Two-colour QCD at $T > 0$
in the presence of a strong magnetic field

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with collaborators

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Outline:

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2. The lattice model
3. How to couple an external constant magnetic field B
4. The influence of an external magnetic field
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5. The chiral condensate in the chiral limit
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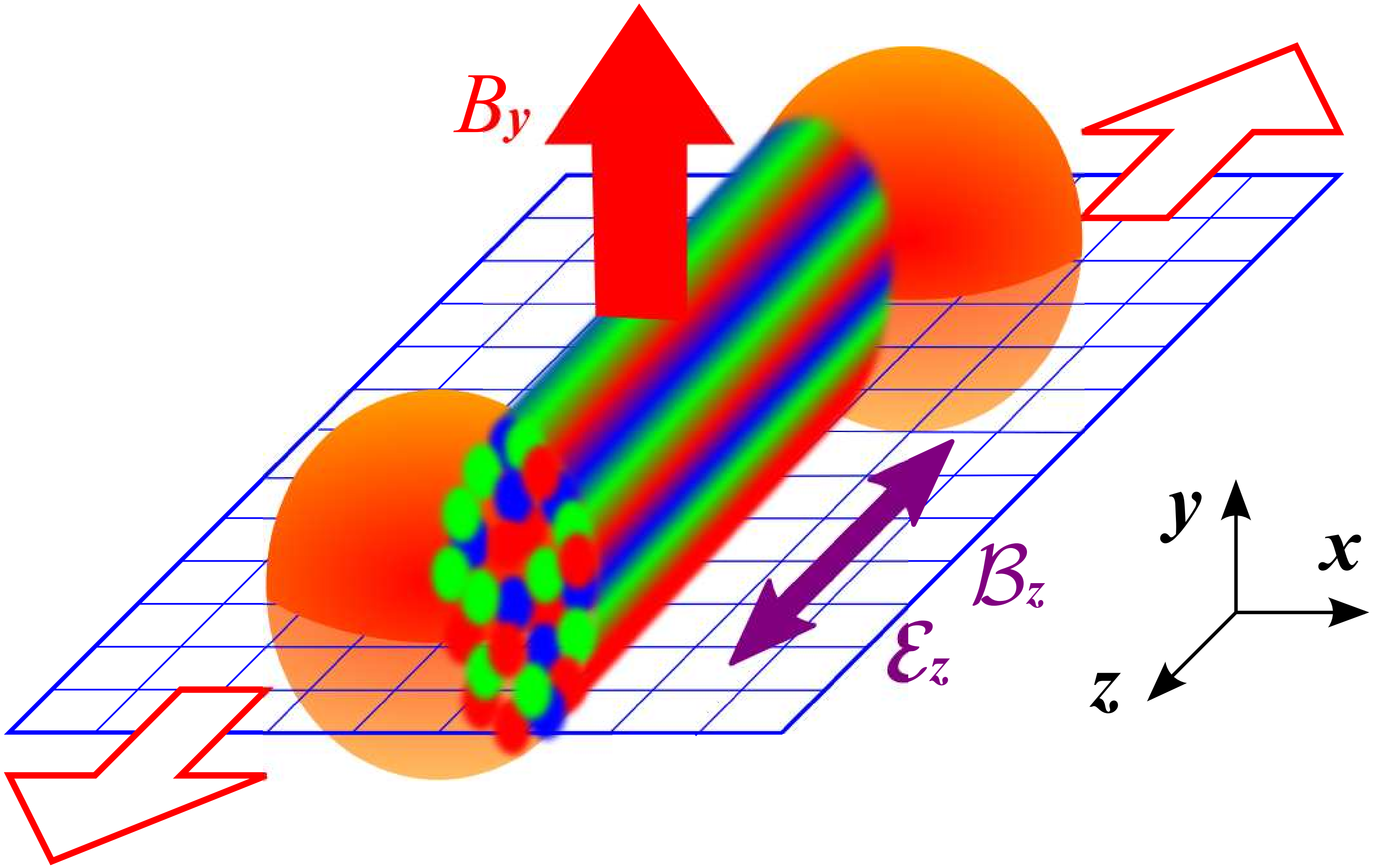
1. Introduction

Very strong magnetic fields may exist (or have existed)

- **during the electroweak phase transition ($\sqrt{eB} \sim 1-2$ GeV),**
- **in the interior of dense neutron stars (magnetons) ($\sqrt{eB} \sim 1$ MeV),**
- **in noncentral heavy ion collisions at RHIC ($\sqrt{eB} \sim 100$ MeV) and LHC ($\sqrt{eB} \sim 500$ MeV), because antiparallel currents of the spectators create a strong magnetic field.**

Non-central heavy ion collision **Kharzeev, McLerran, Warringa, '08**



Such strong magnetic fields may lead to

- **a strengthening of the chiral symmetry breaking (increase of the chiral condensate, increase of F_π, decrease of M_π),**
- **a change of the finite temperature chiral transition both in temperature and in strength,**
- **the chiral magnetic effect (CME), leading to an event by event charge asymmetry in peripheral heavy ion collisions.**

Chiral model at $T = 0$ (Shushpanov, Smilga, '97)

$$<\bar{\psi}\psi>_B = <\bar{\psi}\psi>_0 \left(1 + \frac{1}{F_\pi^2}\frac{(eB)^2}{96\pi^2 M_\pi^2} + \mathcal{O}\left(\frac{(eB)^4}{F_\pi^4 M_\pi^4}\right)\right)$$

In the chiral limit, $M_\pi \ll \sqrt{eB} \ll 2\pi F_\pi \sim \Lambda_{hadr}$**:**

from J. Schwinger's ('51) solution

$$<\bar{\psi}\psi>_B = <\bar{\psi}\psi>_0 \left(1 + \frac{1}{F_\pi^2}\frac{(eB)\log 2}{16\pi^2} + \mathcal{O}\left(\frac{(eB)^2}{F_\pi^4}, \frac{(eB)^2}{\Lambda_{hadr}^4}\right)\right)$$

$$M_{\pi^0}(B) = M_{\pi^0}(0)\left(1 - \frac{1}{F_\pi^2}\frac{(eB)\log 2}{16\pi^2} + \ldots\right)$$

$$F_\pi(B) = F_\pi(0)\left(1 + \frac{1}{F_\pi^2}\frac{(eB)\log 2}{8\pi^2} + \ldots\right)$$

$$M_{\pi^+}(B) = M_{\pi^-}(B) \propto \sqrt{eB}$$

Strong fields $\sqrt{eB} \gg F_\pi, M_\pi, \Lambda_{hadr}$ **or deconfined phase** $(T > T_c)$

$< \bar{\psi}\psi >_B \sim |eB|^{3/2} \implies$ **e**B **the only scale**

Dyson-Schwinger equations suggest a selfconsistent quark mass:

$$m_q(B) \sim \sqrt{|eB|} \exp\left[-\sqrt{\pi/(\alpha_s c_F)}\right]$$

$$< \bar{\psi}\psi >_B \sim |eB|^{3/2} \exp\left[-\frac{\pi}{2}\sqrt{\pi/(2\alpha_s c_F)}\right]$$

where $\alpha_s \equiv \alpha_s(|eB|)$

2. The lattice model

Our simplified quark-gluon matter:

- colour $SU(2)$,
- staggered fermions without rooting of fermionic determinant, i.e $N_f = 4$ flavours,
- unique e.-m. charge.

Why this model?

- Very similar chiral behaviour as in $SU(3)$ colour.
- Can be extended to finite baryon chemical potential without sign problem.
- Topology (important also for the CME) can be studied in a more simple case.
- Much faster to simulate. Can take the chiral limit.
  Use a farm of PC's (and recently GPU's).
- University requirement: nice model to be proposed for master students.

Pioneering calculations (quenched SU(2)): Buividovich, Lushchevskaya, Polikarpov,...

Full QCD (conflict): D'Elia et al ⇔ Bali, Bruckmann, Endrodi, Fodor, Schäfer,...

Lattice gauge action: **from elementary closed (Wilson) loops (“plaquettes”)**

$$U_{n,\mu\nu} \equiv U_{n,\mu}\, U_{n+\hat{\mu},\nu}\, U^{\dagger}_{n+\hat{\nu},\mu}\, U^{\dagger}_{n,\nu}\,, \qquad U_{n,\mu} \in SU(N_c)$$

$$\begin{aligned} S_G^W &= \beta \sum_{n,\mu<\nu} \left(1 - \frac{1}{N_c}\,\mathrm{Re}\,\mathrm{tr}\,U_{n,\mu\nu}\right), \qquad \beta = \frac{2N_c}{g_0^2} \\ &= \frac{1}{2}\sum_n a^4\,\mathrm{tr}\,G^{\mu\nu}G_{\mu\nu} + O(a^2), \\ &\to \frac{1}{2}\int d^4x\,\mathrm{tr}\,G^{\mu\nu}G_{\mu\nu}. \end{aligned}$$

Continuum limit:

$$a(g_0) = \frac{1}{\Lambda_{Latt}} (\beta_0 g_0^2)^{-\frac{\beta_1}{2\beta_0^2}} \exp\left(-\frac{1}{2\beta_0 g_0^2}\right)(1 + O(g_0^2)).$$

$\Longrightarrow$ $a \to 0$ **for** $g_0 \to 0$ **(or** $\beta \to \infty$**),** *asymptotic freedom*.

For $SU(N_c)$ **and** N_f **massless fermions, independent on renormalization scheme:**

$$\beta_0 = \frac{1}{(4\pi)^2}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right), \qquad \beta_1 = \frac{1}{(4\pi)^4}\left(\frac{34}{3}N_c^2 - \frac{10}{3}N_c N_f - \frac{N_c^2-1}{N_c}N_f\right).$$

Staggered fermion action

Kogut, Susskind, '75

- **Use naive discretization and diagonalize action w.r.t. to spinor degrees of freedom.**
- **Neglect three of four degenerate Dirac components.**
- **Attribute the 16 fermionic degrees of freedom localized around one elementary hypercube to four *tastes* with four Dirac indices each.**

Chiral symmetry restored $\Longleftrightarrow$ flavor symmetry broken.

Naturally the mass-degenerated four-flavor case is described.

Path integral quantization for Euclidean time $\Longrightarrow$ 'statistical averages'.

Fermions as anticommuting Grassmann variables

$\Longrightarrow$ analytically integrated $\Rightarrow$ non-local effective action $S^{eff}(U)$.

'Partition function' describing $N_f = 4$ mass-degenerate staggered flavors:

$$
\begin{aligned}
Z &= \int [dU][d\psi]\,[d\bar{\psi}]\ e^{-S^G(U)\ +\ \bar{\psi}M(U)\psi} \\
&= \int [dU]\ e^{-S^G(U)}\ \mathrm{Det}M(U) \\
&= \int [dU]\ e^{-S^{eff}(U)}, \qquad S^{eff}(U) = S^G(U) - \log(\mathrm{Det}M(U))
\end{aligned}
$$

with $M(U) \equiv D_{\mathrm{Latt}}(U) + m$**.**

Simulation on a finite lattice $N_t \times N_s^3$,

with (anti-) periodic boundary conditions for gluons (quarks).

Rooting prescription:

for $N_f = 2+1\ (+1)$ 4th-root of the fermionic determinant is taken.

$\Longrightarrow$ **Locality violated (??)**

Non-zero temperature $T \equiv 1/L_t = 1/(N_t\ a(\beta))$ **:**

T **varied by changing** β **at fixed** N_t **(changing** N_t **at fixed** β**).**

Order parameters:

Polyakov loop: $L(\vec{x}) \equiv \frac{1}{N_c}\mathrm{tr}\prod_{x_4=1}^{N_t} U_4(\vec{x}, x_4)$, $\langle L(\vec{x})\rangle = \exp(-\beta F_Q)$,

F_Q $=$ **free energy of an isolated infinitely heavy quark.**

$\Longrightarrow$ $F_Q \to \infty$**, i.e.** $\langle L(\vec{x})\rangle \to 0$ **within the confinement phase (**$T < T_c$**).**

$\Longrightarrow$ $\langle L(\vec{x})\rangle$ **order parameter for the deconfinement transition (**$T = T_c$**).**

Chiral condensate: $\langle\bar{\psi}\psi\rangle$

order parameter for chiral symmetry breaking ($T < T_c$**) and restoration (**$T > T_c$**)**

Find critical T_c **(or** β_c**) from maxima of susceptibilities of** $L(\vec{x})$ **and/or** $\bar{\psi}\psi$**.**

Fixing the physical scale:

$T > 0$ **calculations done on lattices of size:** $16^3 \times 6$ $(24^3 \times 6)$

$T = 0$ **calculations:** $16^3 \times 32$

The lattice unit scale $a(\beta)$ **fixed via scale parameter** r_0 **[R. Sommer, '94], assumed to be the same as in real QCD:**

Compute static force $F(r) = dV/dr$ **phenomenologically well-known from** $\bar{c}c$**- or** $\bar{b}b$**-potential** $V_{\bar{Q}Q}$**:**

$$F(r_0)\ r_0^2 \ \equiv\ 1.65 \quad \leftrightarrow \quad r_0 \simeq 0.5\ \text{fm}$$

Then determine e.g. the pion mass m_π**.**

For $T = 0$, $ma = 0.01$, $B = 0$ **we obtain at** $\beta = 1.80$ **(**$\simeq \beta_c$ **for** $N_t = 6$**).**

$$a \ = \ 0.17(1)\ \text{fm}$$

$$m_\pi \ = \ 330(25)\ \text{MeV}$$

$$T_c \ = \ 193(13)\ \text{MeV}$$

preliminary

3. How to couple an external constant magnetic field B

$\bar{B} = (0, 0, B)$ $\bar{A}(\bar{r}) = \frac{B}{2}(-y, x, 0)$

On the lattice we use the compact formulation. Constant magnetic field $\equiv$ constant magnetic flux $\phi = a^2(eB)$ through all (x, y) plaquettes.

On the links define $U(1)$ elements coupled to quark fields in lattice covariant derivative.

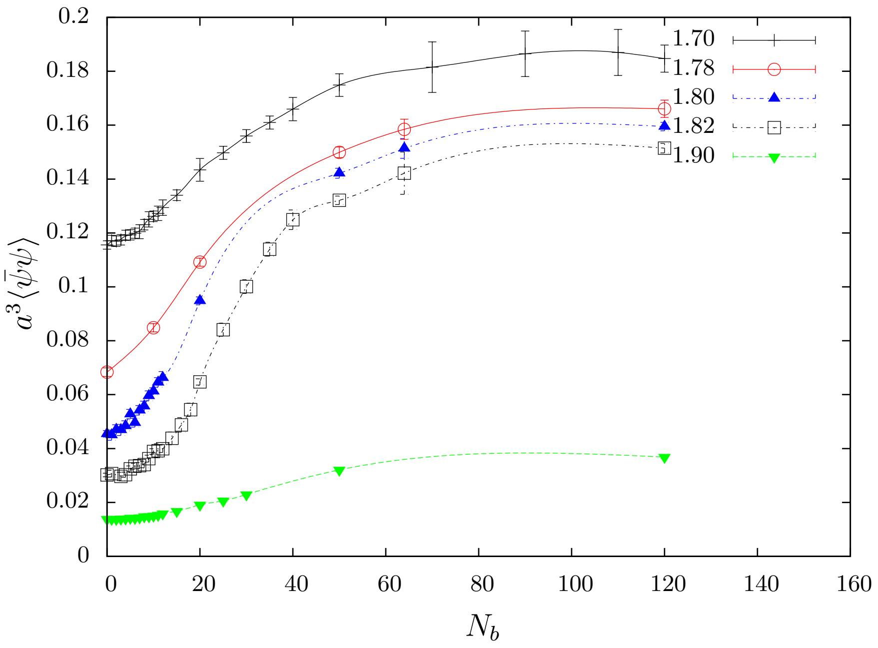
$$V_x(\bar{r}, \tau) = e^{-i\phi y/2}$$
$$V_y(\bar{r}, \tau) = e^{i\phi x/2}$$
$$V_x(N_s, y, z, \tau) = e^{-i\phi(N_s+1)y/2}$$
$$V_y(x, N_s, z, \tau) = e^{i\phi(N_s+1)x/2}$$

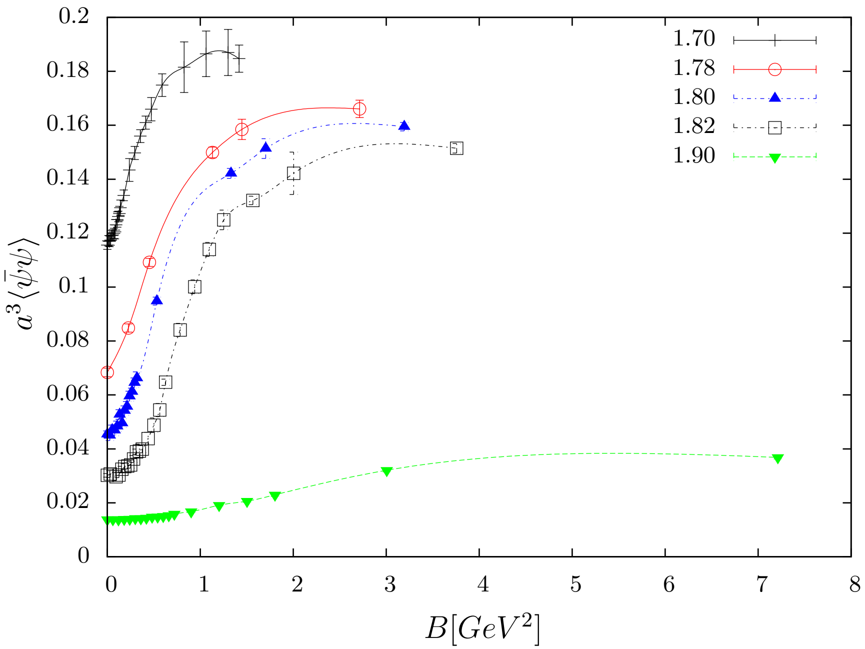
Flux will be quantized: $\phi = \frac{2\pi N_b}{N_S^2}$ $N_b = 1, 2, \ldots$ **DeGrand, Toussaint '80**

Typical field strength for $\beta = 1.80 \simeq \beta_c$ **,** $N_b = 50$ $\Longleftrightarrow$ $\sqrt{(eB)} \simeq O(1 \text{ GeV})$

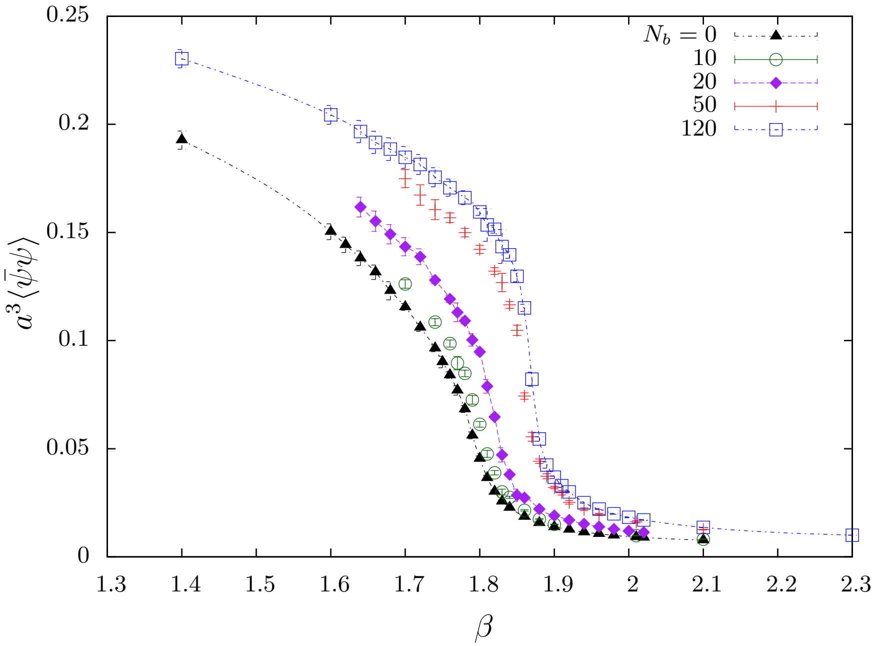
4. The influence of an external magnetic field on the chiral condensate and on the critical temperature

Saturation behavior for various β (V_μ periodic in ϕ):

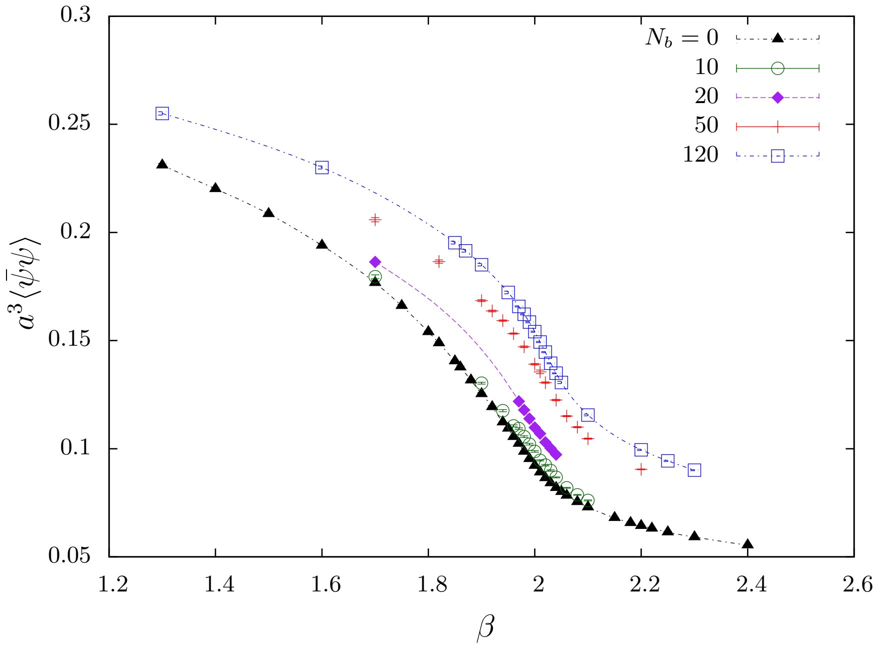




β-dependence ($\equiv$ T dependence) of the bare chiral condensate



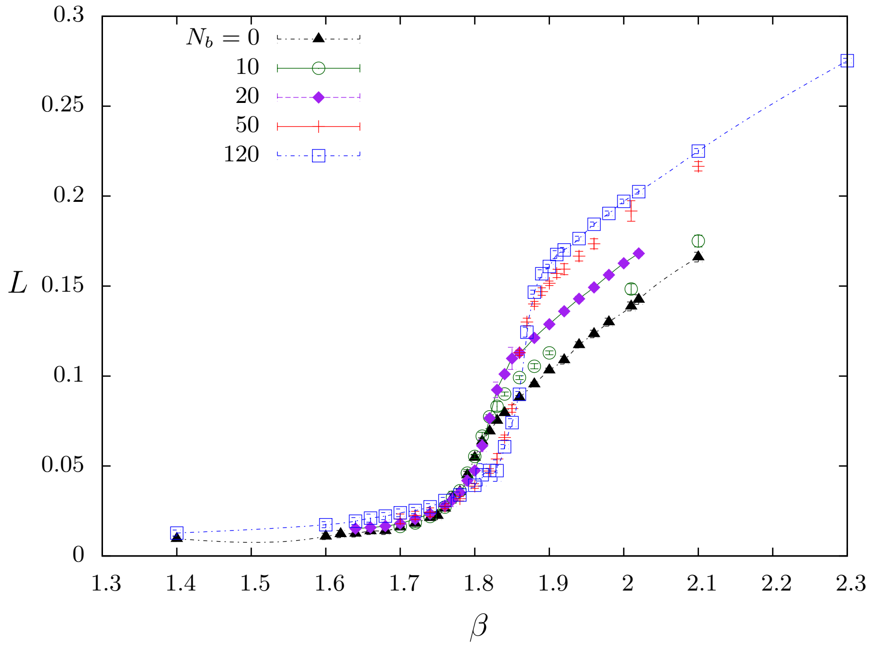
$ma = 0.01$



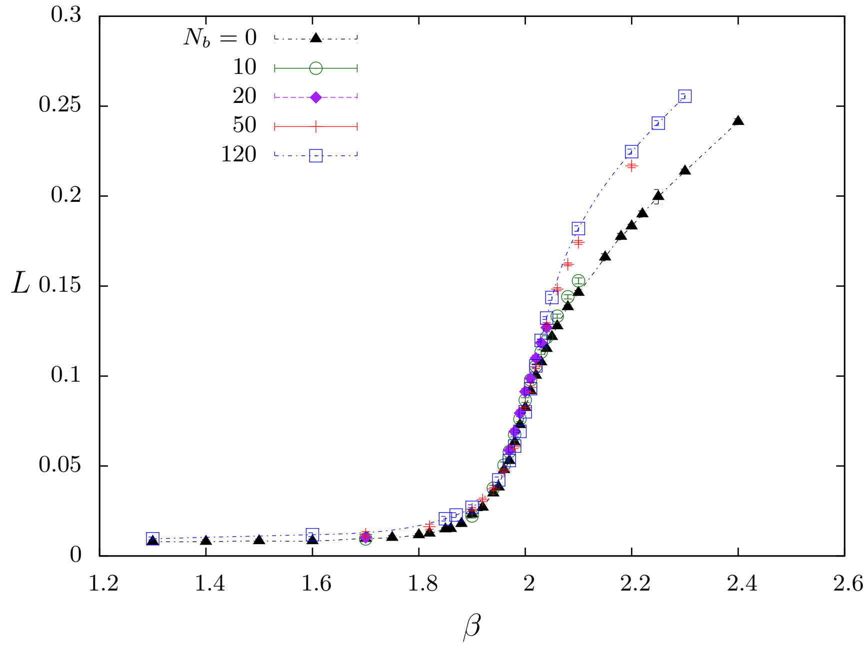
$ma = 0.1$

$<\bar{\psi}\psi>$ **increases with** B **for all** β $\Longrightarrow T_c$ **increases**

Polyakov loop

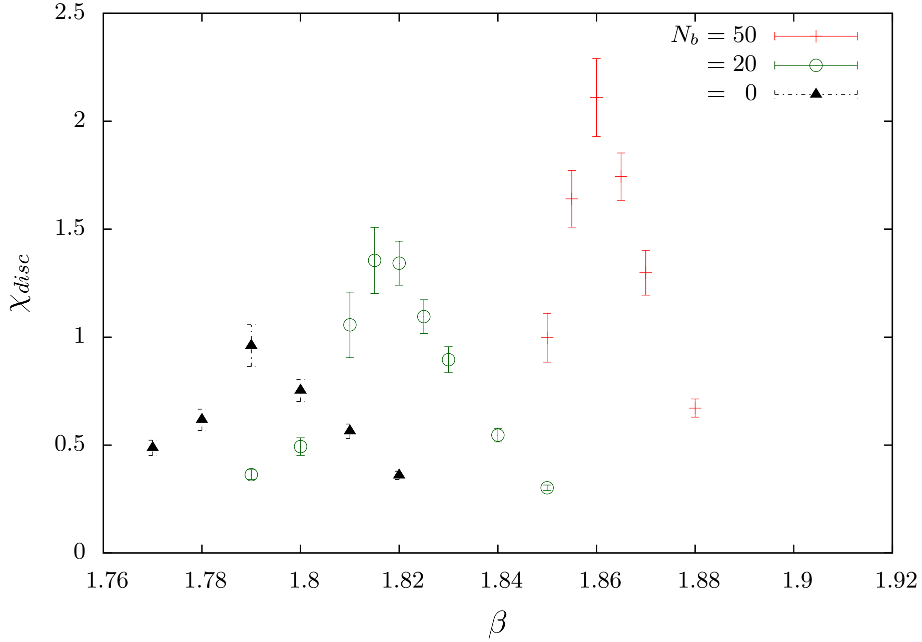


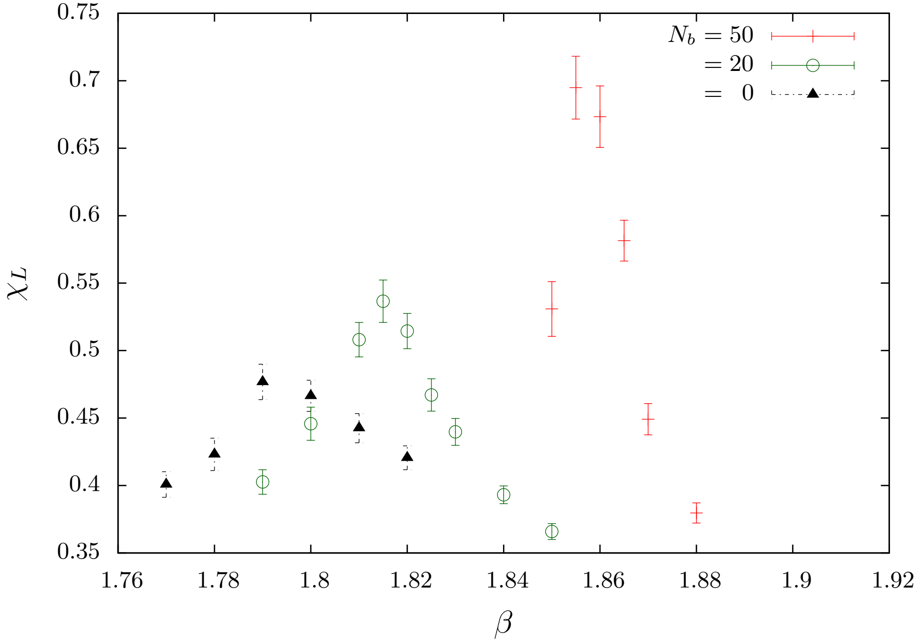
$ma = 0.01$



$ma = 0.1$

Susceptibilities





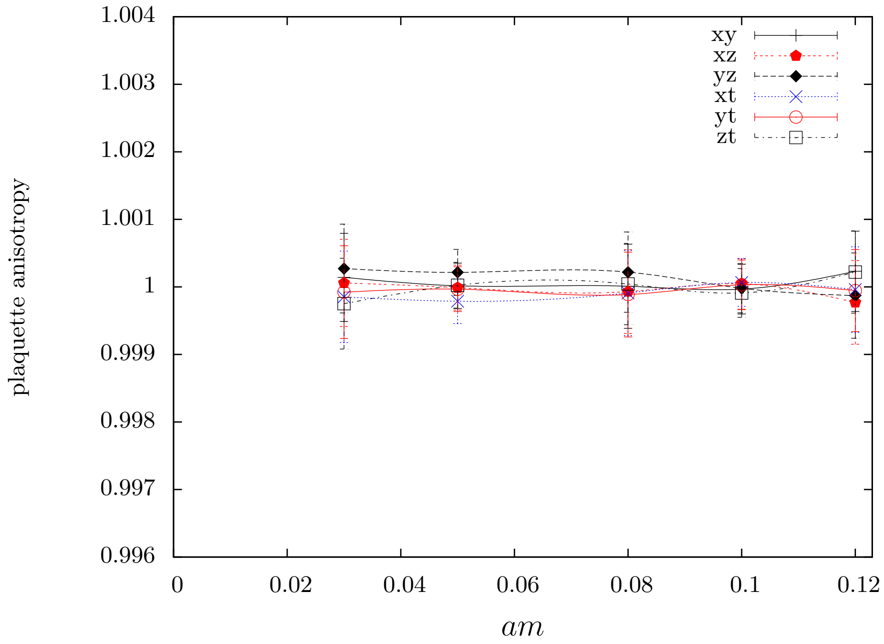
$ma = 0.01$ $\qquad$ $ma = 0.01$

$B \nearrow \;\Rightarrow\; T_c \nearrow$

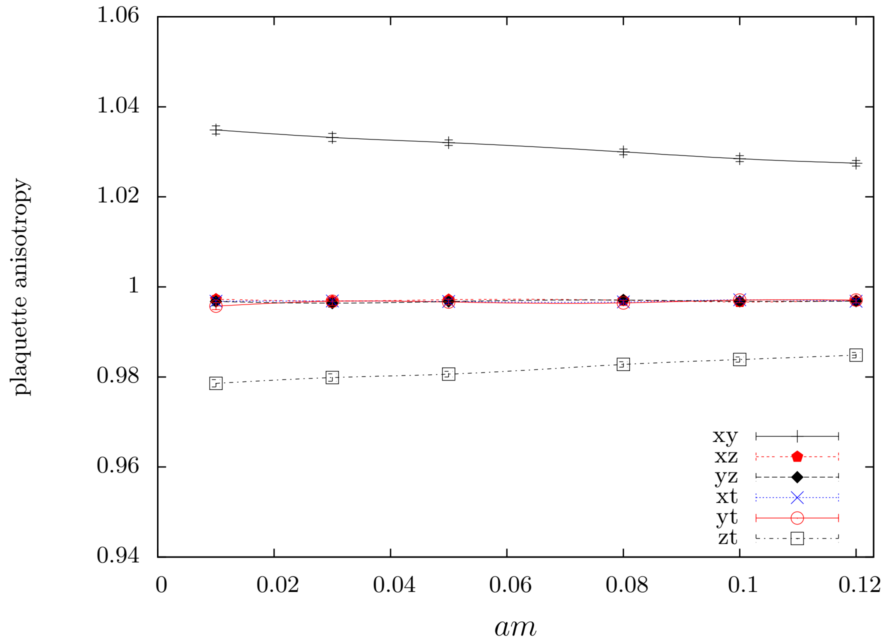
no splitting of the transition

Spatial anisotropy of plaquette averages:

confined phase, $\beta = 1.7$

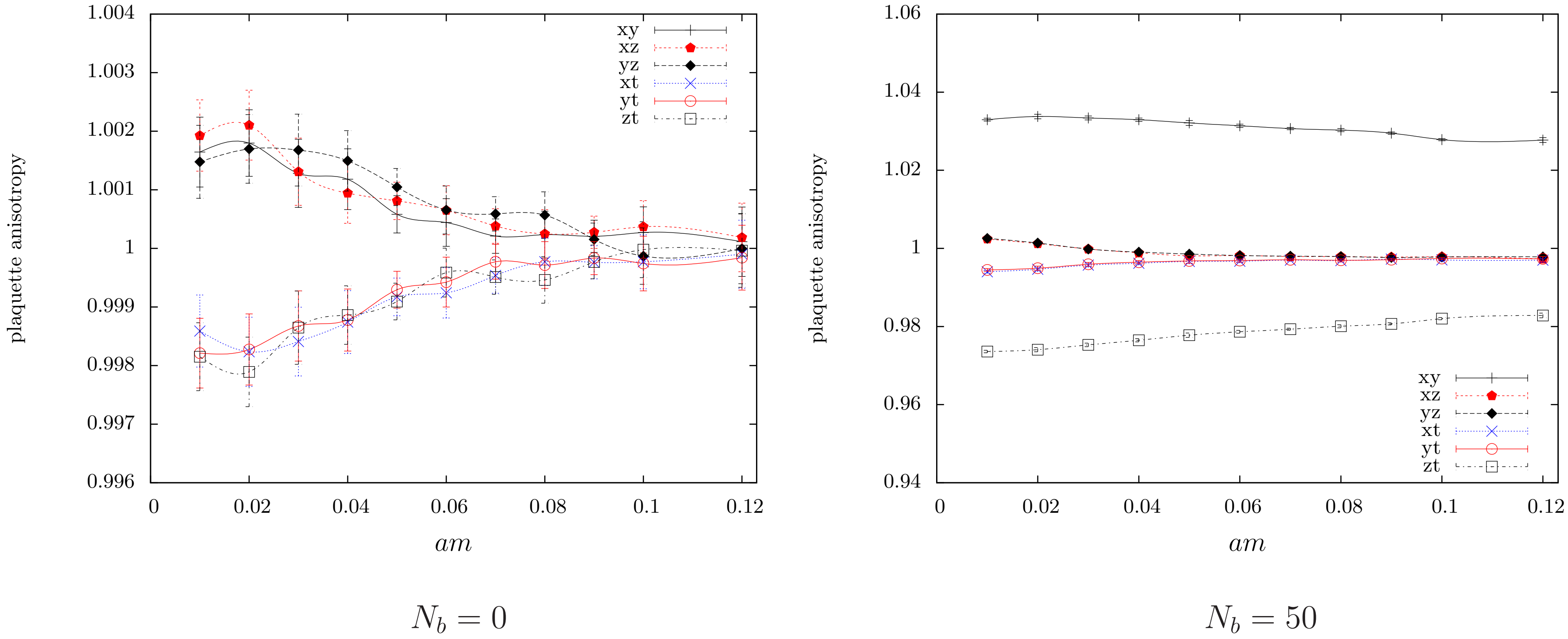


$N_b = 0$



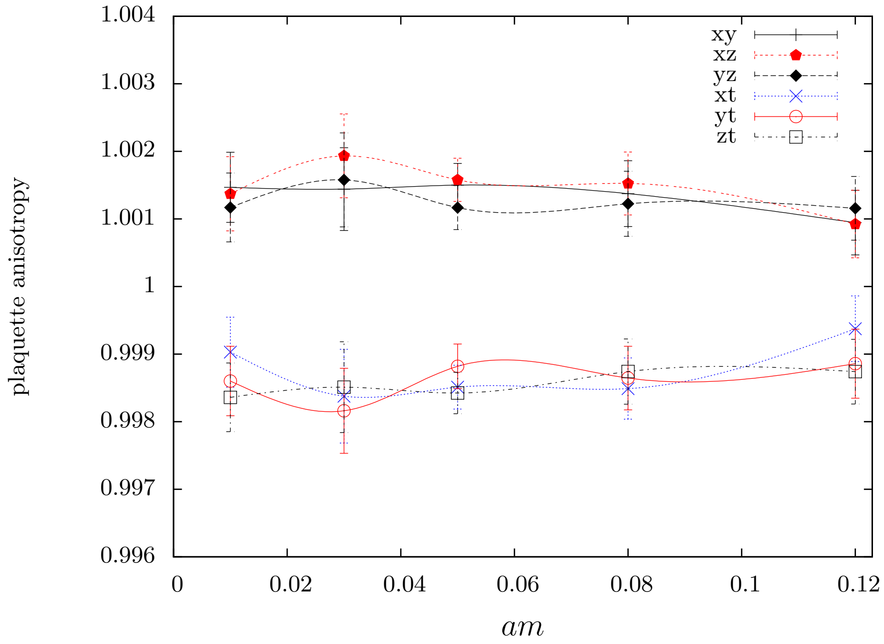
$N_b = 50$

transition region, $\beta = 1.9$

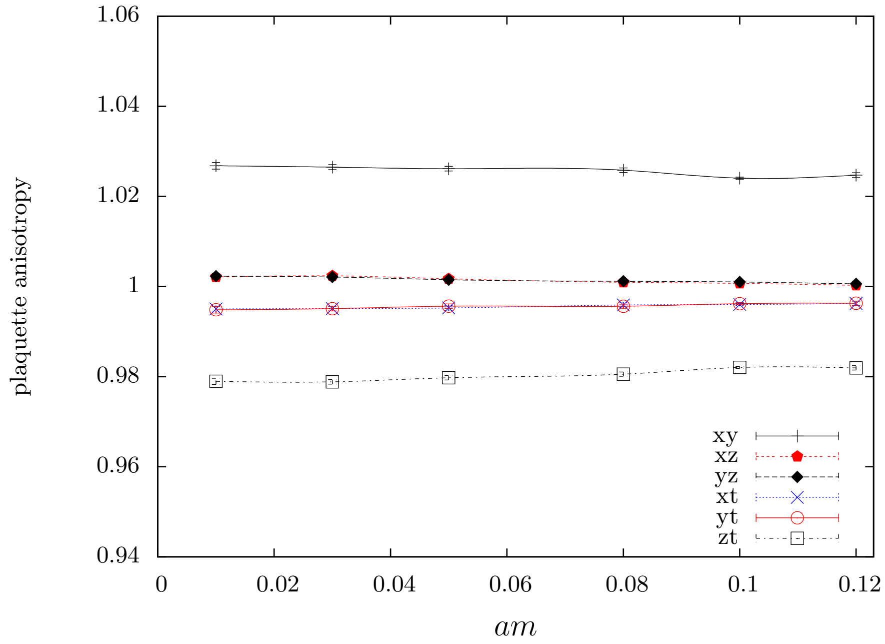


Spacelike-timelike plaquette differences $\propto$ energy density

deconfined phase, $\beta = 2.1$



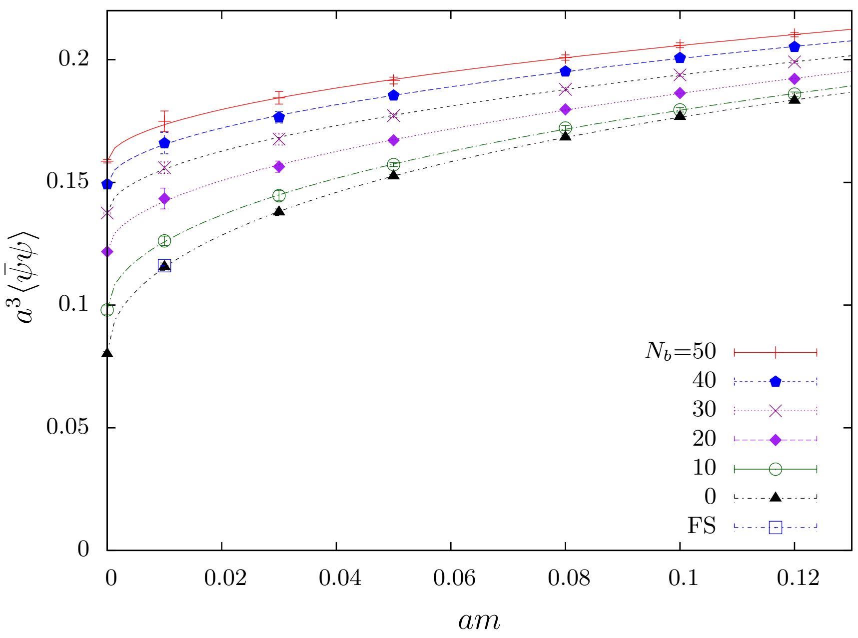
$N_b = 0$

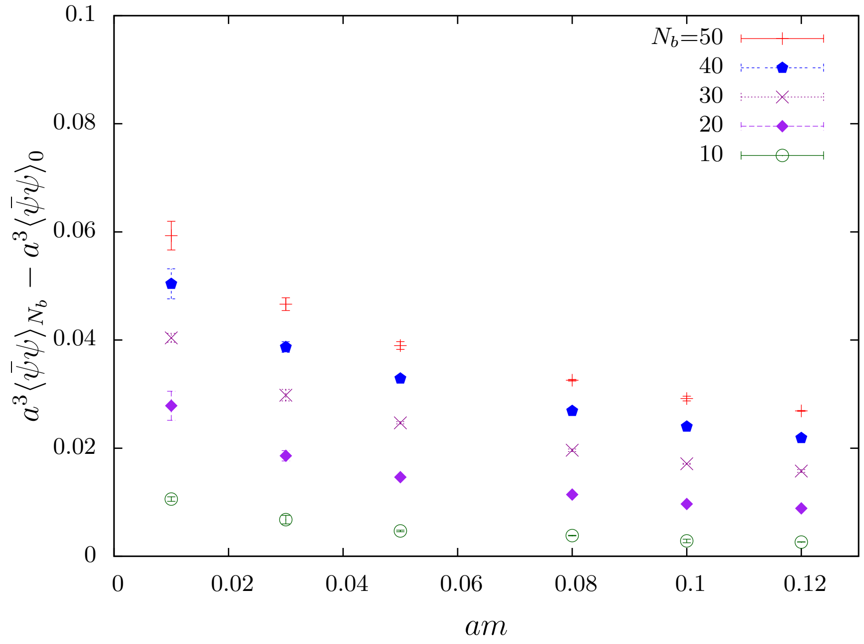


$N_b = 50$

4. The chiral condensate in the chiral limit

Confined phase, $\beta = 1.7$

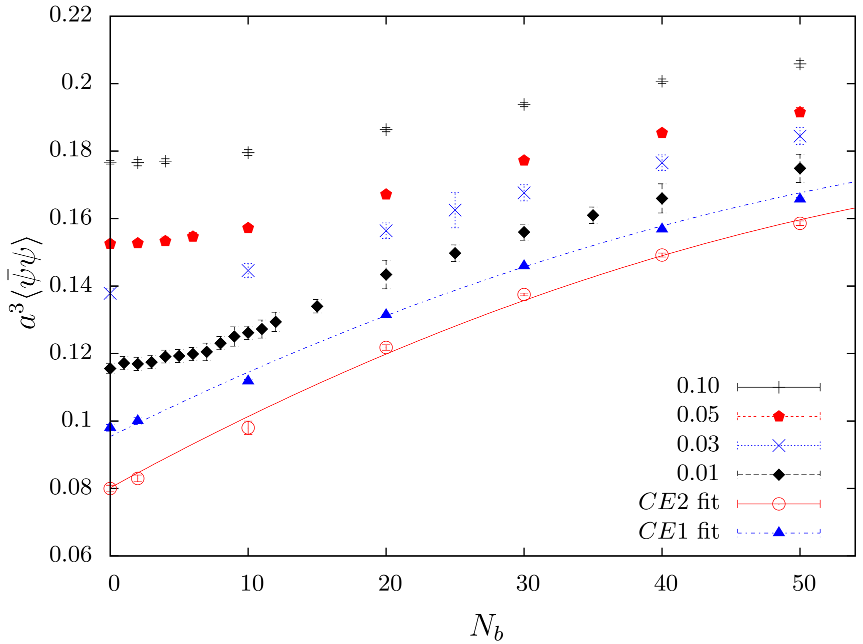




Fit: $a^3 < \bar{\psi}\psi >= a_0 + a_1\sqrt{ma} + a_2 ma$ **(other fit:** $a^3 < \bar{\psi}\psi >= b_0 + b_1 ma \log ma + b_2 ma$**)**

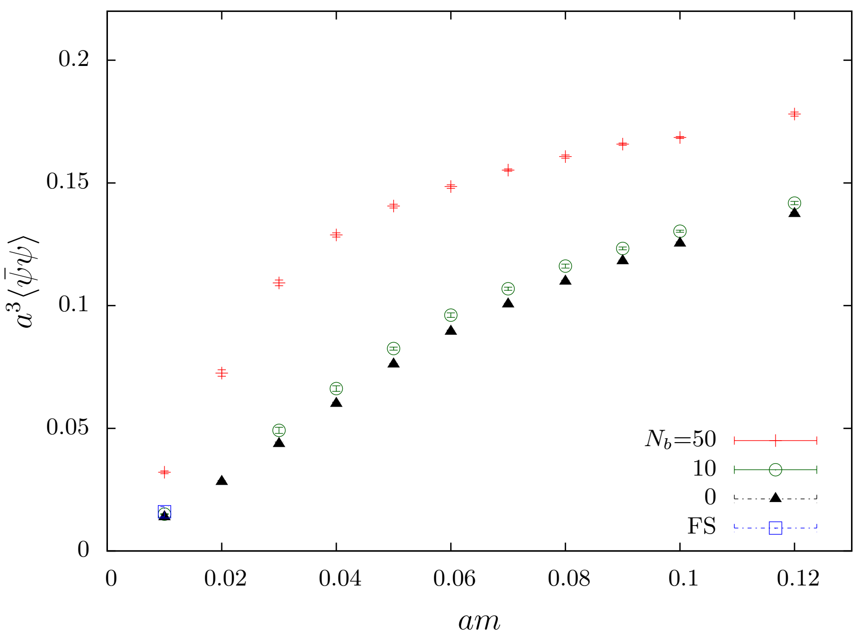
FS $=$ **check for finite-size effects with** $24^3 \times 6$**.**

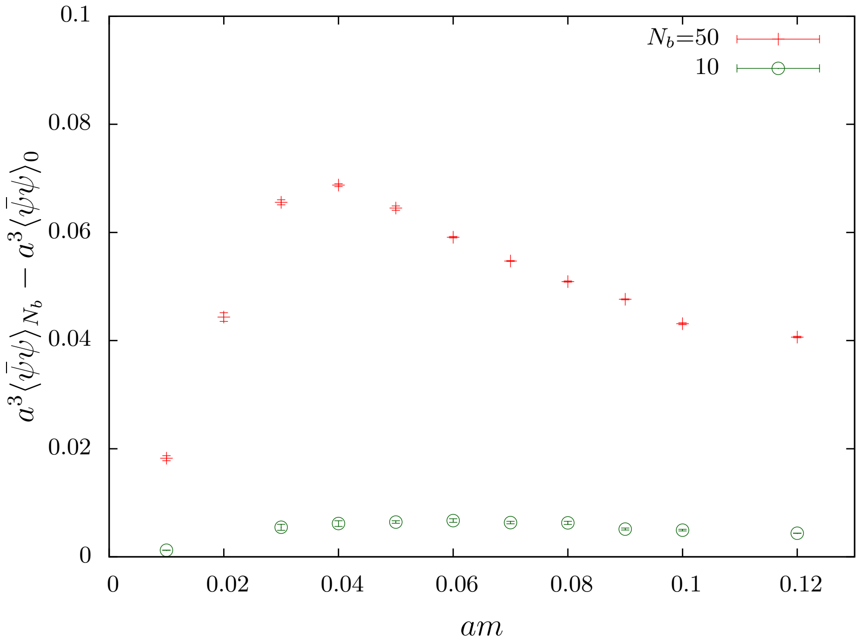
The chiral condensate as a function of the flux for various values of ma



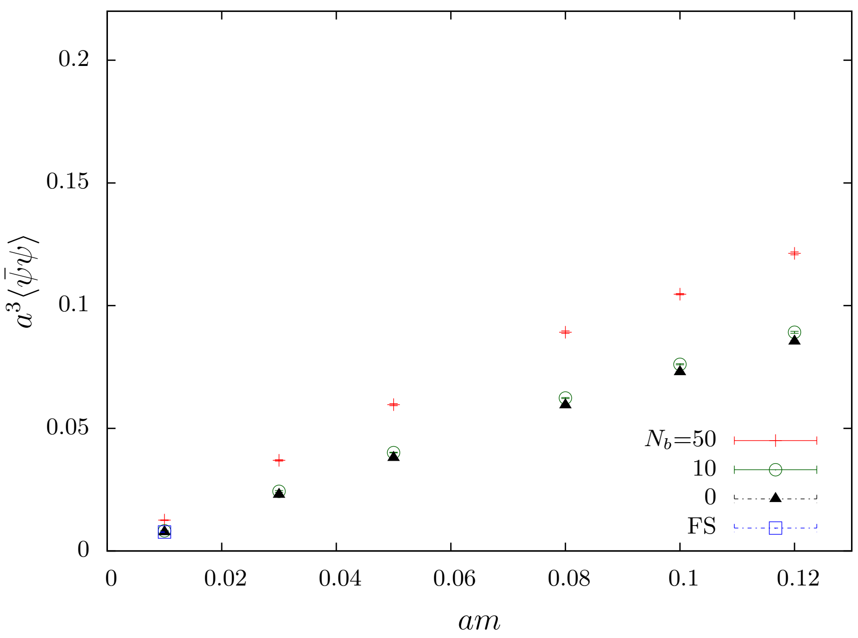
The slope at $ma = 0$ can be compared with chiral model $\Rightarrow F_\pi \approx 60\text{MeV}$

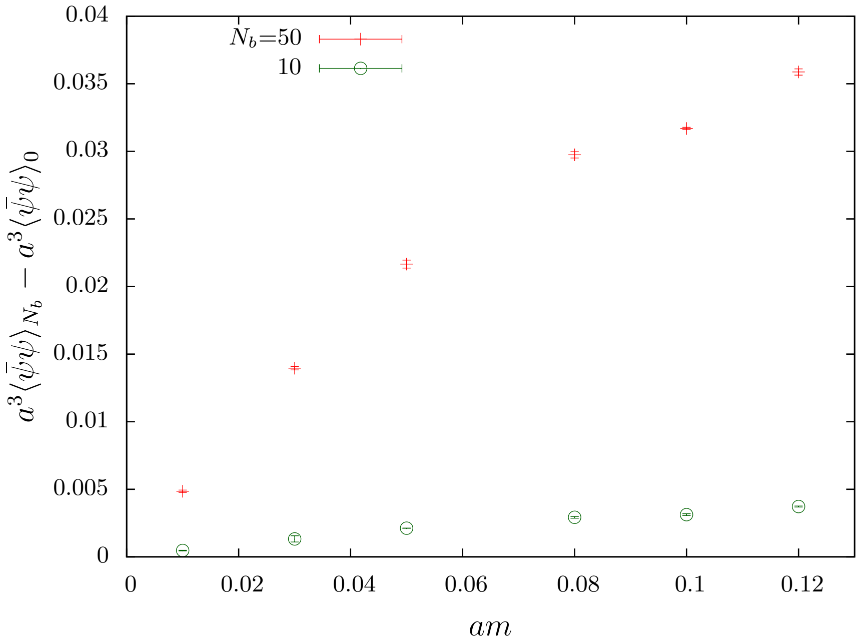
The chiral condensate, transition region, $\beta = 1.9$





The chiral condensate, deconfined phase, $\beta = 2.1$





6. Conclusions and outlook

- We have investigated how a finite temperature system reacts to a constant external magnetic field, in two-colour QCD.
- In the confined phase the chiral condensate increases with the magnetic field strength as predicted by a chiral model, also semi-quantitative agreement.
- The transition temperature increases with the magnetic field strength.
- The chiral condensate goes to zero in the deconfined region for all values of the magnetic field.
- Still much to do...