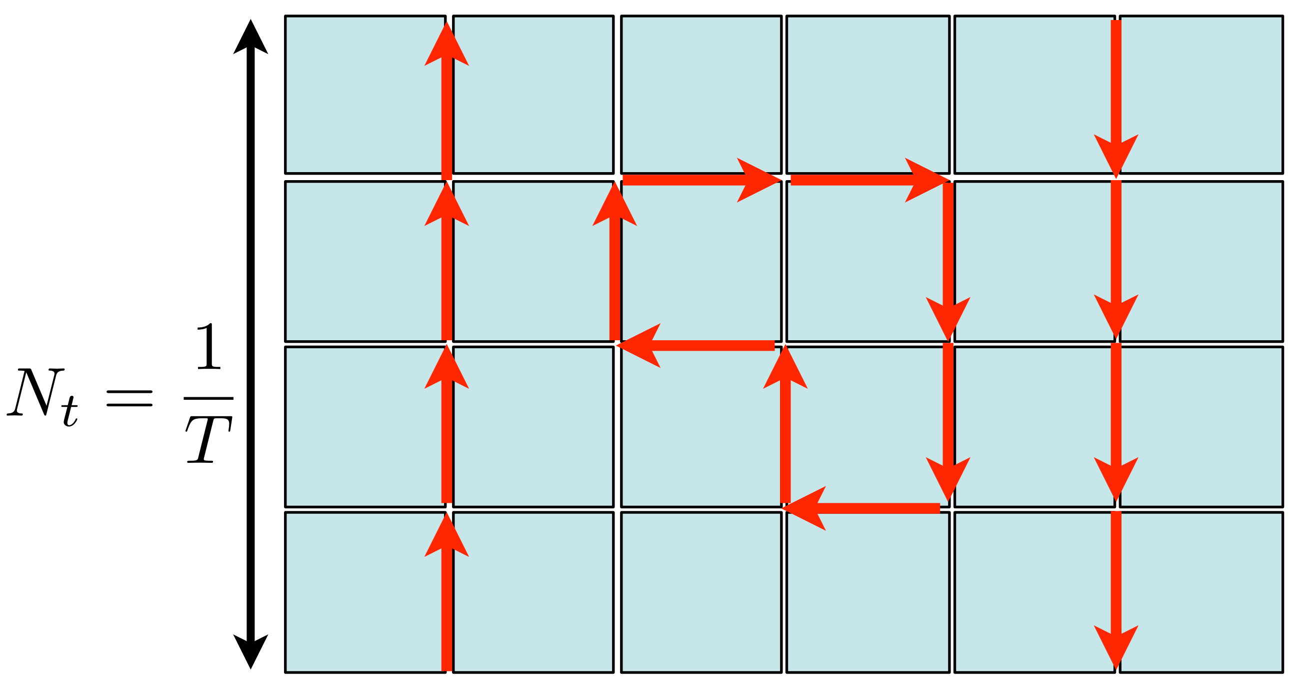
$$\kappa^{N_t} e^{\mu N_t} \mathrm{Tr}(Q^+ \cdots Q^+)$$

$$= * * \kappa^{N_t} e^{\mu/T} \mathrm{Tr} L$$

$$\kappa^{N_t} e^{-\mu N_t} \mathrm{Tr}(Q^- \cdots Q^-)$$

$$= * * \kappa^{N_t} e^{-\mu/T} \mathrm{Tr} L^\dagger$$

$\mathrm{Tr}L$: Polyakov Loop



Add the both

$$* * \kappa^{N_t} \left(\cosh \frac{\mu}{T} \Re \mathrm{Tr} L + i \sinh \frac{\mu}{T} \Im \mathrm{Tr} L \right)$$

There are several cases with no Sign Problem

- Pure Imaginal chemical potential
  - $(\det\Delta(\mu))^* = \det\Delta(-\mu^*)$

$$\mu = i\mu_I \quad \Rightarrow \quad (\det\Delta(\mu_I))^* = \det\Delta(\mu_I)$$

- Color SU(2)
  - $U_\mu^* = \sigma_2 U_\mu \sigma_2$

$$\begin{aligned}\det\Delta(U,\gamma_\mu)^* &= \det\Delta(U^*,\gamma_\mu^*) = \det\sigma_2\Delta(U,\gamma_\mu^*)\sigma_2 \\ &= \det\Delta(U,\gamma_\mu)\end{aligned}$$

- Finite iso-spin

$\mu_d = -\mu_u$

$$\begin{aligned}&\det\Delta(\mu_u)\det\Delta(\mu_d) = \det\Delta(\mu_u)\det\Delta(-\mu_u) \\ &= \det\Delta(\mu_u)\det\Delta(\mu_u)^* = |\det\Delta(\mu_u)|^2\end{aligned}$$

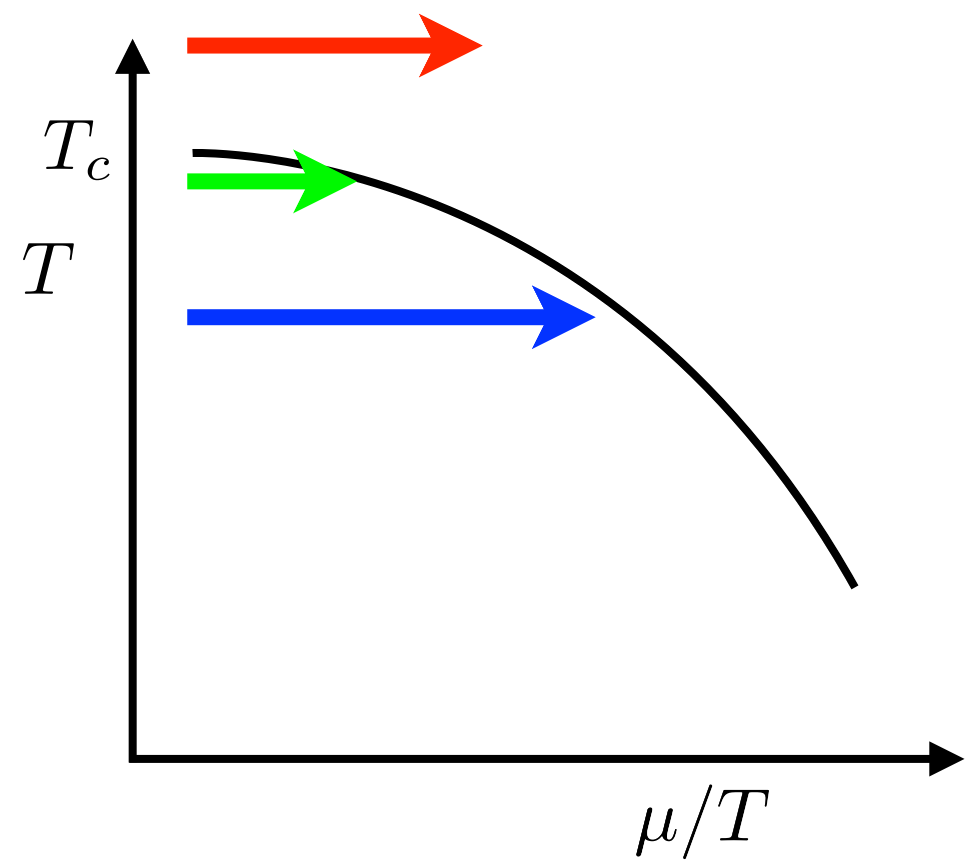
(Phase Quench)

Our Objective: Determine QCD Phase Structure by Lattice QCD

- All methods so far can not calculate large density resins either because it uses Taylor Expansion or it suffers from the overlap problem.

- Color SU(2) or other QCD-like theories are useful, but at the end they are not QCD.

Pessure



Not Grand Canonical
Partition Functions $Z(\mu, T)$?

They are equivalent
and related as

$$Z(\xi, T) = \sum_n Z_n(T)\, \xi^n$$

$\xi \equiv e^{\mu/T}$ Fugacity

Let us prove it !

$$Z(\mu, T) \Leftrightarrow Z_n(T)$$

Grand Canonical **Canonical**

$$Z(\mu, T) = \mathrm{Tr}\, e^{-(H-\mu\hat{N})/T}$$

$$= \sum_n \langle n | e^{-(H-\mu\hat{N})/T} | n \rangle$$

If $[H, \hat{N}] = 0$

$$= \sum_n \langle n | e^{-H/T} | n \rangle \, e^{\mu n/T}$$

$$= \sum_n Z_n(T) \xi^n \qquad \left(\xi \equiv e^{\mu/T} \right)$$

Fugacity

$$Z_n \equiv \langle n | e^{-H/T} | n \rangle$$

→

I. Z_n is only a function T and n

II. If H has CP symmetry, $Z_n = Z_{-n}$

$$Z(\xi, T) = \sum_n Z_n(T)\, \xi^n$$

$$\xi \equiv e^{\mu/T}$$

Is this useful ?

Yes, because

1) We can calculate Z at any ξ (i.e., μ)

2) We can calculate Z even at complex ξ

How to calculate Z_n by Lattice QCD ?

I. Fugacity Expansion
II. Hasenfratz-Toussant

And Sign Problem on Lattice ?

I. Fugacity Expansion of $\det \Delta(\mu)$

- Reduction formula
  - $\det \Delta = \det(\tilde{\Delta})$, but $\tilde{\Delta}$ is smaller matrix than Δ
- Reduced determinant

PRD82, 094027('10), 1009.2149, see refs. therein

  - $\det \Delta(\mu)$ is an analytic function of μ .

➡ fugacity polynomial

Gauge dependent parts

$$\xi = e^{-\mu/T} = e^{-\mu a N_t}$$

$$\det \Delta(\mu) = \xi^{-N_r/2} C_0 \det(\xi + Q)$$

Fugacity Expansion !

Diagonalize Q

$$\det \Delta(\mu) = C_0 \sum_{-2N_c N_s^3}^{2N_c N_s^3} c_n \xi^n$$

Canonical Zn in Glasgow method

$$Z_{GC}(\mu, T) = \int \mathcal{D}U \; [\det \Delta(\mu)]^{N_f} e^{-\beta S_G}.$$

$$\det \Delta(\mu) = C_0 \sum c_n \xi^n$$

$$Z_{GC}(\mu) = \int DU \left[\frac{C_0 \sum c_n \xi^n}{\det \Delta(0)} \right]^{N_f} (\det \Delta(0))^{N_f} e^{-S_G}$$

$$Z_{GC}(\mu) = \sum_{n=-N_q}^{N_q} Z_C(n) e^{-n\mu/T} \qquad Z_n \equiv Z_C(n) = \left\langle \frac{C_0^2 d_n}{(\det \Delta(0))^2} \right\rangle_0$$

convergence radius of fugacity polynomial
roots of polynomial (Lee-Yang zeros)

Simulation Setup
clover-Wilson + RG-gauge(Nf=2)
Volume : 8^3x4
quark mass : mps/mV ~ 0.8
Configurations : HMC at mu=0
Eigen values : 400 configs.

PhysRev *D.83.114507, JHEP, 1204, 092 (2012).*

II. Hasenfratz-Toussant + Zn-Collaboration

$$\det \Delta = \exp\left(e^{+\mu/T}Q^{+} + e^{-\mu/T}Q^{-} + \cdots\right)$$

If μ is pure imaginary, $\det \Delta$ real.

A.Hasenfratz and Toussant, 1992

$$Z_C(n,T) = \int \frac{d\theta}{2\pi} e^{i\theta n} Z_{GC}(\theta \equiv \frac{\mathrm{Im}\mu}{T}, T)$$

Great Idea ! But practically it did not work.

Why ?

Zn Collaboartion Method:

$$Z_C(n,T) = \int \frac{d\theta}{2\pi}\; e^{in\theta} \int \mathcal{D}U e^{-(Gluon Action)}$$

$$\times \frac{\det\Delta(\theta)}{\det\Delta(\theta_0)}\, \underbrace{\det\Delta(\theta_0)}_{\text{Real}}$$

$$\theta \equiv \frac{\mathrm{Im}\,\mu}{T}$$

$$\det\Delta = \det\left(I - \kappa Q(\mu)\right)$$

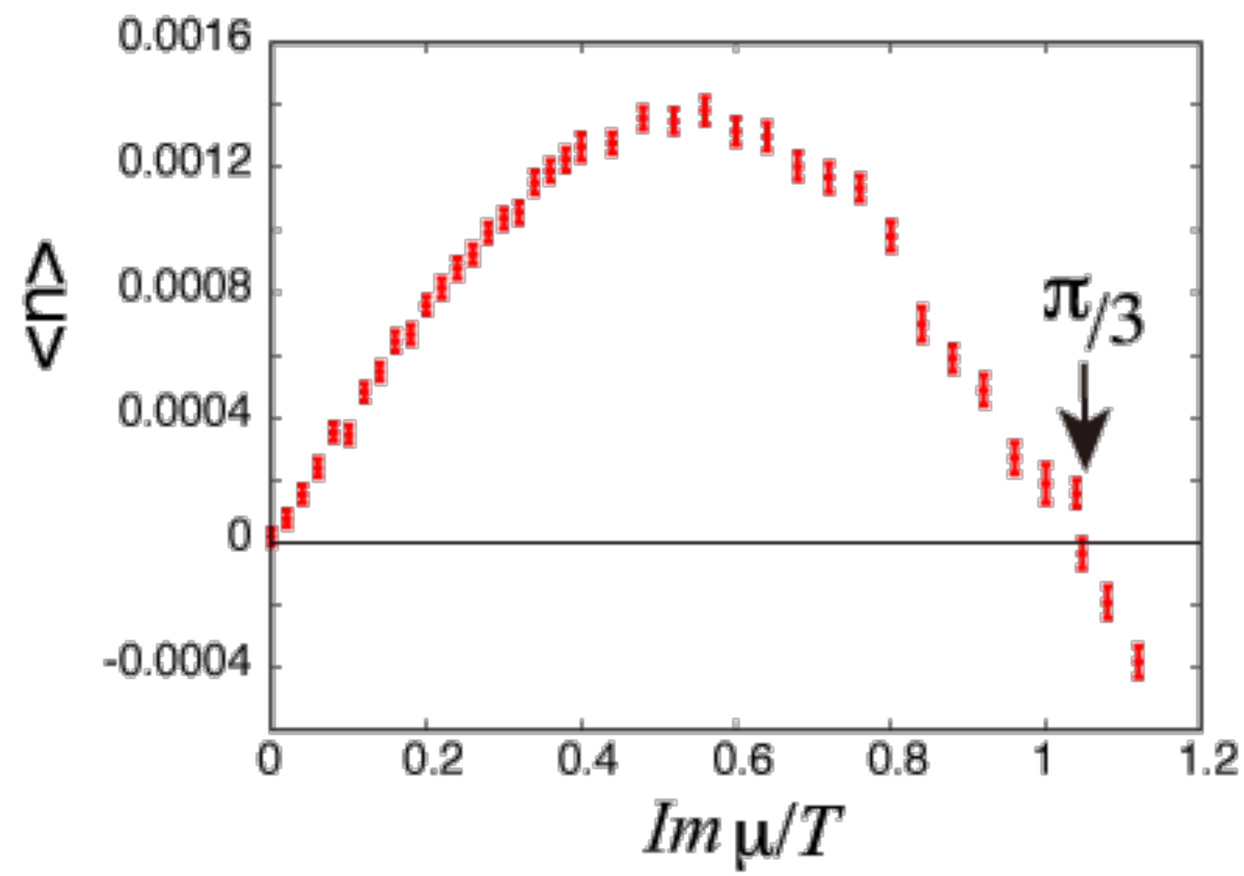
$$= e^{\mathrm{Tr}\log(I-\kappa Q(\mu))} = \cdots = \exp\left(\sum_{n=-\infty}^{+\infty} w_n \xi^n\right)$$

$$\xi \equiv e^{\mu/T} = e^{i\mathrm{Im}\,\mu/T}$$

FEFU Strategy

Evaluate Number density Numerically at imaginary chamical potential

$$\langle n \rangle = T \frac{\partial}{\partial \mu} \log Z(\mu, T)$$



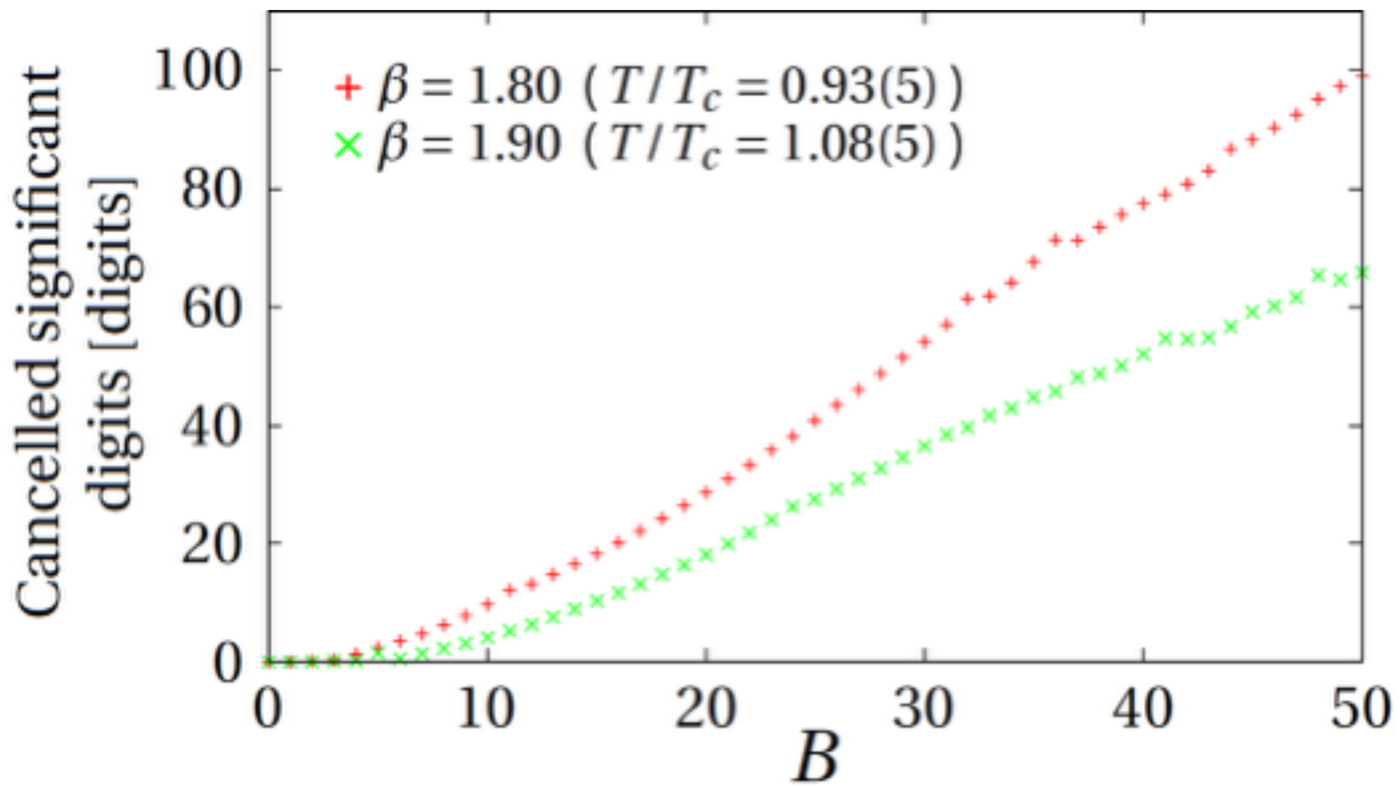
Integrate it

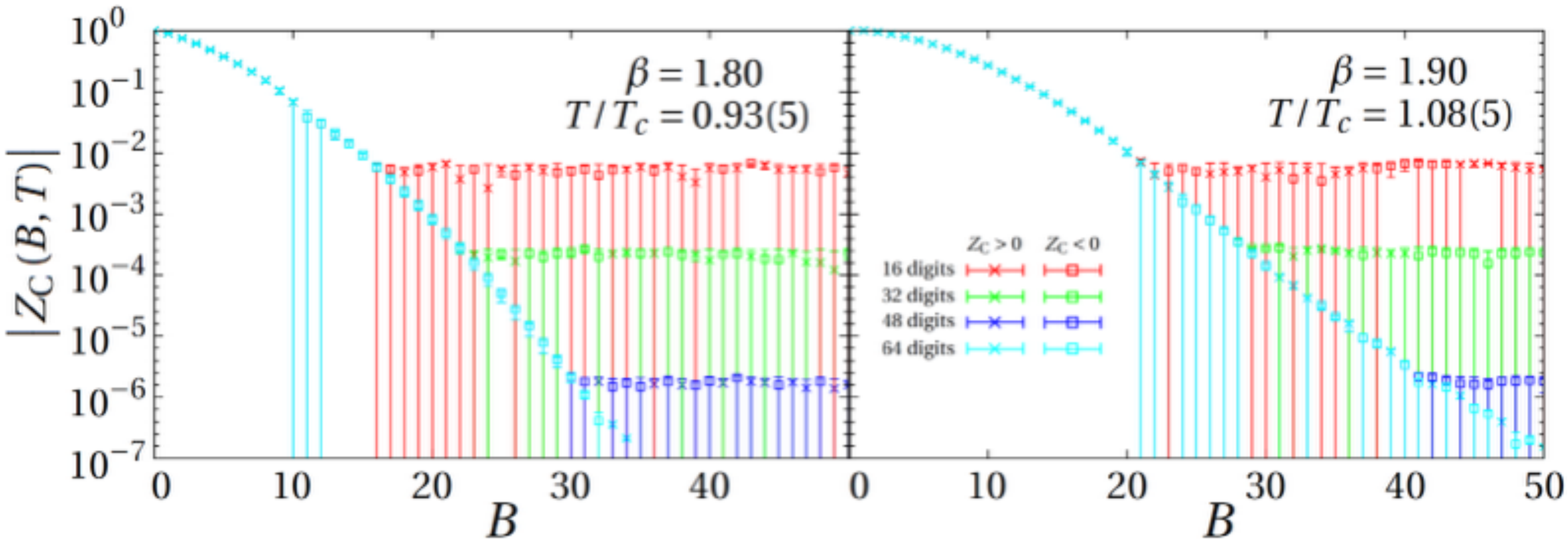
$$\log Z(\theta) = \int_0^{\theta} d\theta \, \langle n \rangle (\theta')$$

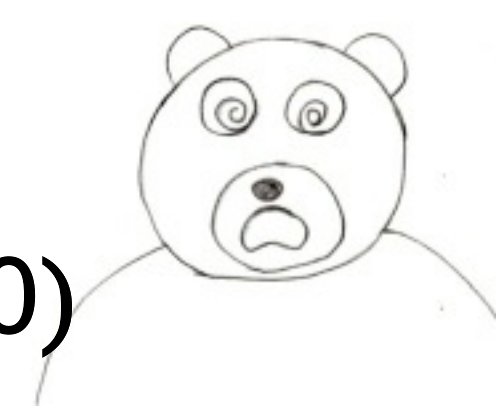
$$\theta = \frac{\mathrm{Im}\, \mu}{T}$$

$$Z_n = \int_0^{2\pi} d\theta e^{in\,\theta} Z(\theta)$$

Big Cancellation in FFT !







θintegration → Multi-Precision (50 - 100)

$$Z(\xi, T) = \sum_n Z_n(T)\,\xi^n$$

$$\xi \equiv e^{\mu/T}$$

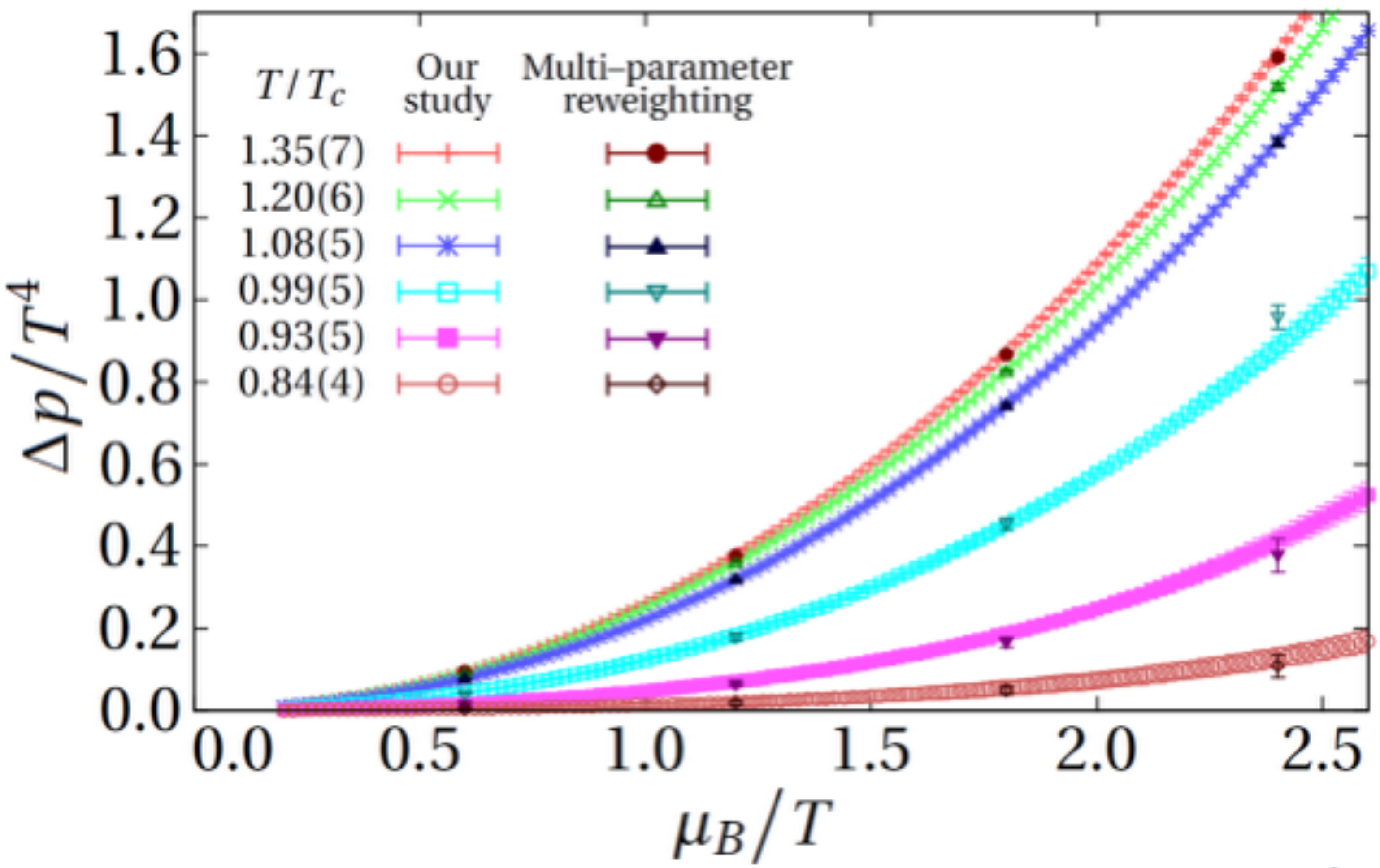
For $\mu/T > 1$, ξ^n become large as *n* increases.

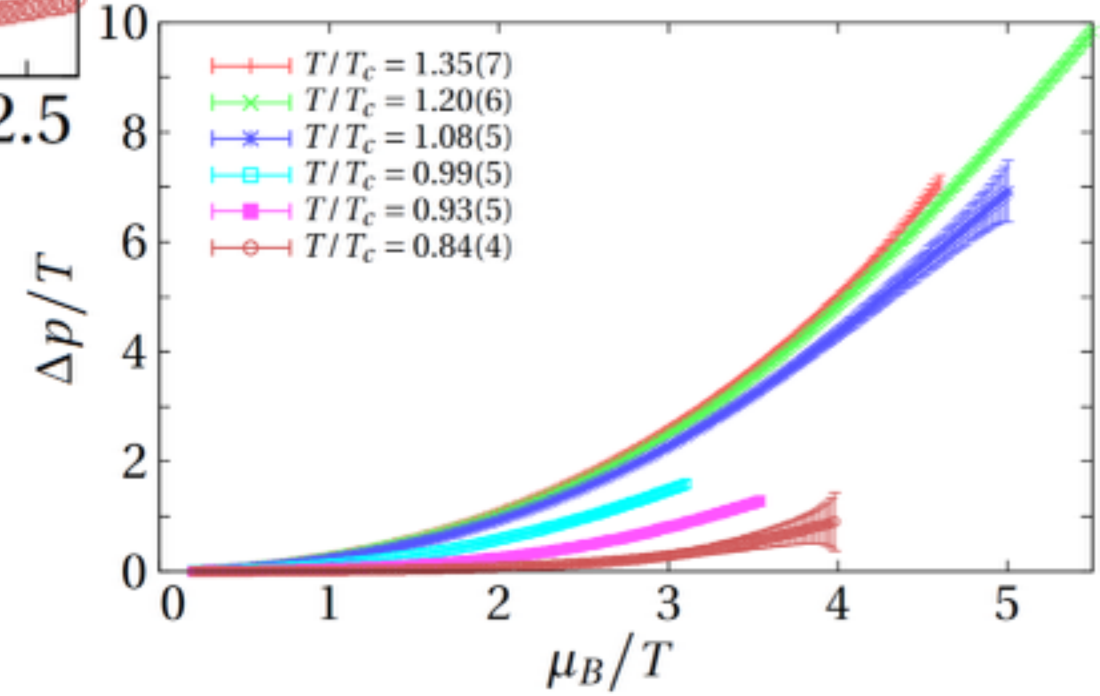
Zn drops very fast, which we must evaluate precisely.

$$Z_C(n, T) = \int \frac{d\theta}{2\pi} e^{i\theta n} Z_{GC}(\theta \equiv \frac{\mathrm{Im}\mu}{T}, T)$$

Large *n* corresponds to High frequency Oscillation.

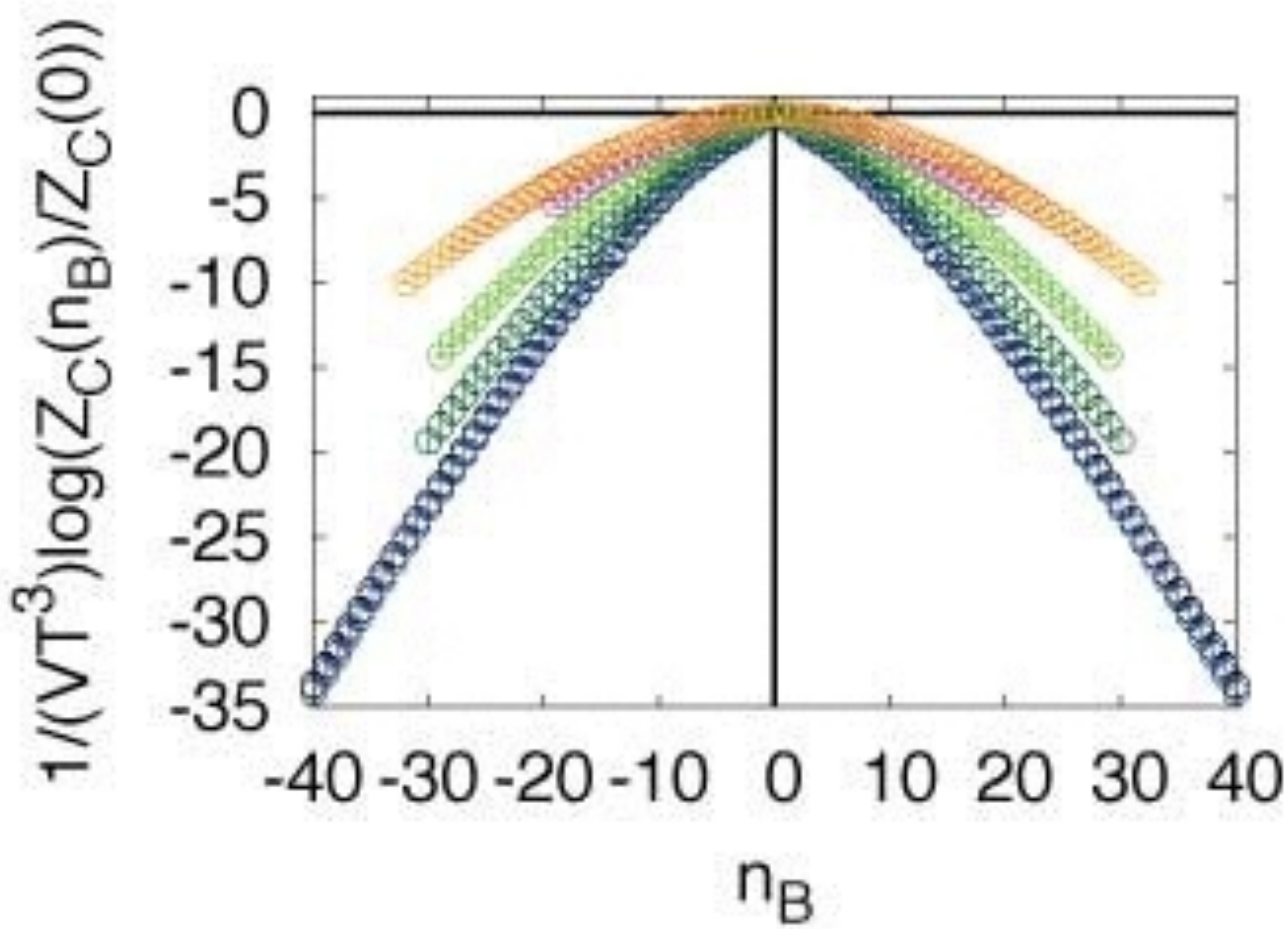
Canonical Approach gives the same results as Multi-Parameter-Reweighting ?





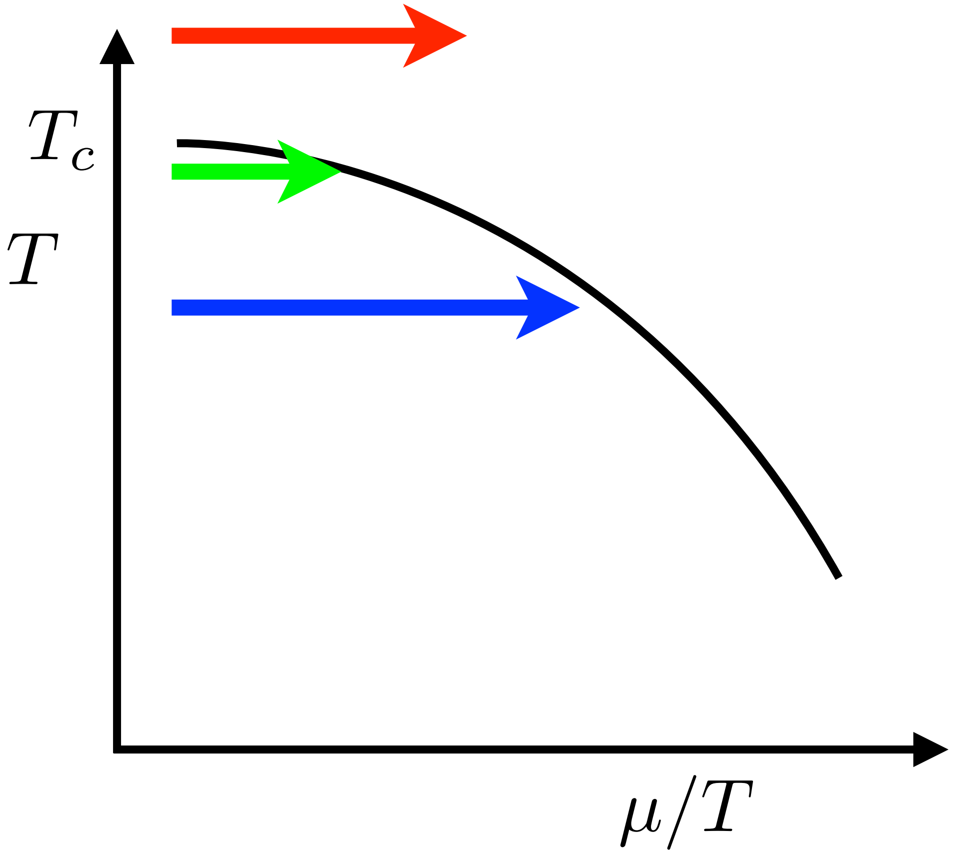
Yes, and even higher density.

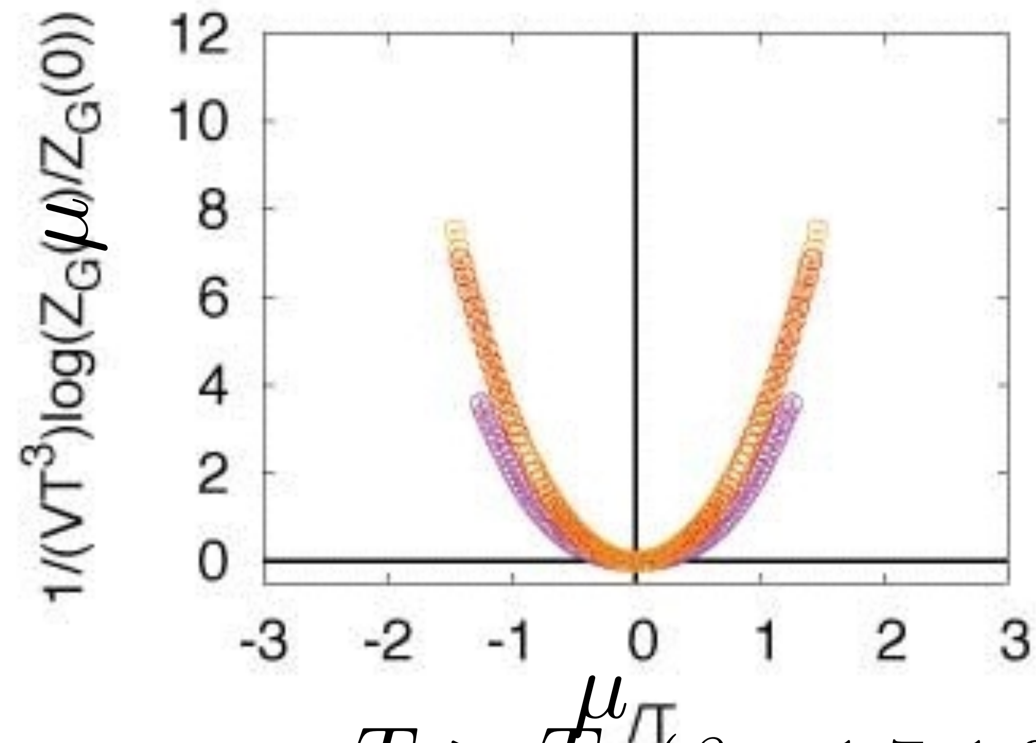
They look similar. Can I see Difference?



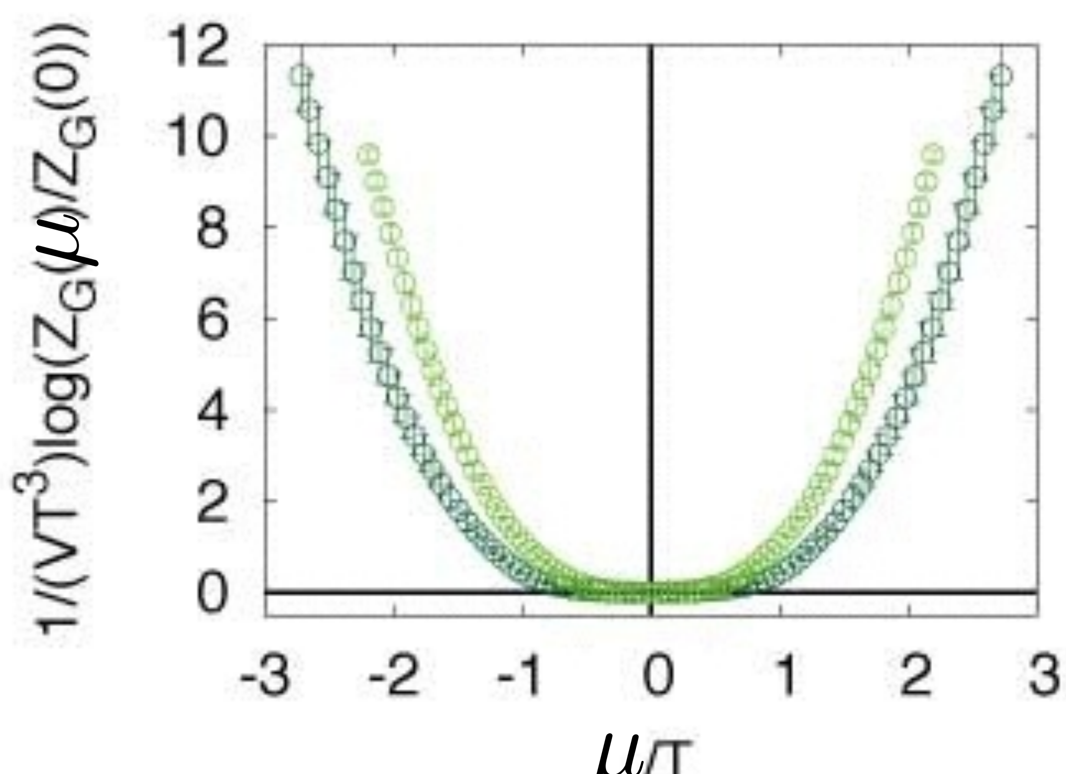
Yes, You Can !
Wait a moment.

Pessure $\frac{P(\mu/T) - P(0)}{T^4}$

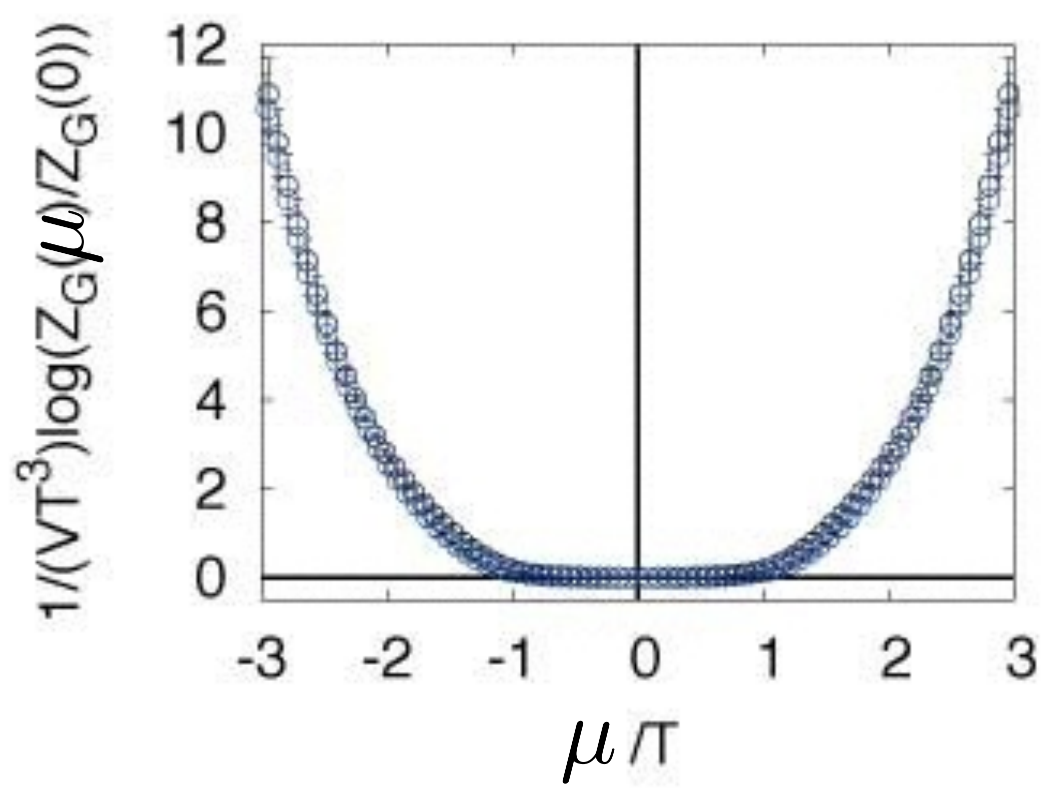




$T > T_c \, (\beta = 1.7, 1.9)$

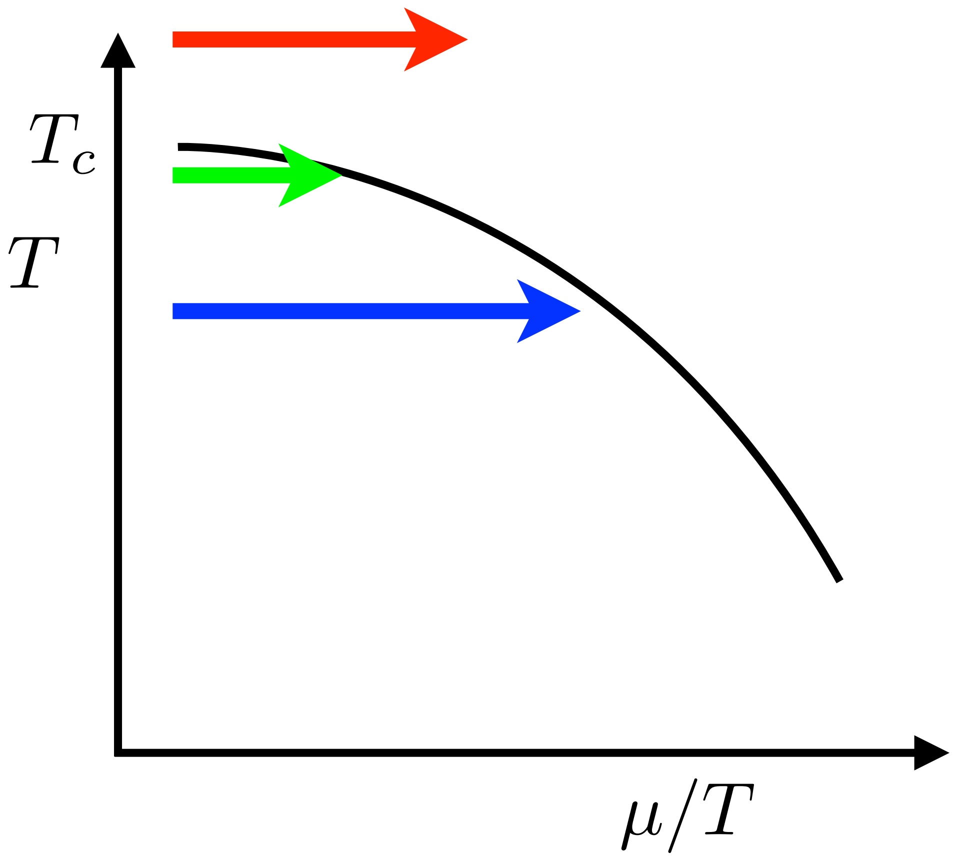


$T \leq T_c \, (\beta = 1.3, 1.5)$

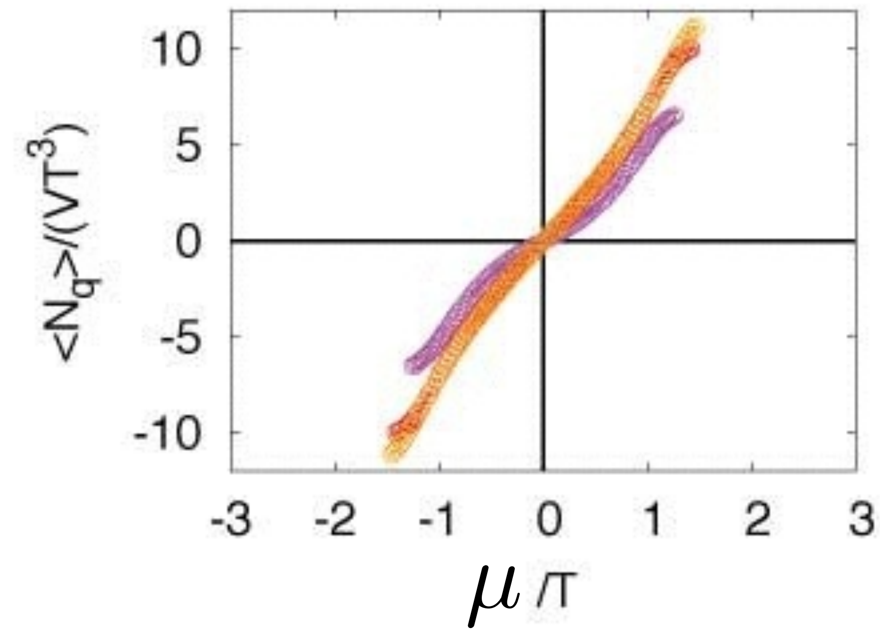


$T < T_c \, (\beta = 0.9, 1.1)$

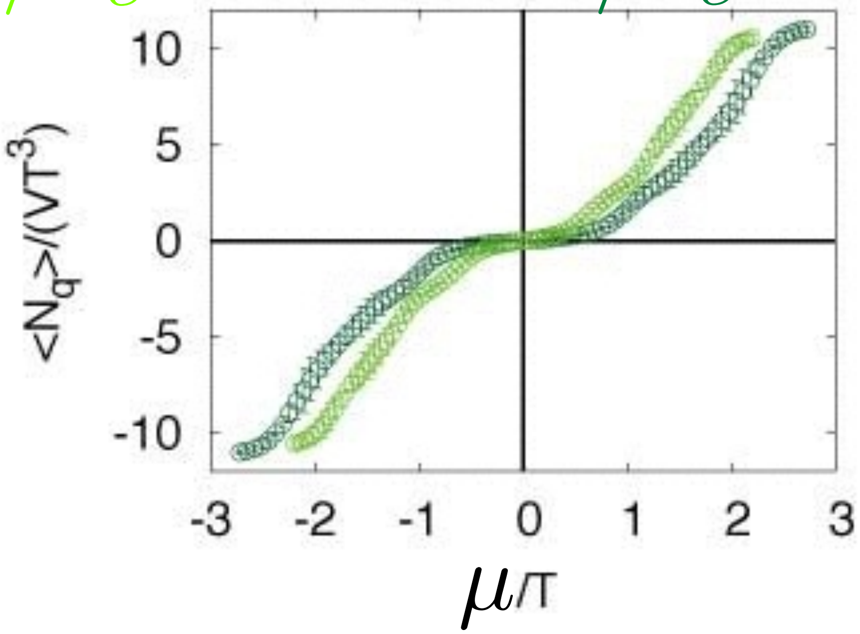
Number Density



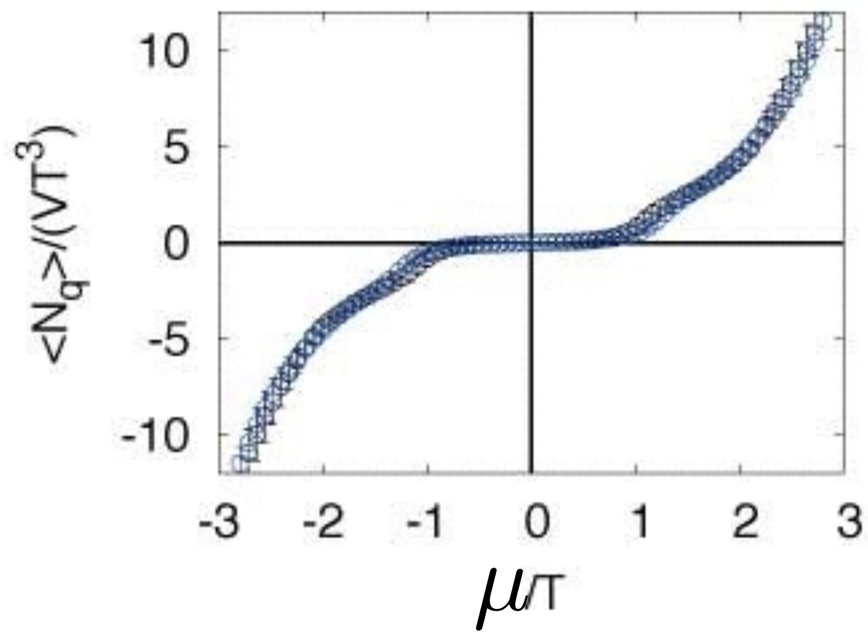
$T/T_c = 3.62$ $T/T_c = 1.77$



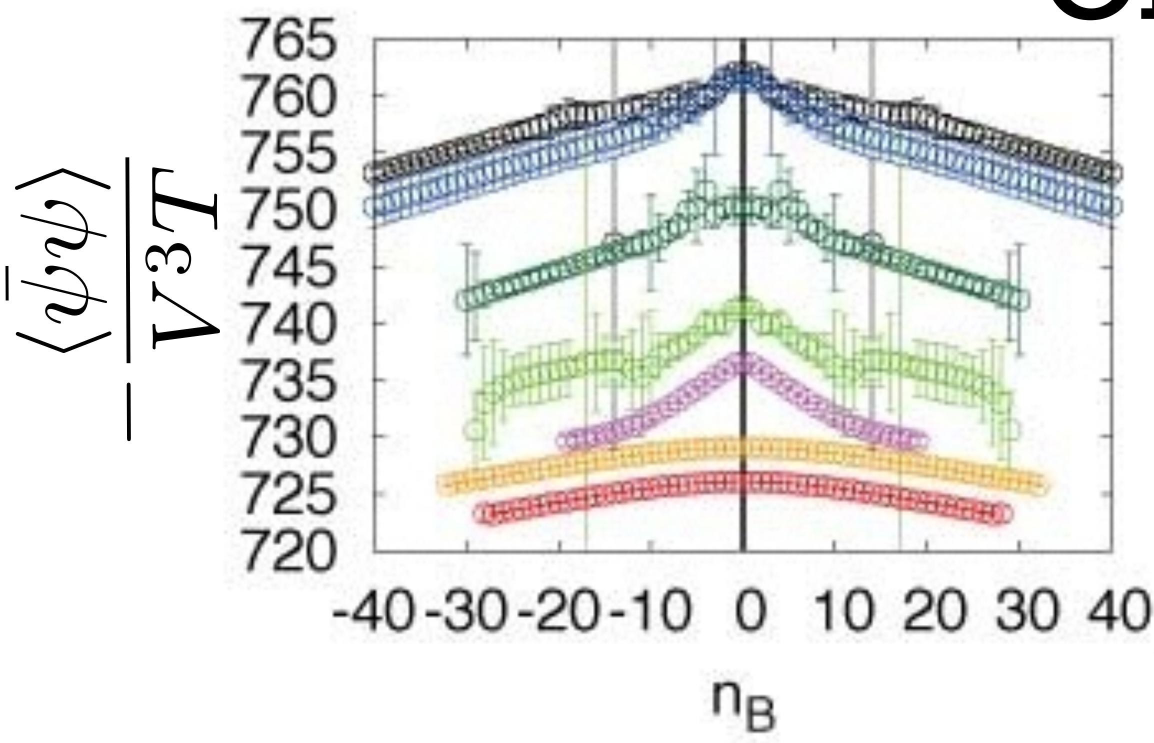
$T/T_c = 0.83$ $T/T_c = 0.72$

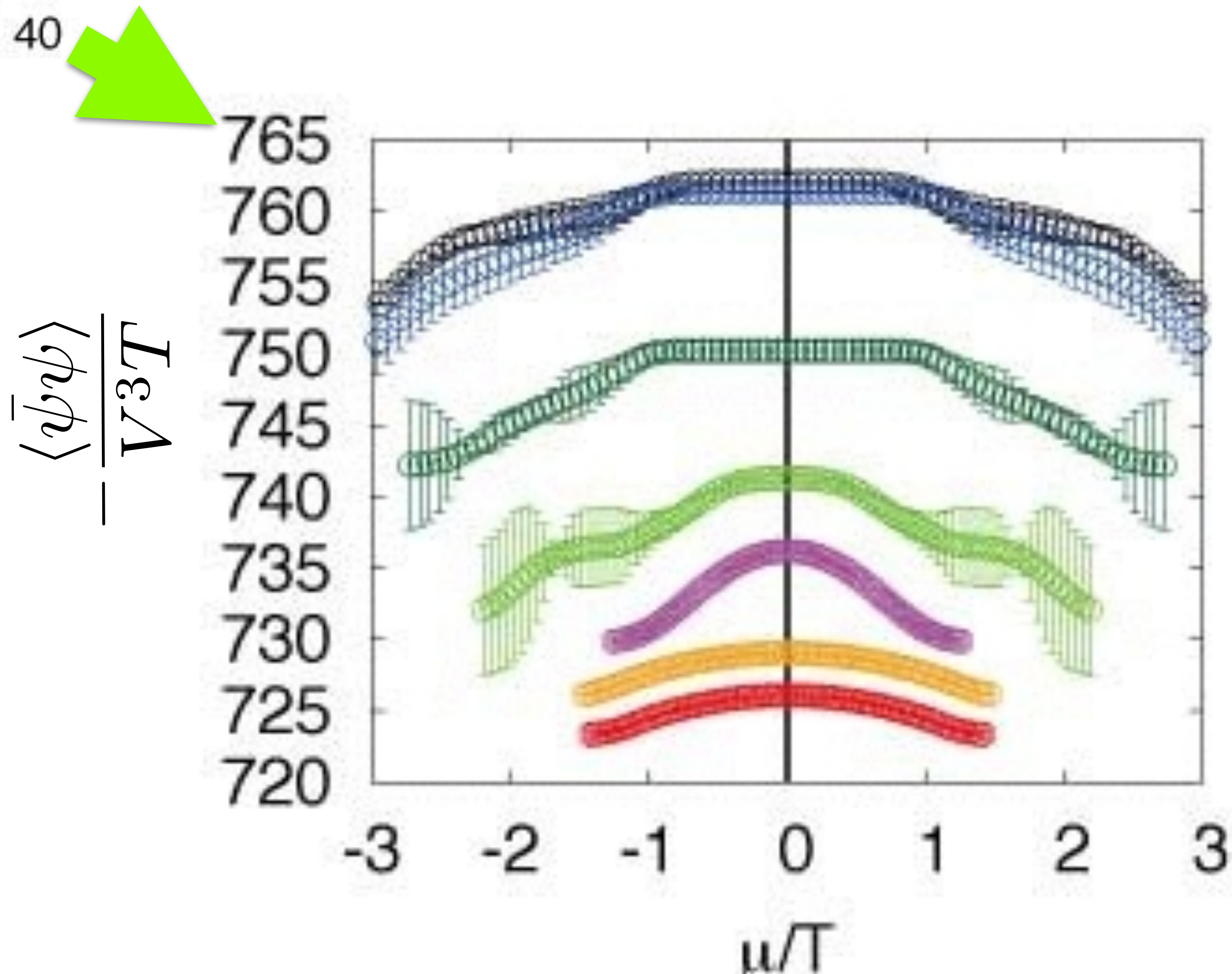


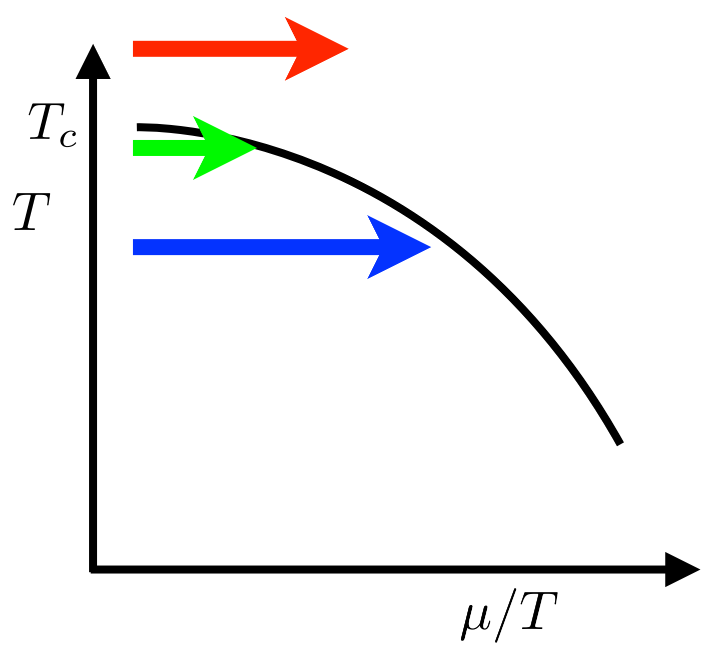
$T/T_c = 0.65$



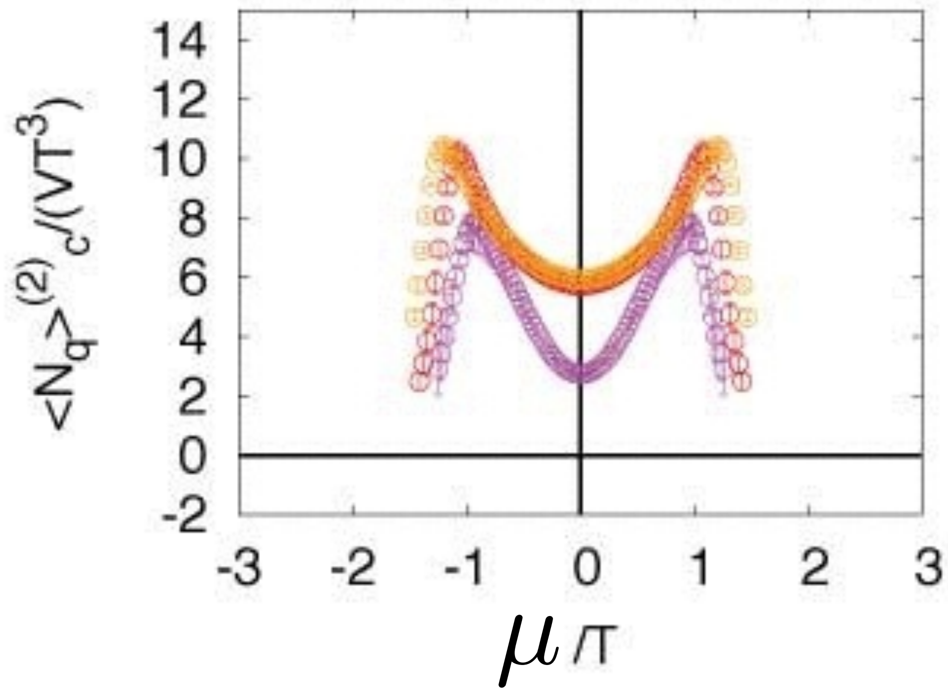
Chiral Condensate





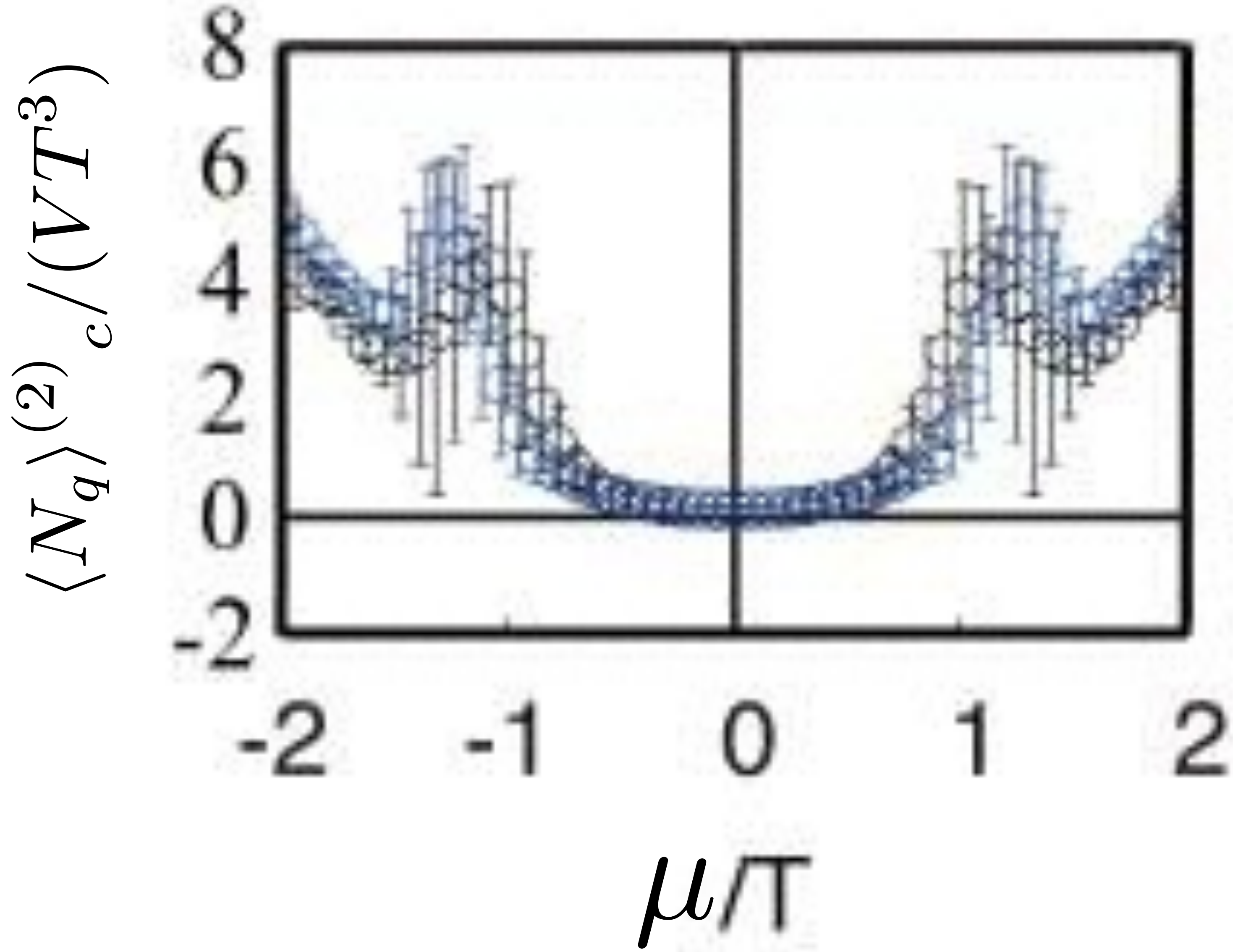


Second Cummulant



$T > T_c$

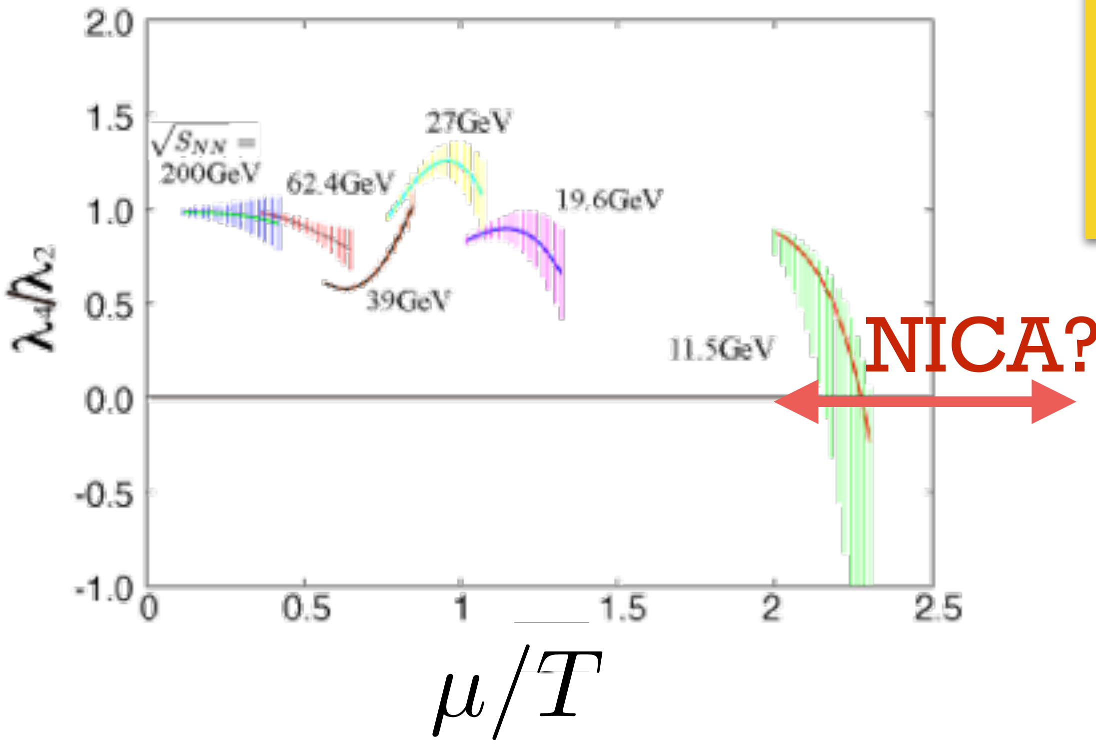
$T < T_c$

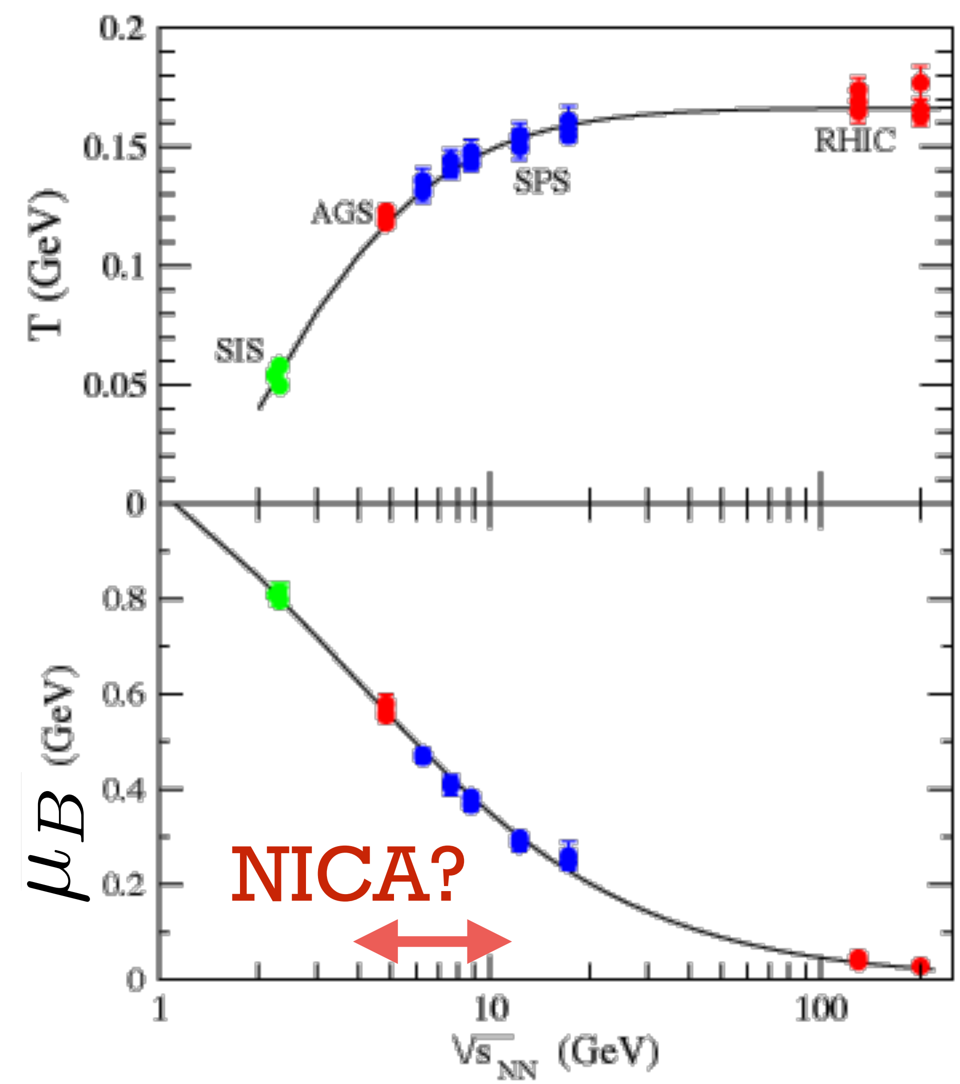


Summary of Data Analysis
or
NICA will bless us

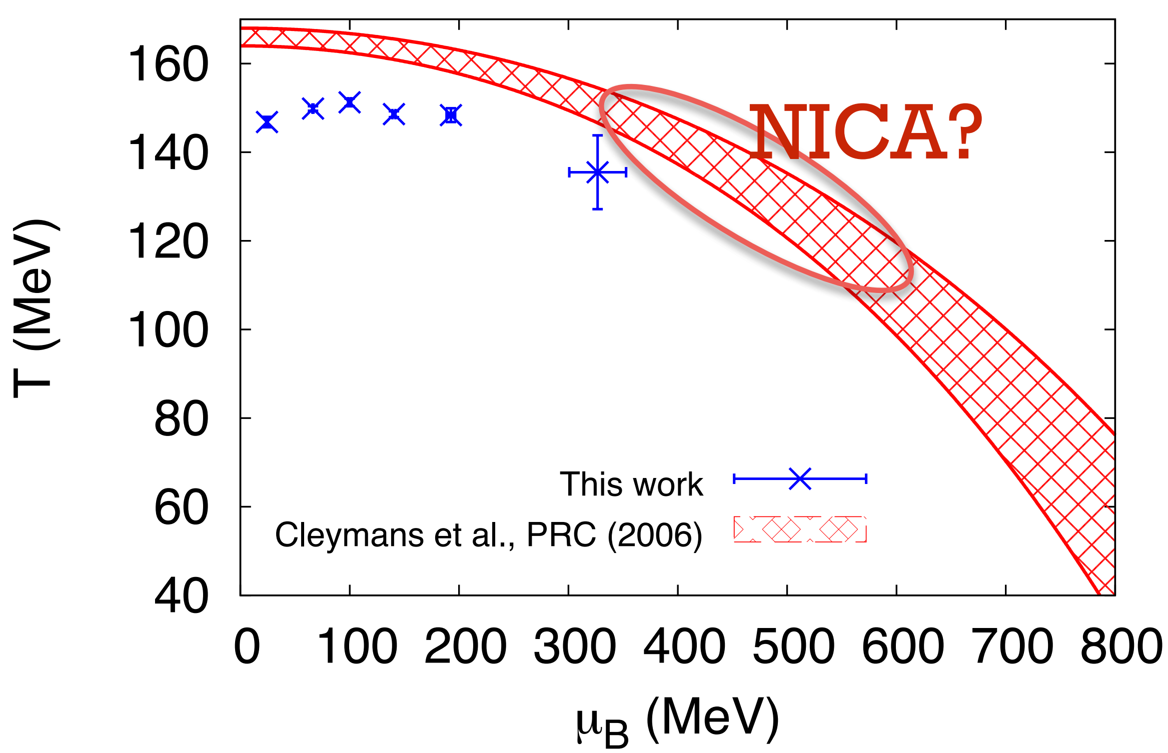
Not too High Energy ⇒ High Density

Kurtosis





J.Cleymans et al.,
Phys. Rev. C73, (2006) 034905.



Alba et al., arXiv:1403.4903

NICA $\sqrt{s_N} = 4 \sim 11$ GeV

J-PARC $\sqrt{s_N} = 2 \sim 5(?)$ GeV

Lee-Yang Zeros Experimental Data (RHIC)

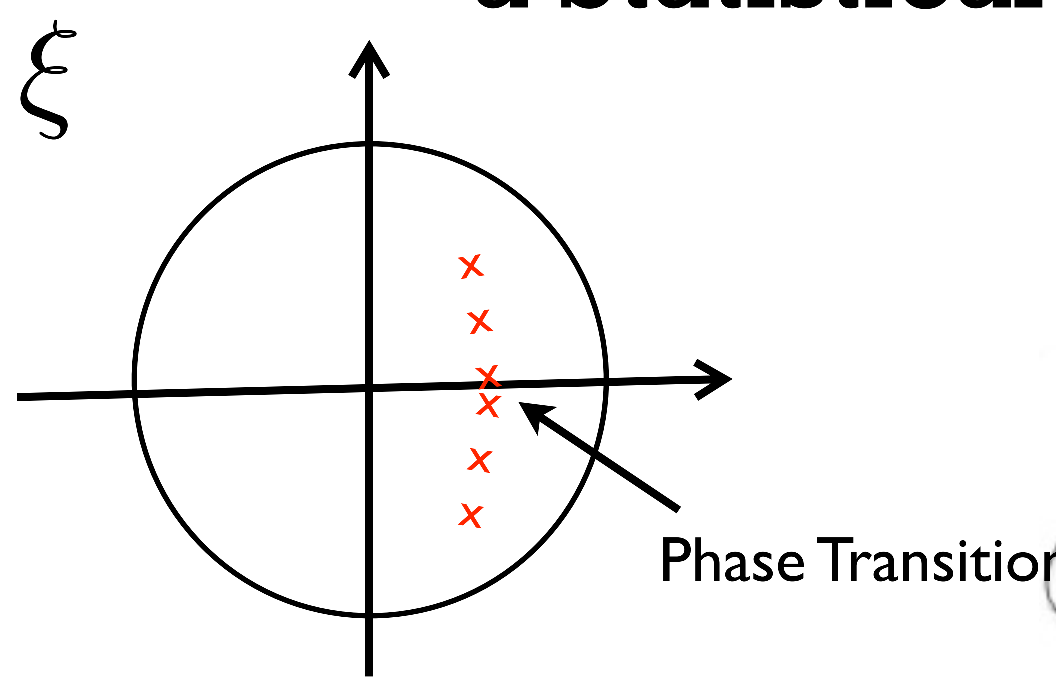
$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\, \xi^n$$

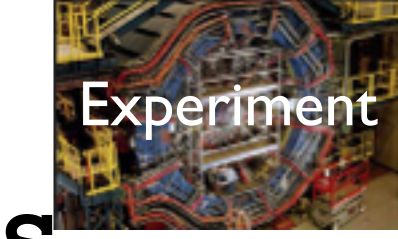
Lee-Yang Zeros (1952)

Zeros of $Z(\xi)$ in Complex Fugacity Plane.

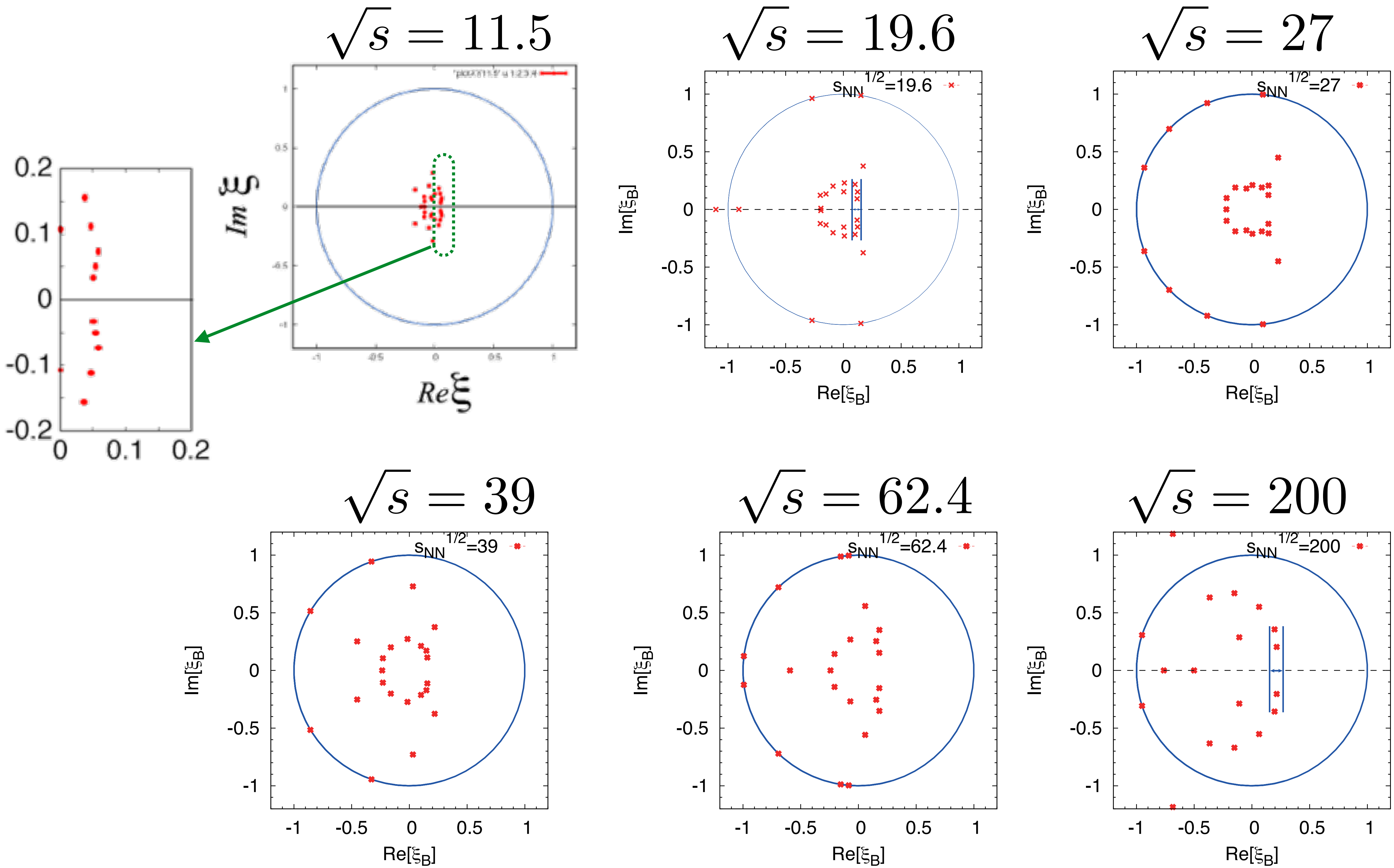
$$Z(\alpha_k) = 0$$

Great Idea to investigate a Statistical System

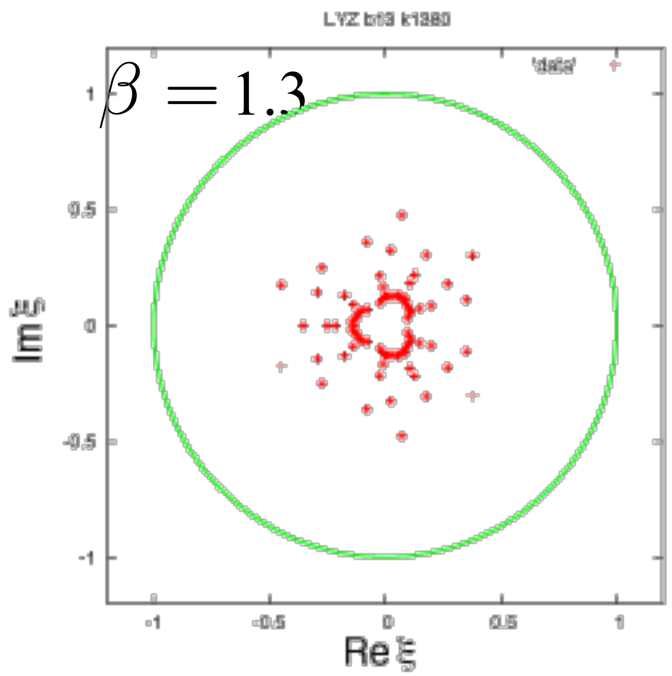


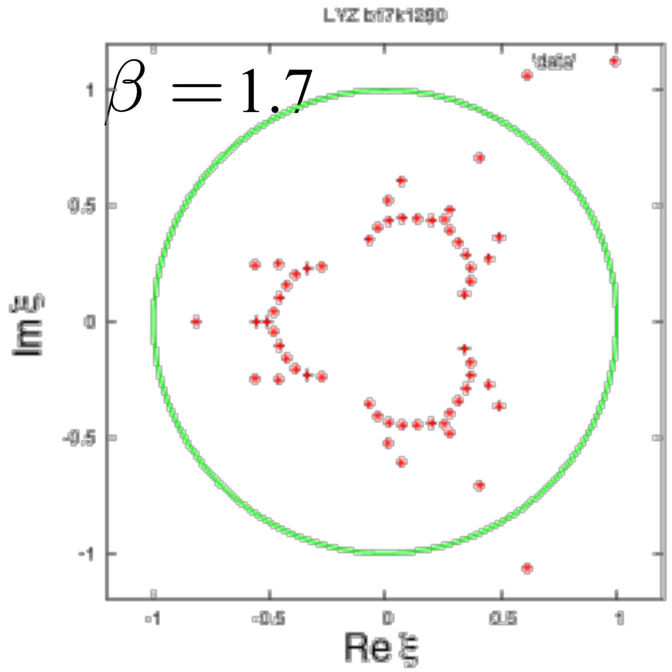


Lee-Yang Zeros: RHIC Experiments



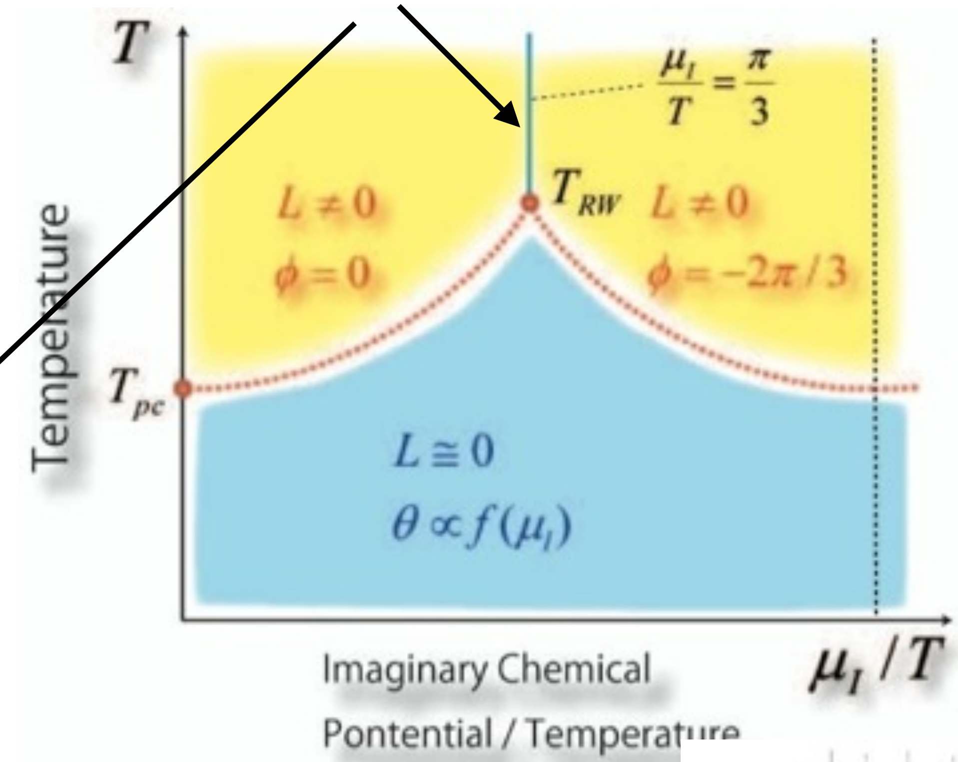
Lee-Yang Zeros:Lattice Simulations

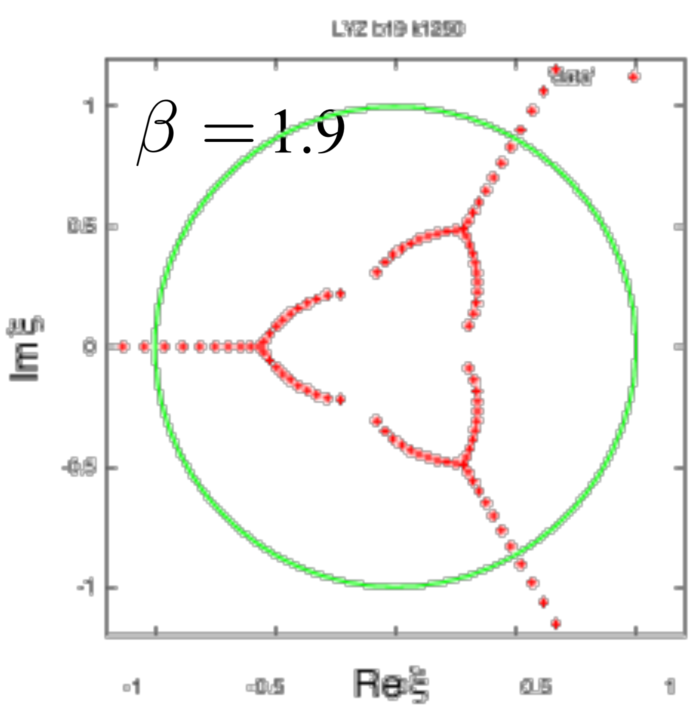


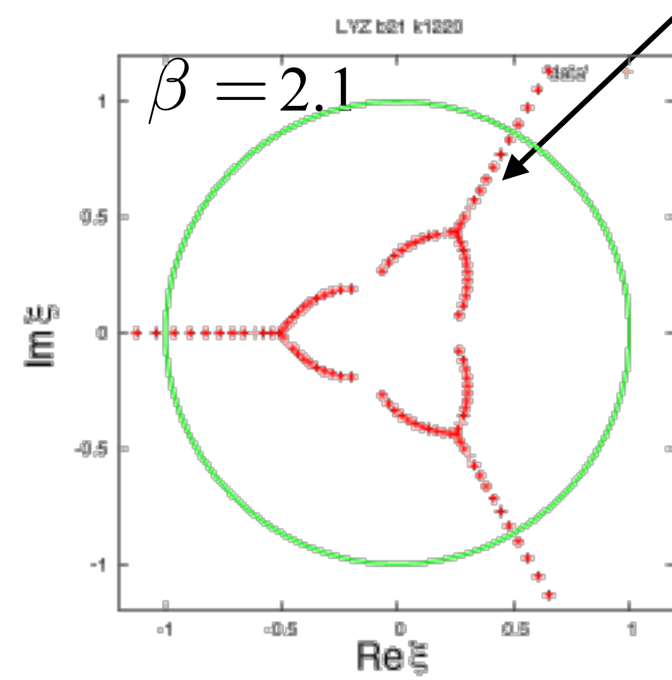


$T \leq T_c$

Roberge-Weise Transition !







$T > T_c$

Summary

- Viscosity (more general Transport coefficients)
  - It tells us deeper information about QGP/QCD than simple EoS.
  - Its evaluation requires
    - (i) Noise reduction and (ii) Knowledge of Spectral functions. We will try any possible approaches.
- Finite density
  - Canonical approach may solve Sign Problem.
    - We are testing it on relatively small lattices and heavy quark mass regions to reveal possible difficulty
    - Then, we study QCD phase diagram near the continuum and physical mass (and with s-quark)
  - From an experimental data at T and μ/T, we can predict at the same T and larger μ/T ,

$$Z(\mu, T) = \sum_n Z_n(T)(\exp(\mu/T))^n$$

Back-up Slides

$\rho : e^{-A+B}$: non-equilibrium statistical operator

$$A = \int d^3x \beta(x,t) u^{\nu} T_{0\nu}(x,t)$$

$$B = \int d^3x \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1 - t)} T_{\mu\nu}(x,t) \partial^{\mu}(\beta(x,t) u^{\nu})$$

Using: $e^{-A+B} = e^{-A} + \int_0^1 d\tau\, e^{A\tau} B e^{-A\tau} e^{-A} + \cdots$

$$\rho \approx \rho_{eq} + \int_0^1 d\tau \left(e^{A\tau} B e^{-A\tau} e^{-A} - \langle B \rangle_{eq}\right) \rho_{eq}$$

$$\rho_{eq} \equiv e^{-A} / \mathrm{Tr} e^{-A} \rightarrow \exp(-\beta H) / \mathrm{Tr} e^{-A}$$

in the co-moving frame, $u^{\mu} = (1 \quad 0 \quad 0 \quad 0)$

Energy-Momentum Tensors

$$\langle T_{\mu\nu} \rangle = \langle T_{\mu\nu} \rangle_{eq} +$$

Deviation from Equilibrium

$$+\int d^3x' \int_{-\infty}^{t} dt' e^{\varepsilon(t'-t)} (T_{\mu\nu}(x,t), T_{\rho\sigma}(x',t'))_{eq}\, \partial^{\rho}(\beta u^{\sigma})$$

where $(T_{\mu\nu}(x,t), T_{\rho\sigma}(x',t'))_{eq}$

$$\equiv \int_0^1 d\tau \left\langle T_{\mu\nu}(x,t) \left(e^{-A\tau} T_{\rho\sigma}(x',t') e^{A\tau} - \langle T_{\rho\sigma}(x',t') \rangle_{eq} \right) \right\rangle_{eq}$$

$$\langle T^{ij} \rangle = \eta (\partial^i u^j + \partial^j u^i)/2$$

$$\langle T^{0i} \rangle = -\chi (\beta^{-1}(x,t)\partial^i \beta + \partial_{\alpha} u^{\alpha})$$

$$\langle p \rangle - \langle p \rangle_{eq} = -\varsigma \partial_{\alpha} u^{\alpha}$$

$$p \equiv -\frac{1}{3} T^i{}_i$$

- One can show

$$(T_{\mu\nu}(x,t),T_{\rho\sigma}(x',t'))_{eq} = -\beta^{-1}\int_{-\infty}^{t'} dt''\left\langle T_{\mu\nu}(x,t),T_{\rho\sigma}(x',t'')\right\rangle_{ret}$$

Transport Coefficients are expressed by Quantities at Equilibrium

Energy Momentum Tensors

$$T_{\mu\nu} = 2Tr(F_{\mu\sigma}F_{\nu\sigma} - \frac{1}{4}\delta_{\mu\nu}F_{\rho\sigma}F_{\rho\sigma})$$

$$(T_{\mu\mu} = 0)$$

$$U_{\mu\nu}(x) = \exp(ia^2 gF_{\mu\nu}(x))$$

$$F_{\mu\nu} = \log U_{\mu\nu} / ia^2 g$$

or

$$F_{\mu\nu} = \left(U_{\mu\nu} - U_{\mu\nu}{}^{\dagger}\right) / 2ia^2 g$$

Temperature Green function

$$G_\beta(\tau,\vec{x};\tau',\vec{x}') = \langle\langle T_\tau \phi(\tau,\vec{x})\phi(\tau',\vec{x}')\rangle\rangle$$

$$\phi(\tau,\vec{x}) = e^{\tau H}\phi(0,\vec{x})e^{-\tau H}$$

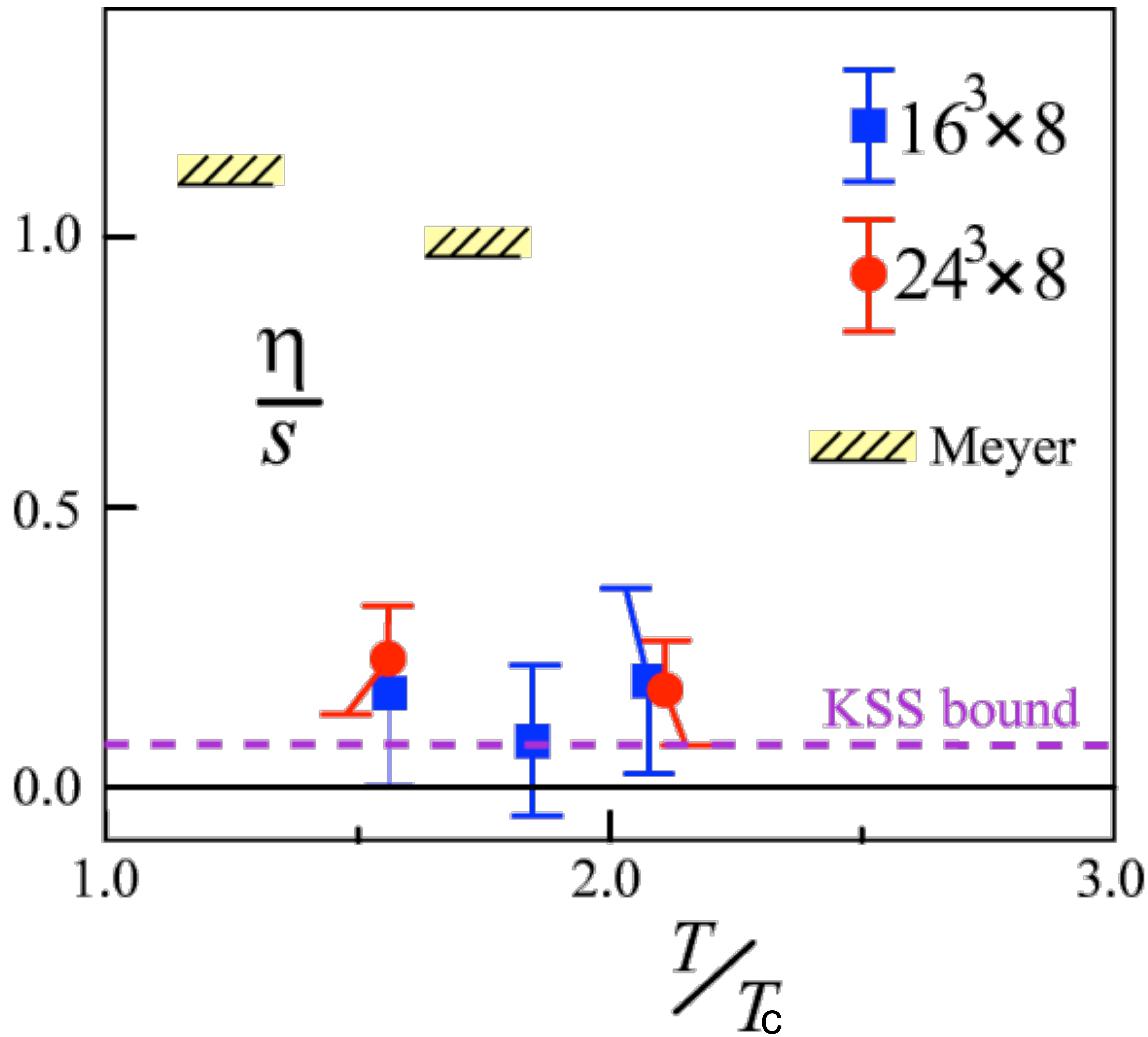
$$G_\beta(\tau,\vec{x};0,\vec{0}) = G_\beta(\tau+\beta,\vec{x};0,\vec{0})$$

$$\hat{K}_\beta(\xi_n,\vec{p}) = F^{-1}\int_0^\beta d\tau e^{-i\xi_n(\tau-\tau')}G_\beta(\tau,\vec{x};\tau',\vec{x}')$$

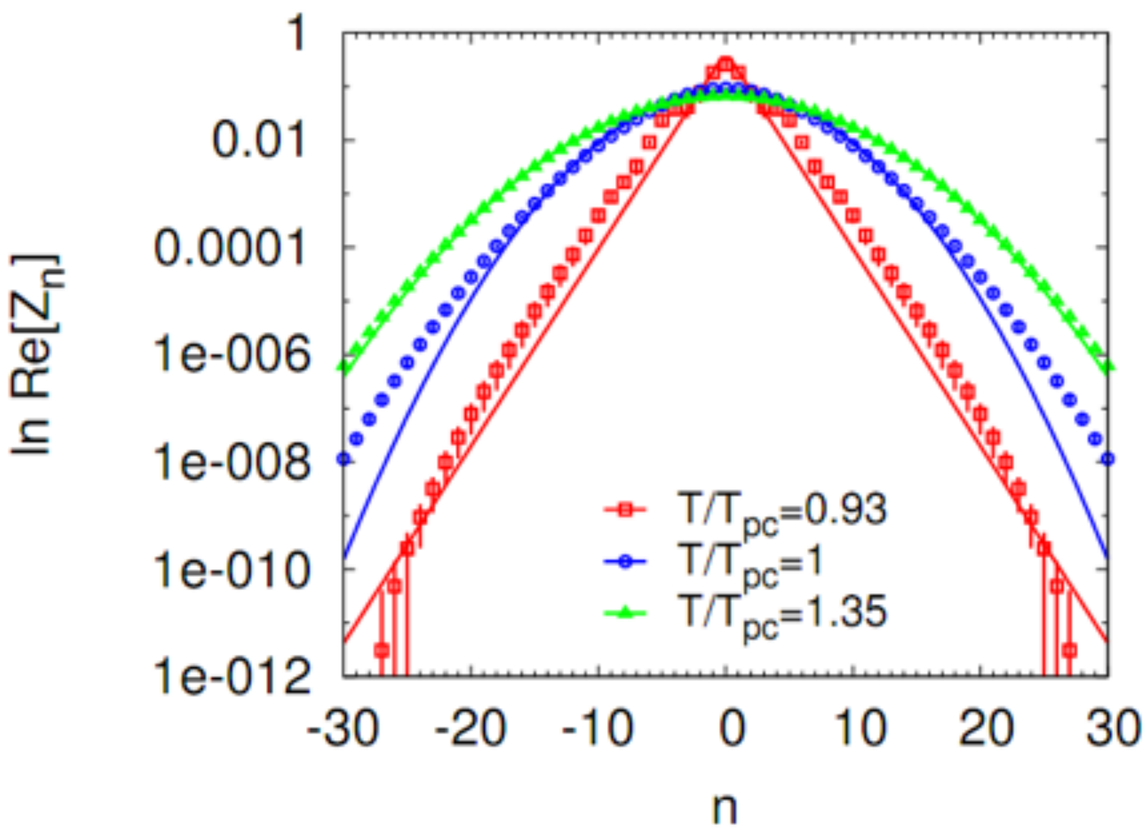
$$\xi_n = \frac{2\pi}{\beta}n, n = 0,\pm 1,\pm 2,,,$$

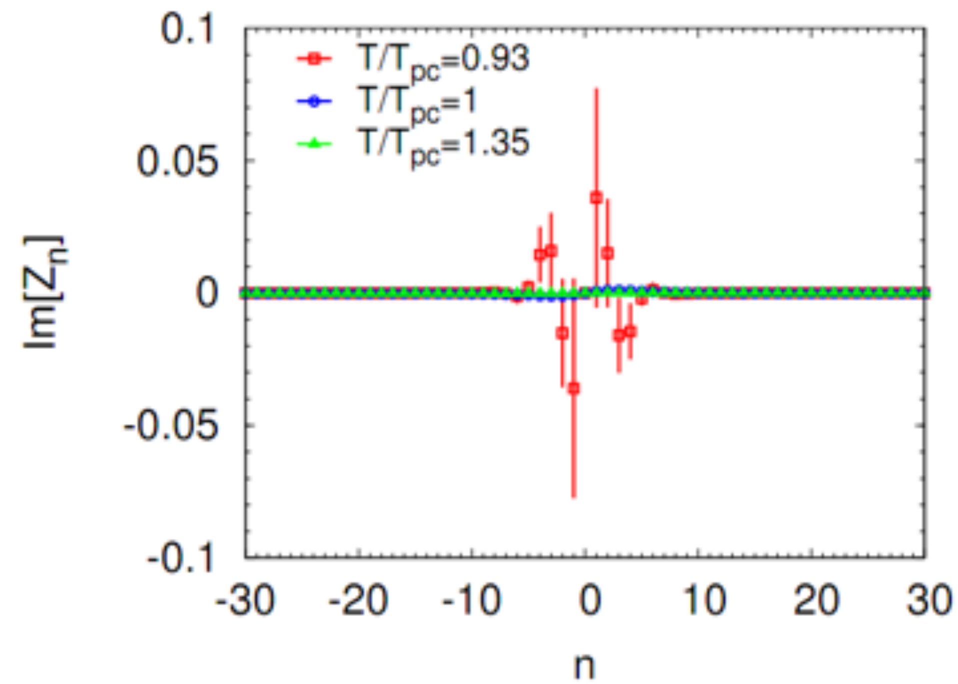
Matsubara-frequencies

Shear Viscosity



Canonical partition function Z(n)

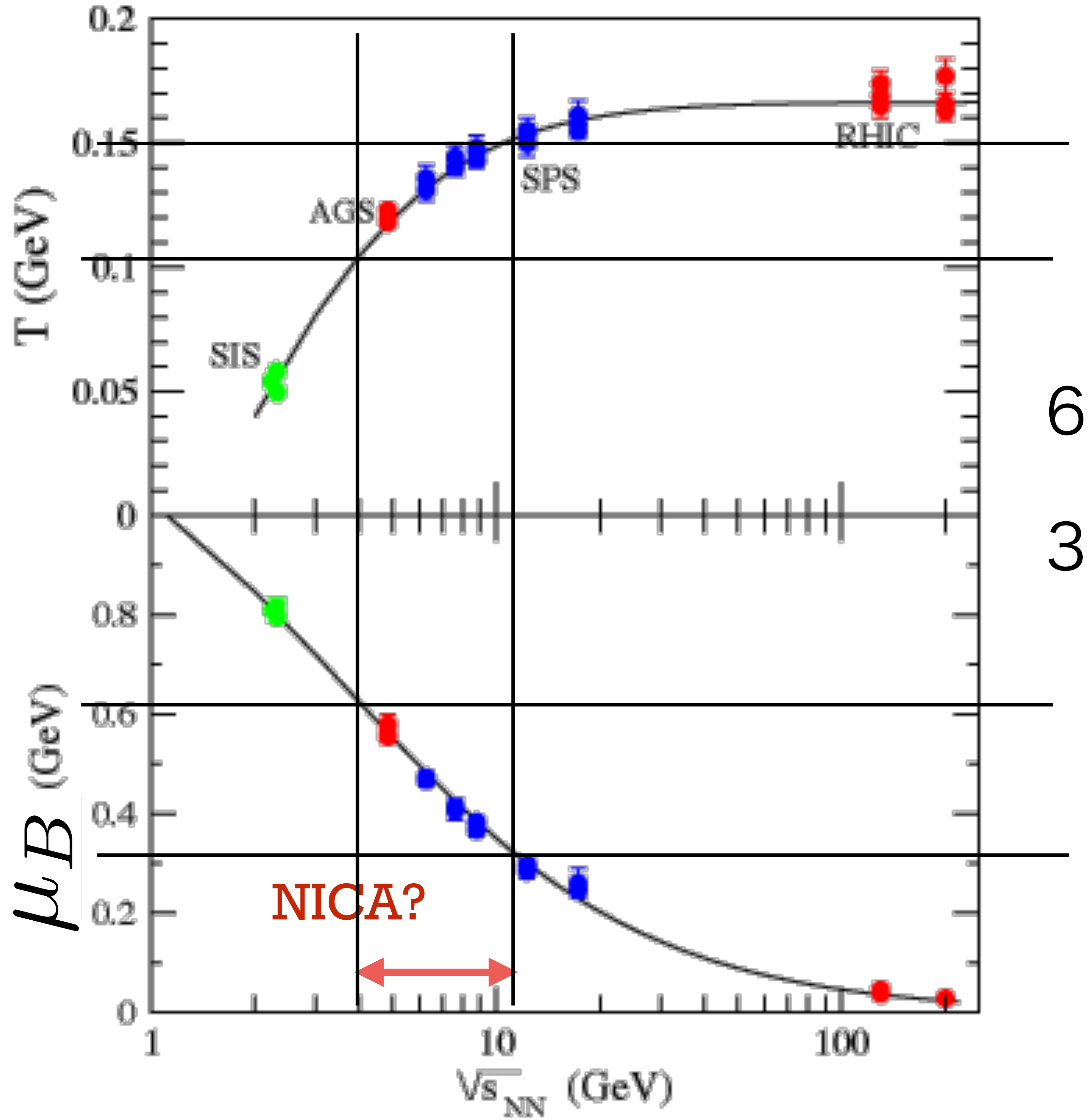




- Distribution changes from high to low temperatures
  - low T : exp(- a |n|)
  - high T : Gaussian
  - increase of effective d.o.f. at high T
  - large fluctuation of Im[Zn] at low T

$$Z_C(n) \sim \begin{cases} e^{-a_{\rm L}|n|}, & (T < T_{\rm pc}), \\ e^{-a_{\rm H}n^2}, & (T > T_{\rm pc}). \end{cases}$$

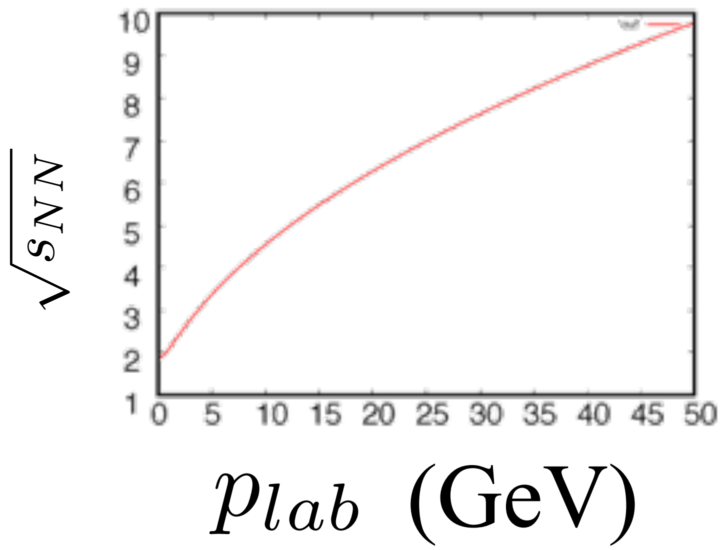
Ejiri , Phys.Rev. D73 (2006) 054502

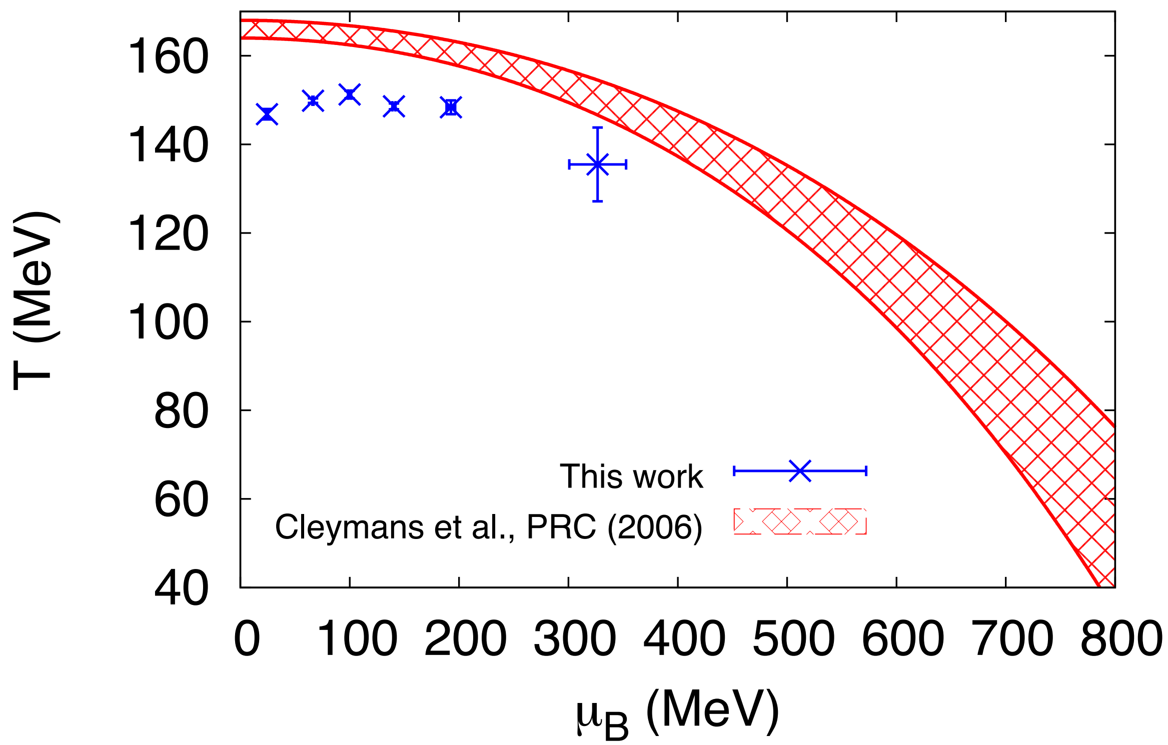


615-102

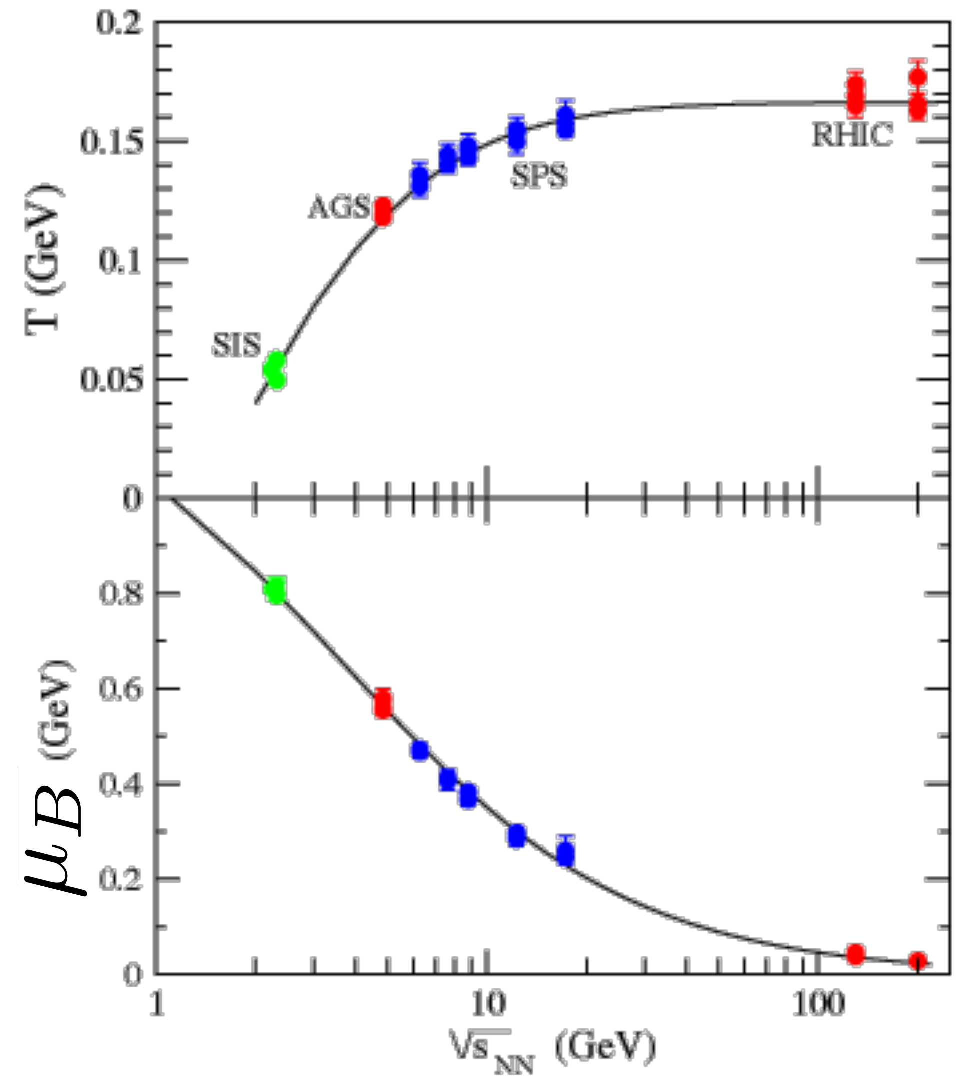
310-150

J-PARC search regions ?





Alba et al., arXiv:1403.4903



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    - Correlation Length << System Size
    - Relaxation time << Macroscopic Characteristic Time
    - Transport Coefficients must be small

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  - Beyond leading log"

Literature (3)

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  - Transport Coefficients Formulation
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- Aarts and Martinez-Resco, JHEP0204 (2002)053
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What is

Z_N ?

Canonical Partition Function.

We will see it later.

Tools

Improved actions: Gauge Iwasaki

Fermions Wilson+Clover, Nf=2

β	κ	m_π/m_ρ
0.9	0.137	$0.8978(55)$
1.1	0.133	$0.9038(56)$
1.3	0.138	$0.809(12)$
1.5	0.136	$0.756(13)$
1.7	0.129	$0.770(13)$
1.9	0.125	$0.714(15)$
2.1	0.122	$0.836(47)$

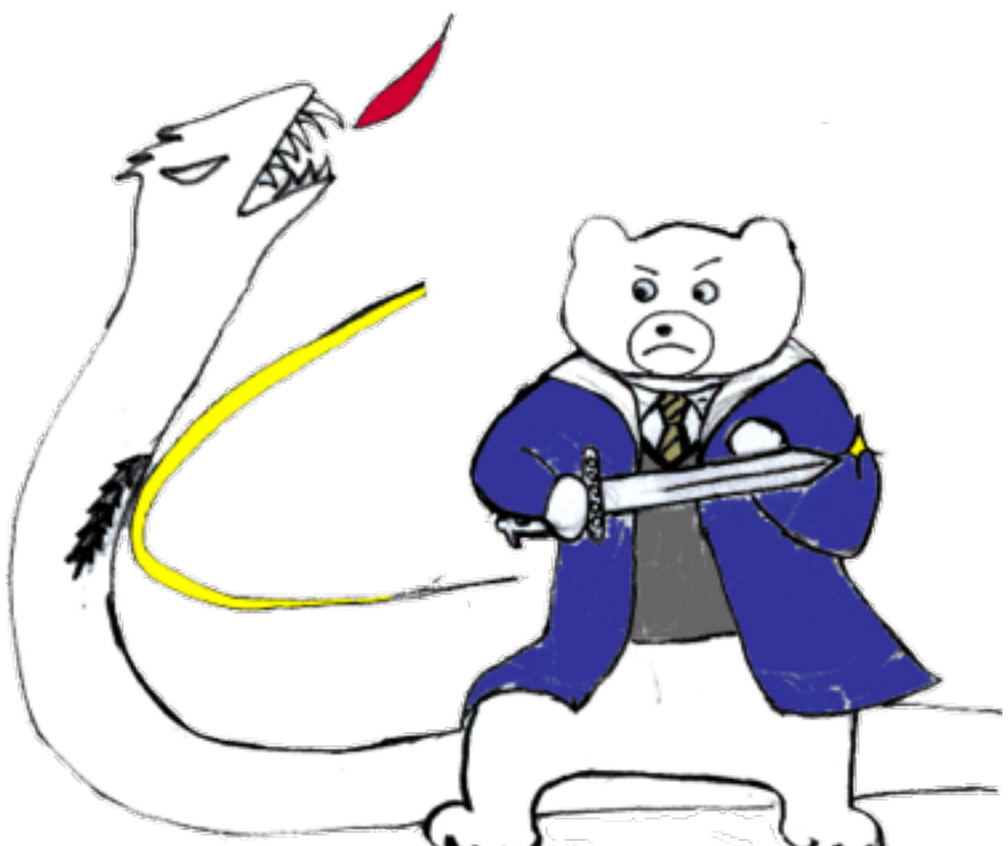
Mainly $8^3 \times 4$

APE-smearing for some data.

Multi-Presision Library: FMlib for Fortran

MPFR for C++

Lattice QCD at finite Density
- Canonical Approach -



Fighting against
Sign Problem