



ρ^0 and A^0 mesons in the background of strong magnetic fields

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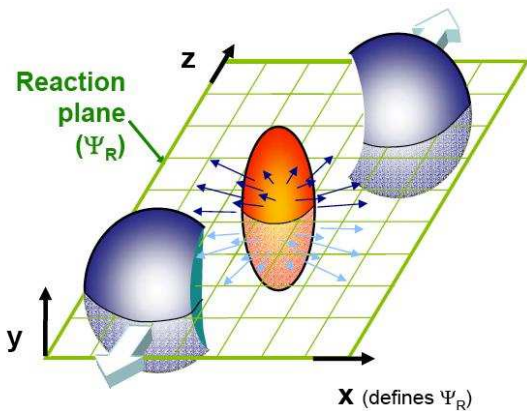
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Introduction

We work in $SU(2)$ gluodynamics with chirally invariant Dirac operator in a constant external abelian magnetic field, directed along the third axes and calculate on the lattice

- 1 the correlator of two vector currents, axial-vector currents and pseudoscalar versus the value of magnetic field in the confinement phase,
- 2 perform mass extrapolation on the lattice,
- 3 explore the dependence of neutral ρ^0 and A^0 meson masses versus the magnetic field.

$$eB \sim \Lambda_{\text{QCD}}^2$$



$$B \sim 10^{15} \text{ T}, \sqrt{eB} \sim 10 \text{ MeV} \dots 400 \text{ MeV}$$

Technical details

Model: $SU(2)$ gluodynamics.

Generation of A_μ : the tadpole-improved Wilson-Symanzik action.

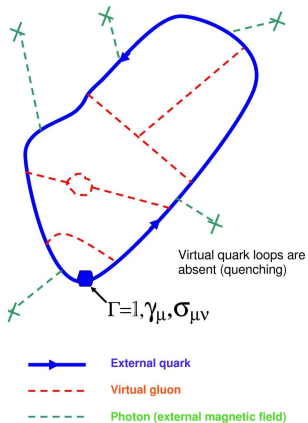
Fermion spectrum: chirally-invariant overlap operator, 50 lowest modes for the inversion, different quark masses
 $am_{lat} = 0.01 \div 0.8$.

$$\langle \psi^\dagger(\mathbf{x}) O_1 \psi(\mathbf{x}) \psi^\dagger(\mathbf{y}) O_2 \psi(\mathbf{y}) \rangle_A =$$

$$\int DA_\mu e^{-S_{YM}[A_\mu]} \text{Tr} \left(\frac{1}{D+m} O_1 \right) \text{Tr} \left(\frac{1}{D+m} O_2 \right) -$$

$$\int DA_\mu e^{-S_{YM}[A_\mu]} \text{Tr} \left(\frac{1}{D+m} O_1 \frac{1}{D+m} O_2 \right)$$

$$O_1, O_2 = \gamma_5, \gamma_5 \gamma_\mu, \gamma_\mu$$



We calculate $\bar{\psi}\Gamma\psi$, where $\gamma_\mu, \gamma_5\gamma_\mu, \gamma_5$ in the external abelian magnetic field and in the presence of the vacuum nonabelian gluon fields.

Technical details

To add magnetic field $F_{12} = B_3 = B$ in the overlap operator we:

- make the exchange

$$A_{\mu ij} \rightarrow A_{\mu ij} + A_{\mu}^{(B)} \delta_{ij}, \quad A_{\mu}^{(B)}(\mathbf{x}) = \frac{B}{2}(x_1 \delta_{\mu,2} - x_2 \delta_{\mu,1})$$

- perform the additional twist for fermions (for p.b.c).

M. H. Al-Hashimi, U. J. Wiese, 2009.

The magnetic field is quantized:

$$qB = \frac{2\pi k}{L^2}, \quad k \in \mathbb{Z}, \quad q = -e/3$$

Correlators and spectral functions

are extracted from the correlator of currents

$$\langle \psi^\dagger(\vec{0}, n_t) O_1 \psi(\vec{0}, n_t) \psi^\dagger(\vec{0}, 0) O_2 \psi(\vec{0}, 0) \rangle_A = \sum_k \langle 0 | O_1 | k \rangle \langle k | O^\dagger | 0 \rangle e^{-n_t E_k}.$$

The main contributions comes from $\langle 0 | O_1 | k \rangle \langle k | O^\dagger | 0 \rangle e^{-n_t E_0}$, fit the correlator by the function $C(n_t) = A \cosh(m_{\text{eff}}(n_t - N_T/2))$ at $4 < n_t < N_T - 4$

$$G(\tau, \vec{p}) = \int d^3 x \langle j_\mu(\tau, \vec{x}) j_\mu^\dagger(0, \vec{0}) \rangle e^{-i\vec{p}\vec{x}}$$

The corresponding spectral function $\rho(\omega, \vec{p})$ is defined by

$$G(\tau, \vec{p}) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau, \omega) \rho(\omega, \vec{p}).$$

$$K(\tau, \omega) = \frac{\omega}{2T} \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh(\frac{\omega}{2T})},$$

Maximal Entropy Method

This procedure is equivalent to the minimization of the free energy $F = L - \alpha S$ or maximization of conditional probability

$$P[\rho|DH\alpha m] = \exp\left(-\frac{1}{2}\chi^2 + \alpha S\right)$$

$$S = \int_0^\infty \frac{d\omega}{2\pi} \left[\rho(\omega) - m(\omega) - \rho(\omega) \ln \frac{\rho(\omega)}{m(\omega)} \right]$$

χ^2 is the likelihood function, α is a parameter, S is the entropy term, H is our hypothesis. The default model is

$$\bar{m}(\omega) = m_a * \omega + m_b, \quad m_a = \frac{G(N_\tau/2)}{9,8696 * T^2}, \quad m_b = a_H \frac{3}{8\pi^2},$$

where $a_H = 1$ for scalar and pseudoscalar channel, $a_H = 2$ for vector and axial vector channel.

Use modified Bryan algorithm. Redefine the kernel and spectral function

$$\bar{K}(\omega, \tau) = \frac{\omega}{2T} K(\omega, \tau), \quad \bar{\rho}(\omega) = \frac{2T}{\omega} \rho(\omega)$$

then apply **SVD** theorem ($K = U_{N_\omega \times N} W_{N \times N} V_{N \times N}^T$) to the modified discretized kernel $\bar{K}(\omega_n, \tau_i)$ to find the spectral function

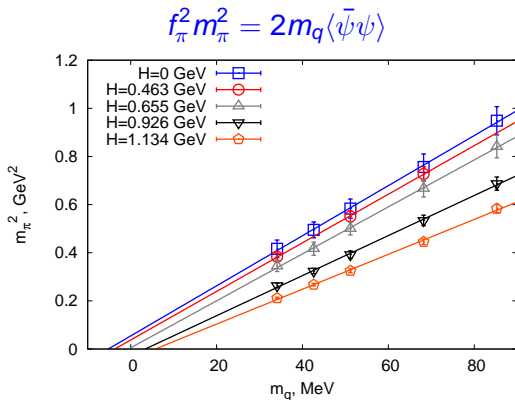
$$\bar{\rho}(\omega) = \bar{m}(\omega) \exp \sum_{i=1}^N \bar{c}_i \bar{u}_i(\omega)$$

The column vectors u_i , ($i = 1, \dots, N$) are normalized

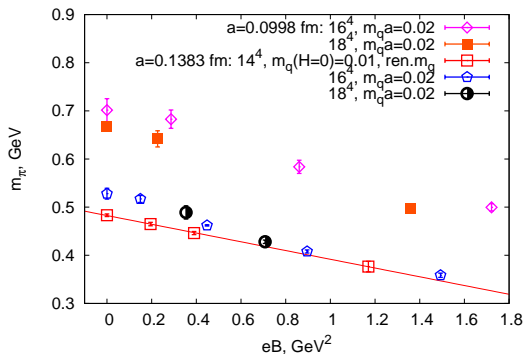
$$\langle u_i | u_j \rangle \equiv \sum_{n=1}^{N_\omega} u_i(\omega_n) u_j(\omega_n) = \delta_{ij},$$

c_i are the coefficients and we set $\bar{K}(0, \tau) = 1$.

Renormalization of quark bare mass on the lattice



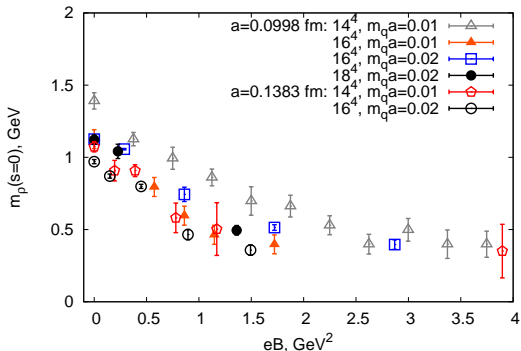
Perform the mass extrapolation to the m_{phys} corresponding to to $m_\pi = 135$ MeV, $m_{lat} = \delta m_{ren} + m_{phys}$. The results are obtained for the 14^4 lattice volume, lattice spacing 0.1383 fm.

Mass of π^0 meson. Fit.

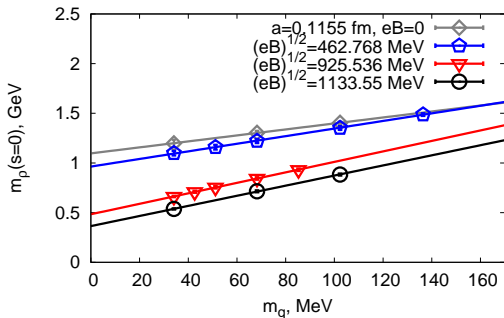
The π^0 - meson mass versus the value of magnetic field.

Mass of $\rho^0(s=0)$ meson. MEM procedure.

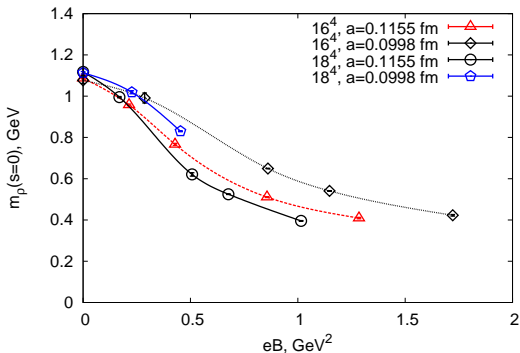
$$C_{zz}^{VV} = \langle \bar{\psi}(\vec{0}, n_t) \gamma_3 \psi(\vec{0}, n_t) \bar{\psi}(\vec{0}, 0) \gamma_3 \psi(\vec{0}, 0) \rangle$$



Mass of $\rho^0(s=0)$ meson versus the magnetic field. Results were obtained with Maximal Entropy method.

Mass extrapolation for $\rho^0(s=0)$ meson. Fit.

The $\rho^0(s=0)$ - meson mass versus the value of lattice quark mass. $m_{lat}(H=0) = -3.25$ MeV, $m_{lat}(H=1) = -1.55$ MeV, $m_{lat}(H=6) = 8.76$ MeV.

Mass of $\rho^0(s=0)$ meson. Fit.

The $\rho^0(s=0)$ - meson mass versus the magnetic field field after mass extrapolation and lattice quark mass renormalization.

Correlators of currents in the directions perpendicular to the magnetic field

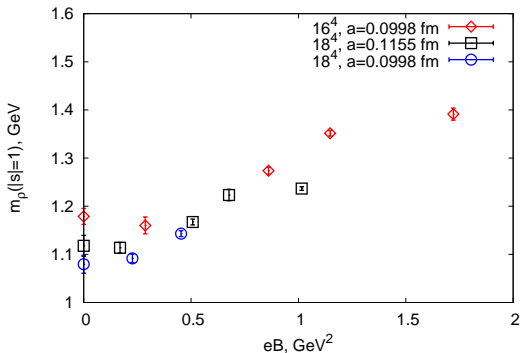
$$C_{yy}^{VV} = \langle \bar{\psi}(\vec{0}, n_t) \gamma_2 \psi(\vec{0}, n_t) \bar{\psi}(\vec{0}, 0) \gamma_2 \psi(\vec{0}, 0) \rangle$$

$$C_{xx}^{VV} = \langle \bar{\psi}(\vec{0}, n_t) \gamma_1 \psi(\vec{0}, n_t) \bar{\psi}(\vec{0}, 0) \gamma_1 \psi(\vec{0}, 0) \rangle$$

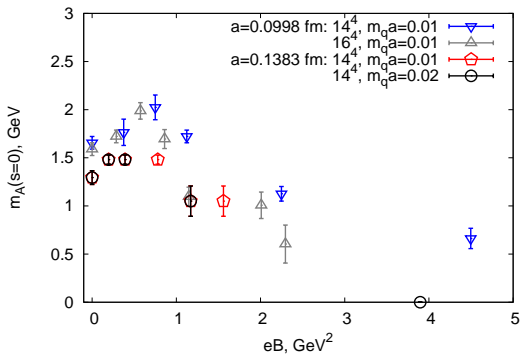
We extract masses with $s = \pm 1$ from the combinations

$$C^{VV}(s = 1) = \frac{1}{\sqrt{2}} (C_{xx}^{VV} + iC_{yy}^{VV})$$

$$C^{VV}(s = -1) = -\frac{1}{\sqrt{2}} (C_{xx}^{VV} - iC_{yy}^{VV})$$

$\rho^0(s=\pm 1)$ meson mass. Fit.

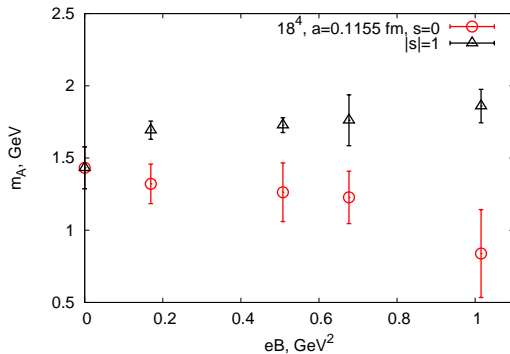
The $\rho^0(s=\pm 1)$ meson mass with nonzero spin projection to the direction of the magnetic field after mass extrapolation and lattice quark mass renormalization.

$A^0(s=0)$ meson mass. MEM procedure.

Mass of $A^0(s=0)$ meson versus the magnetic field for lattice quark mass $m_q a = 0.01$ and for renormalized mass.

A^0 meson mass.

$$C_{ZZ}^{AA} = \langle \bar{\psi}(\vec{0}, n_t) \gamma_5 \gamma_3 \psi(\vec{0}, n_t) \bar{\psi}(\vec{0}, 0) \gamma_5 \gamma_3 \psi(\vec{0}, 0) \rangle$$



The $A^0(s=0)$ meson mass after quark mass extrapolation.

Conclusions

- 1 In $SU(2)$ gluodynamics the masses of ρ^0 and A^0 mesons with spin projection $s = 0$ to the direction of the magnetic field decrease with the value of the field.
- 2 The masses of neutral mesons with spin projection $s = \pm 1$ increase with the magnetic field.