Phase transitions in hot nuclear matter composed of alpha-particles and nucleons

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Recent results -> arXiv: 1811.029024

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- Conclusions and outlook

Previous studies of strongly interacting matter with clusters

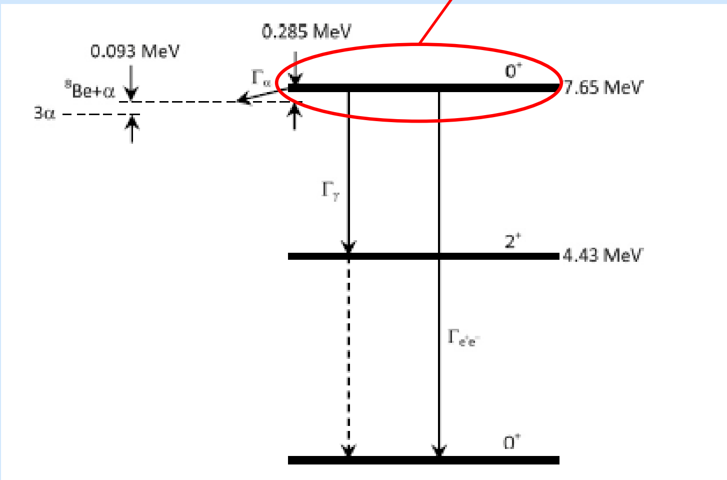
- Statistical models $\rightarrow$ D. Gross; Randrup&Koonin; Bondorf&Mishustin; Stoecker e. a. early works (80-s)
- Generalized liqui-drop model $\rightarrow$ J. Lattimer and F. Swesty, Nucl. Phys. A535 (1991)
- more recent works $\rightarrow$ Botvina&Mishustin (2004, 2010); Hempel&Schaffner-Bielich(2010); Buyukcizmeci e.a. (2013); Furusawa&Mishustin (2018);
- RMF models $\rightarrow$ J. Pais et al., Phys. Rev. C97 (2018); S. Typel, J. Phys. G45 (2018)
- virial EoS $\rightarrow$ C. Horowitz and A. Schwenk, Nucl. Phys. A776 (2006)
  (use information on observed phase shifts of NN, Nα and αα scatterings, the model applicable only at small particle densities)
- multi-component van der Waals model $\rightarrow$ V. Vovchenko et al., Phys. Rev. C96 (2017)
  (Nα and αα attractive interactions are disregarded)
- quasi-particle model $\rightarrow$ X.-H. Wu et al., J. Low Temp. Phys. 189 (2017) [*]
  (only small densities are considered)

All these models (except [*]) disregard the BEC possibility. We are going to fill this gap!

Systems/processes with enhanced formation of α's

- low-density excited states of light nuclei (^{12}C, ^{16}O, ^{20}Ne, … ← large-size isomers)
- periphery of heavy nuclei
- multifragmentation reactions in heavy-ion collisions (spectators}
- outer regions of compact stars
- neutron star mergers, supernovae matter (dilute and warm matter)

The Hoyle (2^{nd} excited) state of ^{12}C → key role in stellar nucleosynthesis



Two-step process:

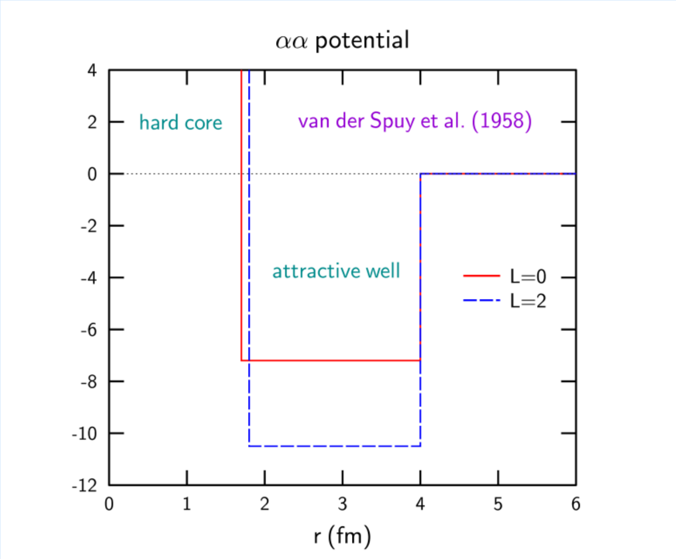
1) $\alpha + \alpha \rightarrow {}^{8}Be$
2) $\alpha + {}^{8}Be \rightarrow {}^{12}C^* \rightarrow {}^{12}C + \gamma$

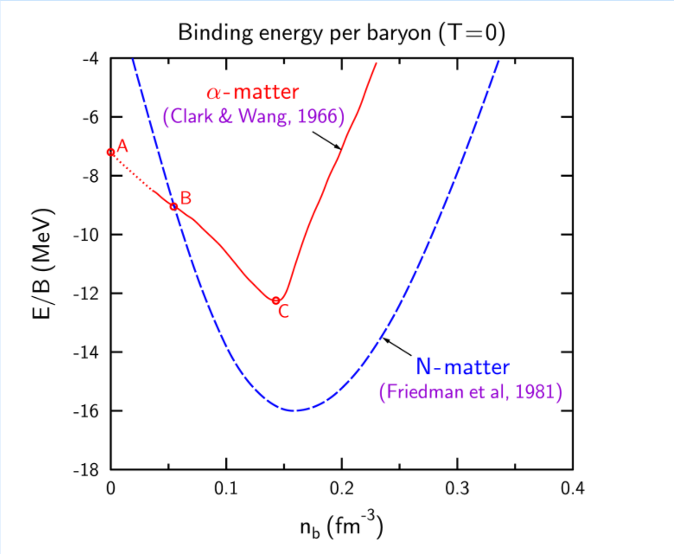
Enhanced rate of ^{12}C formation at $T \gtrsim 200$ keV

Röpke et al. (1998): Hoyle state ≈ BEC state of 3α

Energy per baryon of cold α-matter

Clark & Wang (1966): variational calculation with phenomenological αα potential, comparison with isospin-symmetric nucleon matter





baryon density:

$$n_b = 4n_\alpha = n_N$$

dilute α-matter:

$$\frac{E}{B} = m_\alpha/4 - m_N$$
$$\equiv -B_\alpha \simeq -7.1 \text{ MeV}$$

B_α=binding energy of single α-particle

+ Coulomb potential ($4e^2/r$)

- α-matter is energetically favorable at low baryon densities $n_b \lesssim 0.05 \text{ fm}^{-3}$ (section AB)
- ground state (GS) of pure α-matter at $n_\alpha \simeq 0.036 \text{ fm}^{-3}$ (point C) has higher energy $E/B \simeq -12 \text{ MeV}$ than normal nucleonic matter (-16 MeV)

Our assumptions

- isospin symmetry $(N_p = N_n)$
- homogeneous matter (no surface terms)
- no Coulomb interactions
- no clusters, except α (we neglect d, t, ^{3}He, ^{5}He … and their excited states)
- moderate temperatures $(T \lesssim 30\ \mathrm{MeV})$

  → neglect contributions of mesons (π, ρ, K …), other baryons (Δ, N*, Λ …) and antibaryons
  → nonrelativistic limit is accurate, since $T \ll m_N \simeq 0.939\ \mathrm{GeV}, m_\alpha \simeq 3.727\ \mathrm{GeV}$
- chemical equilibrium with respect to reactions $\alpha \leftrightarrow 4N$
- mean-field approximation for particle interactions
- no in-medium modification of particles masses

Thermodynamic functions for α-N matter

- free energy density $F/V = f(T, n_N, n_\alpha)$ → thermodynamic potential in canonical ensemble
- chemical potentials $\mu_i(T, n_N, n_\alpha) = (\partial f/\partial n_i)_T$ $(i = N, \alpha)$ → (1)
- pressure $p = \mu_N n_N + \mu_\alpha n_\alpha - f$ entropy density $s = -(\partial f/\partial T)_{\{n_i\}}$
- energy density $\varepsilon = Ts + f$ baryon density $n_B = n_N + 4n_\alpha = B/V$
- 'mass' fraction of alphas $\chi = 4n_\alpha/n_B$ → one can use (n_B, χ) instead of (n_N, n_α) $[\chi \leqslant 1]$

condition of chemical equilibrium: $\mu_N = \mu_\alpha/4 \equiv \mu_B$ → (2) (μ_B- baryon chem. potential)

substituting (1) into (2) → isotherms of chem. equilibrium in $(n_N, n_\alpha), (n_B, \chi)$ or (μ_B, p) planes

- $p = p(T, \mu_B), n_B = (\partial p/\partial \mu_B)_T \ldots$ (grand canonical ensemble)

stability condfition with respect to fluctuations of partial densities:

$$\det||\partial^2 f/\partial n_i \partial n_j|| = (\partial \mu_N/\partial n_N)_{\{n_\alpha, T\}} (\partial \mu_\alpha/\partial n_\alpha)_{\{n_N, T\}} - (\partial \mu_N/\partial n_\alpha)^2_{\{n_N, T\}} > 0$$

Bose-Einstein condensation (BEC) in ideal boson gas

condition of BEC: $\mu = m \to T < T_{\rm BEC}(n)$ (T_{BEC} = threshold temperature of BEC)

$$n = \begin{cases} n_{\rm id}(T,\mu)\,, & T > T_{\rm BEC}(n) \\ n_{\rm id}(T,m) + n_{\rm bc}\,, & T < T_{\rm BEC}(n) \end{cases}$$

→ equivalent to $\mu < \mu_{max}$ = m (boson mass)

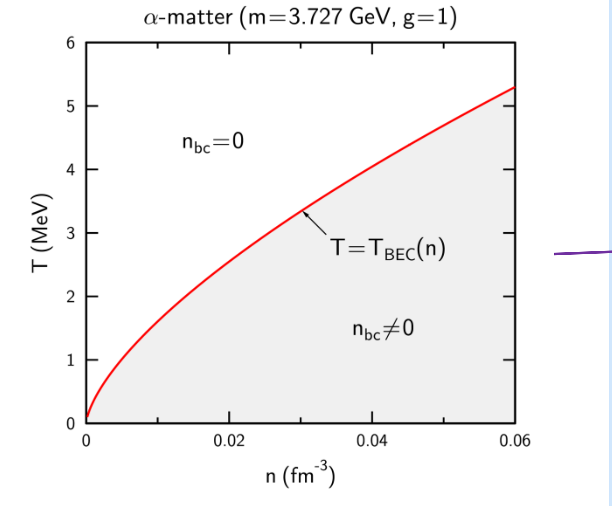
(n_{bc} – density of Bose-condensed particles with zero momenta)

$$n_{\rm id}(T,\mu) = \frac{g}{(2\pi)^3}\int d^3p\left[\exp\left(\frac{\sqrt{m^2+p^2}-\mu}{T}\right)-1\right]^{-1}$$

$$\to n_*(T) \equiv n_{\rm id}(T,m) \simeq g\left(\frac{mT}{2\pi}\right)^{3/2}\zeta(3/2) \quad (T \ll m)$$

g - degeneracy factor, $\zeta(3/2) = \sum_{k=1}^{\infty} k^{-3/2} \simeq 2.612$

$$T_{\rm BEC}(n) \simeq \frac{2\pi}{m}\left[\frac{n}{\zeta(3/2)\,g}\right]^{2/3}$$ $\propto n^{2/3}$



BEC boundary: $n=n_*(T) \to T=T_{BEC}(n)$

BEC occurs at low T or at large n

αN-mixture in mean-field approximation: $T_{BEC}(n_\alpha)$ is the same as in ideal α-gas with density $n=n_\alpha$

Non-interacting α-N mixture in chemical equilibrium

pressure: $p = p_N^{\mathrm{id}}(T,\mu_N) + p_\alpha^{\mathrm{id}}(T,\mu_\alpha)$ partial densities: $n_i = \partial p_i^{\mathrm{id}}/\partial \mu_i \quad (i = N, \alpha)$

(+ n_{bc} at $T<T_{BEC}$ for i=α)

$$p_i^{\mathrm{id}}(T,\mu_i) = \frac{g_i}{(2\pi)^3}\int d^3k \frac{k^2}{3E_i}\left[\exp\left(\frac{E_i-\mu_i}{T}\right)+\eta_i\right]^{-1}$$

$$\left(E_i = \sqrt{m_i^2+k^2},\; g_N = 4,\; g_\alpha = 1,\; \eta_N = 1,\; \eta_\alpha = -1\right)$$

chemical equilibrium: $\mu_N = \mu_\alpha/4$ → isotherms $n_\alpha = n_\alpha$ (T, n_N)

region of BEC states: $\mu_\alpha = m_\alpha$ $\rightarrow \mu_N = m_\alpha/4 \equiv m_N - B_\alpha$

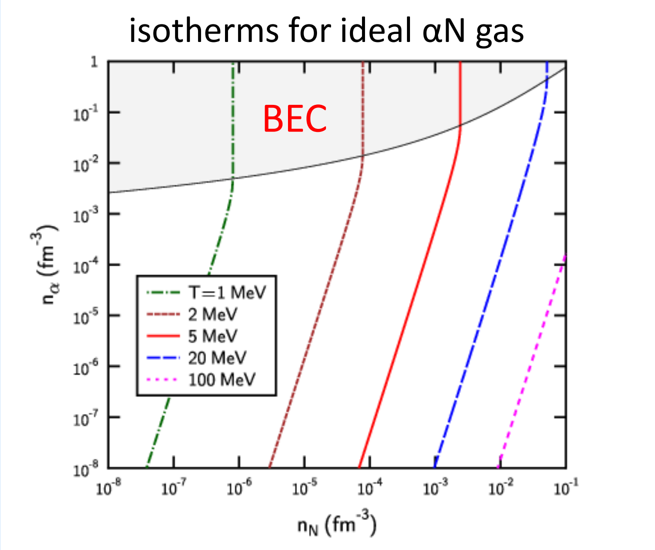
$$T < T_{\mathrm{BEC}} \simeq \frac{2\pi}{m_\alpha}\left[\frac{n_\alpha}{\zeta(3/2)\,g_\alpha}\right]^{2/3}$$

binding energy (per baryon) of single α: $B_\alpha \simeq 7.1$ MeV

in non-relativistic limit

$$n_N|_{T<T_{\mathrm{BEC}}} = g_N\left(\frac{m_N T}{2\pi}\right)^{3/2}\sum_{k=1}^{\infty}(-1)^{k+1}k^{-3/2}e^{-B_\alpha k/T}$$

→ n_N does not depend on n_α for BEC states



⇒ $n_\alpha \gg n_N$ in the BEC region

One-component matter with mean-field interaction

(pure nucleon- or α- matter)

$U = U(n)$ - mean-field potential (depends only on density n, w/o explicit dependence on T)

shift of chemical potential $\mu = \widetilde{\mu} + U(n)$ with respect to the ideal gas

$\widetilde{\mu} = \widetilde{\mu}(T, n)$ - equivalent chemical potential of ideal gas (determined from $n = n_{\text{id}}(T, \widetilde{\mu})$)

pressure $p(T, \mu) = p_{\text{id}}(T, \widetilde{\mu}) + \Delta p(n)$ where $\Delta p(n) = n\,U(n) - \int_0^n dn_1 U(n_1)$

'excess' pressure

We use Skyrme-like parametrization:

$$U(n) = -2an + \frac{\gamma + 2}{\gamma + 1} b n^{(\gamma+1)}$$

attraction

short-range repulsion

parameters of interaction $a, b, \gamma > 0$
from fit of ground state (GS) at T=0

$$\Rightarrow \quad \Delta p(n) = -an^2 + bn^{(\gamma+2)}$$

we compare the results
for soft (γ=1/6) and hard (γ=1)
repulsive interactions

[1] Satarov, Dmitriev, Mishustin, Phys. At. Nucl. 72 (2009) 1390 (iso-symmetric nuclear matter)
[2] Satarov et al, J. Phys. G 44 (2017) 125102 (pure α-matter)

Iso-symmetric nucleon matter with Skyrme interaction

We choose Skyrme parameters a_N, b_N by fitting GS properties of such matter at T=0:

binding energy per baryon $W_N \equiv m_N - \min(\varepsilon/n) = 15.9\ \mathrm{MeV}$

ground-state (saturation) density $n = n_0 = 0.15\ \mathrm{fm}^{-3}$

→ equivalent to $p = 0,\ \mu = \mu_0 = 923\ \mathrm{MeV}$

equations for a_N, b_N : $E_F(n_0) + U(n_0) = \mu_0, \quad p = p_{\mathrm{id}}(T=0, n_0) + \Delta p(n_0) = 0$

($E_F(n_0)$ ← $\tilde{\mu}$)

γ	$a_N\ (\mathrm{GeV\,fm}^3)$	$b_N\ (\mathrm{GeV\,fm}^{3+3\gamma})$	$K_N\ (\mathrm{MeV})$	$T_c\ (\mathrm{MeV})$
1	0.40	2.05	372	21.3
1/6	1.17	1.48	198	15.3

T_c → temperature of critical point

$\gamma = 1$ → hard EoS

$\gamma = 1/6$ → soft EoS

⇒ reasonable values of compressibility $K_N = 9(dp/dn)_{\mathrm{GS}} = 200 - 240\ \mathrm{MeV}$ are obtained for soft Skyrme repulsion (γ=1/6)

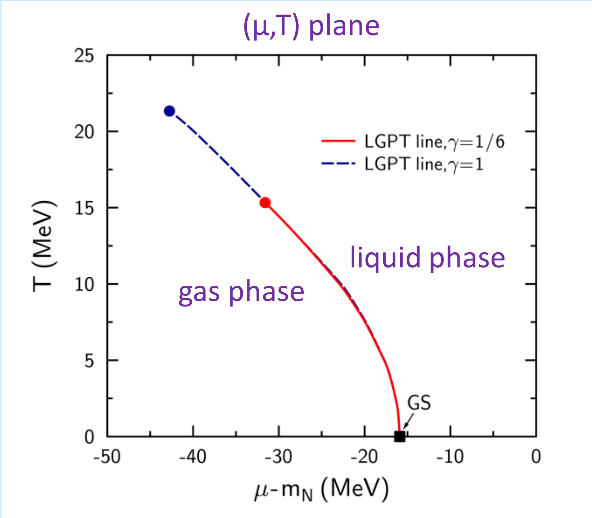
Phase diagram of iso-symmetric nucleon matter

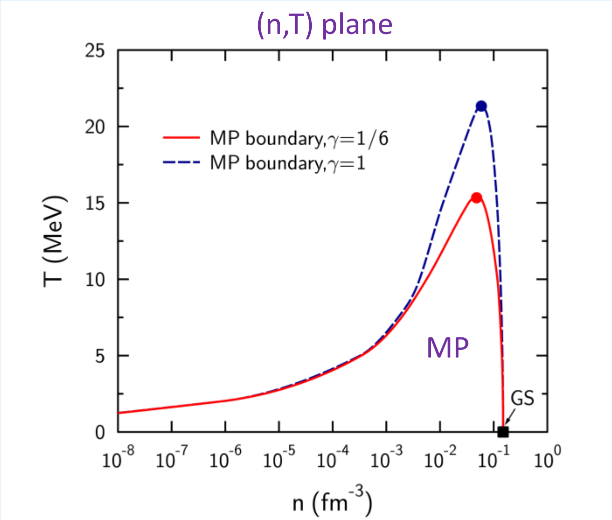
first-order liquid-gas phase transition (LGPT) $\rightarrow$ formation of mixed phase (MP) with coexisting gas ($n=n_g$) and liquid ($n=n_l$) domains ($n_g < n_l$)

exists for any a_N, $b_N > 0$ at $T \leqslant T_{\max} \equiv T_c$

Gibbs conditions of phase equilibrium: $p\,(T, n_g) = p\,(T, n_l), \quad \mu\,(T, n_g) = \mu\,(T, n_l)$

$\rightarrow$ boundaries of MP in (n,T) plane ('binodals')





dots:
critical points

$(\partial p/\partial n)_T = 0$

$(\partial^2 p/\partial n^2)_T = 0$

squares:
ground state

$\varepsilon/n = \min$

temperature of critical point increases with γ

Pure α matter

Clark & Wong (1966) calculated characteristics of GS (T=0) using phenomenological αα-potentials:

binding energy per baryon $W_\alpha \equiv m_N - \min\left(\varepsilon_\alpha/n_B\right) \simeq 12\ \mathrm{MeV}$ ($n_B=4n_\alpha$)

density of GS $n_\alpha = n_{\alpha 0} \simeq 0.036\ \mathrm{fm}^{-3}$ all α's are in BEC state with zero pressure

using $\widetilde{\mu}_\alpha = m_\alpha = 4(m_N - B_\alpha),\ p_\alpha = 0,\ \lim\limits_{T\to 0} p_\alpha^{\mathrm{id}} = 0$ one gets

$$\mu_\alpha = m_\alpha + U(n_{\alpha 0}) = 4(m_N - W_\alpha),\ \Delta p_\alpha(n_{\alpha 0}) = 0$$

$$a_\alpha = b_\alpha n_{\alpha 0}^\gamma = \frac{4(\gamma+1)}{\gamma n_{\alpha 0}}(W_\alpha - B_\alpha)$$

(analytic relations for Skyrme parameters a_α, b_α)

γ	$a_\alpha\,(\mathrm{GeV\,fm}^3)$	$b_\alpha\,(\mathrm{GeV\,fm}^{3+3\gamma})$	$K_\alpha\,(\mathrm{MeV})$	$T_c\,(\mathrm{MeV})$
1	1.09	30.4	354	13.7
1/6	3.83	6.67	207	10.2

smaller critical temperatures as compared to nucleon matter (at the same γ)

Phase diagram of α matter

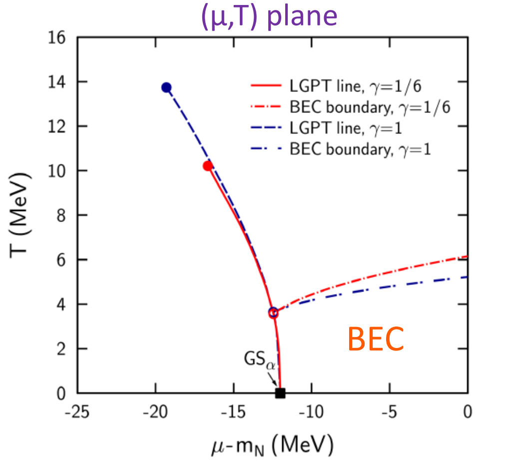
Satarov et al., J. Phys. G44 (2017) 125102 → simultaneous description of LGPT and BEC in pure α matter

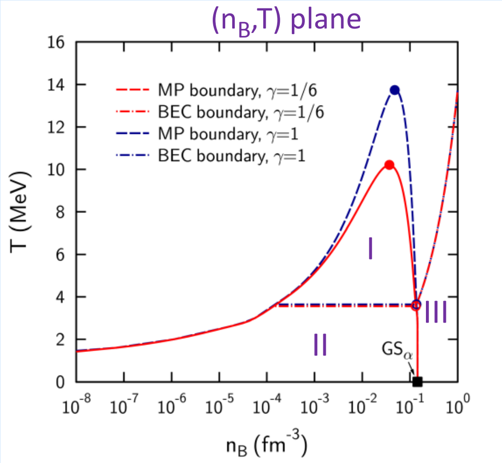
condition of BEC: $\widetilde{\mu}_\alpha(T,n_\alpha) = m_\alpha \to T < T_{\rm BEC} \simeq \frac{2\pi}{m_\alpha}\left[\frac{n_\alpha}{\zeta(3/2)\,g_\alpha}\right]^{2/3}$ (in the MP region $n_\alpha \to n_{\alpha l}$)

➡ BEC boundary in the (n_α,T) plane is not sensitive to interaction (in the mean-field appr.)

triple point (TP)- crossing of BEC line with MP boundary: $T_{\rm TP} \simeq 3.6$ MeV (for γ=1/6, 1)

➡ we obtain phase diagrams similar to those observed for atomic ^{4}He





full dots: critical points

open dots: triple points

squares: ground state

$\mu = \mu_\alpha/4$

$n_B = 4n_\alpha$

region II (MP states with $T<T_{TP}$): gas domains w/o BEC + liquid domains with BEC

Skyrme-like interaction for α-N binary mixture

generalized Skyrme parametrization of excess pressure:

$$\Delta p(n_N,n_\alpha) = p - p_N^{\rm id}(T,n_N) - p_\alpha^{\rm id}(T,n_\alpha) = -\sum_{i,j} a_{ij} n_i n_j + \left(\sum_i B_i n_i\right)^{\gamma+2}$$

(i,j=N,α)

$$\Delta p(n_N,n_\alpha) = -(a_N n_N^2 + 2 a_{N\alpha} n_N n_\alpha + a_\alpha n_\alpha^2) + b_N (n_N + \xi n_\alpha)^{\gamma+2}$$

cross-term coefficient of attraction

$$\xi = (b_\alpha/b_N)^{1/(\gamma+1)} = 2.46\ (\gamma=1), 2.01\ (\gamma=1/6)$$

excess free energy:

$$\Delta f(n_N,n_\alpha) = f - f_N^{\rm id}(T,n_N) - f_\alpha^{\rm id}(T,n_\alpha) = \int_0^1 \frac{d\lambda}{\lambda^2}\,\Delta p(\lambda n_N, \lambda n_\alpha) \quad \rightarrow U_i \equiv \mu_i - \widetilde{\mu}_i = \frac{\partial \Delta f}{\partial n_i}$$

chemical potentials:

$$\mu_N = \widetilde{\mu}_N(T,n_N) - 2(a_N n_N + a_{N\alpha} n_\alpha) + \frac{\gamma+2}{\gamma+1} b_N (n_N + \xi n_\alpha)^{\gamma+1}$$

$$\mu_\alpha = \widetilde{\mu}_\alpha(T,n_\alpha) - 2(a_{N\alpha} n_N + a_\alpha n_\alpha) + \frac{\gamma+2}{\gamma+1} b_N \xi (n_N + \xi n_\alpha)^{\gamma+1}$$

chemical equilibrium:
$\mu_N = \mu_\alpha/4$

below we assume γ=1/6 and study sensitivity of results to cross-term coefficient $a_{N\alpha}$

the only unknown model parameter

Ground state of α-N matter at T=0

Energy per baryon (EPB) as function of n_N, n_α for different α-N couplings ($a_{N\alpha}$)

$E/B \equiv \varepsilon/n_B - m_N \quad (n_B = n_N + 4n_\alpha)$ at T=0 $\rightarrow$ $\varepsilon = \varepsilon_N^{\rm id}(n_N) + m_\alpha n_\alpha + \Delta f(n_N, n_\alpha)$

➡ the EPB surface changes qualitatively at

$$\boldsymbol{a_{N\alpha} = a_*} = \frac{1}{2}\left(\frac{m_\alpha - 4\mu_0}{n_0} + \frac{\gamma+2}{\gamma+1} b_N \xi n_0^\gamma\right) \simeq 2.12 \text{ GeV fm}^3 \quad (\gamma = 1/6)$$

$a_{N\alpha}=a_*$ is found by solving

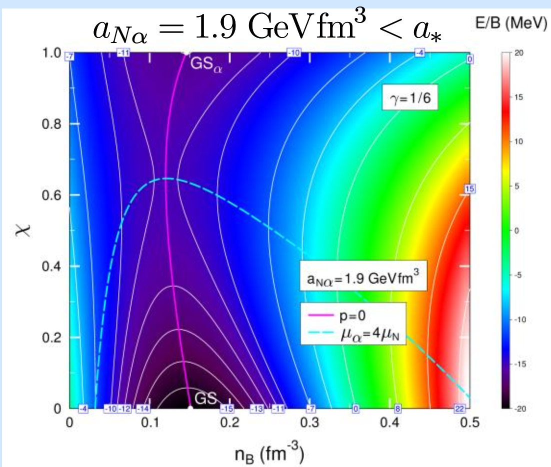
$$\mu_\alpha = m_\alpha + U_\alpha(n_N = n_0, n_\alpha = 0) = 4\mu_0$$

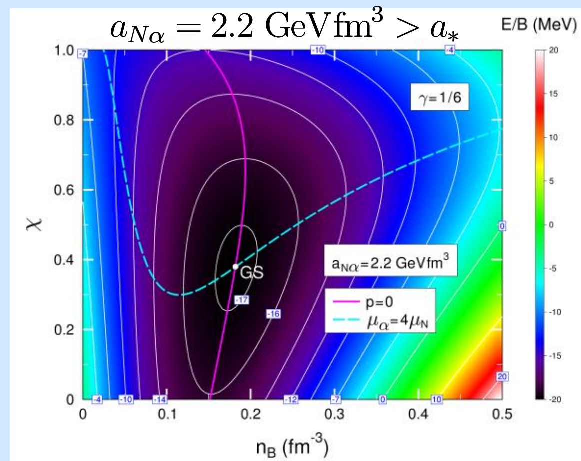
$$\mu_0 = 923 \text{ MeV} = m_N - 15.9 \text{ MeV}$$

$$n_0 = 0.15 \text{ fm}^{-3}$$

$a_{N\alpha} < a_*$ ➡ **two** local **minima** of EPB: 1) stable, without α's + 2) metastable (less bound) without nucleons, separated by energetic barrier $\rightarrow$ GS without α's (i.e. purely nucleonic state)

$a_{N\alpha} > a_*$ ➡ **one** local **minimum** of EPB $\rightarrow$ GS with nonzero fraction of α's





GS: $n_\alpha = 0$ for $a_{N\alpha} < a_*$
$n_\alpha \neq 0$ for $a_{N\alpha} > a_*$

$\rightarrow$ n_α and W increase with $a_{N\alpha}$ (at large $a_{N\alpha}$)

preferrable

➡ further on we assume that $a_{N\alpha} < a_*$ (two parameter sets are considered: $a_{N\alpha}$=1 (A) and 1.9 (B) GeV fm^3)

EoS of α-N matter (numerical scheme)

simultaneously solving the equations:

$\mu_i = \widetilde{\mu}_i(T, n_i) + U_i(n_N, n_\alpha)$ (U_i - mean-filed potentials, $\widetilde{\mu}_i$ – chemical pot. of ideal gas, i=N,α)

$$\widetilde{\mu}_\alpha = \begin{cases} \widetilde{\mu}_\alpha(T, n_\alpha)\,, & n_\alpha < n_*(T) \\ m_\alpha\,, & n_\alpha > n_*(T) \end{cases}$$

→ outside BEC region

→ inside BEC region

$$n_*(T) \equiv g_\alpha \left(\frac{m_\alpha T}{2\pi}\right)^{3/2} \zeta(3/2)$$

$\mu_N = \mu_\alpha/4\ (\equiv \mu)$ → condition of chemical equilibrium

we get $n_N,\, \mu,\, p = p_N^{\rm id} + p_\alpha^{\rm id} + \Delta p$ as functions of n_α, T

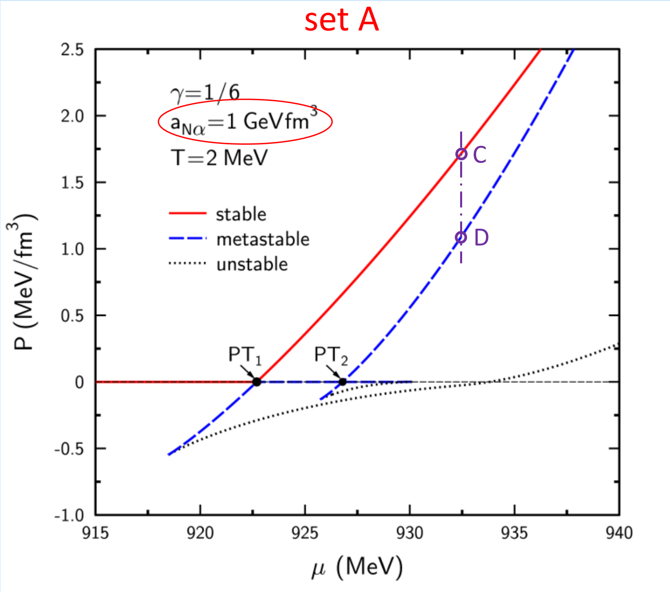
⇒ isotherms in (n_N, n_α), (μ, p) planes

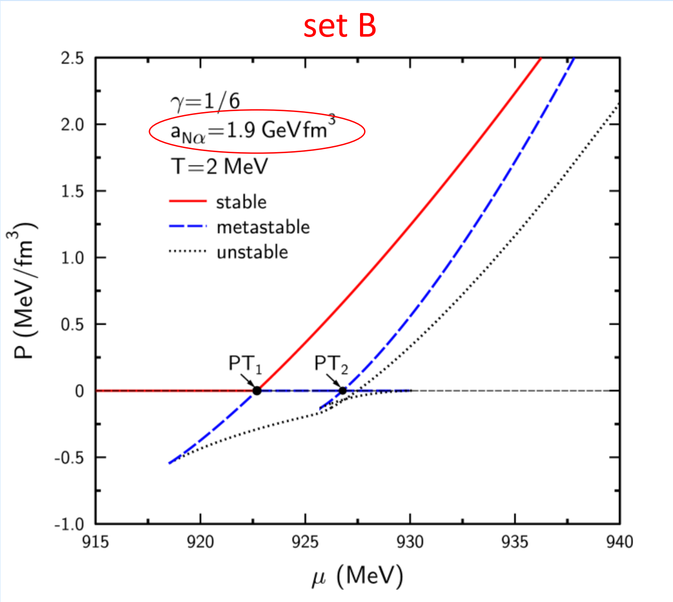
in general, there are several solutions at given T: (in (μ,p) plane) → LGPT

- unstable (spinodal) states $\left(\det||\partial^2 f/\partial n_i \partial n_j|| < 0\right)$
- stable/metastable states with larger/smaller pressure at the same μ

Isotherms T=2 MeV in (μ,p) plane

μ - baryon chem. potential
p - pressure





at given μ states with larger p are favorable (stable)

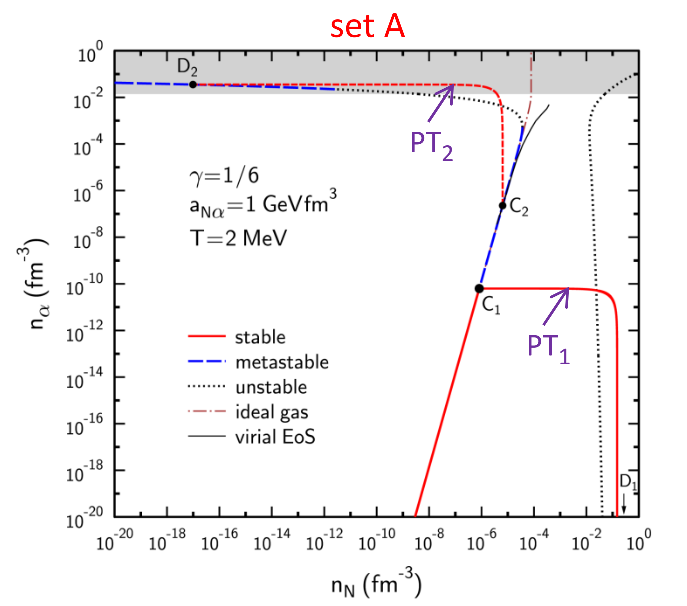
e.g. C → stable state,
D → metastable

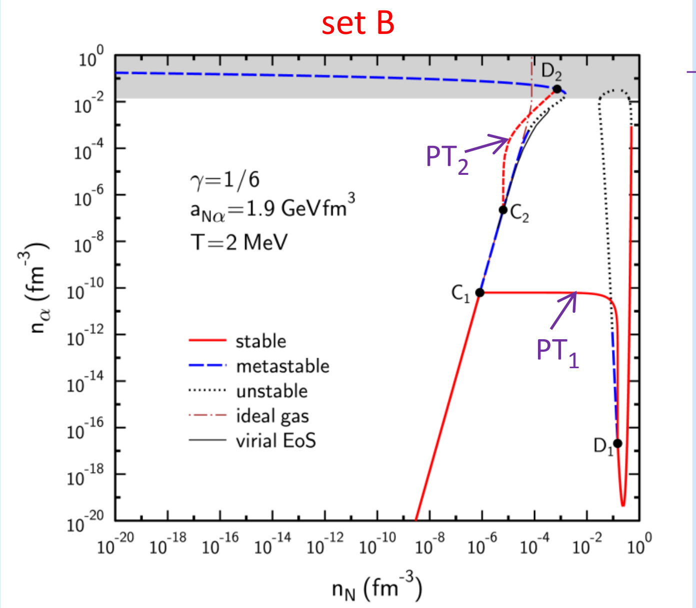
→ jumps of pressure slopes (=n_B) at the LGPT points $PT_{1,2}$

- stable (PT_1) and metastable (PT_2) liquid-gas phase transitions (at T=2 MeV)
- low sensitivity to $a_{N\alpha}$ in the (μ,p) plane

Isotherms T=2 MeV in (n_N, n_α) plane

reasonable values:
$n_\alpha \lesssim 0.1\ \mathrm{fm}^{-3}$





→ shaded domains:
BEC region $n_\alpha > n_*(T)$

C_1D_1→MP states of PT$_1$ (stable, without BEC)

C_2D_2→MP states of PT$_2$ (metastable, D_2=state with BEC)

virial EoS:
Horowitz et al (2006)

- suppression of α's at large nucleon densities → similar to Mott effect
- fractions of α's are small (large) for stable (metastable) LGPT
- states with BEC are metastable
- set B is preferable (closer to virial EoS as compared to set A)

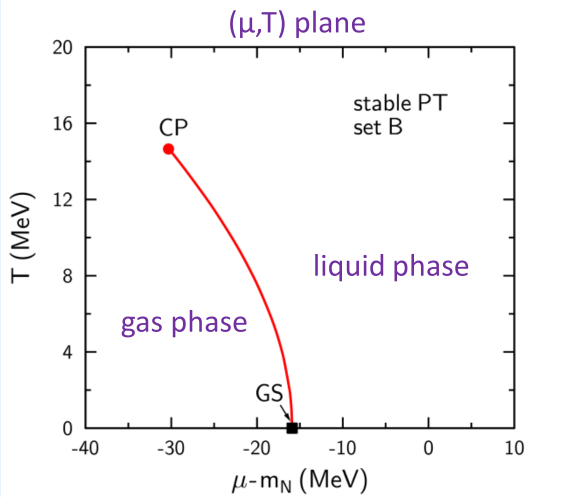
Phase diagram of α-N matter (stable states)

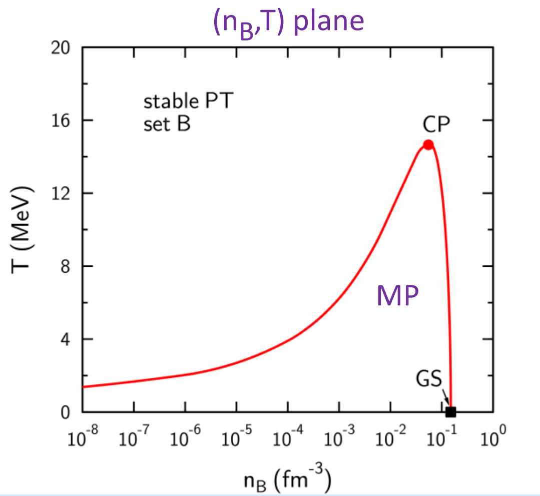
Characteristics of critical point (CP):

	$T_{\rm CP}$(MeV)	$n_{B\rm CP}$ (fm^{-3})	$\chi_{\rm CP}$
set A	15.4	$4.8\cdot10^{-2}$	$2.5\cdot10^{-4}$
set B	14.7	$5.3\cdot10^{-2}$	$6.9\cdot10^{-2}$

→ found from

$$(\partial p/\partial n_B)_T = 0$$
$$(\partial^2 p/\partial n_B^2)_T = 0$$





squares:
ground state (GS)

$T = 0,\ \varepsilon/n_B = \min$

parameters of GS coincide with those for pure nucleon matter (χ=0):

$\mu = m_N - 15.9$ MeV
$n_B = 0.15$ fm^{-3}

position of critical point (CP) only slightly changes with $a_{N\alpha}$

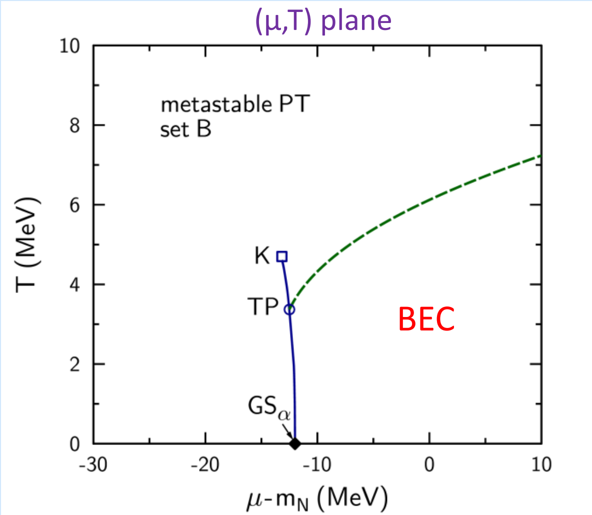
Phase diagram of α-N matter (metastable states)

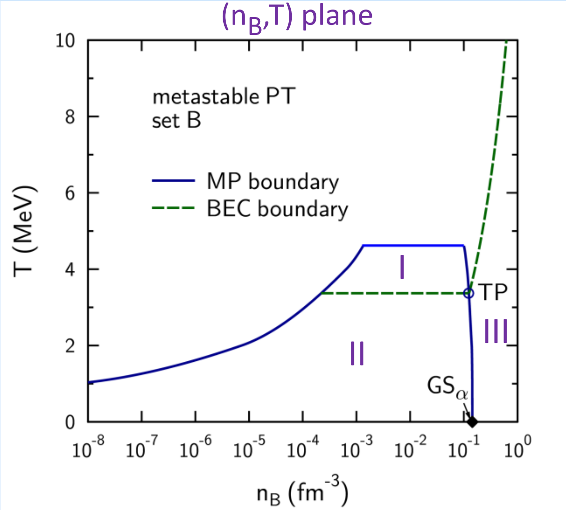
characteristics of metastable PT:

'remnant' of LGPT in pure α -matter

	T_K(MeV)	n_{BK}(fm^{-3})	χ_K	T_{TP}(MeV)
set A	7.6	(1.2-2.6)·10^{-2}	0.14–1.0	3.5
set B	4.6	1.3 ·10^{-3} -0.1	0.46–0.86	3.4

K – end point, TP – triple point (intersection of LGPT and BEC lines)





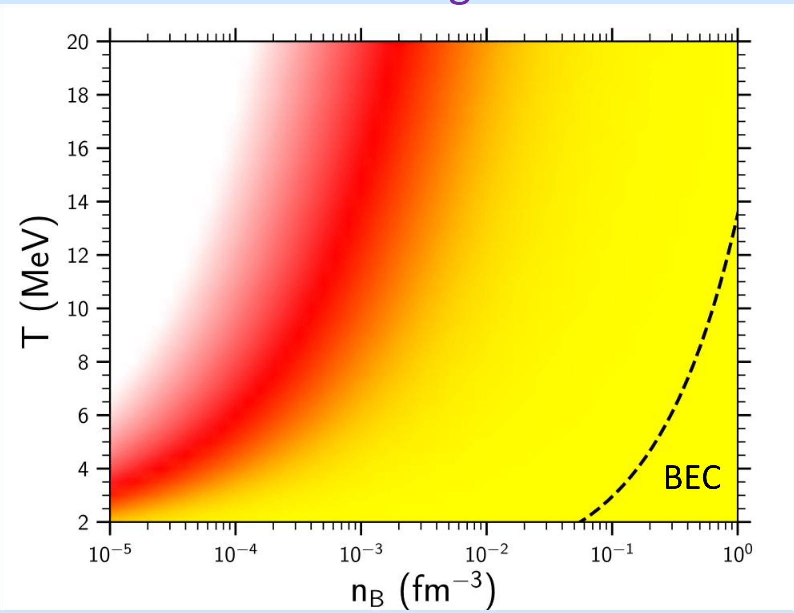
metastable LGPT disappears 'abruptly' at T=TK (with nonzero jump of n_B)

diamonds show GS of pure α-matter

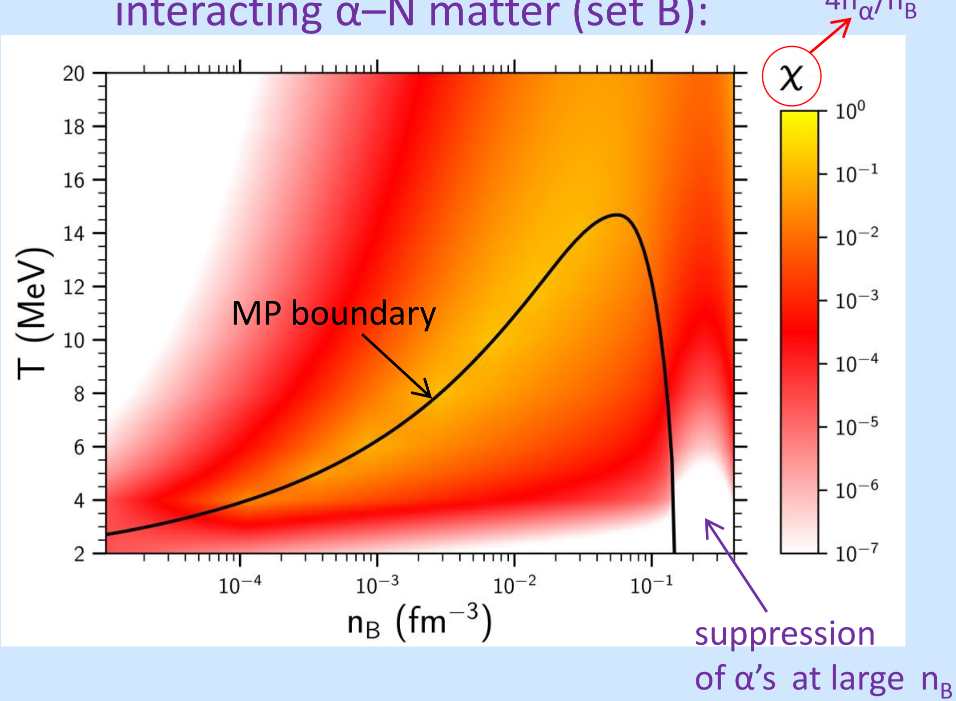
strong sensitivity to $a_{N\alpha}$

Fraction of α in (n_B,T) plane (stable states)

ideal α–N gas:



interacting α–N matter (set B):

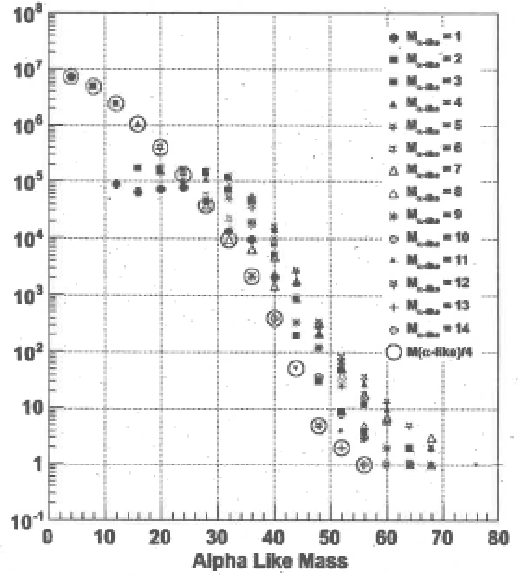


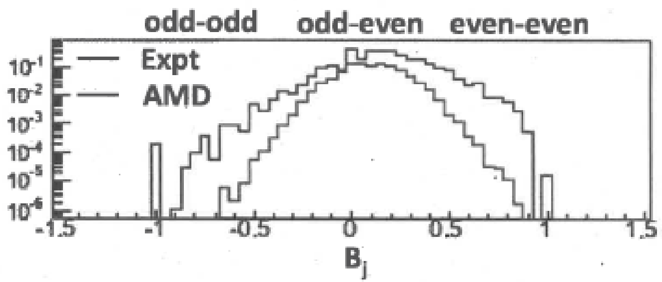
- strong influence of interaction: non-monotonic density behavior of χ
- maximum values of χ (~10-20%) are reached near the left boundary of MP

(larger fractions can be achieved for metastable states)

α-like multiplicities in HI collisions

K. Hagel et al. (Texas A & M University)





$$B_j = \frac{1}{M}\sum_{i=1}^{M}\frac{(-1)^{Z_i}+(-1)^{N_i}}{2}$$

- large number of events with significant yields of α-conjugate nuclei
- larger abundances than predicte by the AMD models

Conclusions

- A simultaneous description of LGPT and BEC in clusterized nuclear matter has been presnted for the first time
- Two L-G phase transitions (stable and metastable) are predicted in interacting α–N matter
- Abundances of α-clusters are maximal at mixed-phase boundary, but strongly suppressed at large baryon densities
- It would be interesting to search for metastable states with α condensates in heavy-ion collisions, both at intermediate and high energies (spectator decay)

Outlook

- Include other light nuclei {d, t, 3He, …) and hypernuclei ($3H_\Lambda$, $4He_\Lambda$ …)
- Study possible α-condensation in isospin-asymmetric (neutron star) matter