Turbulent polarization in QED and QCD plasma

Andrei Leonidov

P.N. Lebedev Physical Institute, Moscow

Theory of hadronic matter under extremal conditions, 02.10.2013

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- Initial conditions for hydro are fixed at time of order of the size of the initial overlap zone at LHC energies
- Initial conditions for hydro are fixed at time less than or of order of the time needed for incoming wavefunctions to decohere into physical modes

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- This talk focuses one one particular segment of these studies: instabilities/turbulence effects in QGP understood as a system of hard particle modes and soft fields.

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- Anomalous viscosity in turbulent QGP
- Turbulence-induced instabilities and damping in weakly turbulent QED and QCD plasma

Anisotropic static HTL non-Abelian plasma

A. Ipp et al., Phys. Rev. D84 (2011), 056003

- Vlasov collisionless evolution

$$D_\mu(A)F^{\mu\nu} - g^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{2|\mathbf{p}|} p^\mu \frac{\partial f(\mathbf{p})}{\partial p^\beta} W^\beta(x;\mathbf{v})$$

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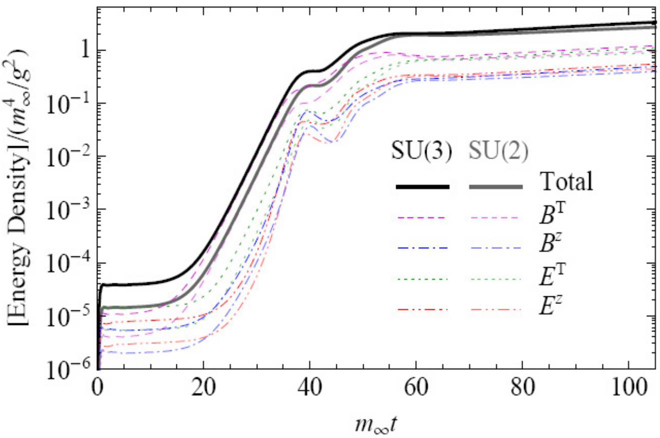
- Early dynamics is dominated by the Weibel instabilities

Anisotropic static HTL Abelian plasma: evolution



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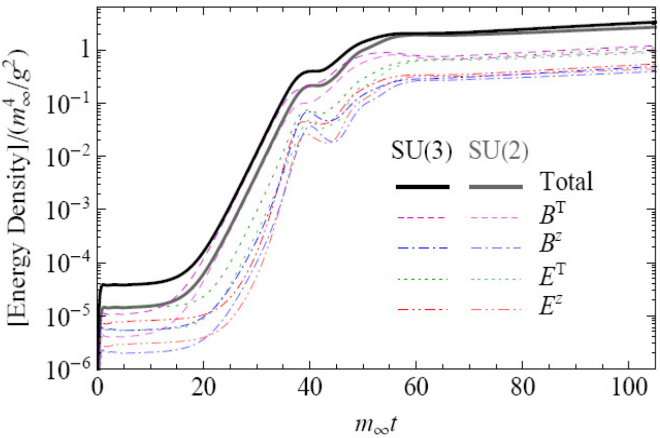

- An approximately linear regime sets in at $tm_\infty \sim 60$

Anisotropic static HTL Abelian plasma: spectra

- Definition of spectra

$$
\begin{aligned}
f_A(k) &= \frac{k}{N_{\rm dof}\, V} \langle \mathbf{A}^2(k) \rangle, \\
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- In the saturated regime the spectra display Kolmogorov turbulent-like scaling $f \sim 1/k^{\nu}$ with $\nu = 2$.

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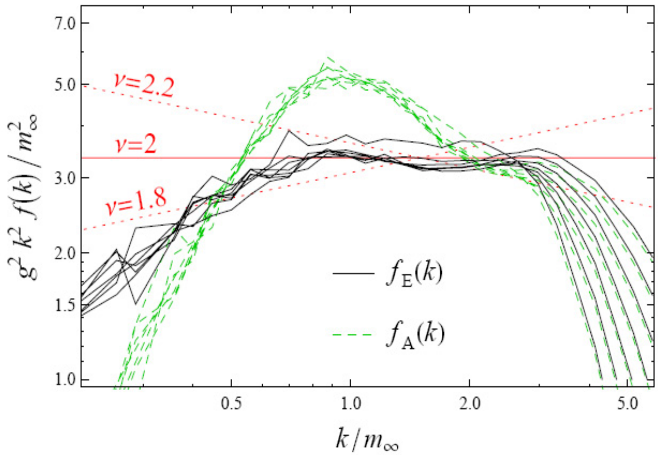

- Evolution of field spectra for $80 \leq m_\infty t \leq 150$.

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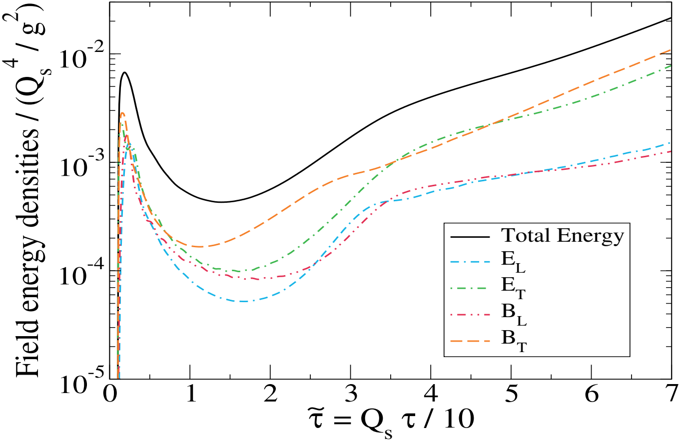
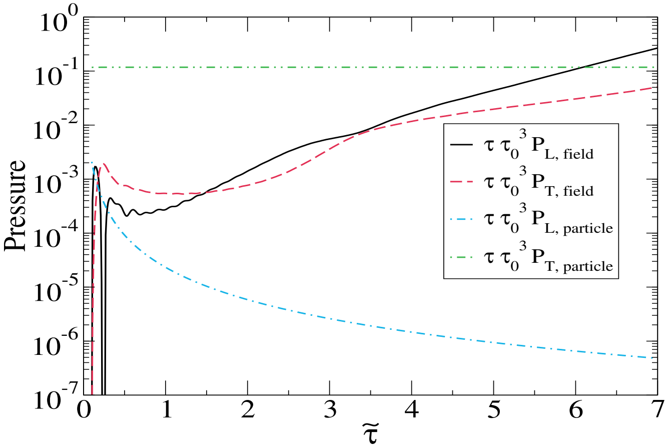
- Initial conditions matched to the CGC description of the initial fields at LHC
- The study describes the QGP evolution for $\tau \lesssim 10\ \mathrm{fm}$

Anisotropic expanding HTL non-Abelian plasma: energy



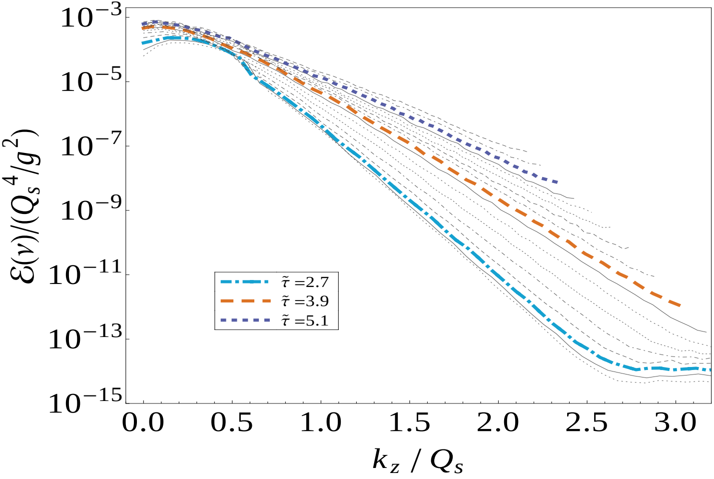
- No saturation of exponential growth

Anisotropic expanding HTL non-Abelian plasma: pressure



- Rapid buildup of longitudinal pressure

Anisotropic expanding HTL non-Abelian plasma: spectra



- The spectra are exponential, not powerlike

Anisotropic bHTL static non-Abelian plasma bHTL

A. Dumitru et al., Phys. Rev. D75 (2007), 025016

- Vlasov collisionless evolution

$$p^{\mu}[\partial_{\mu} - gq^{a}F^{a}_{\mu\nu}\partial^{\nu}_{p} - gf_{abc}A^{b}_{\mu}q^{c}\partial_{q^{a}}]f(x,p,q) = 0$$

$$D_{\mu}F^{\mu\nu} = J^{\nu} = g\int \frac{d^{3}p}{(2\pi)^{3}} dq\, q\, v^{\nu} f(t,\mathbf{x},\mathbf{p},q)$$

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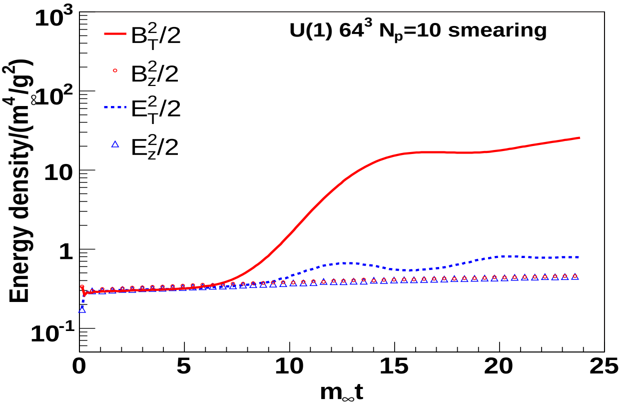
- Characteristic mass scale

$$m^2_\infty = g^2 N_c \int \frac{d^3p}{(2\pi)^3}\frac{f(\mathbf{p})}{|\mathbf{p}|} \sim g^2 N_c \frac{n_g}{p_h}$$

Anisotropic bHTL static Abelian plasma: evolution



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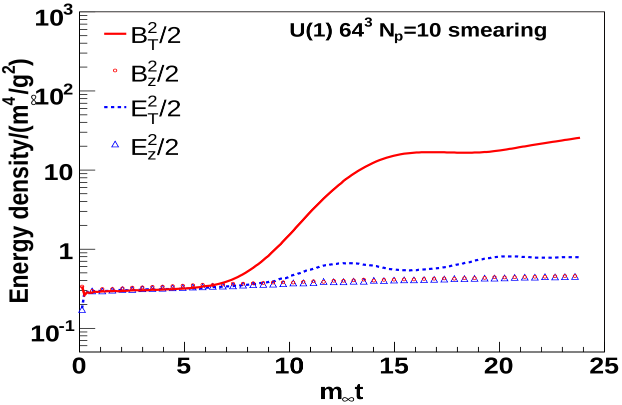

- Weak initial fields $\sim 0.1 m_\infty^4/g^2$

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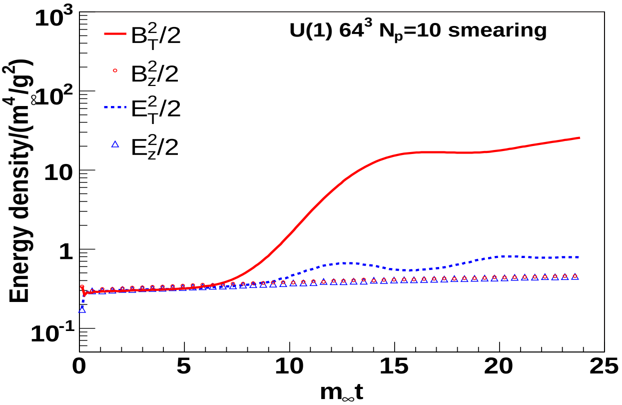
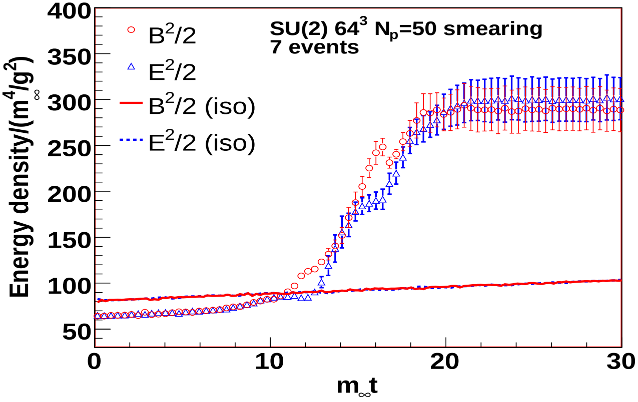


- Weak initial fields $\sim 0.1 m_\infty^4/g^2$
- Slope for B_T^2 growth is half of the HTL value: nonlinear effects are important!

Anisotropic bHTL static non-Abelian plasma: evolution



- The time evolution of non-Abelian fields stronger than $\sim m_\infty^4/g^2$ differs from that in the (effectively Abelian) extreme weak-field limit.

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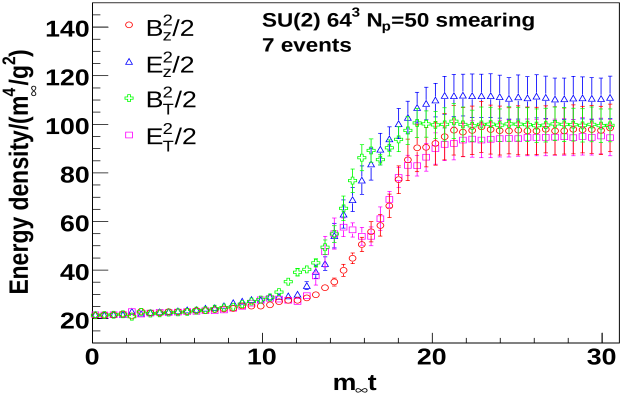
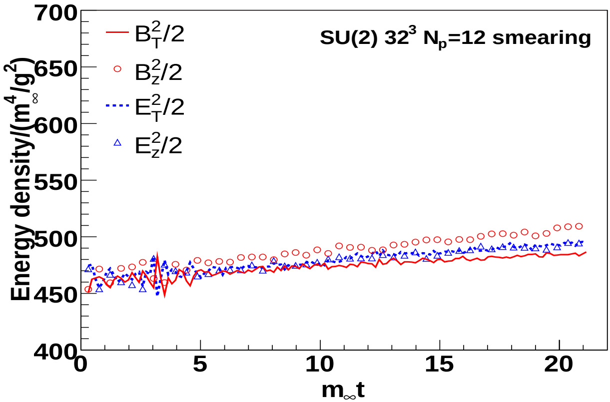
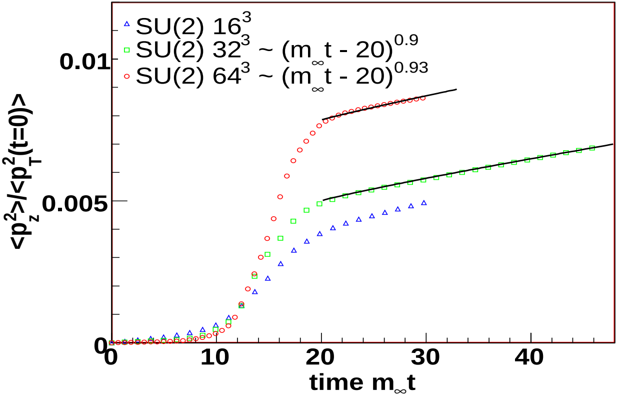


- Distribution of fields gets isotropic.



- For strong initial fields no instability effects can be seen.

Anisotropic bHTL static non-Abelian plasma: particle anisotropy



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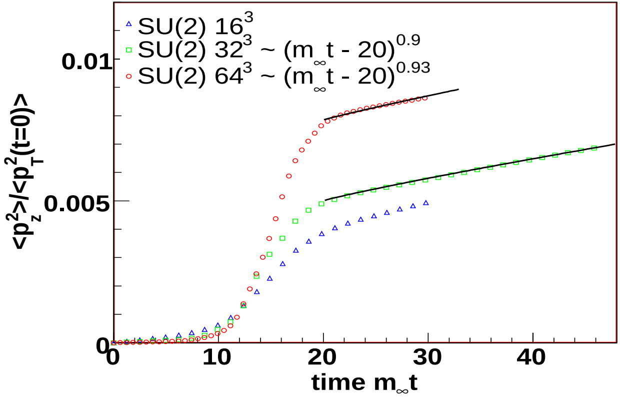

- Particle distribution remains strongly anisotropic when instability-related field evolution is already long over.

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- Collective properties of turbulent plasma are markedly different from those of the ordinary equilibrium one. Turbulent plasmas are characterized, in particular, by anomalously low viscosity and conductivity, dominant effects of coherent nonlinear structures on transport properties, etc.

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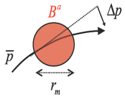
- Main effects to follow: collisions of particles with turbulent fields

Weakly turbulent plasma: anomalous viscosity

M. Asakawa et al. (2006)

- The origin of anomalous viscosity is in deflection of particles in random turbulent fields



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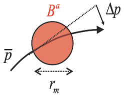

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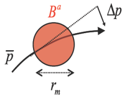

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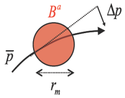

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- $$\eta_{\mathrm{A}} = \frac{n\, \bar{p}^3}{3 g^2\, Q^2 \langle B^2 \rangle\, r_m} \approx \frac{\frac{9}{4} s\, T^3}{g^2\, Q^2 \langle B^2 \rangle\, r_m}$$

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- Anomalous viscosity is a beyond-HTL effect
- The effect does essentially depend on the characteristic scale of turbulent field fluctuations r_m

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- The ensemble of stochastic turbulent fields is assumed to be Gaussian:

$$\langle F^T_{\mu\nu} \rangle = 0$$
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- In the present study we use

$$K^{\mu\nu\mu'\nu'}(x) = K_0^{\mu\nu\mu'\nu'} \exp\left[-\frac{t^2}{2\tau^2} - \frac{r^2}{2a^2}\right].$$

Turbulent polarization: QED plasma

M. Kirakosyan, A.L., B. Muller (2012)

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- Compute the induced current:

$$\langle j^\mu(k\,|F^R, F^T)\rangle_{F^T} = e\sum_{q,s}\int dP p_\nu q\langle\delta f(p, k, q|F^R, F^T)\rangle_{F^T}$$

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- Compute the turbulent polarization (a response to a regular perturbation that depends on turbulent fields):

$$\Pi^{\mu\nu}(k) = \frac{\delta\langle j^\mu(k|F^R,F^T)\rangle_{F^T}}{\delta A^R_\nu}$$

Turbulent polarization: QED plasma

- Kinetic equation:

$$f = f^{eq} + G p^{\mu} F_{\mu\nu} \partial_p^{\mu} f \ , \qquad G \equiv \frac{eq}{\imath((pk) + \imath\epsilon)}$$

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- Let us introduce a generic expansion in $F^R_{\mu\nu}$ (formal expansion in ρ) and $F^T_{\mu\nu}$ (formal expansion in τ):

$$\begin{aligned} \delta f &= \sum_{m=0} \sum_{n=0} \rho^m \tau^n \delta f_{mn} \\ F^{\mu\nu} &= \sum_{m=0} \sum_{n=0} \rho^m \tau^n F^{\mu\nu}_{mn} \end{aligned}$$

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- The leading turbulent contribution for induced current comes from the cubic terms that are of the first order in $F^R_{\mu\nu}$ and of the second order in $F^T_{\mu\nu}$. We are thus interested in computing δf_{12}

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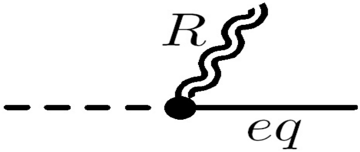
- Explicit expressions for these contributions read:

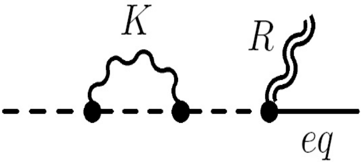
$$\begin{aligned}
\delta f_{\mathrm{HTL}} &= G p_\mu F_{10}^{\mu\nu} \partial_{\mu,p} f^{\mathrm{eq}} \\
\langle \delta f_{12} \rangle_{\mathrm{I}} &= G p_\mu \langle F_{01}^{\mu\nu} \partial_{\nu,p} G p_{\mu'} F_{10}^{\mu'\nu'} \partial_{\nu',p} G p_\rho F_{01}^{\rho\sigma} \rangle \partial_{\sigma,p} f^{\mathrm{eq}} \\
\langle \delta f_{12} \rangle_{\mathrm{II}} &= G p_\mu \langle F_{01}^{\mu\nu} \partial_{\nu,p} G p_{\mu'} F_{01}^{\mu'\nu'} \partial_{\nu',p} G p_\rho F_{10}^{\rho\sigma} \rangle \partial_{\sigma,p} f^{\mathrm{eq}}
\end{aligned}$$

Turbulent polarization: diagrams

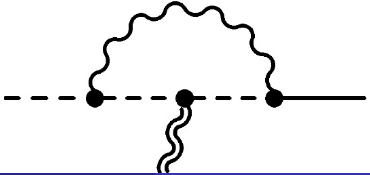
- $\delta f_{\rm HTL}$ $=$



- $\langle \delta f_{12} \rangle_{\rm I}$ $=$



- $\langle \delta f_{12} \rangle_{\rm II}$ $=$



Turbulent polarization: QED plasma

- The evolution of the distribution function due to turbulent fields corresponds to f_{02} and is governed by the (Dupree) equation

$$(p\partial)\langle f \rangle - p_\mu \langle F_{01}^{\mu\nu} \partial_{p\,\nu} p_{\mu'} G F_{01}^{\mu'\nu'} \rangle \partial_{p\,\nu'} \langle f \rangle = 0$$

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- The above equation does not have stationary solutions.

- We will assume that turbulent evolution of the distribution function is slower than that of the probe F_{10} and can be neglected.

Turbulent polarization: QED plasma

- Generic decomposition of polarization tensor ($l \equiv \sqrt{2}(\tau a)/\sqrt{\tau^2 + a^2}$):

$$\Pi_{ij}(\omega, \mathbf{k}|\ l) = \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\Pi_T(\omega, |\mathbf{k}|\ |\ l) + \frac{k_i k_j}{k^2}\Pi_L(\omega, |\mathbf{k}|\ |\ l)$$

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- Separation into HTL and turbulent components:

$$\Pi_{L(T)}(\omega, \mathbf{k}|\ l) = \Pi_{L(T)}^{\mathrm{HTL}}(\omega, \mathbf{k}) + \Pi_{L(T)}^{\mathrm{turb}}(\omega, \mathbf{k}|\ l)$$

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$$\Pi_{L(T)}(\omega, \mathbf{k}|\, l) = \Pi^{\mathrm{HTL}}_{L(T)}(\omega, \mathbf{k}) + \Pi^{\mathrm{turb}}_{L(T)}(\omega, \mathbf{k}|\, l)$$

- Gradient expansion of the turbulent contribution:

$$\begin{aligned} \Pi^{\mathrm{turb}}_{L(T)}(\omega, |\mathbf{k}|\,|\, l) &= \sum_{n=1}^{\infty} \frac{(|\mathbf{k}|\, l)^n}{\mathbf{k}^2} \left[\phi^{(n)}_{L(T)} \left(\frac{\omega}{|\mathbf{k}|} \right) \langle E^2_{\mathrm{turb}} \rangle + \right. \\ &+ \left. \chi^{(n)}_{L(T)} \left(\frac{\omega}{|\mathbf{k}|} \right) \langle B^2_{\mathrm{turb}} \rangle \right] \end{aligned}$$

Turbulent polarization: QED plasma

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- Hard thermal loops contribution:

$$\Pi_L^{\mathrm{H}TL}(\omega, |\mathbf{k}|) = -m_D^2 x^2 \left[1 - \frac{x}{2}\, L(x)\right]$$
$$\Pi_T^{\mathrm{H}TL}(\omega, |\mathbf{k}|) = m_D^2 \frac{x^2}{2} \left[1 + \frac{1}{2x}\, (1 - x^2)\, L(x)\right]$$

- $$L(x) \equiv \ln \left| \frac{1+x}{1-x} \right| - \imath\pi\theta(1-x); \quad m_D^2 = e^2 T^2/3$$

- HTL imaginary part at $x < 1$ corresponds to Landau damping

Turbulent polarization: QED plasma

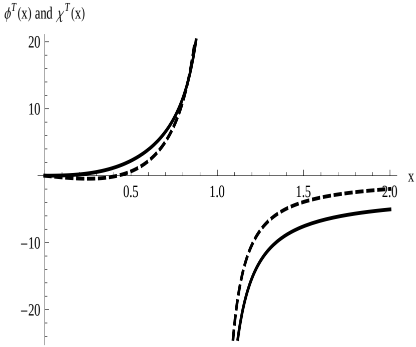
- Transverse polarization:

$$\begin{aligned} \mathrm{Im}\Pi_T(\omega, \mathbf{k}|\ l) &\simeq -\pi m_D^2 \frac{x}{4}(1-x^2)\theta(1-x) \\ &+ \frac{(|\mathbf{k}|\ l)}{\mathbf{k}^2}\left(\langle E^2\rangle\, \mathrm{Im}\phi_{\mathrm{I\ T}}(x) + \langle B^2\rangle\, \mathrm{Im}\chi_{\mathrm{I}}(x)\right) \end{aligned}$$

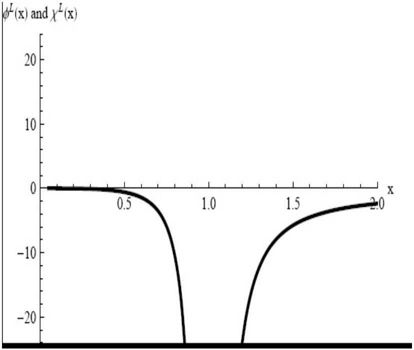
- Longitudinal polarization:

$$\begin{aligned} \mathrm{Im}\Pi_L(\omega, \mathbf{k}|\ l) &\simeq -\pi m_D^2 \frac{x^3}{2}\theta(1-x) \\ &+ \frac{(|\mathbf{k}|\ l)}{\mathbf{k}^2}\left(\langle E^2\rangle\, \mathrm{Im}\phi_{\mathrm{I\ L}}(x) + \langle B^2\rangle\, \mathrm{Im}\chi_{\mathrm{I\ L}}(x)\right), \end{aligned}$$

Turbulent polarization: imaginary part



The functions $\frac{6\pi\sqrt{\pi}}{e^4}\mathrm{Im}\left[\phi_T^{(1)}(x)\right]$ (solid line) and $\frac{6\pi\sqrt{\pi}}{e^4}\mathrm{Im}\left[\chi_T^{(1)}(x)\right]$ (dashed line).



The functions $\frac{6\pi\sqrt{\pi}}{e^4}\mathrm{Im}\left[\phi_L^{(1)}(x)\right]$ (solid line) and $\frac{6\pi\sqrt{\pi}}{e^4}\mathrm{Im}\left[\chi_L^{(1)}(x)\right]$ (dashed line).

Turbulent polarization: spacelike transverse instability

- Spacelike domain $x < 1$, competition between Landau damping and turbulent enhancement for Π_T:

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- For example, for the purely magnetic instability

$$
\begin{aligned}
\mathrm{Im}\Pi_T(\omega, |\mathbf{k}|) &= -\pi \frac{e^2 T^2}{12} x(1-x^2) \left[1 - \frac{4}{\pi^2 \sqrt{\pi}} \frac{(|\mathbf{k}|\, l)}{\mathbf{k}^2} \frac{e^2 \langle B^2 \rangle}{T^2} \Phi(x) \right] \\
\Phi(x) &= \frac{1}{x(1-x^2)} \left[\frac{-4+12x^2}{3(1-x^2)} + 2x \, \ln \left| \frac{1+x}{1-x} \right| \right]
\end{aligned}
$$

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- Instability condition

$$\frac{4}{\pi^2 \sqrt{\pi}} \frac{(|\mathbf{k}|\, l)}{\mathbf{k}^2} \frac{e^2 \langle B^2 \rangle}{T^2} \Phi(x) > 1$$

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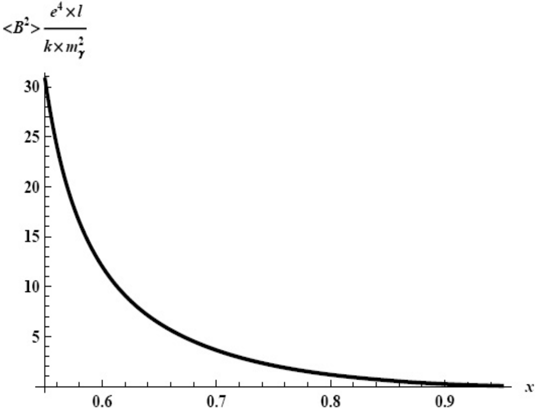
- Instability condition: numerics



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- Possible origin of the spacelike transverse instability is in not taking into account the backreaction of the turbulent field on the thermal distribution of hard modes.

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- The problem of finding the deformed stationary distribution of hard modes corresponding to truly (stochastically) stationary turbulent plasma is very difficult

Turbulent polarization: spacelike transverse instability

- Possible origin of the spacelike transverse instability is in not taking into account the backreaction of the turbulent field on the thermal distribution of hard modes.
- The problem of finding the deformed stationary distribution of hard modes corresponding to truly (stochastically) stationary turbulent plasma is very difficult
- Calculations for the anisotropic case are in progress. It is likely that the turbulent instability will act in addition to the Weibel one. If so, there is an interval (here $\xi > 1$ parametrizes anisotropy)

$$\frac{T}{e} < B < \xi \frac{T}{e}$$

in which turbulent instabilities will play a role.

Turbulent polarization: timelike and spacelike damping

- In the timelike domain $x > 1$ one gets turbulent damping (resonance broadening) of both longitudinal and transverse modes. Because of the absence of HTL timelike damping this is a leading order effect.

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- In the spacelike domain $x < 1$ longitudinal modes get additional turbulent damping in addition to the Landau one

- The corresponding damping rates can be calculated from

$$\Gamma^2 \; = \; -\mathrm{Im}\Pi(\omega, |\mathbf{k}|\, |l)$$

Turbulent polarization: plasmon properties

- Dispersion relations for plasmons, generic equations

$$\mathbf{k}^2 \left(1 - \frac{\Pi_{\rm L}(k^0, |\mathbf{k}|)}{\omega^2} \right) \Big|_{k^0=\omega_{\rm L}(|\mathbf{k}|)} = 0$$

$$\mathbf{k}^2 - (k^0)^2 + \Pi_{\rm T}((k^0, |\mathbf{k}|) \,|_{k^0=\omega_{\rm T}(|\mathbf{k}|)} = 0$$

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- Dispersion equations for plasmons, $|\mathbf{k}| \ll \omega$ $(y_{\rm T(L)} = |\mathbf{k}| / \omega^{\rm turb}_{\rm pl\ T(L)})$

$$\omega^2_{\rm L}(|\mathbf{k}|)_{\rm turb} = (\omega^{\rm turb}_{\rm pl\ L})^2 \left(1 + \frac{3}{5} y^2_{\rm L}\right) - \frac{e^4 l^2}{6\pi^2} \left(\frac{24}{5}\langle E^2\rangle + \frac{64}{15}\langle B^2\rangle\right) y^2_{\rm L} + O\left(y^4_{\rm L}\right)$$

$$\omega^2_{\rm T}(|\mathbf{k}|)_{\rm turb} = (\omega^{\rm turb}_{\rm pl\ T})^2 \left(1 + \frac{3}{5} y^2_{\rm T}\right) - \frac{e^4 l^2}{6\pi^2} \left(\frac{24}{7}\langle E^2\rangle + \frac{32}{15}\langle B^2\rangle\right) y^2_{\rm T} + O\left(y^4_{\rm T}\right)$$

- Turbulent corrections to plasma frequencies:

$$(\omega^{\rm turb}_{\rm pl\ L})^2 = \omega^2_{\rm pl\ L} - \frac{e^4 l^2}{6\pi^2} \left(\frac{16}{3}\langle E^2\rangle + \frac{8}{3}\langle B^2\rangle\right)$$

$$(\omega^{\rm turb}_{\rm pl\ T})^2 = \omega^2_{\rm pl\ T} - \frac{e^4 l^2}{6\pi^2} \left(\frac{128}{15}\langle E^2\rangle + \frac{8}{3}\langle B^2\rangle\right)$$

Turbulent QCD plasma

I.M. Dremin, M. Kirakosyan, A.L.(2013)

The generalization to the non-Abelian case reads

$$p^{\mu}\left[\partial_{\mu} - gf_{abc}A_{\mu}^{b}Q^{c}\frac{\partial}{\partial_{Q^a}} - gQ_{a}F_{\mu\nu}^{a}\frac{\partial}{\partial_{p_\nu}}\right] = 0,$$

where the fields $F_{\mu\nu}$ satisfy the Yang-Mills equations

$$D^{\mu}F_{\mu\nu}^{a} = j_{\nu}^{a}$$

The main distinction from the Abelian case is the dependence of the distribution function on the color spin Q, where for $SU(3)$ $Q = (Q^1, Q^2, ..., Q^8)$, so that $f(x, p, Q)$. The components of color spin $Q = (Q^1, Q^2, ..., Q^8)$ are dynamic variables satisfying the Wong equation

$$\frac{dQ^a}{d\tau} = -gf^{abc}p^{\mu}A_{\mu}^{b}Q^{c}$$

Turbulent QCD plasma

Separation of regular and turbulent contributions:

$$f = f^R + f^T, \quad A^a_\mu = A^{Ra}_\mu + A^{Ta}_\mu, \quad \left\langle A^{Ta}_\mu \right\rangle = 0$$

Gauge fixing making $\langle A^{Ta}_\mu \rangle = 0$ gauge invariant:

$$\begin{aligned} \delta A^{Ra}_\mu &= \partial_\mu \alpha^a + g f^{abc} A^{Rb}_\mu \alpha^c \\ \delta A^{Tb}_\mu &= g f^{abc} A^{Tb}_\mu \alpha^c \end{aligned}$$

This leads to gauge field strength decomposition:

$$F^a_{\mu\nu} = F^{Ra}_{\mu\nu} + \mathbf{F}^{Ta}_{\mu\nu} + \mathfrak{F}^{Ta}_{\mu\nu}$$

where:

$$\begin{aligned} F^{Ra}_{\mu\nu} &= \partial_\mu A^{Ra}_\nu - \partial_\nu A^{Ra}_\mu + g f^{abc} A^{Rb}_\mu A^{Rc}_\nu \\ \mathfrak{F}^{Ta}_{\mu\nu} &= \partial_\mu A^{Ta}_\nu - \partial_\nu A^{Ta}_\mu + g f^{abc} A^{Tb}_\mu A^{Tc}_\nu \\ \mathbf{F}^{Ta}_{\mu\nu} &= g f^{abc} \left(A^{Tb}_\mu A^{Rc}_\nu + A^{Rb}_\mu A^{Tc}_\nu \right) \end{aligned}$$

Turbulent QCD plasma

Turbulent contributions to the distribution function:

$$f^{R(1)} = HTL + I_1 + I_2$$

where

$$\begin{aligned} I_1 &= g^3 p^\mu f^{abc} \left\langle A_\mu^{Tb} Q^c \frac{\partial}{\partial Q^a} p^{\mu'} \frac{1}{p^\mu \partial_\mu} f^{def} A_{\mu'}^{Re} Q^f \frac{\partial}{\partial Q^d} \frac{1}{p^\mu \partial_\mu} p^{\mu'} Q_g \mathfrak{F}_{\mu'' \nu} \right\rangle \frac{\partial}{\partial p_\nu} f^{R(0)} \\ I_2 &= g^2 p^\mu f^{abc} \left\langle A_\mu^{Tb} Q^c \frac{\partial}{\partial Q^a} \frac{1}{(p^\mu \partial_\mu)} p^{\mu'} Q_d \mathbf{F}_{\mu' \nu}^{Td} \right\rangle \frac{\partial}{\partial p_\nu} f^{R(0)} \end{aligned}$$

Turbulent QCD plasma

$$\left\langle A_{\mu}^{Ta}(x)A_{\nu}^{Tb}(y)\right\rangle = G_{\mu\nu}^{ab}(x,y)$$

$$\left\langle \mathcal{F}_{\mu\nu}^{Ta}(x)U^{ab}(x,y)\mathcal{F}_{\mu'\nu'}^{Tb}(y)\right\rangle = K_{\mu\nu\mu'\nu'}^{ab}(x,y)$$

$$K_{\mu\nu\mu'\nu'}^{Ta}(x,y) = K_{\mu\nu\mu'\nu'}^{Ta}(x-y)$$

$$G_{\mu\nu}^{ab} = \delta_{ab}\left[g_{\mu\nu}g_{\nu 0}\left\langle A_0^2\right\rangle + \frac{1}{3}\hat{\delta}_{\mu\nu}\left\langle \mathbf{A}^2\right\rangle\right]\exp\left[-\frac{r^2}{2a^2}-\frac{t^2}{2\tau^2}\right]$$

Defining $f^{Ra(1)}(x,p) = \int Q^a\, d\, Q\; f^{R(1)}(x,p,Q)$, we get

$$[(p^{\mu}\partial_{\mu}) + p\gamma]f^{Rl(1)} = \int Q^l\, d\, Q\; (HTL + I_1 + I_2)\,,$$

$$\gamma = g^2\frac{N^2-1}{4N}\sqrt{\pi}l\left[\left\langle A_0^2\right\rangle + \left\langle\frac{1}{3}\mathbf{A}^2\right\rangle\right]$$

Turbulent polarization: non-Abelian contributions

- Leading order contribution:

$$\Pi_L = m_g^2 x^2 \left[-1 + x \, \mathrm{Arcth}[y] - \imath \frac{\gamma x}{|\mathbf{k}|} \frac{1}{1-y^2} \right]$$

$$\Pi_T = m_g^2 \frac{x}{2} \left[y + \left(1 - y^2\right) \mathrm{Arcth}[y] - \imath \frac{\gamma x}{|\mathbf{k}|} \left(2 - 2y \, \mathrm{Arcth}[y]\right) \right]$$

$$y = \frac{k^0 + \imath\gamma}{|\mathbf{k}|}, \qquad \gamma = g^2 \frac{N^2 - 1}{4N} \sqrt{\pi} I \left[\langle A_0^2 \rangle + \left\langle \frac{1}{3} \mathbf{A}^2 \right\rangle \right]$$

- No new contributions to the previously discussed instabilities/damping phenomena

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- We have several beautiful theoretical schemes, theoretical progress is very rapid
- Relation of these constructions to experimental data is unclear
- What is however clear that the system we are studying is, in contrast to early Universe, expanding so fast that its phenomenology should be sensitive, perhaps even anomalously sensitive, to event-by-event details of its early dynamics.