Confinement viewed with caloron, dyon and dimeron ensembles

M. Müller-Preussker

Humboldt-University Berlin, Institute of Physics

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Collaborators and coauthors since 2002:

- HU Berlin:
  P. Gerhold, S. Dinter, H. Forkel, E.-M. Ilgenfritz, B. Maier,
  D. Peschka, S. Shcheredin, M. Wagner, F. Zimmermann

- ITEP Moscow:
  S. Morozov, B. Martemyanov, A. Veselov †

- IHEP Protvino:
  V. Bornyakov

- Univ. Leiden:
  P. van Baal, D. Nógrádi

- Univ. Regensburg:
  F. Bruckmann, C. Gattringer, R. Pullirsch,
  S. Schaefer, A. Schäfer, S. Solbrig

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1. Introduction, motivation

Reinvestigation of an old issue: semiclassical approach to QCD path integral

['t Hooft, '76; Callan, Dashen, Gross, '78; Gross, Pisarski, Yaffe, '81;

Ilgenfritz, M.-P., '81; Shuryak, '82; Diakonov†, Petrov, '84]

$T = 0$: Belavin-Polyakov-Shvarts-Tyupkin (BPST) instantons
$T > 0$: Harrington-Shepard (HS) calorons

$$\text{BPST}: \quad A^{inst}_{a,\mu}(x) = R_{a\alpha}\bar{\eta}^{\alpha}_{\mu\nu}\frac{x_\nu}{(x^2+\rho^2)x^2}, \qquad A^{antiinst}: \bar{\eta} \leftrightarrow \eta$$

superpositions of "pseudo-particles"

$$A^{class}(z = \{\rho^{(i)}, x^{(i)}, R^{(i)}\}) = \sum_{i=1}^{N_+ + N_-} A^{(i)}_{a,\mu}(x - x^{(i)}, \rho^{(i)}, R^{(i)}),$$

in order to approximate the functional integral by $A = A_{class}(z) + \varphi$

$$\int DA \exp(-S[A]) \simeq \sum_{class} \int [dz] \exp(-S[A^{class}]) \int D\varphi \exp\left(-\frac{1}{2!}\int \varphi \frac{\delta^2 S}{\delta A^2}|_{A^{class}} \varphi\right) + \cdots$$

$\int [dz]$ – modular space integration ("collective coordinates").

Turns path integral into stastistical mechanics partition function.

- Powerful approach for non-perturbative phenomena like chiral symmetry breaking and $U_A(1)$ problem $\Longleftrightarrow$ confinement hard to explain
  [Reviews by Schäfer, Shuryak, '98; Forkel, '00; Diakonov†, '03;...]
- (Anti)selfdual solutions seen on the lattice with “cooling”, “smoothing”,...
  [Teper, '86; Ilgenfritz, Laursen, M.-P., Schierholz, Schiller, '86; Polikarpov, Veselov†, '88; ...]

Old idea to implement confinement $\Longrightarrow$ increase entropy by “dissociation” of instantons into constituents (“instanton quarks”).

- $T = 0$: “meron” mechanism [Callan, Dashen, Gross, '77 - '79].
  [Lenz, Negele, Thies, '03-'04; M. Wagner, '06]
- $T > 0$: KvBLL (multi-) calorons with non-trivial holonomy – “dyons”
  [Kraan, van Baal, '98 - '99; Lee, Lu, '98; Diakonov et al., '04 - '09]

Here simulate caloron, dyon ensembles – for $0 < T < T_c$,
meron pair (“dimeron”) ensembles – for $T = 0$.

2. Instantons at $T > 0$: calorons with non-trivial holonomy

Partition function

$$Z_{\mathrm{YM}}(T,V) \equiv \mathrm{Tr}\, e^{-\frac{\hat{H}}{T}} \propto \int DA\, e^{-S_{\mathrm{YM}}[A]} \quad \text{with } A(\vec{x}, x_4 + b) = A(\vec{x}, x_4), \quad b = 1/T\,.$$

Old treatment with HS caloron solutions

$\equiv$ x_4-periodic instanton chains Gross, Pisarski, Yaffe, '81

$$A_\mu^{a\,\mathrm{HS}} = \bar{\eta}^a_{\mu\nu}\partial_\nu \log(\Phi(x))$$

$$\begin{aligned} \Phi(x) &= 1 + \sum_{k\in\mathbf{Z}} \frac{\rho^2}{(\vec{x}-\vec{z})^2 + (x_4 - z_4 - kb)^2} \\ &= 1 + \frac{\pi\rho^2}{b|\vec{x}-\vec{z}|}\, \frac{\sinh\left(\frac{2\pi}{b}|\vec{x}-\vec{z}|\right)}{\cosh\left(\frac{2\pi}{b}|\vec{x}-\vec{z}|\right) - \cos\left(\frac{2\pi}{b}(x_4 - z_4)\right)} \end{aligned}$$

't Hooft symbols: $\eta_{a\mu\nu} = \varepsilon_{a\mu\nu}$, $\eta_{a\mu 4} = -\eta_{a4\mu} = \delta_{a\mu}$ for $\mu,\nu = 1,2,3$, $\eta_{a44} = 0$;
$\bar{\eta}_{a\mu\nu} = (-1)^{\delta_{\mu 4} + \delta_{\nu 4}}\, \eta_{a\mu\nu}$.

HS (anti)caloron exhibits trivial holonomy, i.e. Polyakov loop behaves as:

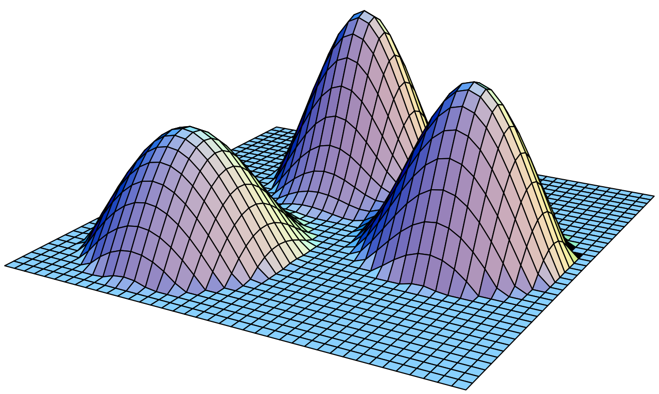
$$\frac{1}{2}\mathrm{tr}\,\mathbf{P}\exp\left(i\int_0^{b=1/T} A_4(\vec{x},t)\; dt\right) \overset{|\vec{x}|\to\infty}{\Longrightarrow} \pm 1.$$

Kraan - van Baal - Lee - Lu solutions (KvBLL)

= (multi-) calorons with non-trivial asymptotic holonomy ($SU(2)$)

$$P(\vec{x}) = \mathbf{P}\exp\left(i\int_0^{b=1/T} A_4(\vec{x},t)\ dt\right) \overset{|\vec{x}|\to\infty}{\Longrightarrow} \mathcal{P}_\infty = e^{2\pi i\omega\tau_3} \notin \mathbf{Z}(2)$$

Holonomy parameter: $0 \leq \omega \leq \frac{1}{2}$, $\omega = \frac{1}{4}$ – maximally non-trivial holonomy.



Action density of an $SU(3)$ caloron (van Baal, '99)

$\Longrightarrow$ not a simple $SU(2)$ embedding into $SU(3)$!!

$SU(2)$ calorons with non-trivial holonomy

[K. Lee, Lu, '98, Kraan, van Baal, '98 - '99, Garcia-Perez et al. '99]

- x_4-periodic, (anti)selfdual solutions from ADHM formalism,
- generalize Harrington-Shepard calorons (i.e. x_4 periodic BPST instantons).

holonomy parameter $\bar{\omega} = 1/2 - \omega, \quad 0 \leq \omega \leq 1/2$

$$
\begin{aligned}
A_\mu^C &= \frac{1}{2}\bar{\eta}^3_{\mu\nu}\tau_3\partial_\nu \log\phi + \frac{1}{2}\ \phi\ \mathrm{Re}\left((\bar{\eta}^1_{\mu\nu} - i\bar{\eta}^2_{\mu\nu})(\tau_1 + i\tau_2)(\partial_\nu + 4\pi i\omega\delta_{\nu,4})\tilde{\chi}\right) \\
&\qquad\qquad + \delta_{\mu,4}\ 2\pi\omega\tau_3\,, \\
\phi(x) &= \frac{\psi(x)}{\hat{\psi}(x)}\,, \qquad x = (\vec{x}, x_4 \equiv t)\,, \qquad r = |\vec{x} - \vec{x}_1|, \quad s = |\vec{x} - \vec{x}_2|\,, \\
\psi(x) &= -\cos(2\pi t) + \cosh(4\pi r\bar{\omega})\cosh(4\pi s\omega) + \frac{r^2 + s^2 + \pi^2\rho^4}{2rs}\sinh(4\pi r\bar{\omega})\sinh(4\pi s\omega) \\
&\quad + \frac{\pi\rho^2}{s}\sinh(4\pi s\omega)\cosh(4\pi r\bar{\omega}) + \frac{\pi\rho^2}{r}\sinh(4\pi r\bar{\omega})\cosh(4\pi s\omega), \\
\hat{\psi}(x) &= -\cos(2\pi t) + \cosh(4\pi r\bar{\omega})\cosh(4\pi s\omega) + \frac{r^2 + s^2 - \pi^2\rho^4}{2rs}\sinh(4\pi r\bar{\omega})\sinh(4\pi s\omega)\,, \\
\tilde{\chi}(x) &= \frac{1}{\psi}\left\{e^{-2\pi i t}\frac{\pi\rho^2}{s}\sinh(4\pi s\omega) + \frac{\pi\rho^2}{r}\sinh(4\pi r\bar{\omega})\right\}.
\end{aligned}
$$

Properties:

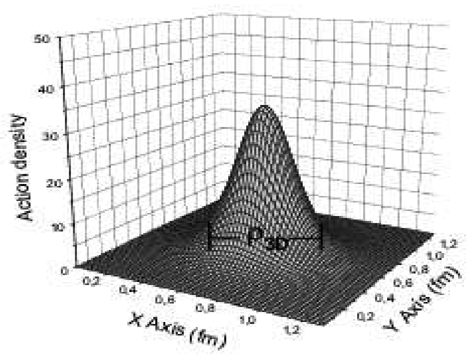
- (anti)selfdual with topological charge $Q_t = \pm 1$,
- has two centers at $\vec{x}_1, \vec{x}_2 \quad \Rightarrow \quad$ "instanton quarks", carrying opposite magnetic charges (visible in maximally Abelian gauge),
- scale-size ρ versus distance d: $\quad \pi\rho^2 T \; = |\vec{x}_1 - \vec{x}_2| = d$,
- limiting cases:
  - $\omega \to 0 \quad \Longrightarrow \quad$ 'old' HS caloron,
  - $|\vec{x}_1 - \vec{x}_2|$ large $\quad \Longrightarrow \quad$ two static BPS monopoles or "dyon pair" (DD) with topological charges ($\sim$ masses) $q_t^{\text{dyon}} \; = \; 2\omega, \;\; 1 - 2\omega$,
  - $|\vec{x}_1 - \vec{x}_2|$ small $\quad \Longrightarrow \quad$ non-static single caloron (CAL).

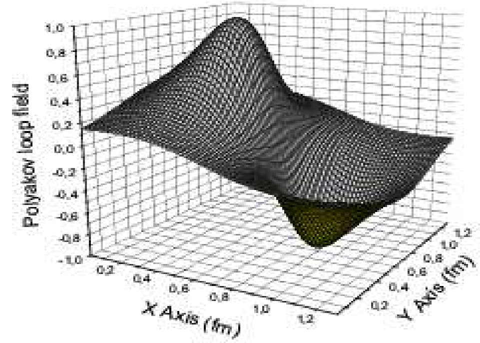
- $L(\vec{x}) = \frac{1}{2}\mathrm{tr}P(\vec{x}) \to \pm 1$ close to $\vec{x} \; \simeq \; \vec{x}_{1,2} \quad \Longrightarrow \quad$ "dipole" structure

$SU(2)$ KvBLL caloron

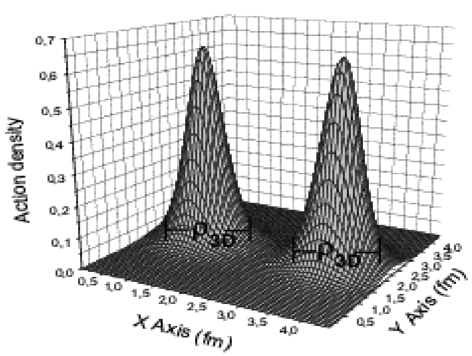
Action density | Polyakov loop

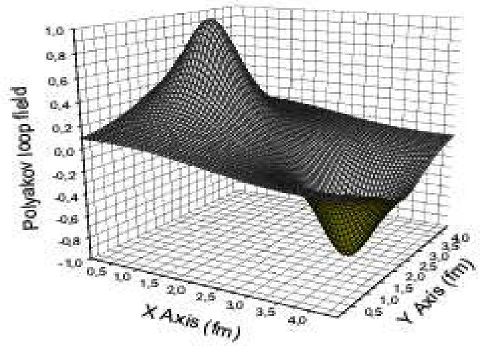
singly localized caloron (CAL)





caloron dissociated into dyon-dyon pair (DD)





Seen by cooling also on the lattice.

[E.-M. Ilgenfritz, B. Martemyanov, M. M.-P., S. Shcheredin, A. Veselov, PRD 66 (2002) 074503]

- Localization of the zero-mode of the Dirac operator:

- time-antiperiodic b.c.:
  around the center with $L(\vec{x}_1) = -1$,

$$|\psi^-(x)|^2 = -\frac{1}{4\pi}\partial_\mu^2 \left[\tanh(2\pi r\bar{\omega})/r\right] \quad \text{for large} \quad d,$$

- time-periodic b.c.:
  around the center with $L(\vec{x}_2) = +1$,

$$|\psi^+(x)|^2 = -\frac{1}{4\pi}\partial_\mu^2 \left[\tanh(2\pi s\omega)/s\right] \quad \text{for large} \quad d.$$

- Lattice search for KvBLL configurations:

- Caloron constituents seen in MC generated lattice gauge fields by cooling, smearing, and filtering with eigenmodes of the (overlap) lattice Dirac operator.
- Their occurence at $T < T_c$ significantly differs from $T > T_c$.

[V. Bornyakov, E.-M. Ilgenfritz, B. Martemyanov, M. M.-P., ..., '02 - '09]

3. Simulating caloron ensembles

[HU Berlin master thesis by P. Gerhold, '06; Gerhold, Ilgenfritz, M.-P., NPB 760, 1 (2007)]

Model based on random superpositions of KvBLL calorons.

Superpositions made in the algebraic gauge – A_4-components fall off.
Gauge rotation into periodic gauge

$$A_\mu^{per}(x) = e^{-2\pi i x_4 \vec{\omega}\vec{\tau}} \cdot \sum_i A_\mu^{(i),alg}(x) \cdot e^{+2\pi i x_4 \vec{\omega}\vec{\tau}} + 2\pi\vec{\omega}\vec{\tau} \cdot \delta_{\mu,4}.$$

First important check: study the influence of the holonomy

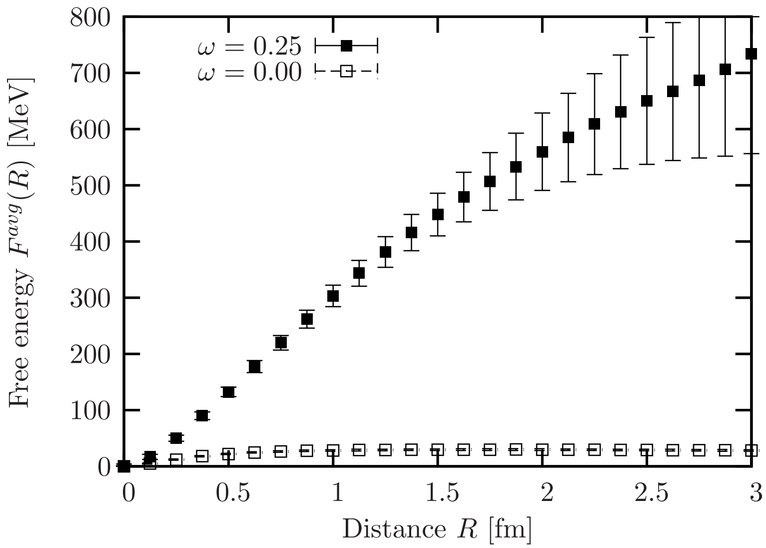
- same fixed holonomy for all (anti)calorons: $\mathcal{P}_\infty = \exp 2\pi i \omega \tau_3$,
  $\omega = 0$ – trivial versus $\omega = 1/4$ – maximally non-trivial,
- put equal number of calorons and anticalorons randomly in a 3d box with open b.c.'s,
- for measurements use a $32^3 \times 8$ lattice grid and lattice observables,
- fix parameters and lattice scale:
  temperature: $T = 1\ \text{fm}^{-1} \simeq T_c$, density: $n = 1\ \text{fm}^{-4}$,
  scale size: compare fixed $\rho = 0.33$ fm
  with distribution $D(\rho) \propto \rho^{7/3} \exp(-c\rho^2)$, such that $\overline{\rho} = 0.33$ fm.

Polyakov loop correlator $\rightarrow$ quark-antiquark free energy

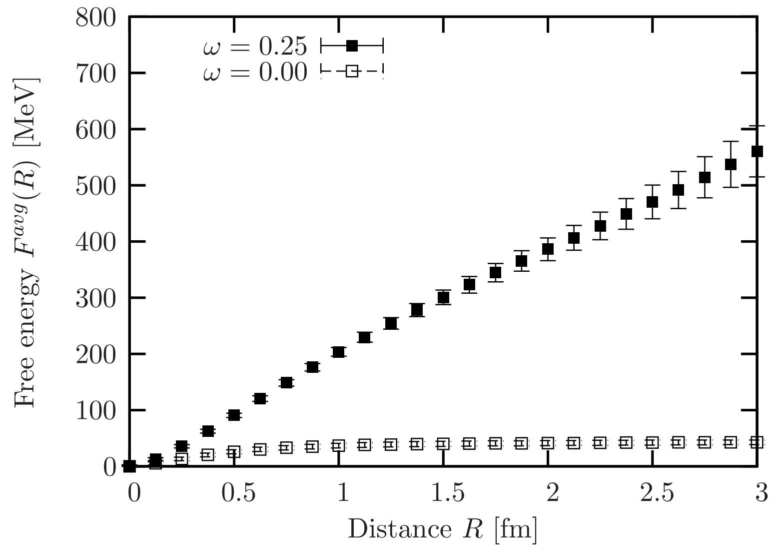
$$F(R) = -T \log \langle L(\vec{x}) L(\vec{y}) \rangle, \quad R = |\vec{x} - \vec{y}|$$

with trivial ($\omega = 0$) and maximally non-trivial holonomy ($\omega = 0.25$).

ρ fixed



ρ sampled with distribution



$\Rightarrow$ Non-trivial (trivial) holonomy creates long-distance coherence (incoherence) and (de)confines for standard instanton or caloron liquid model parameters.

Building a more realistic, T-dependent model:

Main ingrediences:

- Holonomy parameter: $\omega = \omega(T)$
  from lattice results for the (renormalized) average Polyakov loop.
  Digal, Fortunato, Petreczky, '03; Kaczmarek, Karsch, Zantow, Petreczky, '04
  $\Rightarrow \quad \omega = 1/4$ for $T \leq T_c$, $\quad \omega$ smoothly decreasing for $T > T_c$.

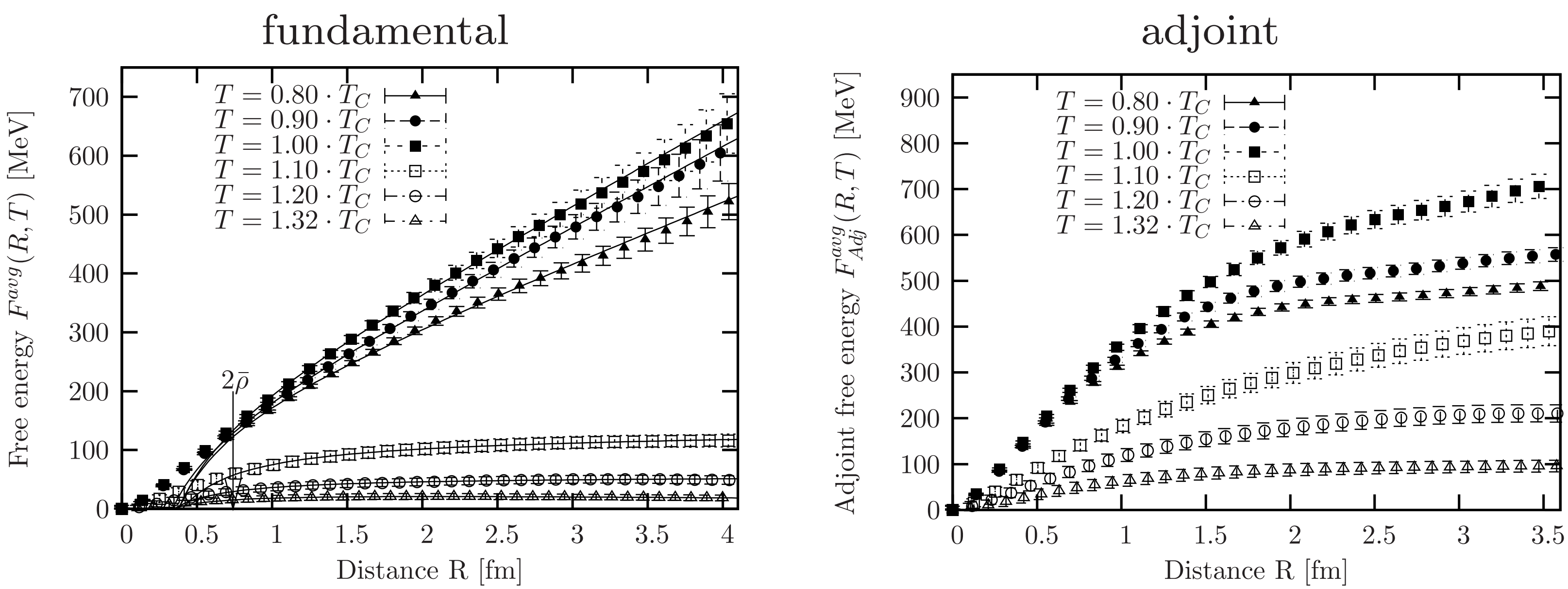
- Density parameter: $n = n(T)$ for uncorrelated caloron gas to be identified with top. susceptibility $\chi(T)$ from lattice results
  Alles, D'Elia, Di Giacomo, '97

- ρ-distribution:
  $T = 0$: Ilgenfritz, M.-P., '81; Diakonov, Petrov, '84
  $T > 0$: Gross, Pisarski, Yaffe, '81

$T < T_c$	$D(\rho,T) = A \cdot \rho^{7/3} \cdot \exp(-c\rho^2)$	$\int D(\rho,T)d\rho = 1,\quad \bar{\rho}$ fixed
$T > T_c$	$D(\rho,T) = A \cdot \rho^{7/3} \cdot \exp(-\frac{4}{3}(\pi\rho T)^2)$	$\int D(\rho,T)d\rho = 1,\quad \bar{\rho}$ running

Distributions sewed together at $T_c \quad \Longrightarrow \quad$ relates $\overline{\rho}(T=0)$ to T_c,
$\overline{\rho}(T=0)$ to be fixed from known lattice space-like string tension
$T_c/\sqrt{\sigma_s(T=0)} \simeq 0.71$: $\quad \Longrightarrow \quad \overline{\rho} = 0.37$ fm.

Color averaged free energy versus distance R at different temperatures from Polyakov loop correlators.



$\Longrightarrow$ successful description of the deconfinement transition,

$\Longrightarrow$ but still no realistic description of the deconf. phase.

4. Dyon gas ensembles and confinement

Working hypothesis (cf. Polyakov, '77):
Confinement evolves from magnetic monopoles effectively in 3D.
Here: monopoles = dyons (KvBLL caloron constituents) for $0 < T < T_c$.

Assume:
integration measure over KvBLL caloron moduli space
rewritten in terms of dyon degrees of freedom,
$\Rightarrow$ difficult task for (still unknown) general multi-caloron solutions.

Diakonov, Petrov, '07 :
proposed integration measure for all kind dyons
(Abelian fields; no antidyons, i.e. CP is violated).
Dyon ensemble statistics analytically solved $\Longrightarrow$ confinement.

However, observation from numerical simulation:
Moduli space metric satisfies positivity only for a small fraction
of dyon configurations and only for low density $\Longrightarrow$ inconsistent metric.
[Bruckmann, Dinter, Ilgenfritz, M.-P., Wagner, PRD 79, 034502 (2009)].

Simplify the model:

- Far-field limit, i.e. purely Abelian monopole fields, non-trivial holonomy.
- Neglect moduli space metric and describe random monopole gas.
- Compute free energy of a static quark-antiquark pair from Polyakov loop correlators.

  [Bruckmann, Dinter, Ilgenfritz, Maier, M.-P., Wagner, PRD 85, 034502 (2012)]

Monopole field:

$$a_0(\mathbf{r};q) = \frac{q}{r}, \quad a_1(\mathbf{r};q) = -\frac{qy}{r(r-z)}, \quad a_2(\mathbf{r};q) = +\frac{qx}{r(r-z)}, \quad a_3(\mathbf{r};q) = 0,$$

$\mathbf{r} = (x,y,z)$, $r = |\mathbf{r}|$ - 3d distance to the dyon center, q - magnetic charge.

With 't Hooft's symbol write:

$$a_\mu(\mathbf{r};q) = -q\ \bar{\eta}^3_{\mu\nu}\ \partial_\nu \ln(r-z).$$

Holonomy to be added to superponed monopole fields:

$$A_0 \quad \to \quad 2\pi\ \omega\ T\ \tau_3,$$

$$P(\mathbf{r}) \quad \equiv \quad \frac{1}{2}\mathrm{Tr}\left(\exp\left(i\int_0^{1/T} dx_0\, A_0(x_0,\mathbf{r})\right)\right) \to \frac{1}{2}\mathrm{Tr}\left(\exp\left(2\pi i\omega\tau_3\right)\right) = \cos(2\pi\omega),$$

Superposition of gauge fields of $2K$ dyons:

$$A_\mu(\mathbf{r}) = \left(\delta_{\mu 0} 2\pi\omega T + \frac{1}{2} \sum_{i=1}^{K} \sum_{m=1}^{2} a_\mu(\mathbf{r} - \mathbf{r}_i^m; q_m) \right) \tau_3 ,$$

$\mathbf{r}_i^m$ and $q_m = -(-1)^m$ – positions and magnetic charges of i-th dyon ($m = 1$), antidyon ($m = 2$), respectively.

Local Polyakov loop $P(\mathbf{r})$:

$$P(\mathbf{r}) = \cos\left(2\pi\omega + \frac{1}{2T}\Phi(\mathbf{r}) \right) , \qquad P(\mathbf{r})\Big|_{\omega=1/4} = -\sin\left(\frac{1}{2T}\Phi(\mathbf{r}) \right),$$

where

$$\Phi(\mathbf{r}) \equiv \sum_{i=1}^{K} \sum_{m=1}^{2} \frac{q_m}{|\mathbf{r} - \mathbf{r}_i^m|} = \sum_{i=1}^{K} \left[\frac{1}{|\mathbf{r} - \mathbf{r}_i^1|} - \frac{1}{|\mathbf{r} - \mathbf{r}_i^2|} \right].$$

Compute free energy of a static $\bar{Q}Q$ pair from Polyakov loop correlator

$$F_{\bar{Q}Q}(d) = -T \ln \left\langle P(\mathbf{r}) P^\dagger(\mathbf{r}') \right\rangle , \quad d \equiv |\mathbf{r} - \mathbf{r}'| .$$

Expectation value:

$$\left\langle O \right\rangle = \int \prod_{i=1}^{K} d\mathbf{r}_i^1 d\mathbf{r}_i^2 \, O\Big(\{\mathbf{r}_i^1, \mathbf{r}_i^2\}\Big) \Big/ \int \prod_{i=1}^{K} d\mathbf{r}_i^1 d\mathbf{r}_i^2$$

$$= \int \prod_{i=1}^{K} d\mathbf{r}_i^1 d\mathbf{r}_i^2 \, O\Big(\{\mathbf{r}_i^1, \mathbf{r}_i^2\}\Big) \Big/ V^{2K},$$

V – spatial volume $\Longrightarrow$ $\rho = \frac{2K}{V}$ – density of monopole gas.

ω, T, and ρ – basic parameters of the model.

Expectation values can be computed numerically generating random distributions of the dyon positions in the volume V.

Monopole fields have infinite range $\Longrightarrow$ strong finite size effects, well-known in plasma and statistical physics.

Solution: Ewald's method [P. Ewald, Ann. Phys. (1921)]

- use infinite identical replica of a basic 3d cell ("super cell") with randomly distributed monopoles:

$$\Phi(\mathbf{r}) = \sum_{\mathbf{n}\in\mathbb{Z}^3} \sum_{j} \frac{q_j}{|\mathbf{r}-\mathbf{r}_j-\mathbf{n}L|},$$

$j=(i,m)$ running over all dyons and antidyons coming in equal number (j takes $n_D = 2K$ different values).

- Split phase $\Phi(\mathbf{r})$ into short-distance and long-distance contributions

$$\begin{aligned}
\Phi(\mathbf{r}) &= \Phi^{\text{short}}(\mathbf{r}) + \Phi^{\text{long}}(\mathbf{r}) \\
\Phi^{\text{short}}(\mathbf{r}) &\equiv \sum_{\mathbf{n}\in\mathbb{Z}^3} \sum_{j} \left(1 - \operatorname{erf}\left(\frac{|\mathbf{r}-\mathbf{r}_j-\mathbf{n}L|}{\sqrt{2}\lambda}\right)\right) \frac{q_j}{|\mathbf{r}-\mathbf{r}_j-\mathbf{n}L|} \\
\Phi^{\text{long}}(\mathbf{r}) &\equiv \sum_{\mathbf{n}\in\mathbb{Z}^3} \sum_{j} \operatorname{erf}\left(\frac{|\mathbf{r}-\mathbf{r}_j-\mathbf{n}L|}{\sqrt{2}\lambda}\right) \frac{q_j}{|\mathbf{r}-\mathbf{r}_j-\mathbf{n}L|},
\end{aligned}$$

where erf(.) – error function produces smeared charges cancelling with point charges at rising distances, λ – certain cutoff scale.

Short-distance contribution converges exponentially.

Long-distance contribution converges in its Fourier transform
(for charge-neutral system only $\Longrightarrow$ for dyon gas satisfied).

$$
\begin{aligned}
\Phi^{\rm long}(\mathbf{r}) &= \frac{4\pi}{L^3} \sum_{\mathbf{n}\in\mathbb{Z}^3\setminus\vec{0}} e^{+i\mathbf{k}(\mathbf{n})\mathbf{r}} \; \frac{e^{-\lambda^2\mathbf{k}(\mathbf{n})^2/2}}{\mathbf{k}(\mathbf{n})^2} \left(\sum_{j=1}^{N} q_j\, e^{-i\mathbf{k}(\mathbf{n})\mathbf{r}_j} \right), \\
\mathbf{k}(\mathbf{n}) &= \frac{2\pi}{L}\,\mathbf{n}.
\end{aligned}
$$

Cutoff scale λ chosen such that CPU time is optimized.

Model can be solved also analytically:

Infinite-volume limit and for large distance $d = |\mathbf{r} - \mathbf{r}'|$

$$\left\langle P(\mathbf{r}) P(\mathbf{r}') \right\rangle = \frac{1}{2} \exp\left(- \frac{\pi d \rho}{2T^2} + \text{const.} \right)$$

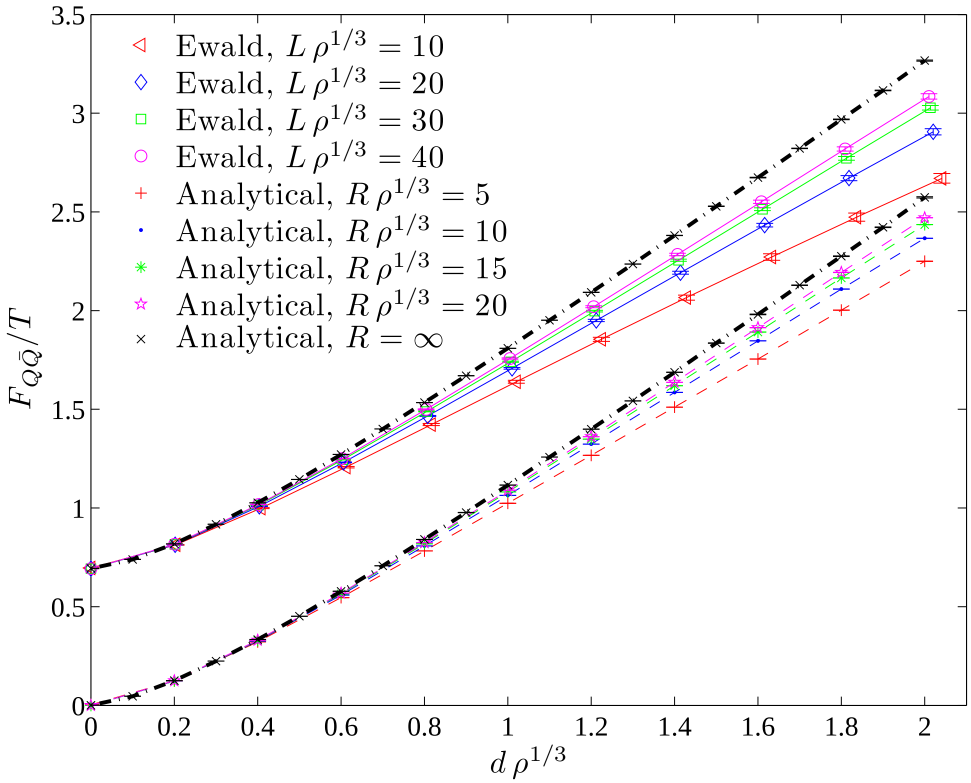
$\Longrightarrow$ free energy $F_{\bar{Q}Q}(d) = \sigma\, d + \text{const.}$

$\Longrightarrow$ string tension $\sigma = \frac{\pi}{2} \frac{\rho}{T}$.

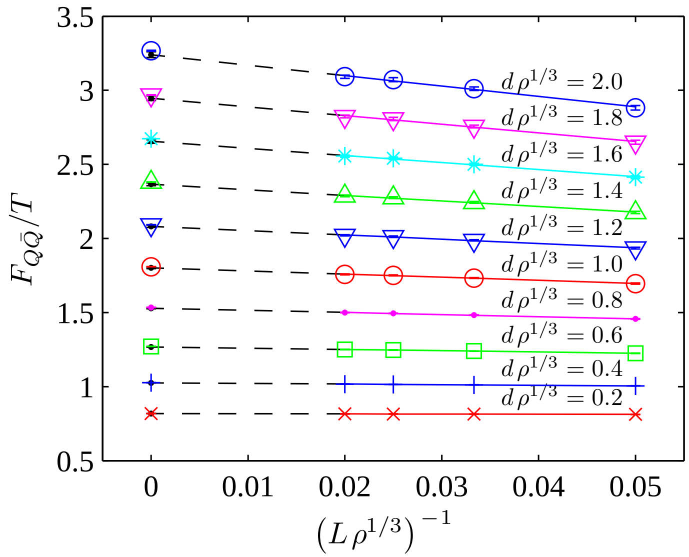
At arbitrary finite distance d :

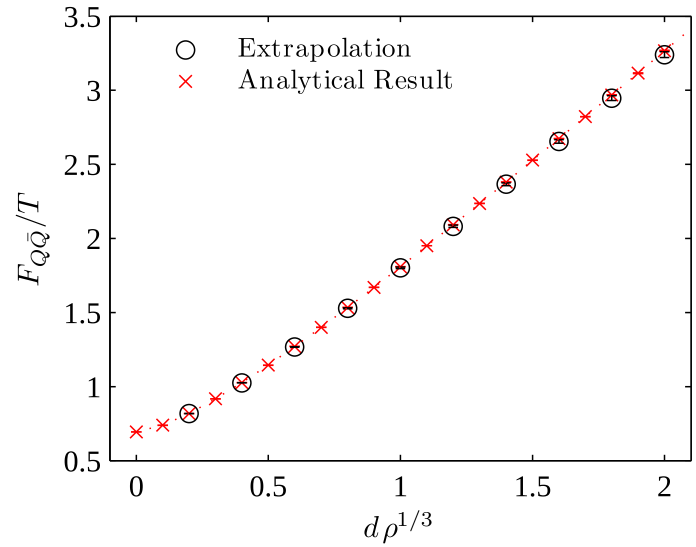
Polyakov loop correlator can be obtained by numerical integrations.

Compare Ewald's method with analytical results:



$$F_{\bar{Q}Q}(d)/T \quad \text{vs.} \quad d \quad (\rho/T^3 = 1.0)$$





Ewald's result extrapolated to $V \to \infty$ $\quad$ $F_{\bar{Q}Q}(d)/T$ vs. d for $V \to \infty$

$\Longrightarrow$ excellent agreement !!

$\Longrightarrow$ simplest dyon gas model provides confinement
(in terms of Polyakov loop correlator).

5. $T = 0$: Simulating dimeron ensembles

[F. Zimmermann, H. Forkel, M. M.-P., PRD 86, 094005 (2012)]

– KvBLL-like solutions in Euclid. 4d Yang-Mills theory unknown.
– Possibility: Dimeron (DM) configurations [De Alfaro, Fubini, Furlan, '76 - '77]

single (anti)merons: $Q_t = \pm 1/2, \quad S \to \infty.$

(anti)dimeron = (anti)meron pair ($r \equiv$ regular gauge, $SU(2)$): $Q_t = \pm 1$

$$A_\mu^{(DM,r)}(x; \{x_0, a, u\}) = \left[\frac{(x - x_0 + a)_\nu}{(x - x_0 + a)^2} + \frac{(x - x_0 - a)_\nu}{(x - x_0 - a)^2} \right] u^\dagger \sigma_{\mu\nu} u \,,$$

$\sigma_{\mu\nu} := \eta_{a\mu\nu}\ \tau_a/2$, for $DM \leftrightarrow \overline{DM}$ replace $\eta \leftrightarrow \bar{\eta}$, u – colour rotations.

Limiting cases:
$a \to 0 \quad \Rightarrow$ regular gauge instanton,
$a \to \infty \quad \Rightarrow$ well-separated merons with 'locked' colour orientation.

DM has 11 collective coordinates $z \equiv \{x_0, a, u\}$

(instead of 8 for one instanton, 14 for two single, colour-unlocked merons).

$\Rightarrow$ corresponding increase of entropy in the path integral.

To superpone (anti)dimerons put them into singular gauge ("s")

$\Longrightarrow \quad A_\mu^{(DM,s)} \sim \frac{1}{x^3}$ for $|x| \gg |a|$, i.e. better localized.

For numerical integrations (action, top. charge, parallel transporters etc.) meron and gauge singularities have to be regularized.

Superpositions: (anti)dimerons superponed (neglect meron-antimeron pairs)

$$A_\mu(x, \{z_I, \bar{z}_{\bar{I}}\}) = \sum_I^{N_{DM}} A_\mu^{(DM,s)}(x, \{z_I\}) + \sum_{\bar{I}}^{N_{\overline{DM}}} \bar{A}_\mu^{(\overline{DM},s)}(x, \{\bar{z}_{\bar{I}}\}).$$

Partition function:

$$Z = \int \prod_{I,\bar{I}}^{N_{DM}, N_{\overline{DM}}} dz_I d\bar{z}_{\bar{I}} \ \exp\left\{-S\left[A_\mu\left(x, \{z_I, \bar{z}_{\bar{I}}\}\right)\right]\right\},$$

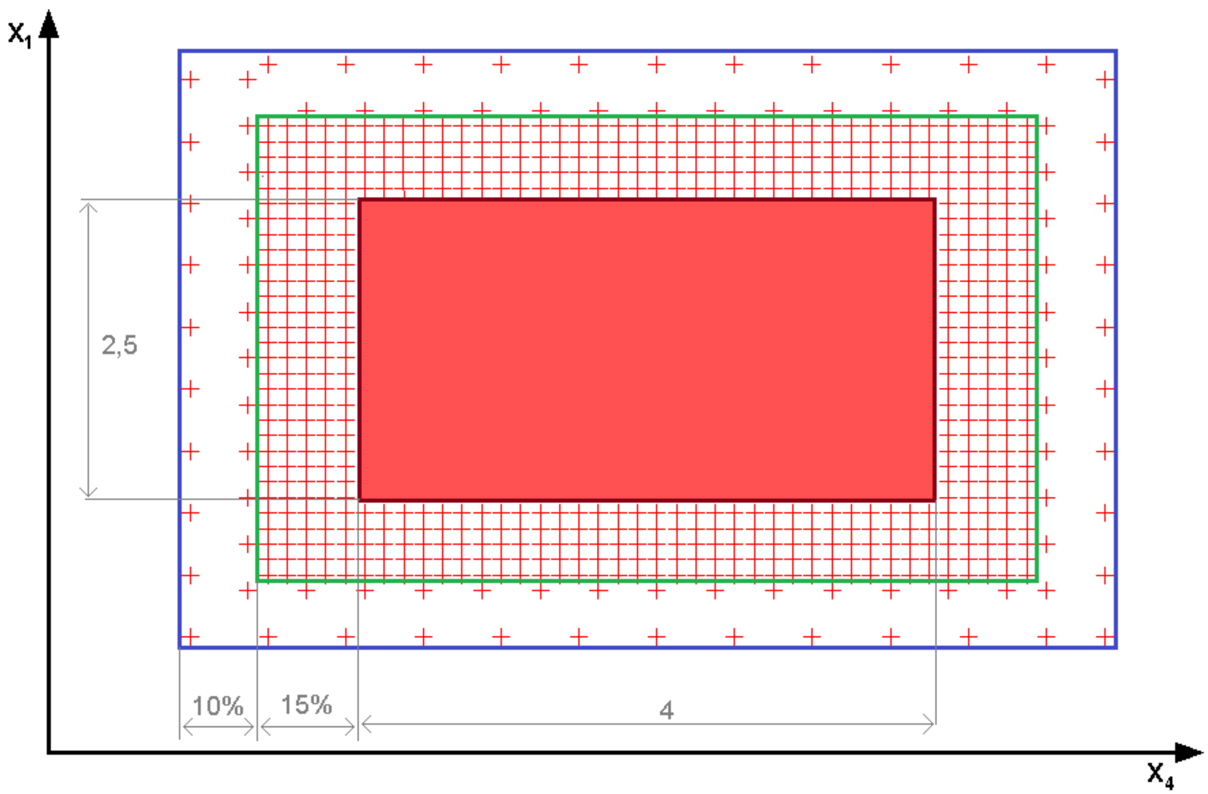
with

$$\begin{aligned} S\left[A\right] &= \frac{1}{2\ g^2} \int d^4x \ tr\left\{F_{\mu\nu} F_{\mu\nu}\right\} =: \int d^4x \ s\left(x\right), \\ F_{\mu\nu}\left(x\right) &= \partial_\mu A_\nu\left(x\right) - \partial_\nu A_\mu\left(x\right) - i\left[A_\mu\left(x\right), A_\nu\left(x\right)\right]. \end{aligned}$$

In the following study g^2 dependence ("temperature").

Simulating the dimeron gas:

- Take statistical weight of dimeron confs. $\exp -S[A]$ into account.
- Estimate $S[A]$ on a lattice grid.



Multi-layered multigrid to control boundary effects.

Inner measurement volume $\Leftrightarrow$ approximate constant action density.

- MC method: Metropolis.
- For approaching to equilibrium adapt step-sizes and grid resolution.
- Ensemble: $N_{DM} = 243, N_{\overline{DM}} = 244$ in whole volume.
- Bare coupling ("temperature"): $g^2 \in \{1, 25, 100, 1000, \infty\}$.

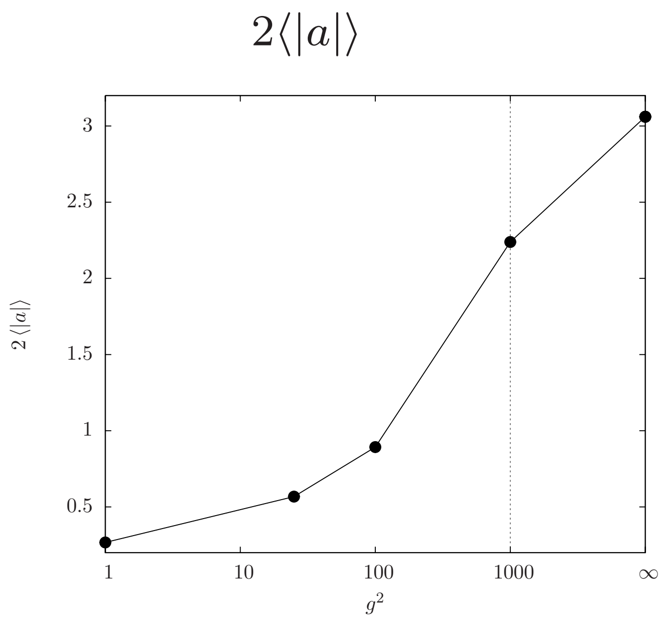
Measurements:

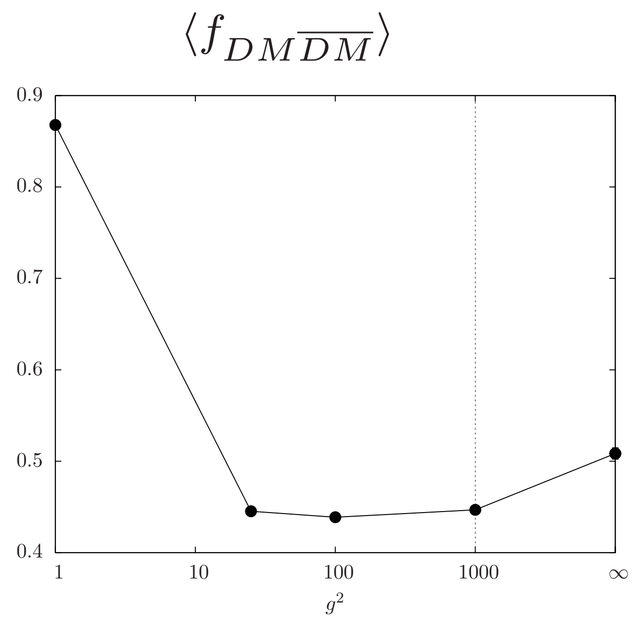
- Ensemble parameters: intermeron distances, neighbour densities, colour correlations,...
- Topology: spatial top. charge distribution, topological susceptibility.
- Wilson loops: static $\bar{Q}Q$-potential, string tension.

Results:

Left: Average inter-meron distance $2\langle|a|\rangle$ versus g^2.

Right: Probability for the nearest neighbour of a dimeron to have opposite topological charge $\langle f_{DM\overline{DM}}\rangle$ versus g^2.





$\Longrightarrow$ Dimerons dissociate into their constituents with increasing g^2.

$\Longrightarrow$ Topological order at $g^2 = 1 \iff$ disorder for $g^2 > 10$.

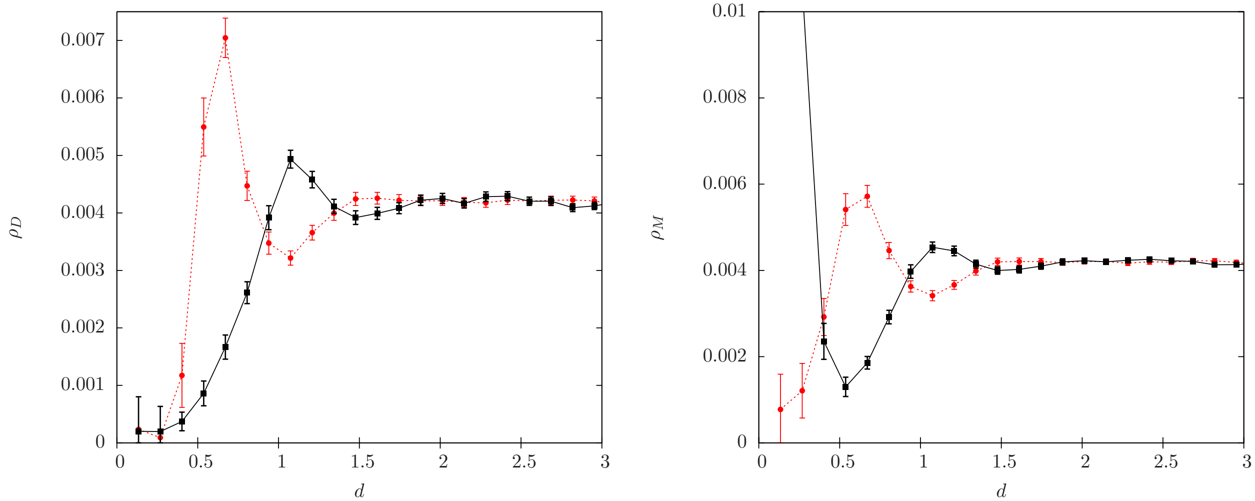
Spatial topological order / disorder:

average radial density of neighbour pseudoparticles vs. distance d

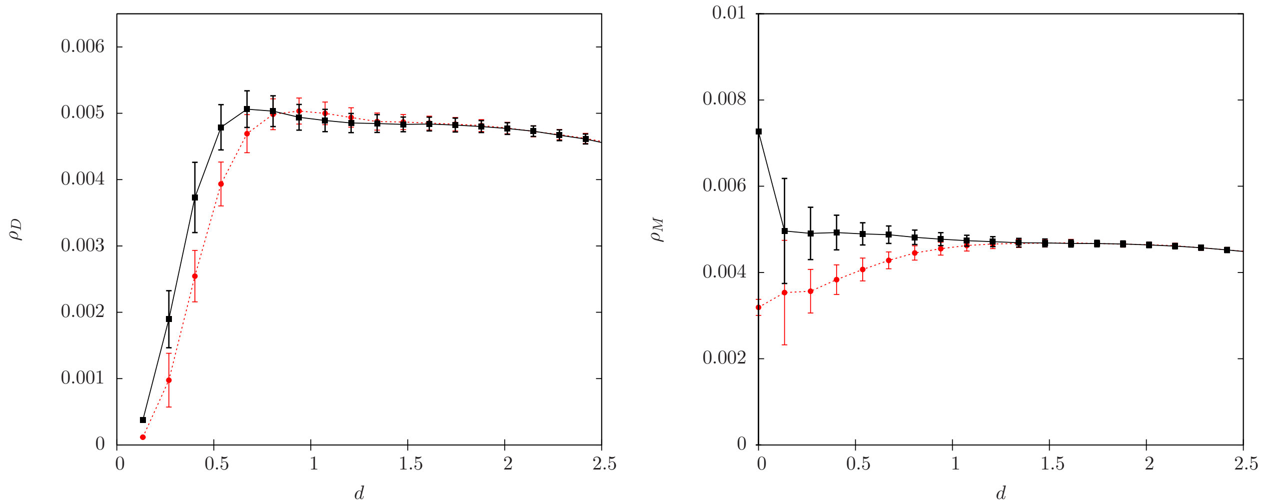
equal (black), opposite (red) sign topological charges

dimeron density

meron density



$g^2 = 1$



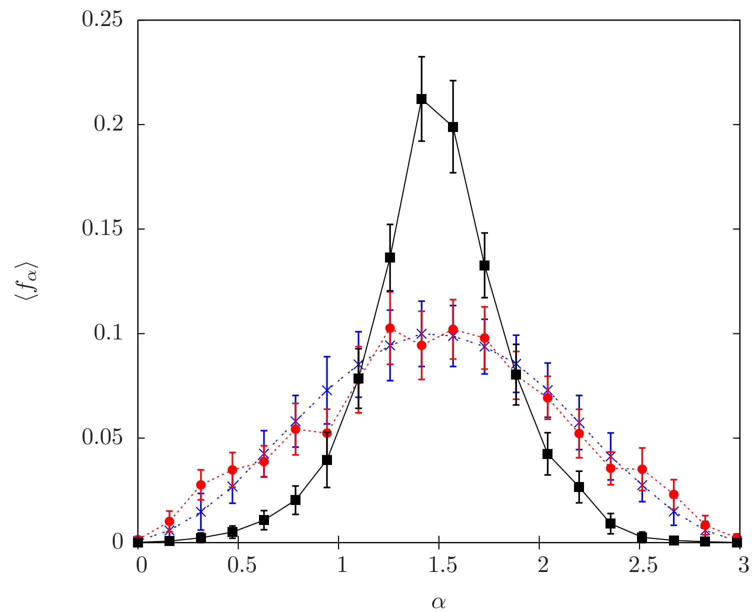
$g^2 = 100$

Nearest neighbour dimeron colour-angle distribution $\langle f_\alpha \rangle$

black squares: equal top. charge,

red bullets: opposite top. charge,

blue crosses: random distribution ($g^2 = \infty$).



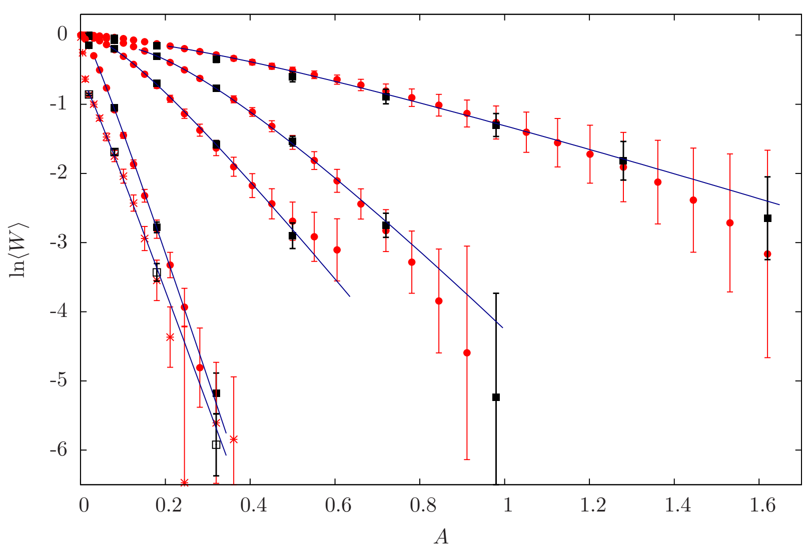
$g^2 = 1$

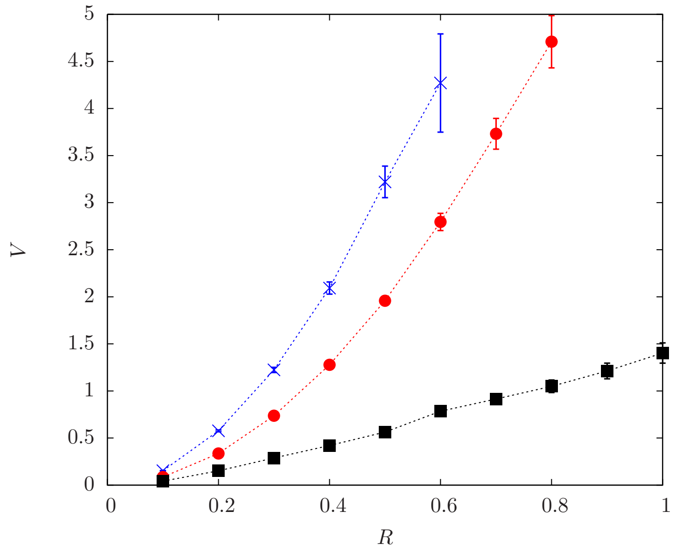
$\Longrightarrow$ $\alpha \simeq \pi/2$ dominant for $DM - DM$ pairs,

$DM - \overline{DM}$ pairs randomly mutually orientated.

$\Longrightarrow$ qualitatively understood from two-dimeron ($\simeq$ two-instanton) interactions.

Wilson loops $\log < W >$ vs. area A static potential $V(R)$:



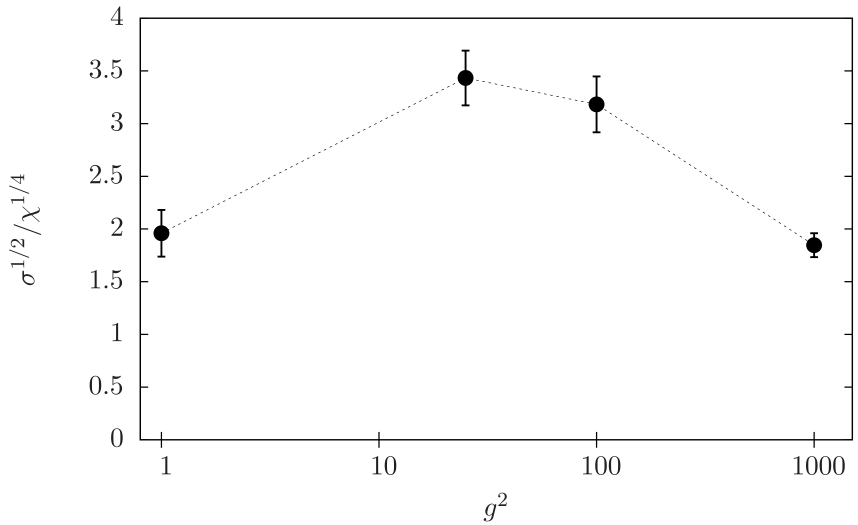


$g^2 = 1, 25, 100, 1000, \infty$

$g^2 = 1, 25, 100$

Dimensionless ratio:

string tension / top. susceptibility $\sigma^{1/2}/\chi^{1/4}$ vs. g^2.



$\Longrightarrow$ $\chi^{1/4}/\sigma^{1/2} \simeq 0.30, \ldots, 0.55.$

$\Longrightarrow$ Compatible with lattice ($SU(2)$): $\chi^{1/4}/\sigma^{1/2} = .483 \pm .006$

[Lucini, Teper, 01]

$\Longrightarrow$ Compatible also with simulations of single meron and regular instanton ensembles [Lenz, Negele, Thies, 08]

7. Summary

- Topological aspects in QCD occur naturally and have phenomenological impact.
  Standard instanton gas/liquid remains phenomenologically important: chiral symmetry breaking, solution of $U_A(1)$, ...,
  but fails to explain confinement.

- $0 < T < T_c$: KvBLL caloron gas model with non-trivial holonomy very encouraging for description of confinement. Model can be improved.

- Non-interacting Abelian dyon gas model analytically solvable
  $\Rightarrow$ confinement.
  Ewald's method allows to keep finite-size effects under control and provides same infinite volume result.
  Full modular space metric (?) should be taken into account.

- $T = 0$: Dimerons play similar role as KvBLL calorons for $0 < T < T_c$.
  Shows Callen-Dashen-Gross mechanism of meron disorder at strong coupling.
  Reasonable results for topological susceptibility in units of the string tension obtained.

Thank you for your attention

Bol'shoye spasibo za vnimaniye