Non-equilibrium Dynamics of the Chiral/Deconfinement Phase Transition

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FIAS Frankfurt Institute for Advanced Studies

HIC for **FAIR**
Helmholtz International Center

NATIONAL RESEARCH CENTER
KURCHATOV INSTITUTE

Contents

- **Introduction: Effects of fast dynamics**
- **Effective potential and fluctuations of order parameter**
- **Chiral fluid dynamics with damping and noise**
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- **Conclusions**

Recent publications

M. Nahrgang, C. Herold, S. Leupold, I. Mishustin, M. Bleicher, The impact of dissipationand noise on fluctuations in chiral fluid dynamics, J. Phys. G 40 (2013) 055108;

C. Herold, M. Nahrgang, I. Mishustin, M. Bleicher, Chiral fluid dynamics with explicit propagation of the Polyakov loop, Phys. Rev. C 87 (2013) 014907;

C. Herold, M. Nahrgang, I. Mishustin, M. Bleicher, Formation of droplets with high baryon density at the QCD phase transition in expanding matter, Nucl. Phys. A 925 (2014) 14;

I. Mishustin, T. Koide, G. Danicol, G. Torrieri, Dynamics and stability of chiral fluid, Phys. Atom. Nucl. 77, 1130-1144, arXiv:1401.4103.

Phase diagram of strongly-interacting matter
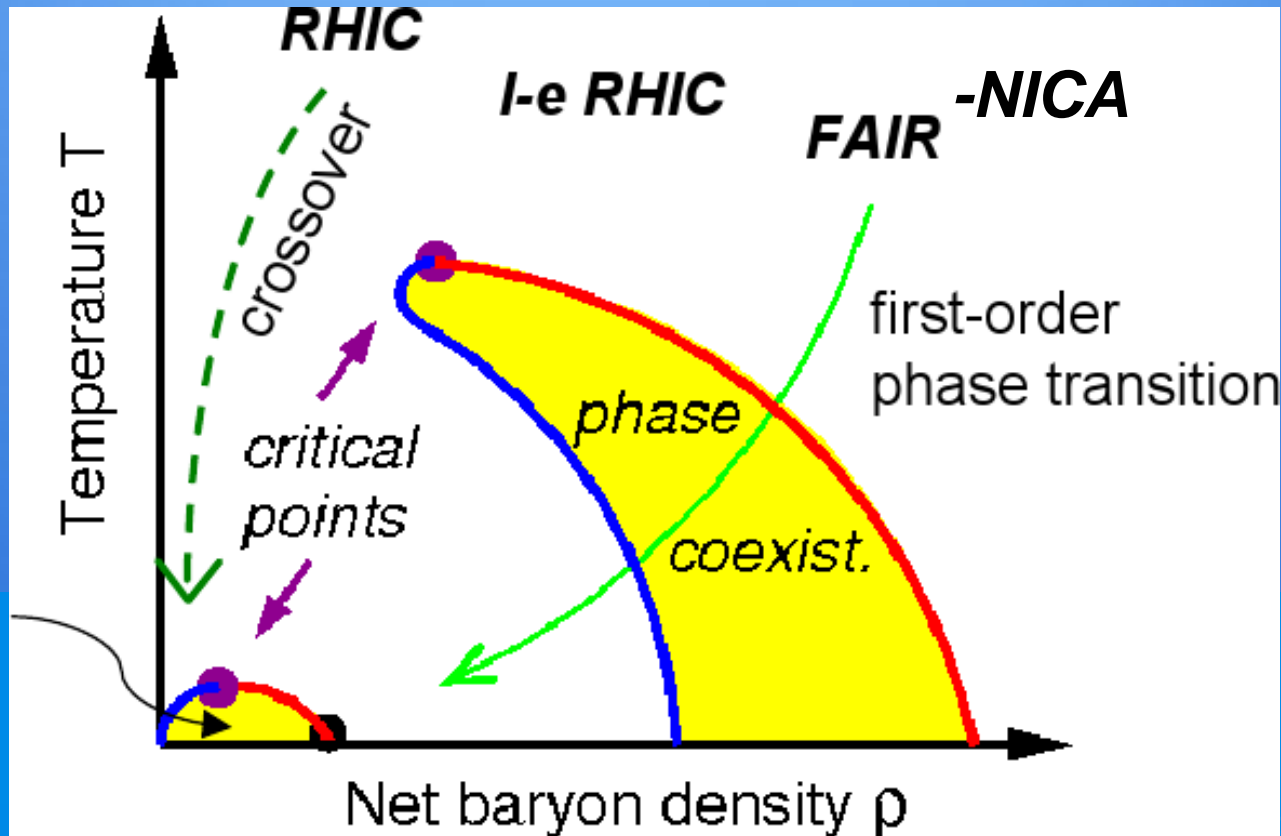
Such a phase diagram is still a beautiful dream! We hope that future FAIR-NICA experiments will help to establish what is the reality.
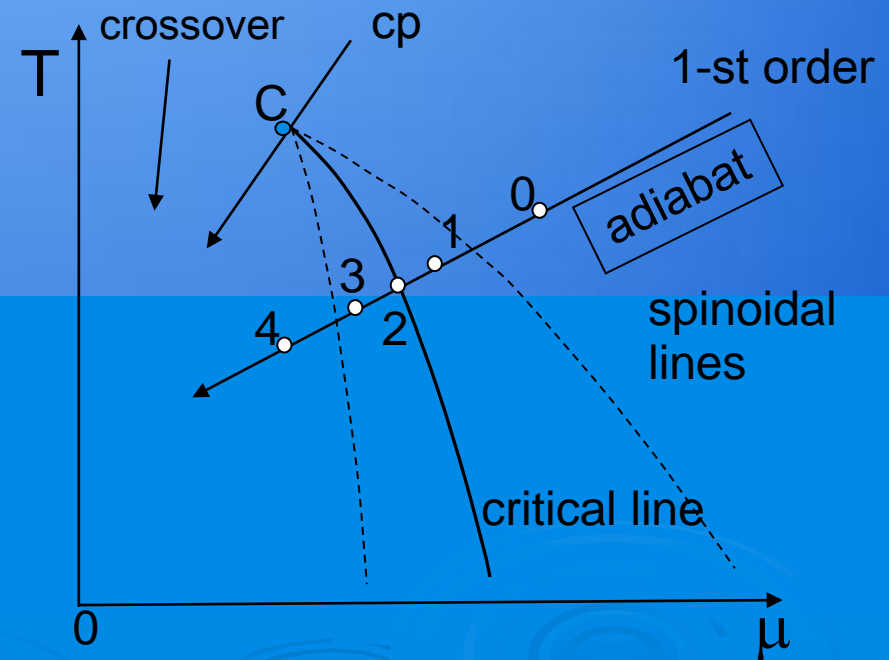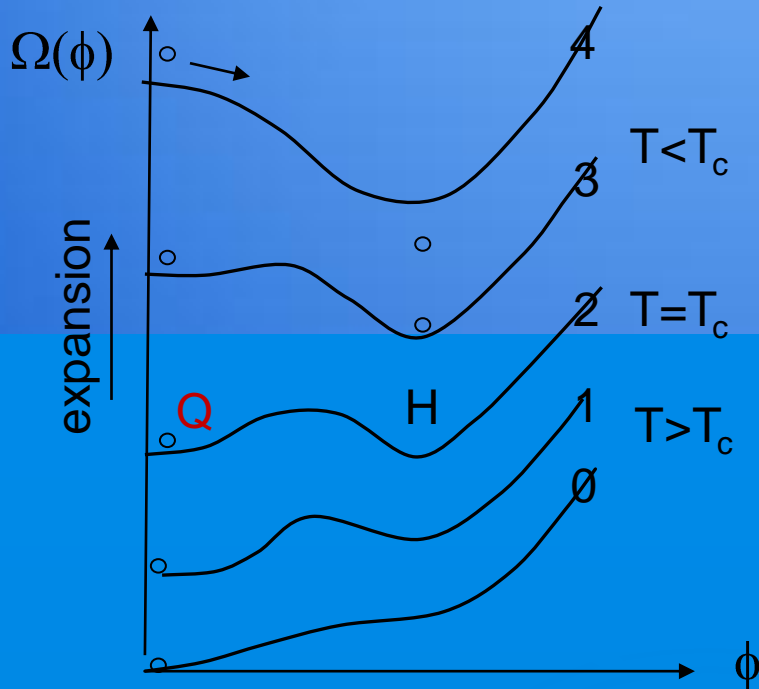
Effects of fast dynamics

Effective thermodynamic potential for a 1st order transition

$$\Omega(\phi;T,\mu) = \Omega_0(T,\mu) + \frac{a}{2}\phi^2 + \frac{b}{4}\phi^4 + \frac{c}{6}\phi^6$$

a,b,c are functions of T and μ

Equilibrium ϕ is determined by

$$\frac{\partial\Omega}{\partial\phi} = 0 \Rightarrow P = -\Omega(\phi_{eq})$$

$\Omega(\phi)$

4

T<T_c

3

2   T=T_c

Q   H

T>T_c

1

0

expansion

ϕ

T

crossover   cp

1-st order

C

0

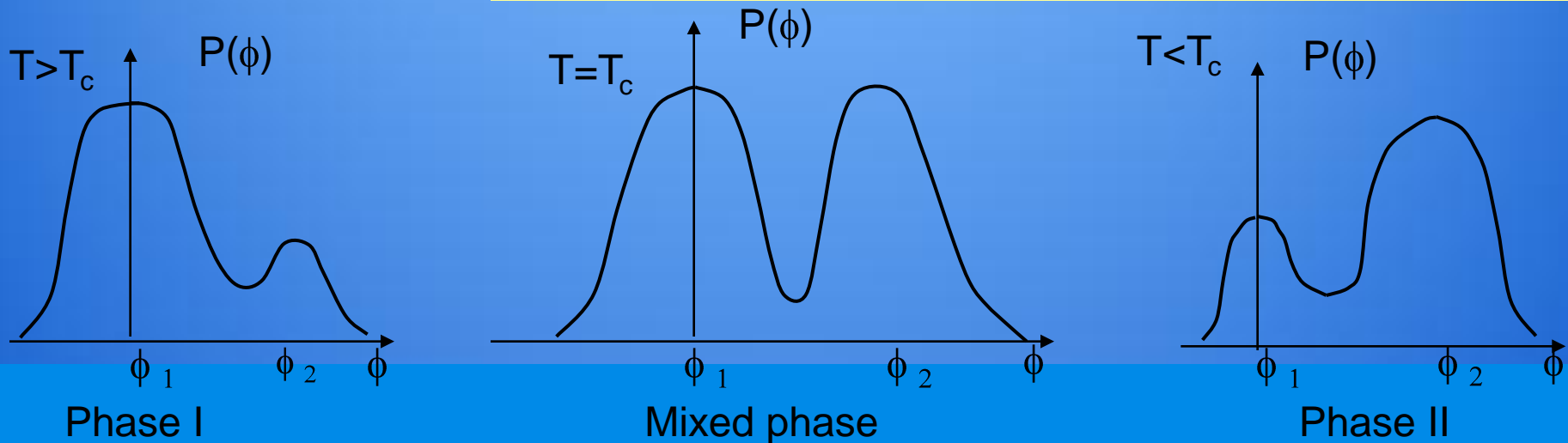adiabat

1

3

4   2

spinoidal lines

critical line

0

μ

In rapidly expanding system 1-st order transition is delayed until the barrier between two competing phases disappears - spinodal decomposition

I. Mishustin, Phys. Rev. Lett. 82 (1999) 4779; Nucl. Phys. A681 (2001) 56

Equilibrium fluctuations of order parameter in 1st order phase transition

Probability distribution for fluctuations

$$P(\delta\phi) \square \exp\left(-\frac{\Delta\Omega(\delta\phi)V}{T}\right), \quad \delta\phi = \phi - \langle\phi\rangle$$

$T > T_c$ $P(\phi)$

ϕ_1 ϕ_2 ϕ

Phase I

$T = T_c$ $P(\phi)$

ϕ_1 ϕ_2 ϕ

Mixed phase

$T < T_c$ $P(\phi)$

ϕ_1 ϕ_2 ϕ

Phase II

➡ **In an equilibrated system fluctuations of the order parameter, i.e. Polyakov loop, should demonstrate bi-modal distributions (lattice calculations?);**

➡ **In a rapidly evolving system these fluctuations will be out of equilibrium;**

➡ **During supercooling process strong fluctuations may develop in the form of droplets** of a metastable phase.

Rapid expansion through a 1ˢᵗ order phase transition
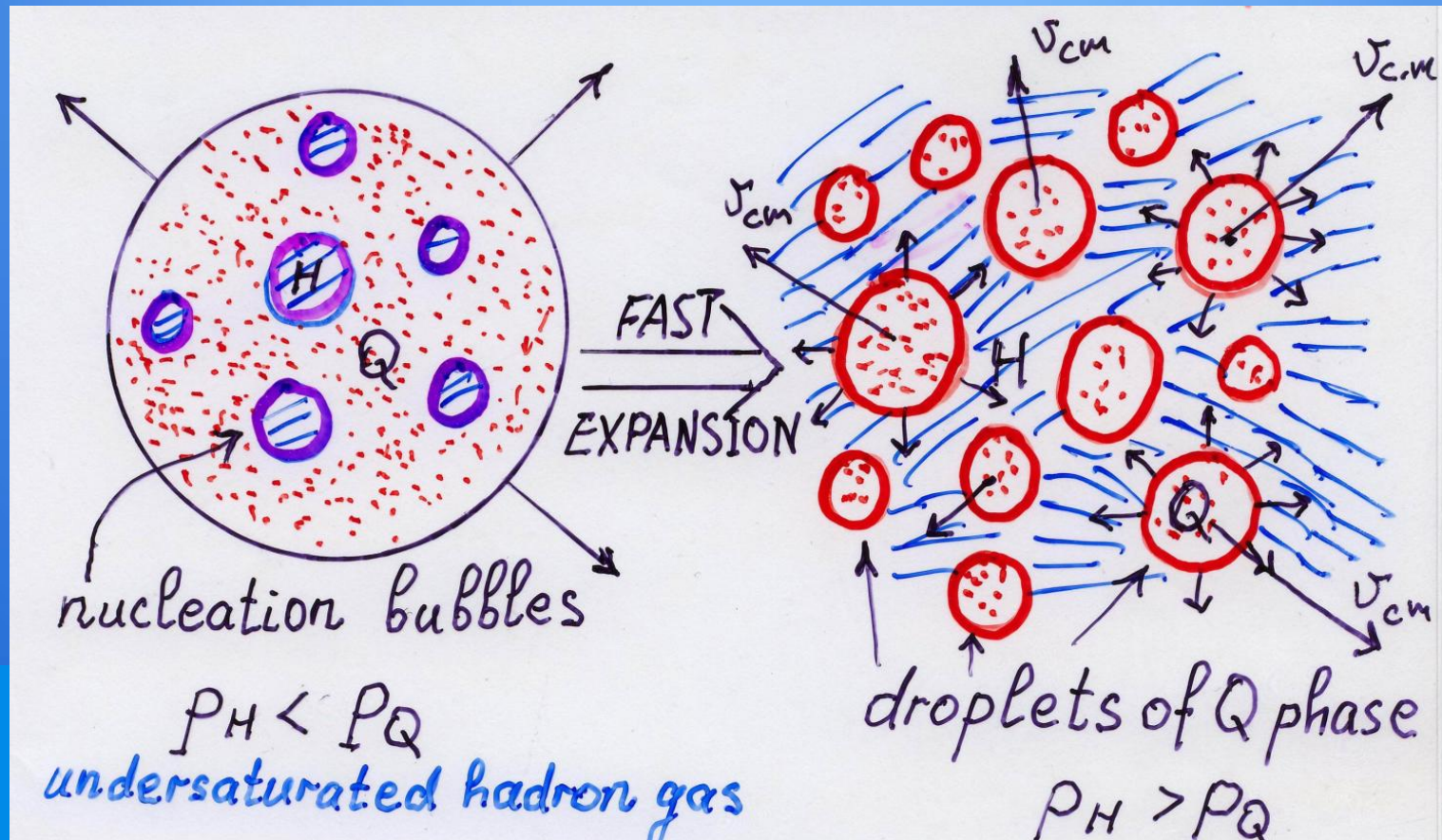
The system is trapped in a metastable state until it enters the spinodal instability region, when Q phase becomes unstable and splits into droplets
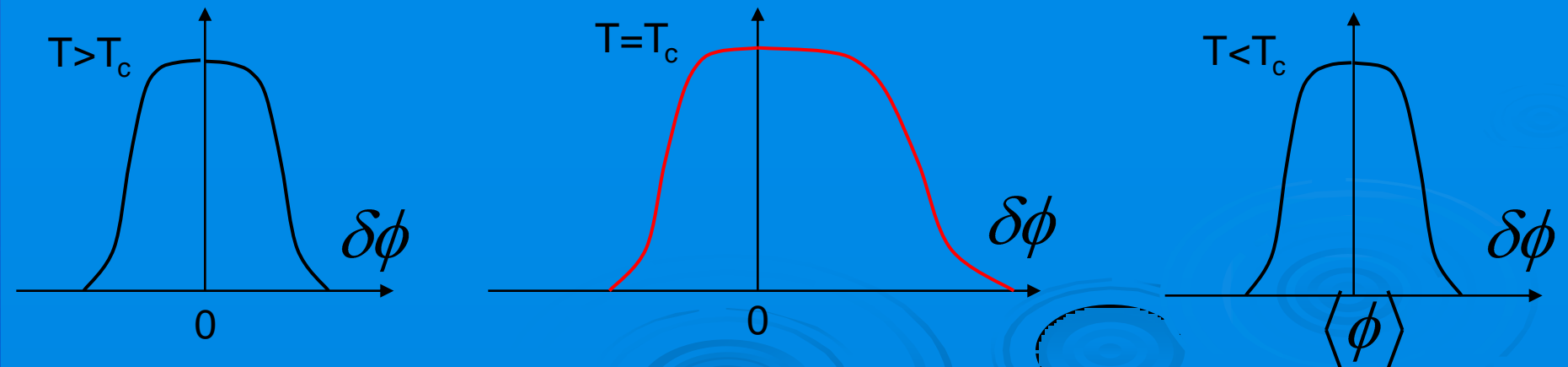
Csernai&Mishustin, 1995; Mishustin, 1999; Rafelski et al. 2000; Randrup, 2003; Steinheimer&Randrup 2013; …

Evolution of equilibrium fluctuations in 2nd order phase transition

$$\Omega(\phi) = \frac{1}{2}a(T)\phi^2 + \frac{1}{2}b(\nabla\phi)^2 + \frac{\lambda}{4}\phi^4, \quad a(T) = a_0(T - T_c)$$

$$\langle\phi\rangle = \frac{a(T)}{\lambda}, \quad T < T_c \text{ and } \langle\phi\rangle = 0, \quad T > T_c, \quad \delta\phi = \phi - \langle\phi\rangle$$

Distribution of fluctuations $\quad P(\delta\phi) \, \square \, \exp\left[-\frac{\Delta\Omega(\delta\phi)V}{T}\right]$

T>T_c $\delta\phi$ 0

T=T_c $\delta\phi$ 0

T<T_c $\delta\phi$ $\langle\phi\rangle$

In rapidly expanding system critical fluctuations have not sufficient time to develop

Critical slowing down in the 2nd order phase transition

$U_{eff}(\phi)$

T>T$_c$

T=T$_c$

T=0

B

0

f_π

ϕ

Fluctuations of the order parameter evolve according to the relaxation equation

$$\frac{d\delta\phi}{dt} = -\gamma\frac{\partial\Omega}{\partial\phi} \approx -\frac{\delta\phi}{\tau_{\rm rel}}$$

In the vicinity of the critical point the relaxation time for the order parameter diverges - no restoring force

$$\tau_{\rm rel}(T) \, \square \, \frac{1}{\left|T - T_c\right|^\nu} \rightarrow \infty, \quad \nu \, \square \, 2$$
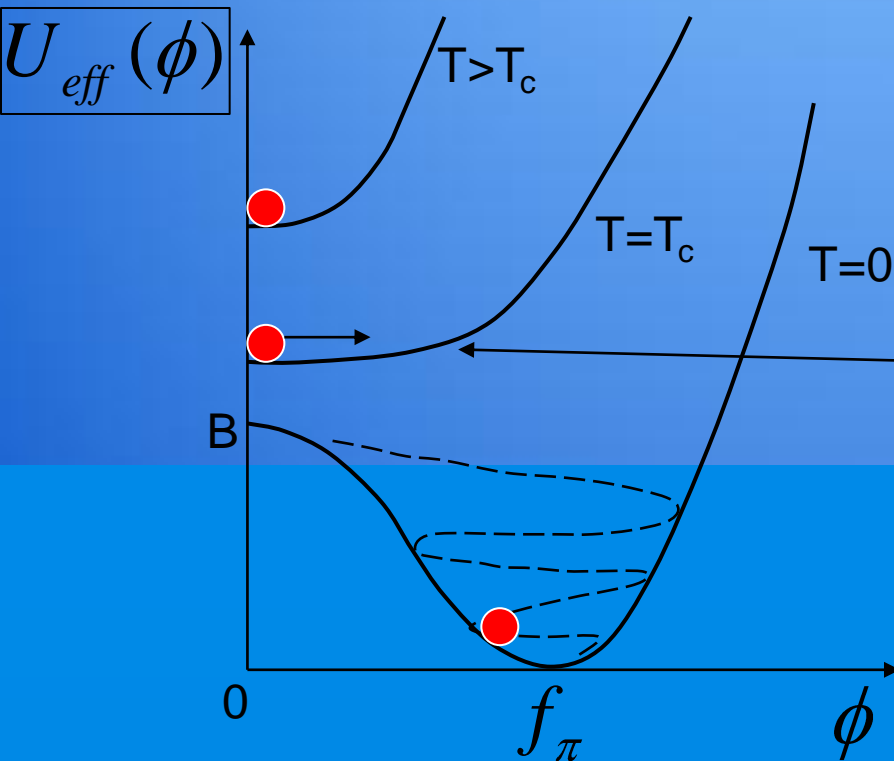
"Rolling down" from the top of the potential is similar to spinodal decomposition (Csernai&Mishustin 1995)

(Landau&Lifshitz, vol. X, Physical kinetics)

Critical slowing down 2

B. Berdnikov, K. Rajagopal, Phys. Rec. D61 (2000)
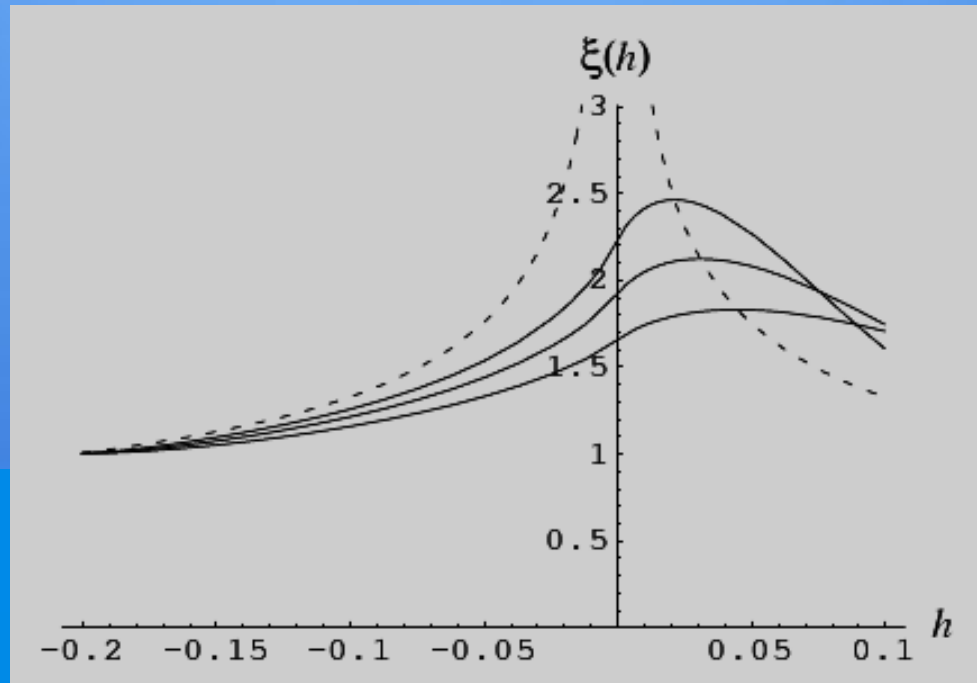
Critical fluctuations have not enough time to build up. One can expect only a factor 2 enhancement in the correlation length even for slow cooling rate, dT/dt=10 MeV/fm.

Simple model for chiral phase transition

Linear sigma model (LσM) with constituent quarks

$$L = \bar{q}[i\gamma\partial - g(\sigma + i\gamma_5\boldsymbol{\tau\pi})]q + \frac{1}{2}[\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}] - U(\sigma,\boldsymbol{\pi}),$$

$$U(\sigma,\vec{\pi}) = \frac{\lambda^2}{4}(\sigma^2 + \boldsymbol{\pi}^2 - v^2)^2 - H\sigma, \quad <\sigma>_{vac} = f_\pi \rightarrow H = f_\pi m_\pi^2$$

Effective thermodynamic potential contains contributions of mean field σ and quark-antiquark fluid:

$$U_{eff}(\sigma;T,\mu) = U(\sigma,\boldsymbol{\pi}) + \Omega_q(m;T,\mu)$$

$$m^2 = g^2(\sigma^2 + \boldsymbol{\pi}^2)$$

CO, 2nd and 1st order chiral transitions are obtained in T-μ plane.

Phase diagram
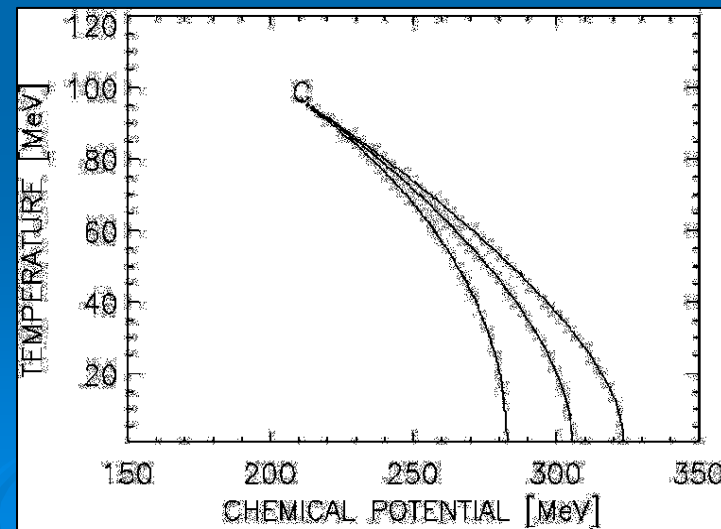
Effective thermodynamic potential

$$\Omega_q(m;T,\mu) = -v_q T \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\left\{\ln\left[1+\exp\left(\frac{\mu-\sqrt{m^2+p^2}}{T}\right)\right]+(\mu\to-\mu)\right\},\ \ v=2N_f N_c$$
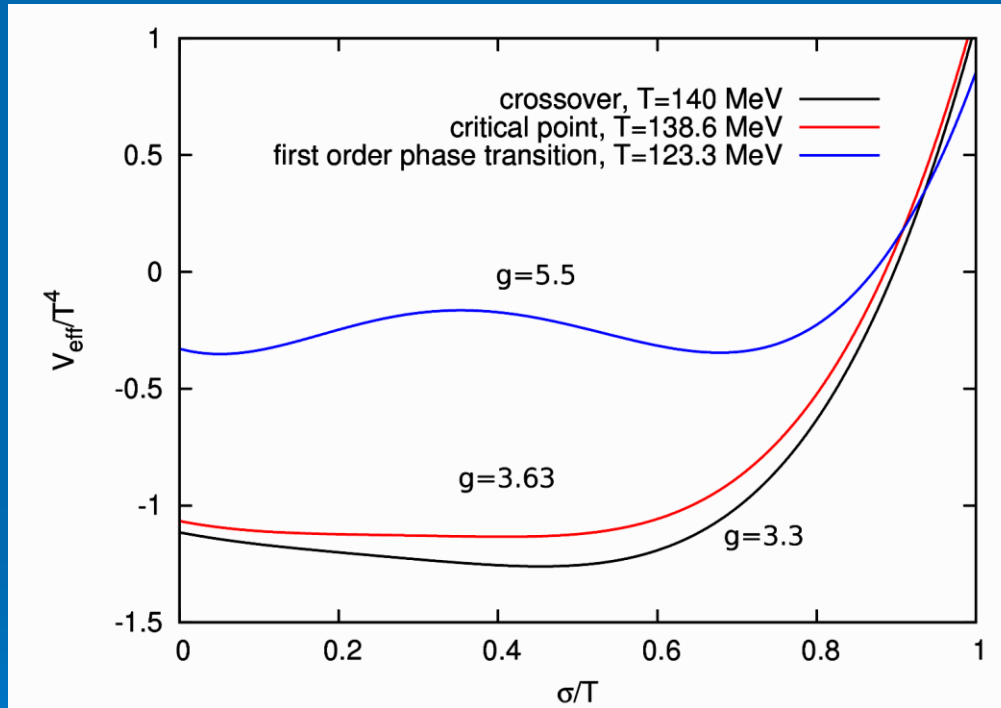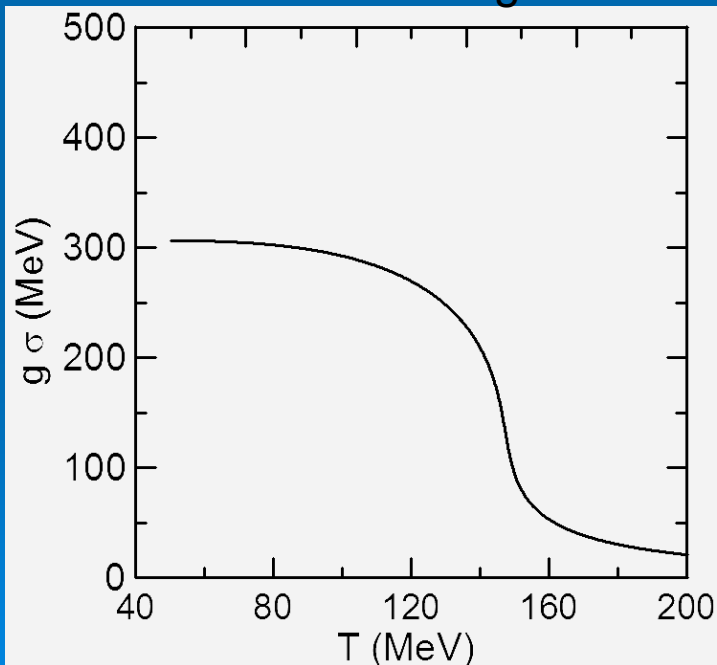

First we consider μ=0 system but tune the order of the chiral phase transition by changing the coupling g.
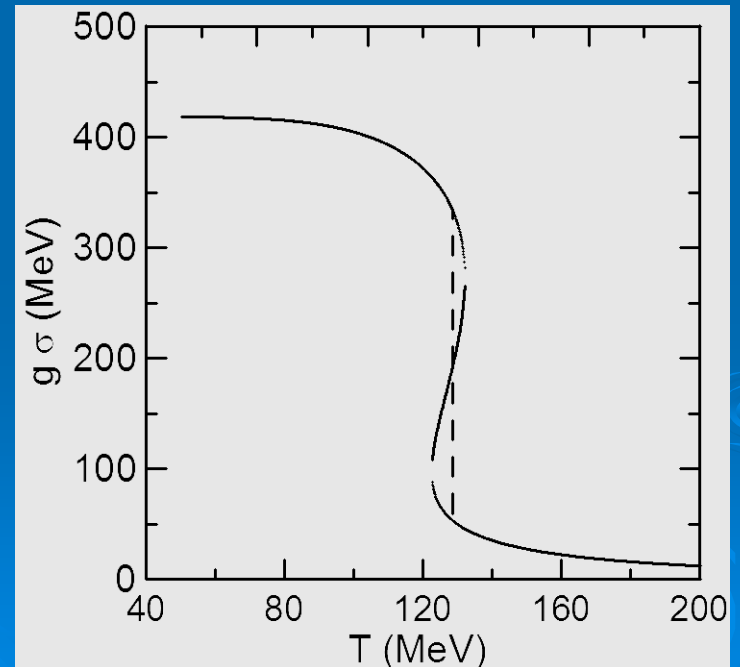
Equilibrium order parameter field

$$\lambda\left(\sigma^2 - v^2\right)\sigma + g\rho_s(\sigma) = 0, \ \rho_s(\sigma) = \frac{\partial\Omega_q(m;T,\mu)}{\partial\sigma}$$

$$m_\sigma^2(\tau) = \frac{\partial^2 U_{eff}}{\partial\sigma^2} = \lambda^2(3\sigma^2 - \sigma^2{}_0) + g\frac{\partial\rho_s}{\partial\sigma}$$

crossover   g=3.3

1-st order   g=4.5

Only 1 equilibrium solution at each T

3 solutions at 122 MeV<T<132 MeV
unstable states - spinodal instability

Spectrum of plane-wave fluctuations

I. Mishustin, T. Koide, G. Danicol, , G. Torrieri, Phys. Atom. Nucl.; arXiv:1401.4103.

σ-meson excitations

sound waves
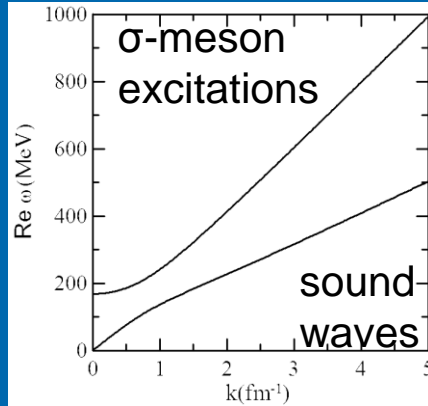
T=120 MeV

$$\delta\sigma(x) = \delta\sigma(\omega,k)e^{i\omega t - ikx}$$

Generally two branches: 1) sound branch
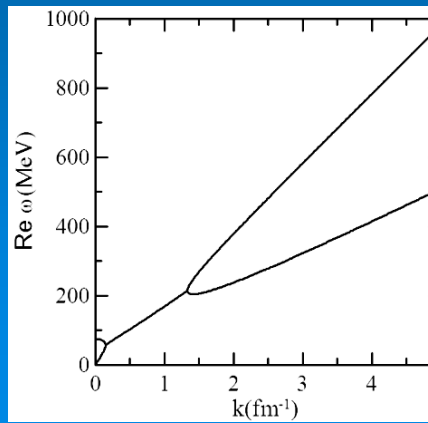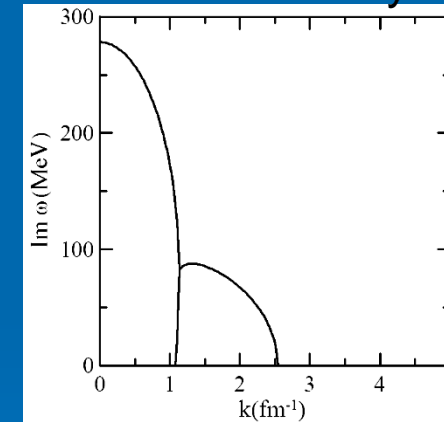2) sigma branch

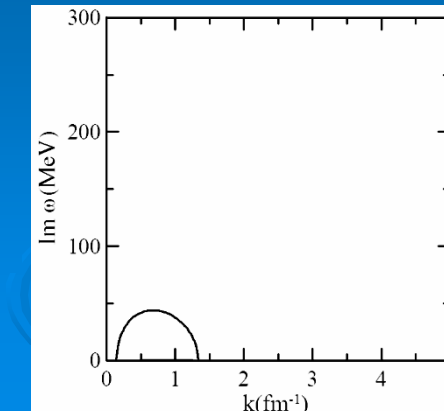$$\omega^2 \approx c_s^2 k^2$$
$$\omega^2 \approx m_\sigma^2 + k^2$$

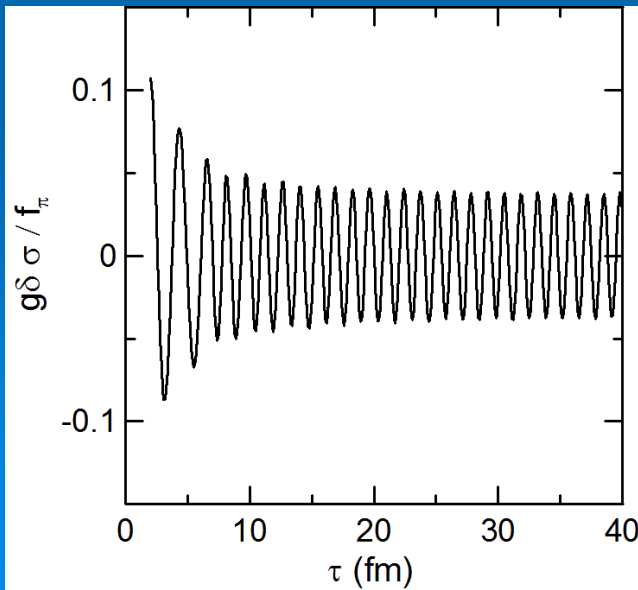Solutions with $\omega^2 < 0$ indicate instability
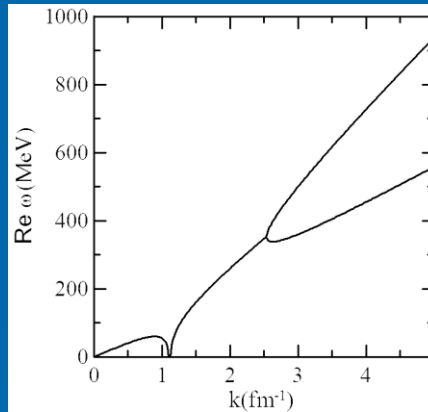
T=125 MeV

T=131 MeV

Fluctuations in Bjorken background

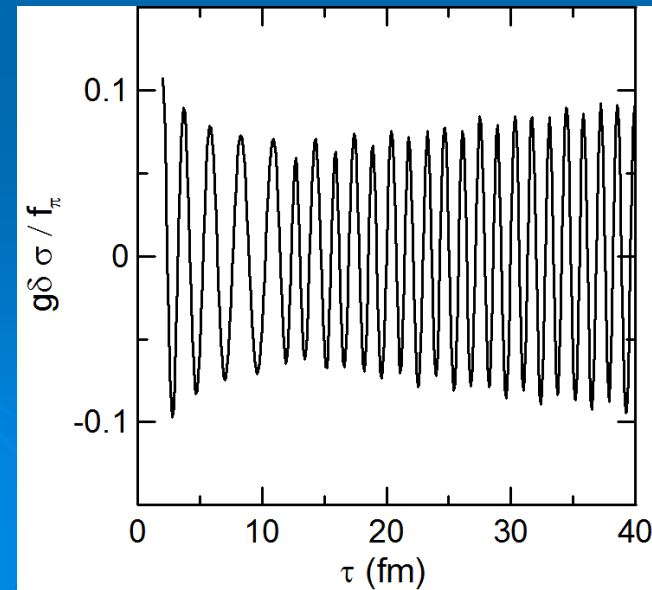$$\delta\sigma(\eta,\tau) = \delta\sigma_k(\tau)e^{ik\eta}, \ \tau = \sqrt{t^2 - z^2}$$

$$\left[\frac{1}{\tau}\frac{\partial}{\partial\tau}\left(\tau\frac{\partial}{\partial\tau}\right) + \frac{k^2}{\tau^2} - \Gamma^2(\tau)\right]\delta\sigma_k(\tau) = f(T)$$

$$\Gamma^2(\tau) = -m_\sigma^2(\tau) + g\frac{\partial s/\partial\sigma}{\partial s/\partial T}\frac{\partial\rho_s}{\partial T}$$

Crossover transition (g=3.3)

1st order transition (g=4.5)

Non-equilibrium Chiral Fluid Dynamics

I.N. Mishustin, O. Scavenius, Phys. Rev. Lett. 83 (1999) 3134;
K. Paech, H. Stocker and A. Dumitru, Phys. Rev. C 68 (2003) 044907;
M. Nahrgang, C. Herold, S. Leupold, , C. Herold, M. Bleicher, Phys. Rev. C 84 (2011) 024912;
M. Nahrgang, C. Herold, S. Leupold, I. Mishustin, M. Bleicher, J. Phys. G40 055108.

Fluid is formed by constituent quarks and antiquarks which interact with the chiral field via quark effective mass $m = g\sigma$

CFD equations are obtained from the energy momentum conservation for the coupled system fluid+field

$$\partial_\nu (T_{\text{fluid}}{}^{\mu\nu} + T_{\text{field}}{}^{\mu\nu}) = 0 \Rightarrow \partial_\nu T_{\text{fluid}}{}^{\mu\nu} = -\partial_\mu T_{\text{field}}{}^{\mu\nu} \equiv S^\nu$$

$$S^\nu = -(\partial^2 \sigma + \frac{\partial U_{\text{eff}}}{\partial \sigma})\partial^\nu \sigma = (g \rho_s + \eta \partial_t \sigma)\partial^\nu \sigma$$

We solve generalized e. o. m. with friction (η) and noise (ξ):

$$\partial_\mu \partial^\mu \sigma + \frac{\partial U_{\text{eff}}}{\partial \sigma} + g <\bar{q}q> + \eta \partial_t \sigma = \xi$$

Langevin equation for the order parameter

$$<\xi(t,\vec{r})> = 0, \quad <\xi(t,r)\xi(t',r')> = \frac{1}{V} m_\sigma \eta \delta(t-t')\delta(r-r')\coth\left(\frac{m_\sigma}{2T}\right)$$

Calculation of damping term

T.Biro and C. Greiner, PRL, 79. 3138 (1997)

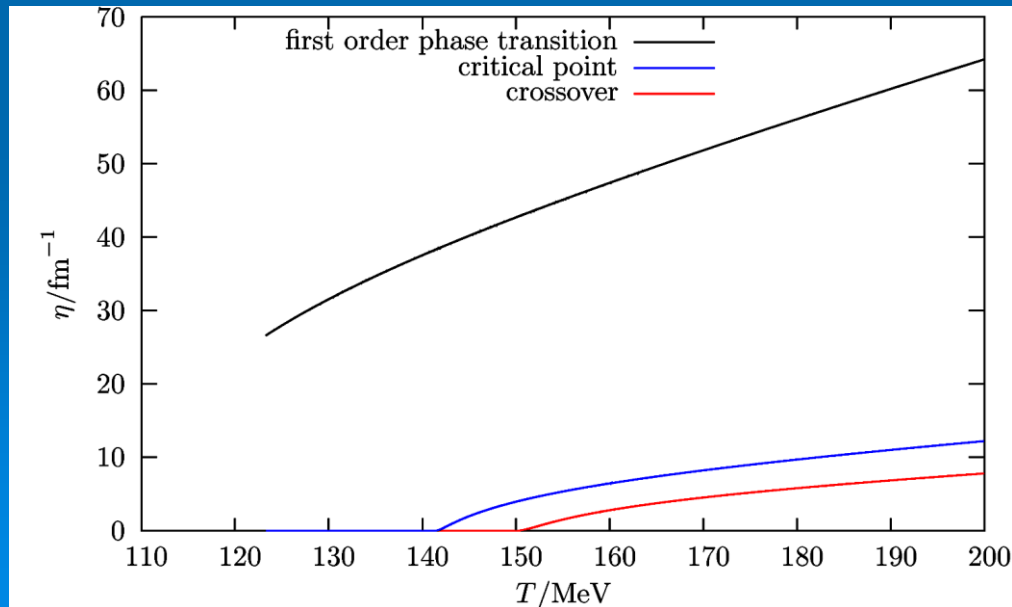**M. Nahrgang, S. Leupold, C. Herold, M. Bleicher, PRC 84, 024912 (2011)**

The damping is associated with the processes:

$$\sigma \to \bar{q}q, \ \ \sigma \to \pi\pi$$

It has been calculated using 2PI effective action

$$\eta = g^2 \frac{\nu_q}{\pi m_\sigma^2} \left[1 - 2n_F\left(\frac{m_\sigma}{2}\right) \right] \left(\frac{m_\sigma^2}{4} - m_q^2 \right)^{3/2}$$

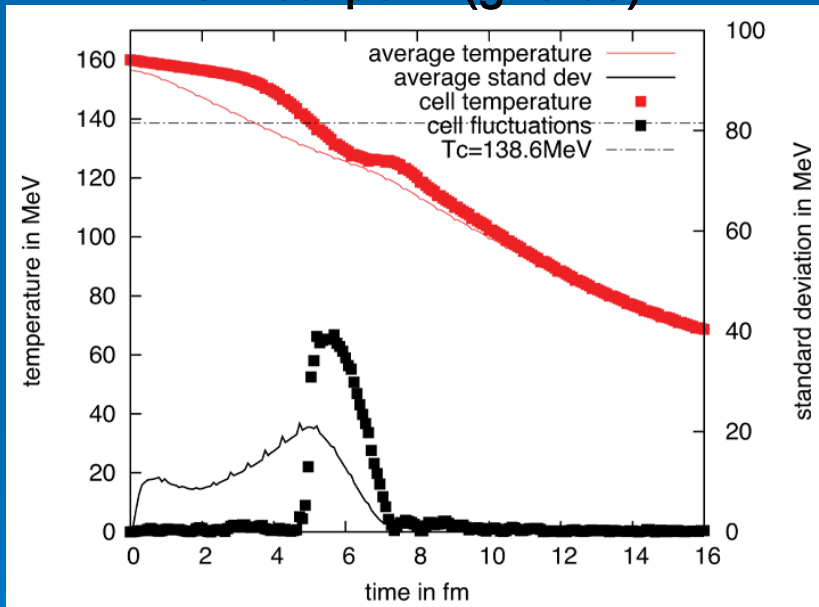Around Tc the damping is due to the pion modes, η=2.2/fm
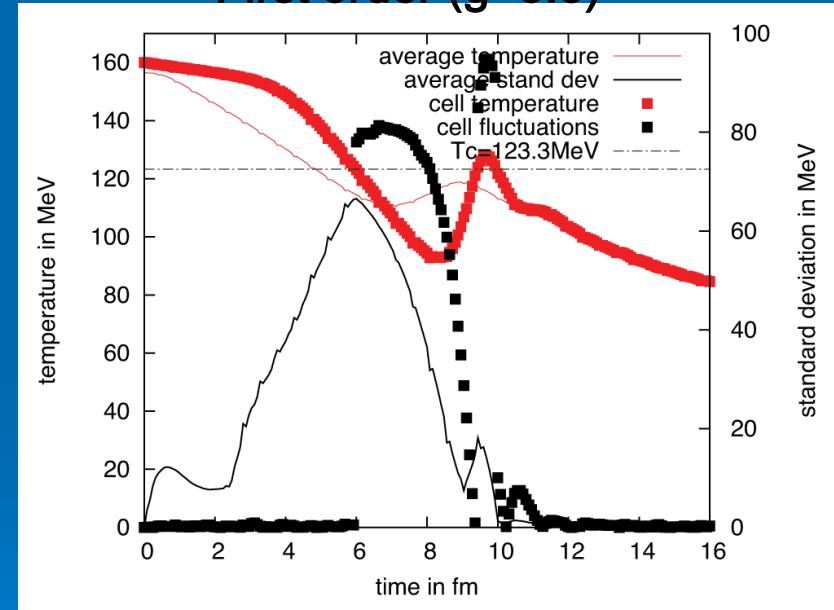
Dynamic simulations: Bjorken-like expansion

Initial state: cylinder of length L in z direction, with ellipsoidal cross section in x-y direction

$$\text{At } t = 0: \; v(z) = \frac{2z}{L}0.2c, \quad -\frac{L}{2} < z < \frac{L}{2}; \; v_x = v_y = 0; \; T = 160 \, MeV$$
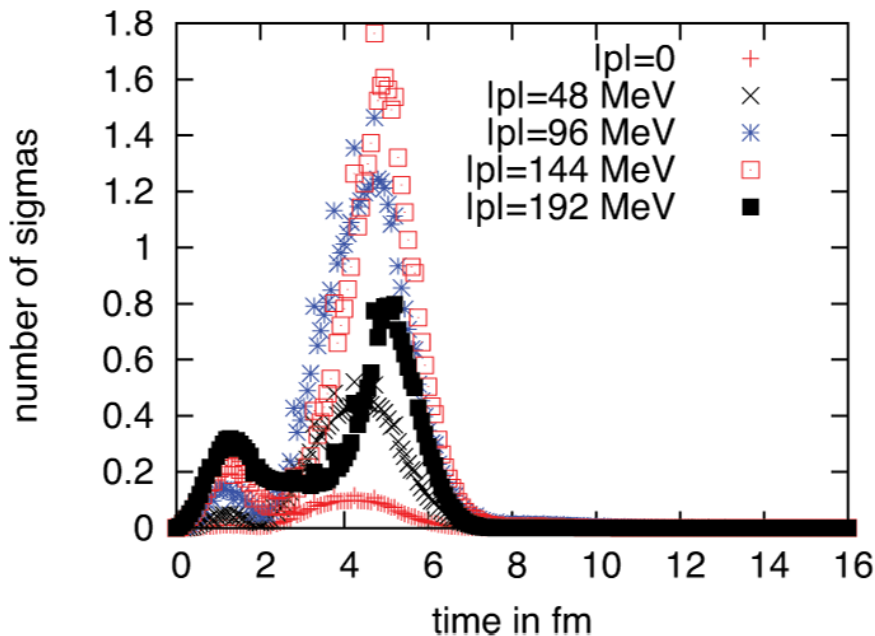
Critical point (g=3.63)　　　　　　　　First order (g=5.5)

Mean values and standard deviation of T for the whole system and for a central cell (1 fm³) are shown as a function of time.
Supercooling and reheating effects are clearly seen in the 1-st order transition, fluctuations become especially strong after 4 fm/c.
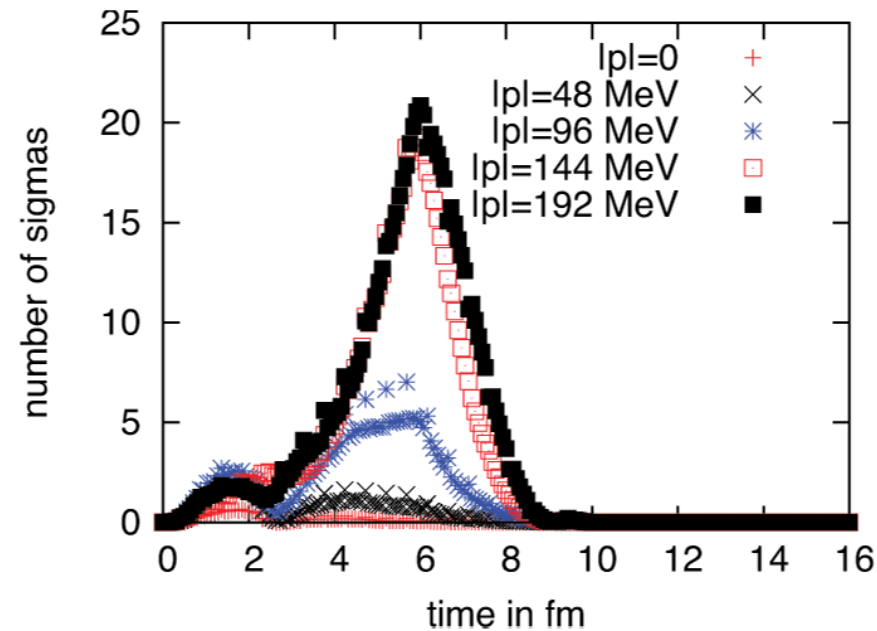
Sigma fluctuations in expanding fireball

$$\frac{dN_\sigma}{d^3k} = \frac{1}{(2\pi)^3}\frac{1}{2\omega_k}[\omega_k^2 \mid \sigma_k \mid^2 + \mid \dot{\sigma}_k \mid^2], \quad \omega_k = \sqrt{m_\sigma^2 + k^2}, \quad m_\sigma^2 = \frac{\partial^2 U_{\text{eff}}}{\partial \sigma^2}\bigg|_{\sigma=\sigma_{\text{eq}}}$$

Critical point (g=3.63)

First order (g=5.5)

Fluctuations are rather weak at critical point (left), but increase strongly at the 1st order transition (right) after 4 fm/c

Extension to finite baryon densities

➢Include μ-dependence in Polyakov loop potential,
  (cf. Schäfer, Pawlowski, Wambach Fukushima)

$$\mathcal{U}(\ell, T, T_0) , \quad T_0 \to T_0(\mu)$$

➢Calculate grand canonical potential for finite chemical potential

$$\Omega_{q\bar{q}} = -2N_f T \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \left\{ \left(\ln \left[1 + 3\ell e^{-\beta(E-\mu)} + 3\ell e^{-2\beta(E-\mu)} + e^{-3\beta(E-\mu)} \right] + (\mu \to -\mu) \right\}$$
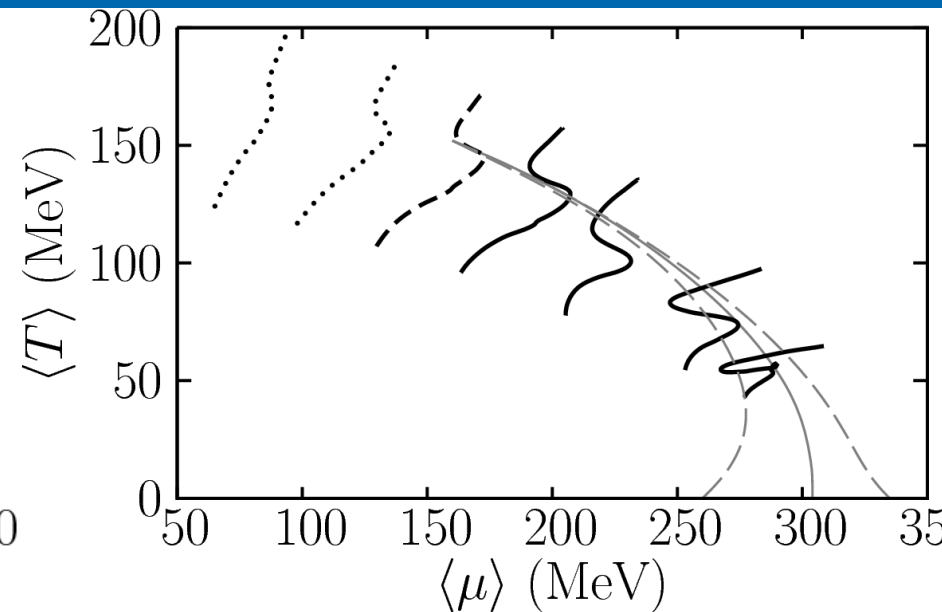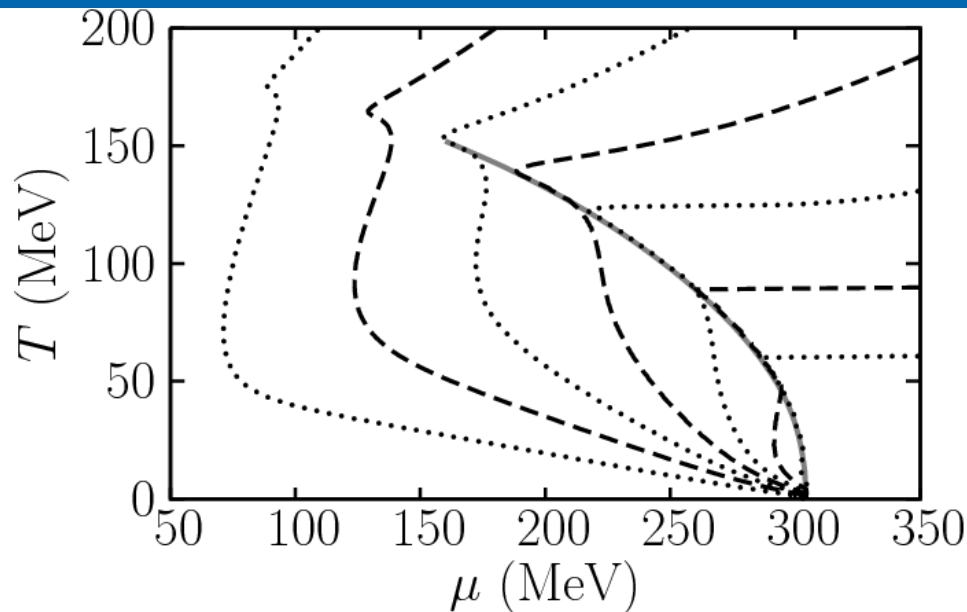
➢Propagate (net) baryon density in the hydro sector

$$\partial_\mu n^\mu = 0 , \quad n^\mu = \rho u^\mu$$

Trajectories on the T-µ plane

CFD calculations are done for spherical fireball of R=4 fm
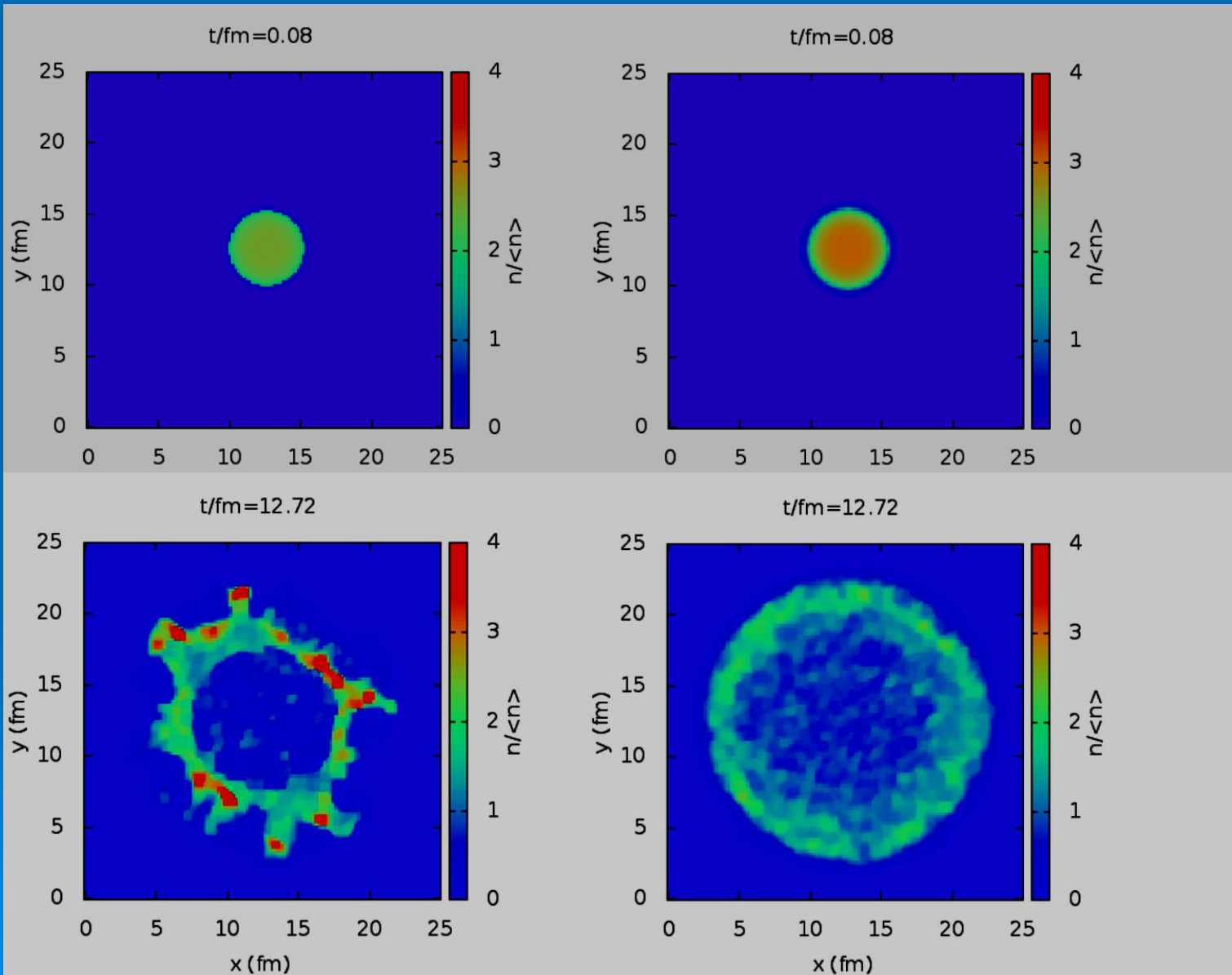
Isentropic expansion   Hydrodynamic evolution

➤ **Trajectories are close to isentropes for crossover and CP;**
➤ **Non-equilibrium "back-bending" is clearly seen in FO case;**
➤ **In the case of strong FO transition (solid lines) the system is trapped in spinodal region for a significant time**
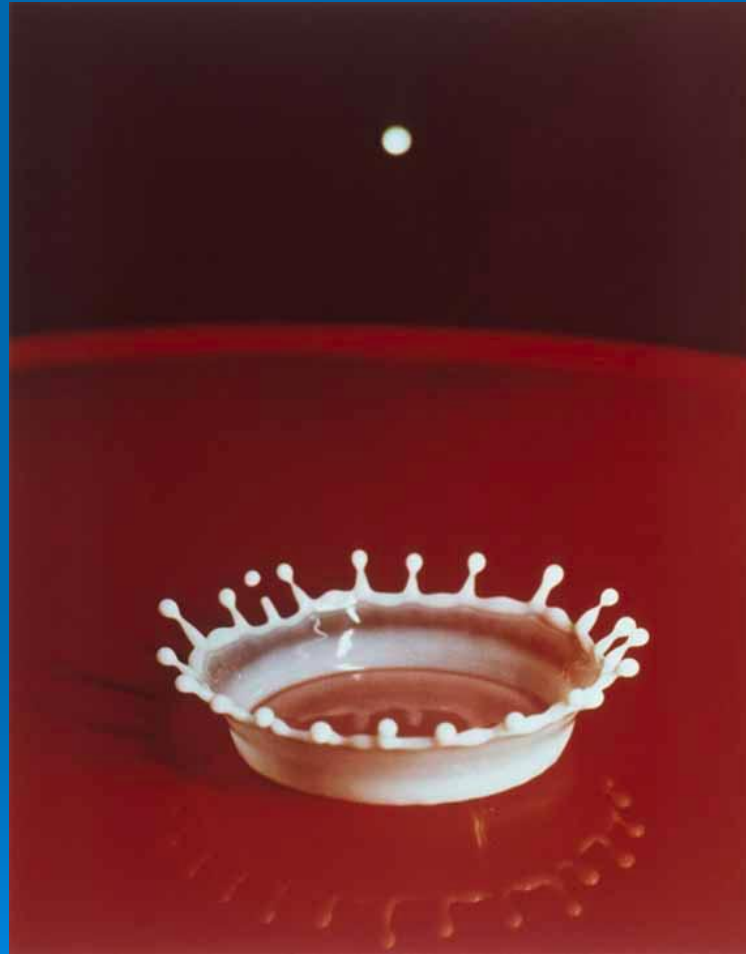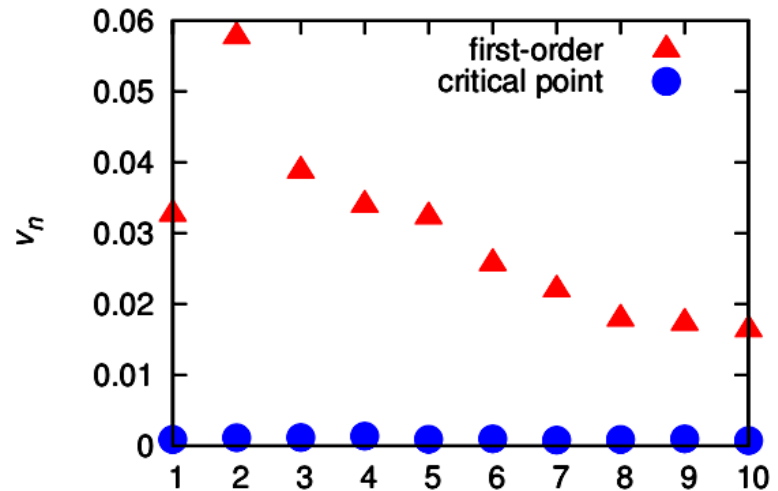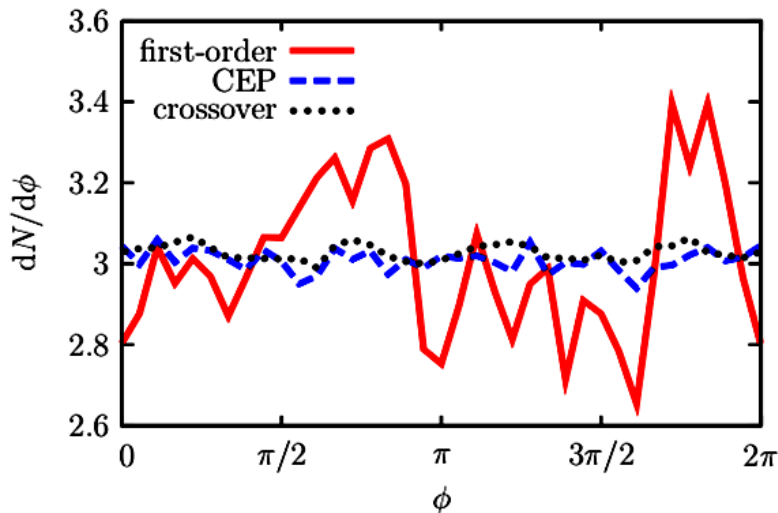
Dynamical droplet formation

Splash of a milk drop

Observable signatures of high-density domains

Azimuthal fluctuations of net-B In single events: strong enhancement at first order PT

High harmonics of baryonic flow (averaged over many events):
$$v_n = \langle \cos[n(\varphi - \varphi_n)] \rangle$$

New developments

➢ **In the previous calculations the EOS had a P=0 point at a finite baryon density (like the MIT bag model), that makes possible stable quark droplets**

➢ **It is interesting to see what happens in a more realistic case when quark droplets are unstable at zero pressure (J. Steinheimer et al, PRC 89 (2014) 034901)**

➢ **There exist several models which have such a property, in particular so called Quark-Hadron Model (S. Schramm et al.) or Quark-Dilaton Model (C. Sasaki et al.).**

SU(3) chiral quark-hadron (QH) model

Includes: a) 3 quarks (u,d,s) plus baryon octet,
b) scalar mesons (σ, ς), vector meson (ω)
c) Polyakov loop (l)

$$\mathcal{L} = \sum_i \overline{\psi}_i \left(i\gamma^\mu \partial_\mu - \gamma^0 g_{i\omega}\omega - M_i \right) \psi_i + \frac{1}{2} \left(\partial_\mu \sigma \right)^2 - U(\sigma, \zeta, \omega) - \mathcal{U}(\ell)$$
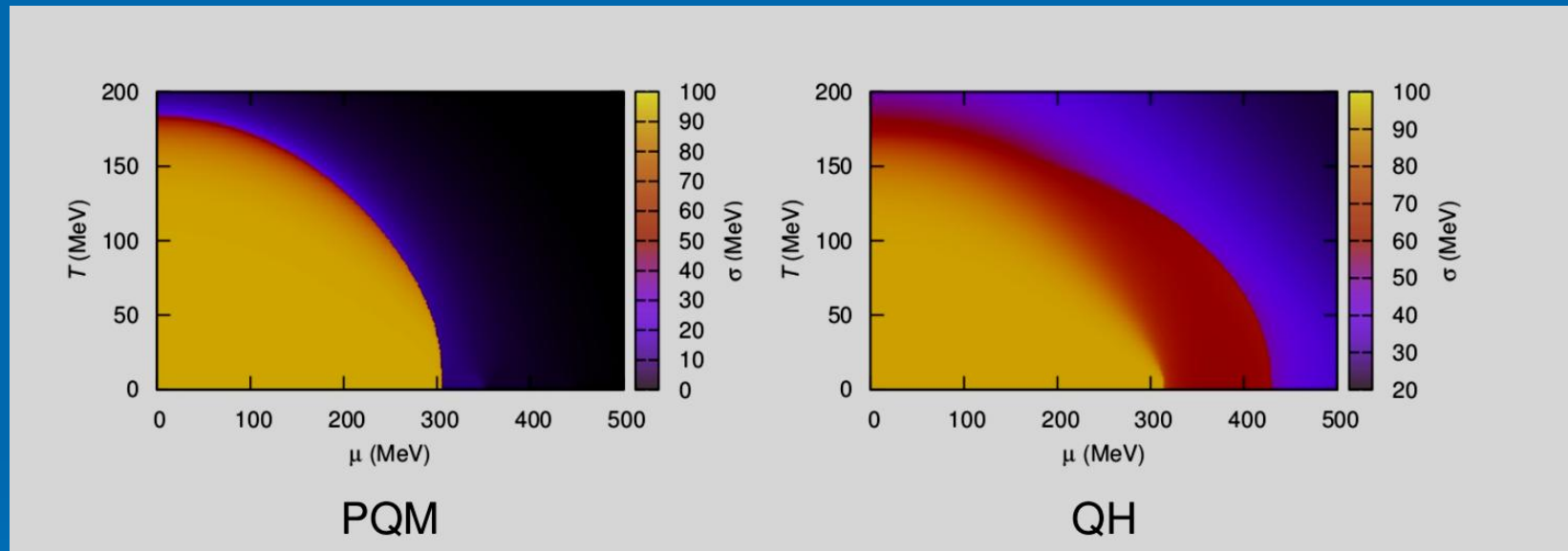
Effective masses:

$$M_q = g_{q\sigma}\sigma + g_{q\zeta}\zeta + M_{0q} + g_{q\ell}(1 - \ell)$$
$$M_B = g_{B\sigma}\sigma + g_{B\zeta}\zeta + M_{0B} + g_{B\ell}\ell^2$$
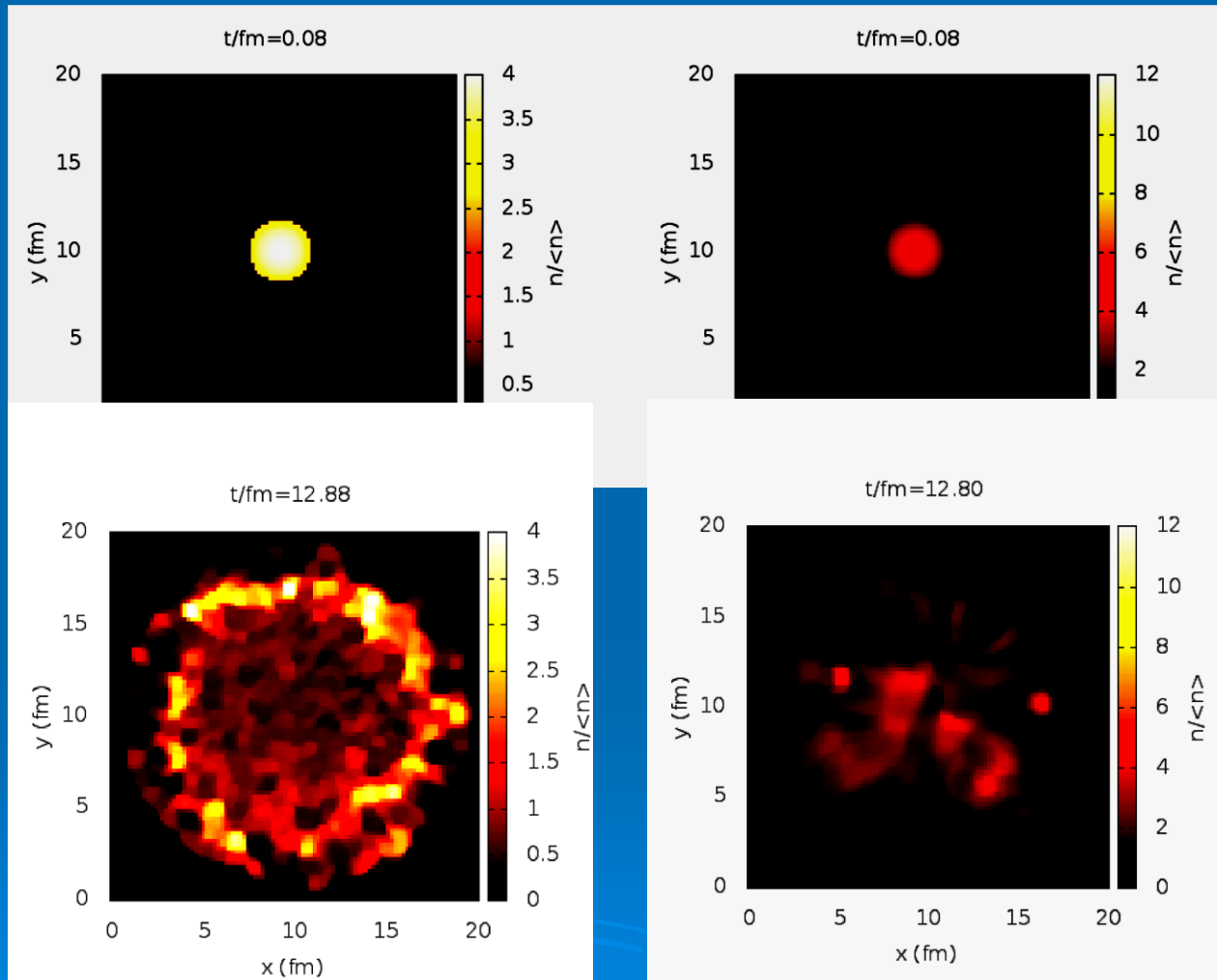
PQM vs. QH: phase diagram

Herold, Limphirat, Kobodaj, Yan, Seam Pacific Conference 2014

PQM   QH

Nuclear ground state at $\mu_N = 3\mu \approx m_N$ is reproduced correctly
QH predicts two phase transitions:
1) **liquid-gas PT at $\mu \approx 300$ MeV, and**
2)   **deconfinement/chiral PT at higher $\mu \approx 450$ MeV**
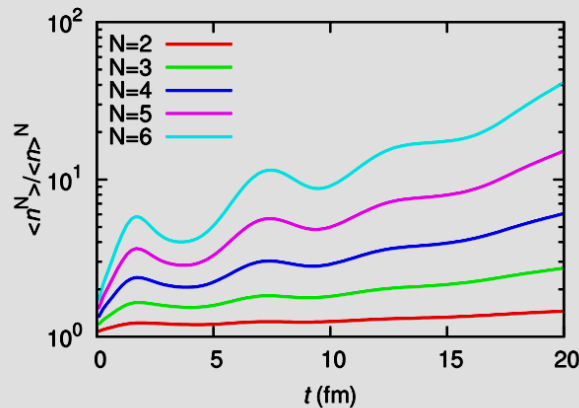
PQM vs. QH: domain formation

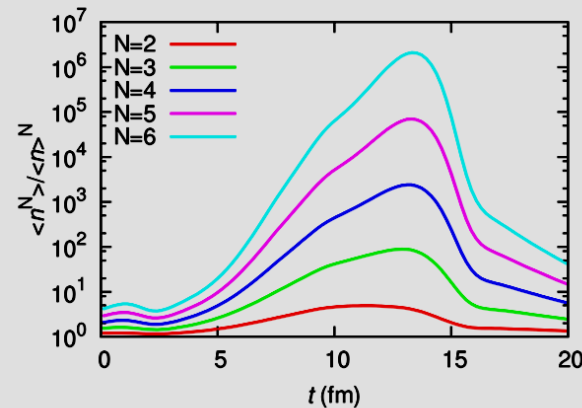**Herold, Limphirat, Kobodaj, Yan, Seam Pacific Conference 2014**

QH predicts domains with much higher densities!

PQM vs. QH: density moments

$$\langle n^N \rangle = \int d^3x \, n(x)^N P_n(x) \quad \text{with} \quad P_n(x) = \frac{n(x)}{\int d^3x \, n(x)}$$
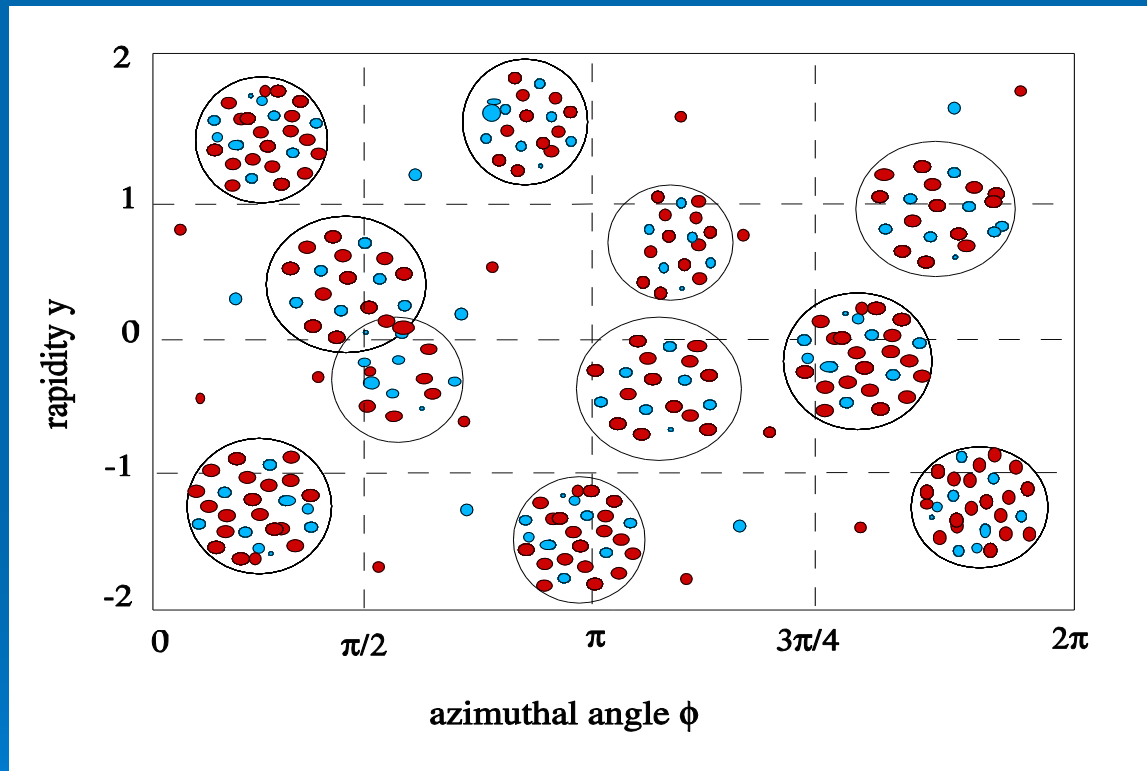
PQM eos

QH eos

Strong clustering effect survives even at late times, t>15 fm/c

Experimental signatures of droplets

Look for bumpiness in distributions of net baryons in individual events, i. e. in azimuthal angle, rapidity, transverse momentum

The bumps correspond to the emission from individual domains.

Conclusions

- Phase transitions in relativistic heavy-ion collisions will most likely proceed out of equilibrium

- 2nd order phase transition (with CEP) is too weak to produce significant observable effects in fast dynamics

- Non-equilibrium effects in a1^{st} order transition (spinodal decomposition, dynamical domain formation) may help to identify the chiral/deconfinement phase transition

- If large QGP domains survive until the freeze-out stage they will show up by large non-statistical fluctuations of observables in single events

- Exotic objects like strangelets have a better chance to be formed in such a non-equilibrium scenario