Causality of polarizeable dissipative fluids

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Research Plan I

► **Effective action of relativistic hydrodynamics by fluctuations of Crooks.**

“*A new effective theory framework for fluctuating hydrodynamics in the relativistic regime is derived using standard thermodynamical principles and general properties of non-equilibrium stochastic dynamics.* “

1. Estimate bottom-up limits to dissipative transport coefficients.
2. Relativistic fluctuations around more general out-of-equilibrium steady states
3. Investigate the property of matter of particles $\hat{J}^{\mu}$ and angular momentum $\hat{\mathcal{J}}$.

Research Plan II

- **Non-linear fluctuations of polarizeable dissipative fluids.**

"*To deal with these nonlinear aspects encoded in a set of partial differential equations, we address hydrodynamics by using an effective field theory language. Performing a causality analysis on the dispersion relation of hydrodynamics with spin well as shear and bulk viscosity*"

1. Investigate the nonlinear transport of hydrodynamical variables.
2. Examine the phenomenological behavior of the viscosity and relaxation time by the interaction of different fluctuation modes.
3. Solving the question: spin-shear coupling term is dissipative or not ?

Research Plan III (in progress)

- **A heavy quark interacting with the ideal fluid, the ergodic limit**

“*We will investigate the problem of a strongly coupled heavy quark in local equilibrium with a hydrodynamic medium.*“

- Modeling the drag on, and energy loss of, heavy quarks with modest rapidity in heavy ion collisions.
- How heavy quarks lose energy and gain momentum as they move through the rapidly expanding quark-gluon plasma.

Research Plan IV (in progress)

- **Gaussian pseudogauge invariant hydrodynamics with spin.**

“*We formulate a fluctuating hydrodynamics with spin which is explicitly covariant with respect to pseudo-gauge transformations and generally covariant with respect to foliations*“

1. Focusing on what to imply for the applicability of hydrodynamics as an effective theory w.r.t. statistical mechanics and in small systems.
2. Clarifying the role of the pseudo-gauge in hydrodynamics with spin.

Introduction

Fluid Dynamics is everywhere

Introduction

Hydrodynamical has a lot of practical application...

- You care about the applications
- You want to understand Hydrodynamics in a deep level

I am interested in extreme matter: high temperature and/or high density. Matter of quark and gluons, both relativistic and quantic

Scales in fluid dynamcs

$$l_{micro} \leq l_{mfp} \leq l_{macro} \tag{1}$$

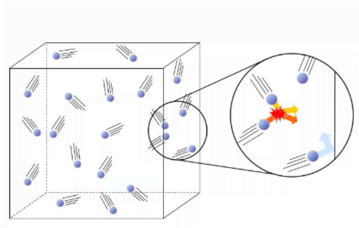
- $l_{macro} \sim \partial_\mu \sim L_{hydro}^{-1}$ (scale over which the fluid fields vary).
- $l_{mfp} \sim (<\sigma> n)^{-1}$ (average distance between collisions for a molecule).
- $l_{micro} \sim \beta \sim \lambda_{th}$ (interparticle distance)

$$\frac{l_{mfp}}{l_{micro}} \sim \frac{(<\sigma> \lambda_{th})^{-1}}{n} \sim \frac{\eta}{s} \tag{2}$$

Scales in fluid dynamcs

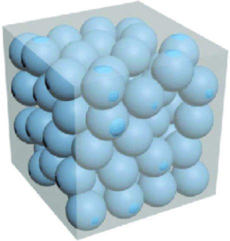
- $\eta/s \to \infty$

Average distance between collision $\gg$ interparticle distance

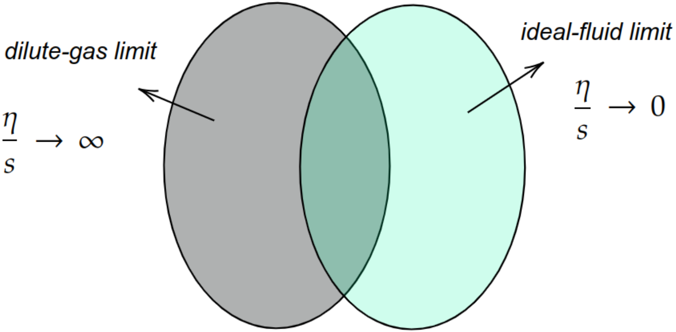


- $\eta/s \to 0$

Interactions are so strong that the fluid assumes locally thermodynamic equilibrium



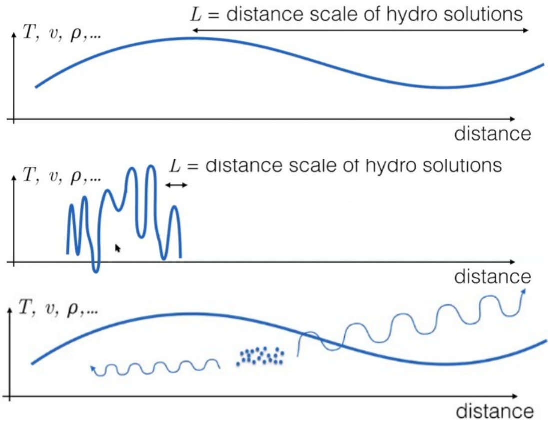
Scales in fluid dynamcs



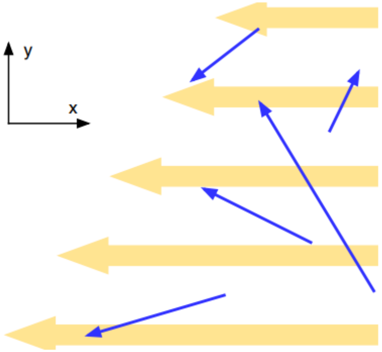
A quick look at the limitations of hydrodynamics

Macroscopic stuff is made out of microscopic stuff.

These sound waves will back-react on the macroscopic physics because hydro is non-linear.



Example: Viscosity



Momentum transfer between layers of fluid.

$$T_{xy} = \eta \partial_y v_x + \mathcal{O}(\partial^2)$$

Related to correlations of stress

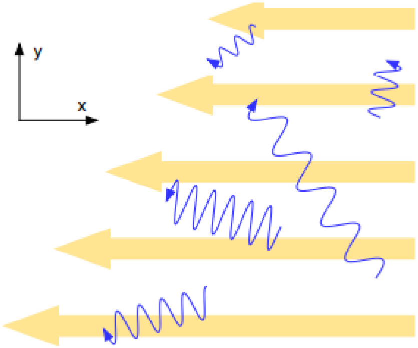
$$T^{xy} T^{xy} = p - i\omega\eta + \mathcal{O}(\omega)^2$$

How to extract more dynamic information ? How to analyze the correlation function behavior in more complex circumstances ?

Does hydrodynamics produces non-linear evolution ?

- Sensible relativistic hydrodynamics seems to require that its eqs contain non-hydrodynamics parameters.
- Microscopic dynamics
  - Thermal fluctuations
  - Collective excitations appear in a region (ω, k), sensitive to microscopic dynamics.

Example: Viscosity



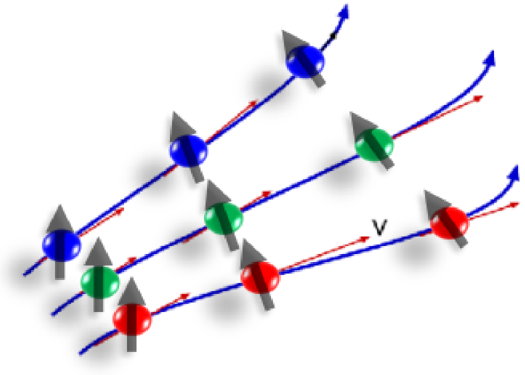
Momentum can also be transferred by collective excitations

Contribution to viscosity

Inhomogeneous velocity profile $v_x(y)$ (shear - thick arrows). Wiggly arrows (sound and shear modes - thermal fluctuations)

Momentum transfer between layers of fluid.

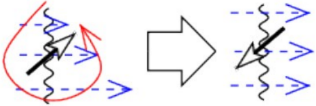
Polarization/Vorticity

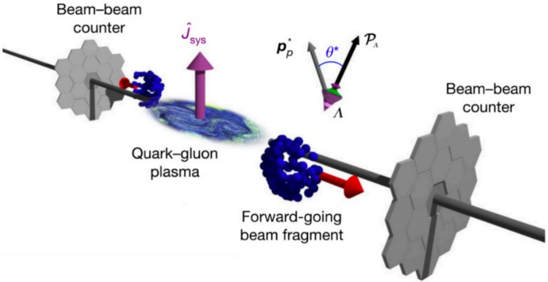


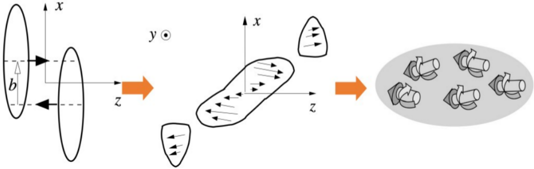
How do the microscale interactions interfere in the macroscopic scale behavior ?

Polarization/Vorticity

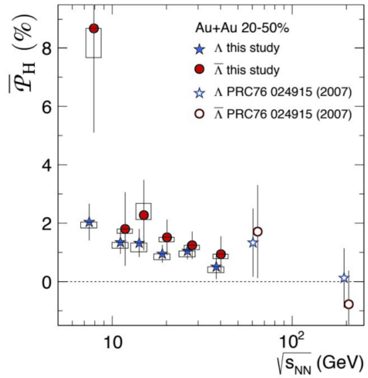
STAR Col. Nature **548**, 62 (2017)







Global polarization: from prediction to measurements



- indicates an extremely large vorticity $\omega(91)10^{21}s^{-1}$
- STAR Collaboration, Nature 548, 62 (2017)

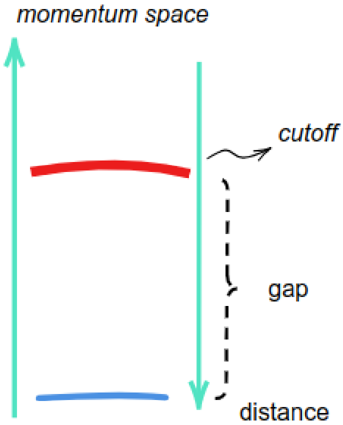
- Polarization estimated at quark level by spin-orbit coupling. *Z. T. Liang, X. N. Wang, Phys. Rev. Lett. 94 (2005) 102301*
- By local thermodynamic equilibrium of the spin degrees of freedom *F. B., F. Piccinini, Ann. Phys. 323 (2008) 2452*

Effective Field Theory

- At low energies by decoupling short distance dof.
- Neglecting/being agnostic microscopic dof.

The generating functional.

$$Z[b, y^2] = \int \mathcal{D}\phi^i \mathcal{D}y^{\mu\nu} \rho(\phi^i, y) e^{i \int d^4x (\mathcal{L}_{free} + \mathcal{L}_{shear} + \mathcal{L}_{pol})}$$



We can separate the contribution in

$$\mathcal{L} \supset \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{self-int}} \tag{3}$$

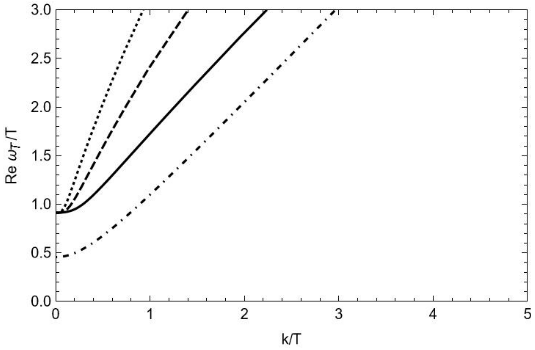
$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{Bulk} + \mathcal{L}_{NS} + \mathcal{L}_{Pol}, \tag{4}$$

$$\mathcal{L}_{free} = F(b) \tag{5}$$

$$\mathcal{L}_{Bulk} = \sum_{i,j,k} h_{ijk}(b^2) K^{\mu I} K^{\nu J} \partial_\mu K_\nu^K \tag{6}$$

$$\mathcal{L}_{NS} = \sum_{i,j,k} z_{ijk}(b^2) b^2 B_{ij}^{-1} \partial^\mu \phi^{iI} \partial^\nu \phi^{jJ} \partial_\mu K_\nu^K \tag{7}$$

$$\mathcal{L}_{Pol} = F(b - c\, b\, y^2) \tag{8}$$



- Hydrodynamic limit increases with shear viscosity.
- gap in the hydrodynamic limit $\lim_{k \to 0} \omega(k) \neq 0$
- It still lack a microscopic description and our approximation breaks down.

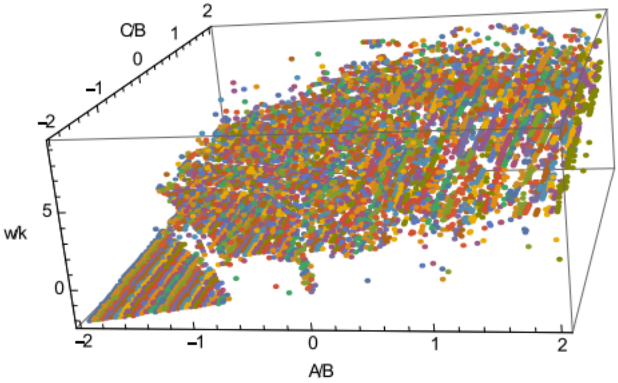
"Every time you break a symmetry, you learn something new"

$$\mathcal{S} = \mathcal{S}_{free} + \mathcal{S}_{shear} + \mathcal{S}_{bulk} + \mathcal{S}_{pol} + S\left(\partial\phi_{\pm}\right)^2 , \tag{9}$$

	parity	time	charge	$SO(3)$	$SDiff\left(\mathbb{R}^{1,3}\right)$
$\mathcal{S}_{free}$	even	even	even	unbroken	unbroken
$\mathcal{S}_{bulk}$	even	even	even	unbroken	unbroken
$\mathcal{S}_{shear}$	even	even	even	unbroken	broken
$\mathcal{S}_{pol}$	odd	odd	even	broken	unbroken

Break of rotation $\rightarrow$ new d.o.f. $\rightarrow$ current of polarization

THE DYNAMICS



(i) Conflict between causality and the non-dissipative regime.

(ii) Quantitative lower limit for dissipation to restore causality.

(iii) We must have a minimum viscosity for polarizable fluids.

How to cure unstable and acausal polarization medium ?

$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{IS-shear} + \mathcal{L}_{IS-Bulk} + \mathcal{L}_{IS-pol}, \tag{10}$$

$$\mathcal{L}_{IS-bulk} = \frac{\tau_\xi}{2}\left(\Pi_- u_+^\alpha \partial_\alpha \Pi_+ - \Pi_+ u_-^\alpha \partial_\alpha \Pi_-\right) + \frac{\Pi_\pm^2}{2} + h_{IJK}(b^2) K^{\mu I} K^{\nu J} \partial_\mu K_\nu^K, \tag{11}$$

$$\mathcal{L}_{IS-shear} = \frac{\tau_\eta}{2}\left(\pi_-^{\mu\nu} u_+^\alpha \partial_\alpha \pi_{\mu\nu}^+ - \pi_+^{\mu\nu} u_-^\alpha \partial_\alpha \pi_{\mu\nu}^-\right) + \frac{\pi_\pm^{\mu\nu 2}}{2} + z_{IJK}(b^2) b^2 B_{ij}^{-1} \partial^\mu \phi^{iI} \partial^\nu \phi^{jJ} \partial_\mu K_\nu^K, \tag{12}$$

$$\mathcal{L}_{IS-pol} = \frac{\tau_\chi}{2}\left(Y_-^{\mu\nu} u_+^\alpha \partial_\alpha Y_{\mu\nu}^+ - Y_+^{\mu\nu} u_-^\alpha \partial_\alpha Y_{\mu\nu}^-\right) + \frac{Y_\pm^{\mu\nu 2}}{2} + F(b(1 - cy^2)). \tag{13}$$

Constraint on τ_Y in ideal fluid

$$\tau_Y^2 \geq \chi^2 \left(\frac{dp}{ds} \right)^{-1} \tag{14}$$

- For an unpolarizeable medium ($\chi \to 0$), we have ($\tau_Y^2 \to 0$)
- The numerator is proportional to vorticity absorption by angular momentum.
- Causality condition implies stability

Evolving a small vortex with a finite dissipation time

$$\omega_{\mu\nu}(t) \sim \omega_{\mu\nu}(t = 0) \exp \left[-\frac{t}{\tau_Y} \right] \tag{15}$$

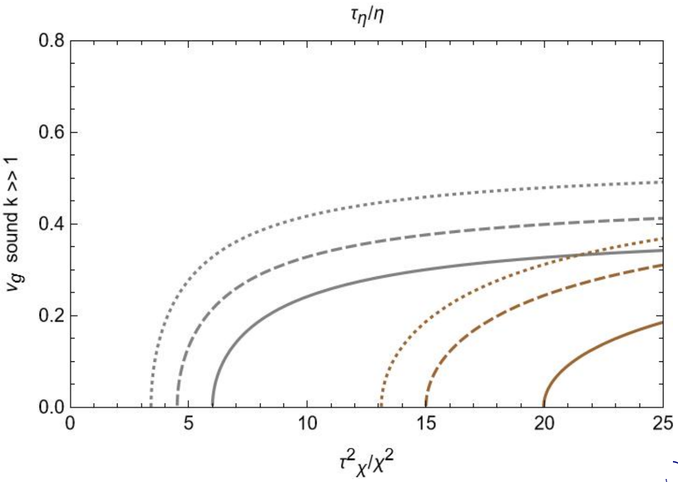
Including shear in polarizable fluid

Clarify how the shear viscosity influence the polarization, and consequently the macroscale structure

Dispersion relations for η and χ

shear $\parallel$ pol $\rightarrow$ shift max. value

shear $\nparallel$ pol $\rightarrow$ shift origin



Constraint on τ_Y in dissipative fluid

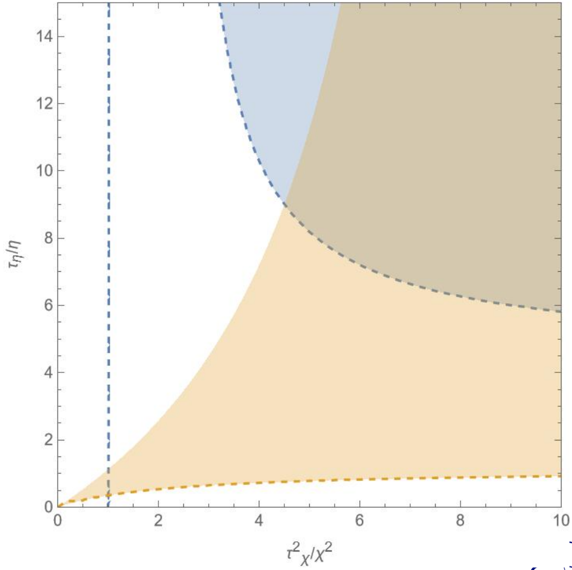
$$\tau_\eta \geq \frac{\eta}{(\frac{ds}{dp}) - c(\frac{\chi}{\tau_Y})^2}. \tag{16}$$

1. Direction of spin increase or decrease the relaxation time of shear viscosity.
2. Anisotropy turning the transport coefficient dependent of angular momentum.

Transient flows $\rightarrow$ a gradient of vortex accumulation $\rightarrow$ spin current (spin-vorticiy coupling)

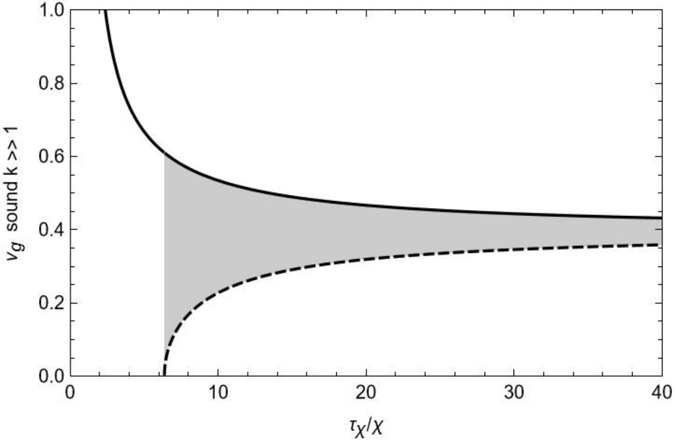
1. The change of spin orientation based on pressure sound waves from transient flows
2. Changes the spatial gradient of spin concentration between the fluid layers.

- Causally allowed regions for spins parallel (lighter) and antiparallel (darker) spin w.r.t. vorticity.
- Note the positiveness of χ only remains valid when we analyze the restrictions of entropy current for a dissipationless polarizeable fluid.
- Polarization lowers the effective viscosity.



Causally region

- The presence of polarization imposes upper and lower constraints on the propagation of hydrodynamic perturbations.



Thank you for your attention